



Variational Methods in Shape Optimization Problems

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Variational Methods
in
Shape Optimization Problems

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Preface

The study of shape optimization problems is a very wide field, both classical, as the isoperimetric problem and the Newton problem of the best aerodynamical shape show, and modern, for all the recent results obtained in the last two, three decades.

The fascinating feature is that the competing objects are *shapes*, i.e. domains of \mathbb{R}^N , instead of functions, as it usually occurs in problems of the calculus of variations. This constraint often produces additional difficulties that lead to a lack of existence of a solution and to the introduction of suitable *relaxed* formulations of the problem. However, in some few cases an optimal solution exists, due to the special form of the cost functional and to the geometrical restrictions on the class of competing domains.

This volume started as a collection of the lecture notes of two courses given in the academic year 2000-2001 by the authors at the Dipartimento di Matematica of University of Pisa and at Scuola Normale Superiore di Pisa respectively. The courses were mainly addressed to Ph. D. students and required as a background the topics in functional analysis that are usually taught in undergraduate courses. More material has been added later. However, the style of the volume remains quite informal and follows, in a large part, the lectures that have been given.

The volume also contains a wide bibliography, but the list of references is far to be complete and exhaustive, both for our partial knowledge of all the papers published in this field as well as for the reason that the subject is still quickly developing.

Metz and Pisa, July 31, 2004

Dorin Bucur and Giuseppe Buttazzo