Color Image Correction

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Overview

- The problem
- Color representation : perceptual parameters
- Retinex theory and basic experiments
- The manifold of color parameters
- Color contrast enhancement
- Experiments

Color Image Correction: the problem

We perceive color under different illumination conditions, in a stable and robust way in a varying environment

Illumination may be poor, it may have a color dominance, may be night, daylight, etc.

How can we enhance color images ?

How can we eliminate the color cast ?

How can we normalize the color ?

Color image correction is a utility in present digital photography

Color Image Correction: the problem

Figure 1: Courtesy of Frédéric Guichard, DXO

Color Image Correction: the problem

Figure 2: Courtesy of Frédéric Guichard, DXO

Color representation : perceptual parameters

Simplifying, color is basically due to :

Selective emission/reflection of different wavelengths by surfaces in the world

The different response of the eye to the different wavelengths : the eye disposes of three types of cones for vision in photopic conditions

Each cone has its own sensitivity which depends on the wavelength : maximum sensitivity in short, medium and large wavelengths. They are called the cones sensitive to blue, green and red components of light.

Color sensors in the retina

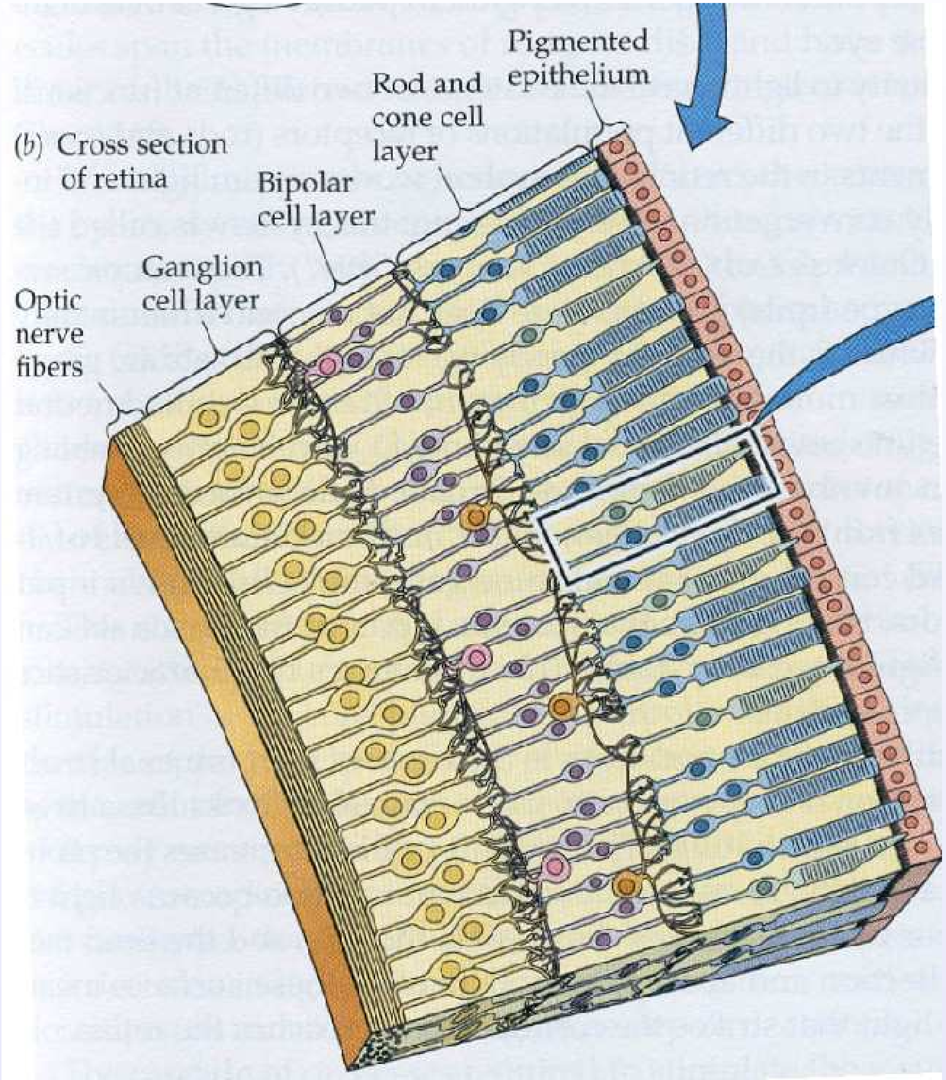


Figure 3: The sensors in the retina: rods and cones

The function of spectral sensitivity of the cones

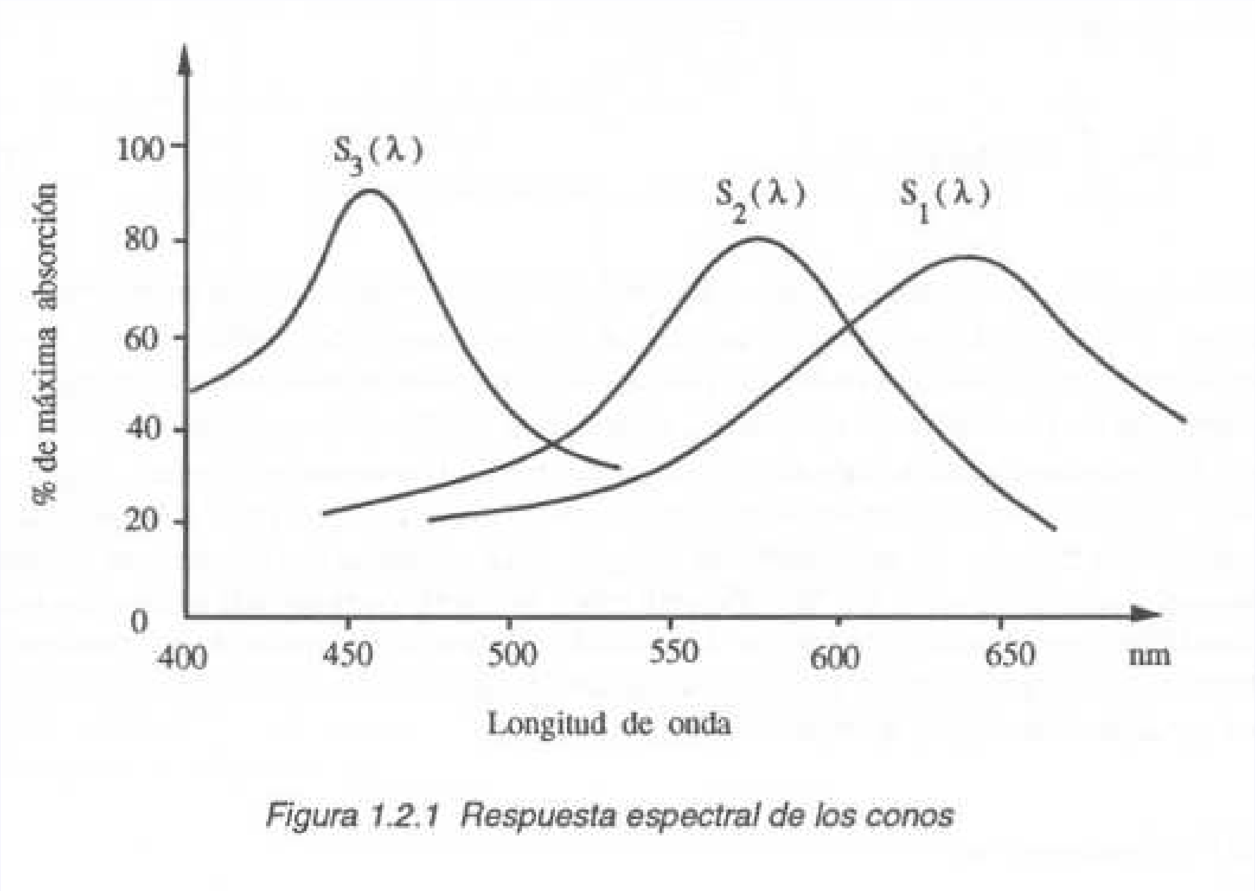


Figure 4: Three types of cones: short (blue), medium (green) and large (ref) wavelengths

The three responses of the cones

- $Il(\lambda)$ illumination
- $R(\lambda)$ reflectance
- $C(\lambda) = Il(\lambda) \times R(\lambda)$ light stimulus arriving at the retina

