

THE NEUTRINO MASS PROBLEM

- Neutrinos $\begin{matrix} \nearrow \text{Unique Probe} \\ \searrow \text{Intrinsic Properties} \end{matrix}$
- S.M. choice of Chiral Fields
- Without $\nu_R \dots$
 - S.M. $\Rightarrow m_\nu = 0$
 - dim. 5 WINDOW to BSM
- With $\nu_R \dots$
 - Both m_D and m_R Masses
 - Global $U(1) \leftrightarrow m_R = 0$?
- Facts ... and Opinions

Neutrinos as a Unique Probe: $10^{-33} - 10^{+28}$ cm

• Particle Physics

- $\nu N, \mu N, eN$ scattering: existence/ properties of quarks, QCD
- Weak decays ($n \rightarrow pe^- \bar{\nu}_e, \mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$): Fermi theory, parity violation, mixing
- Neutral current, Z -pole, atomic parity: electroweak unification, field theory, m_t ; severe constraint on physics to TeV scale
- Neutrino mass: constraint on TeV physics, grand unification, superstrings

• Astrophysics/Cosmology

- Core of Sun
- Supernova dynamics
- Atmospheric neutrinos (cosmic rays)
- AGNs, cosmic rays, violent events, GRB
- Large scale structure (dark matter)
- Nucleosynthesis (big bang - small A ; stellar - to iron; supernova - large N)
- Baryogenesis
- Simultaneous probes of ν and astrophysics

② There was a time ---

1957 → Two-component ν theory

Only ν_L in weak interactions

Argument in favour for $m_\nu = 0$

1958 → (V-A) theory $\Rightarrow$ CHIRAL fields enter for ALL fermions

Nothing special for ν 's $\longleftrightarrow \nu_L, e_L, N_L$

No reason why $m_\nu = 0$

BUT : - e_R, q_R needed $\Rightarrow$ Dirac Mass $\neq 0$
QED, QCD

- ν_R ? at will $\leftrightarrow$ Sterile

1967 → Standard Model "choice" : ν_R

Dirac Mass = 0

{ No $SU(2) \times U(1)$ gauge invariant term for Majorana Mass

$\downarrow$ $m_\nu = 0$
 $\oplus$ Lepton-Flavour Conservation }

BUT : - S.M. with total symmetry (?)

QUARKS $\longleftrightarrow$ LEPTONS

$$SU(2) \times U(1)$$

Chiral Fields

Quarks

	I_3	Y	Q
u_L	$+\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
d_L	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
u_R	0	$\frac{2}{3}$	$\frac{2}{3}$
Have to... d_R	0	$-\frac{1}{3}$	$-\frac{1}{3}$

Leptons

	I_3	Y	Q
ν_L	$+\frac{1}{2}$	$-\frac{1}{2}$	0
e_L	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
e_R	0	-1	-1
??? ν_R	0	0	0

- Follow the fathers of SM :

(AT PRESENT ENERGIES) $SU(2) \times U(1)$

with a particle content WITHOUT ν_R

- Is it possible (with only ν_L) to have $m_\nu \neq 0$?

A priori, YES $\longleftrightarrow$ Majorana

$$\mathcal{L}_{\text{mass}}^{\text{Maj}} = -\frac{1}{2} \bar{\nu}_L M \nu_L^c + \text{h.c.}$$

$SU(3)_{\text{colour}} \otimes U(1)_{\text{e.m.}}$ invariant $\Rightarrow$ legal

$$\mathcal{L}_{\text{Maj}} + \text{Pauli} \Rightarrow M^T = M$$

Diagonalization

$$\downarrow$$

$$M = U m U^T$$

$$\mathcal{L}_{\text{mass}}^{\text{Maj}} = -\frac{1}{2} \bar{X} m X$$

Majorana Field

$$X = X^c = C \bar{X}^T$$

$$X \equiv \nu^+ \nu_L + (\nu^+ \nu_L)^c$$

No phase-freedom to rotate away

$\Rightarrow$ Neutrinos would be "true" neutrals

- NO conserved GLOBAL LEPTON NUMBER

$\Rightarrow \beta\beta_{\nu}$ allowed

BUT ...

- Fermion Field

$$\psi(x) = \int \frac{d^3 p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} \sum_{\lambda} \left\{ c_{\lambda}(p) u_{\lambda}(p) e^{-ipx} + d_{\lambda}^{\dagger}(p) C[\bar{u}_{\lambda}(p)]^T e^{ipx} \right\}$$

- Dirac Propagator



Flavour Oscillations $\neq \langle 0 | T \{ \psi(x) \bar{\psi}(0) \} | 0 \rangle$ 3 mixings, 1 phase
 $\equiv$ for both Dirac & Majorana Neutrinos

$\equiv$ Neutrino propagation

- Majorana Propagator



$$\langle 0 | T \{ \psi(x) \psi(0)^T \} | 0 \rangle \quad \left[\begin{array}{l} 3 \text{ mixings,} \\ 3 \text{ phases} \end{array} \right]$$

2 additional phases

$$\Leftrightarrow \psi \equiv X = C \bar{X}^T$$

$$c_{\lambda}(p) = d_{\lambda}(p)$$

$\equiv$ only for Majoranas

$\equiv$ "Neutrino-antineutrino" propagation

- $\int \mathcal{L}_{\text{mass}}^{\text{Maj}}$ cannot be **GENERATED** by
SSB of a **RENORMALIZABLE** (dim. 4)
 Yukawa interaction! Higgs triplet?

1970's $\rightarrow$ 2000's: Requirement of
RENORMALIZABILITY has a different
PHILOSOPHY:

- Take particle content of SM and ask
 WHAT IS THE LOWEST DIMENSION (**NON-RENORMALIZABLE**) OPERATOR WITH
 $SU(2) \times U(1)$ GAUGE INVARIANCE?

UNIQUE
 dim. 5

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2\Lambda} (\bar{\tilde{e}}_L \psi) F (\tilde{\psi}^+ e_L)$$

$$\tilde{e} = i\tau_2 e^c = i\tau_2 C \bar{e}^T$$

$\updownarrow$

- The **FIRST** window to **BEYOND SM**
 - $F = F^T$ matrix in flavour-space
 - Coupling $\frac{1}{\Lambda} \Leftrightarrow \Lambda \equiv$ New Physics Scale
- $\Rightarrow$
- 1) $\Delta L = 2$
 - 2) $F \Rightarrow$ Mixing, LFV

3) After **SSB** :

$$\mathcal{L}_{\text{eff}} \rightarrow \mathcal{L}_{\text{mass}}^{\text{Maj}}$$

$$M = \frac{v^2}{\Lambda} F$$

"see-saw" type

Conclusion

$\mathcal{L}_{\text{mass}}^{\text{Maj}}$ for "light" neutrinos **GENERATED** from $\mathcal{L}_{\text{eff}}$ (at present energies).

- Origin of Λ ?

1) Heavy mass ν_R ?

2) Fierz-reordering

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4\Lambda} (\bar{\tilde{l}}_L F \tilde{\Sigma} l_L) (\tilde{\varphi}^+ \tilde{\Sigma} \varphi)$$

Higgs triplet?

- A particular example: **SO(10) GUT**

$\Rightarrow$ As part of **SB**, ν_R obtains a large Majorana mass Λ .

- The Dirac mass terms $\nu_R \leftrightarrow \nu_L$ m_D produce mixing of heavy ν_R into $\nu_L \Rightarrow m_\nu = \frac{m_D^2}{\Lambda}$

- ν_R and Yukawa interaction only

$$\mathcal{L}_Y = - \frac{\sqrt{2}}{v} \bar{l}_L M \nu_R \tilde{\varphi} + h.c.$$

$$\tilde{\varphi} = i\tau_2 \varphi^* \xrightarrow{SSB} \begin{Bmatrix} (v+H)/\sqrt{2} \\ 0 \end{Bmatrix}$$

⇒ Total analogy with quarks,

U

CKM-matrix, ----

↓ Pontecorvo-MNS ↓

↓
- ν 's are Dirac particles

- ν_R only in Mass-terms ⇒

⇒ Masses and Mixings

- Mixing relevant for CC

- Mixing irrelevant for NC-Lagrangian

⇒ GIM-suppressed FCNC

- ν_R 's do not appear elsewhere than H

- LFV, but $\mathcal{L}$ still invariant under
GLOBAL U(1)-GAUGE TRANSFORMATION

||
TOTAL LEPTON NUMBER

WHY?
BSM?

③ Is it possible (with only ν_L) to
generate $m_\nu \neq 0$?

YES (a priori) $\longleftrightarrow$ Majorana

The Pontecorvo MNS Matrix

$$\begin{Bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{Bmatrix} = U \begin{Bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{Bmatrix} \quad \Leftrightarrow$$

For Flavour oscillations
 U : 3 mixings, 1 phase

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\approx \text{Atmospheric LBL - beams}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\begin{matrix} ? \\ \text{Atmospheric} \\ \text{(through Matter)?} \\ \text{Reactor?} \\ \text{Appearance } \nu_\mu \rightarrow \nu_e! \end{matrix}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\approx \text{Solar Reactor (if LMA-MSW)}}$$

"Extended" SM

- $SU(2) \times U(1)$ gauge invariance
- SM + ν_R in particle content

$$\mathcal{L}(x) = -\frac{\sqrt{2}}{v} \bar{l}_L m_D \nu_R \tilde{\varphi} + h.c.$$

$SU(2) \times U(1)$
invariant

$$- \frac{1}{2} \bar{\nu}_R m_R (\nu_R)^c + h.c.$$

⇒ Not only L-R Dirac Mass,
R-Majorana too

$$\mathcal{L}^{D-M} = -\bar{\nu}_R m_D \nu_L - \frac{1}{2} \bar{\nu}_R m_R (\nu_R)^c + h.c. \equiv -\frac{1}{2} (\bar{n}_L)^c M n_L + h.c.$$

$$n_L \equiv \begin{Bmatrix} \nu_L \\ (\nu_R)^c \end{Bmatrix}, \quad M = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}$$

⇒ Physical neutrinos $\equiv$ MAJORANA
unless
 $m_R = 0$? Why? Global $U(1)$?
⇔ BSM

FACTS

- SM $\leftrightarrow \nu_L$ only
 $\Rightarrow m_\nu = 0$
- BSM + ν_L only at present
 $\Rightarrow$ Unique window $\equiv \text{dim. } 5$
 $\Rightarrow m_\nu \neq 0$ MAJORANA
 $\Delta L = 2$ interactions at higher energies
- SM + ν_R (light)
 $\Rightarrow m_\nu \neq 0$ MAJORANA
 $\Delta L = 2$ at present energies
- $m_R = 0$? $\Leftrightarrow U(1)$ -global symmetry
 $\Delta L = 0$?
 $m_\nu \neq 0$ DIRAC