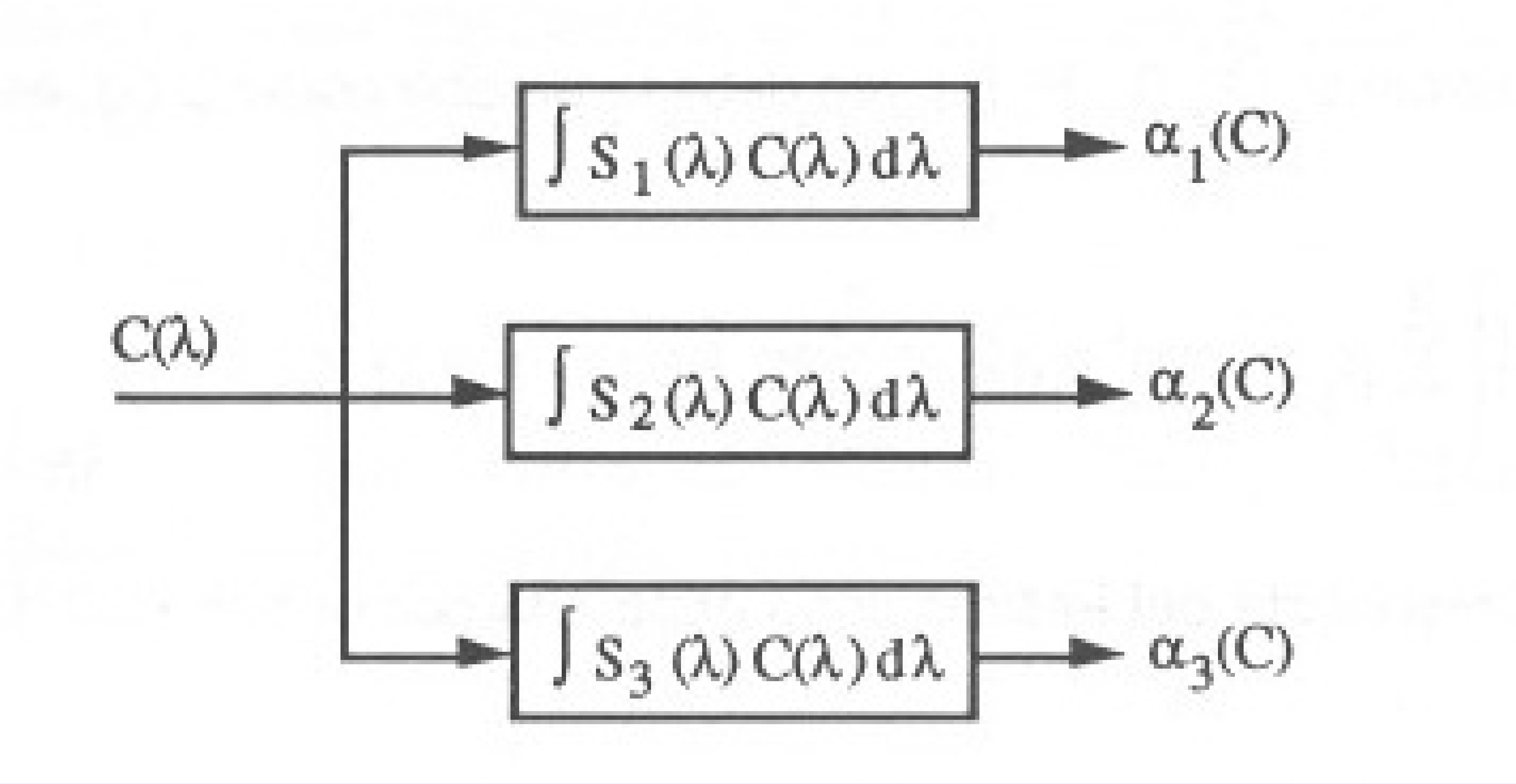


Figure 5: The three cone responses.

Color representation : three dimensional vectors

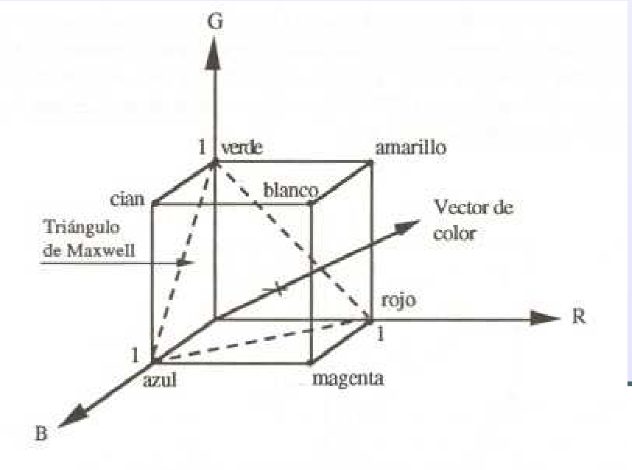
Color representation: basis of colors $P_k(\lambda)$ such that $\sum_{k=1}^{3} \beta_i P_k(\lambda) \equiv C(\lambda)$, i.e.,

$$\langle \sum_{k=1}^{3} \beta_i P_k(\lambda), S_i(\lambda) \rangle = \langle C(\lambda), S_i(\lambda) \rangle \quad i = 1, 2, 3.$$

Normalized coordinates: the white has coordinates $(1, 1, 1)$

Triestimulus functions $T_k(\lambda)$, $k = 1, 2, 3$: they give the coordinates

$$T_k(C) = \beta_k = \langle C(\lambda), T_k(\lambda) \rangle.$$



Triestimulus functions for the RGB system

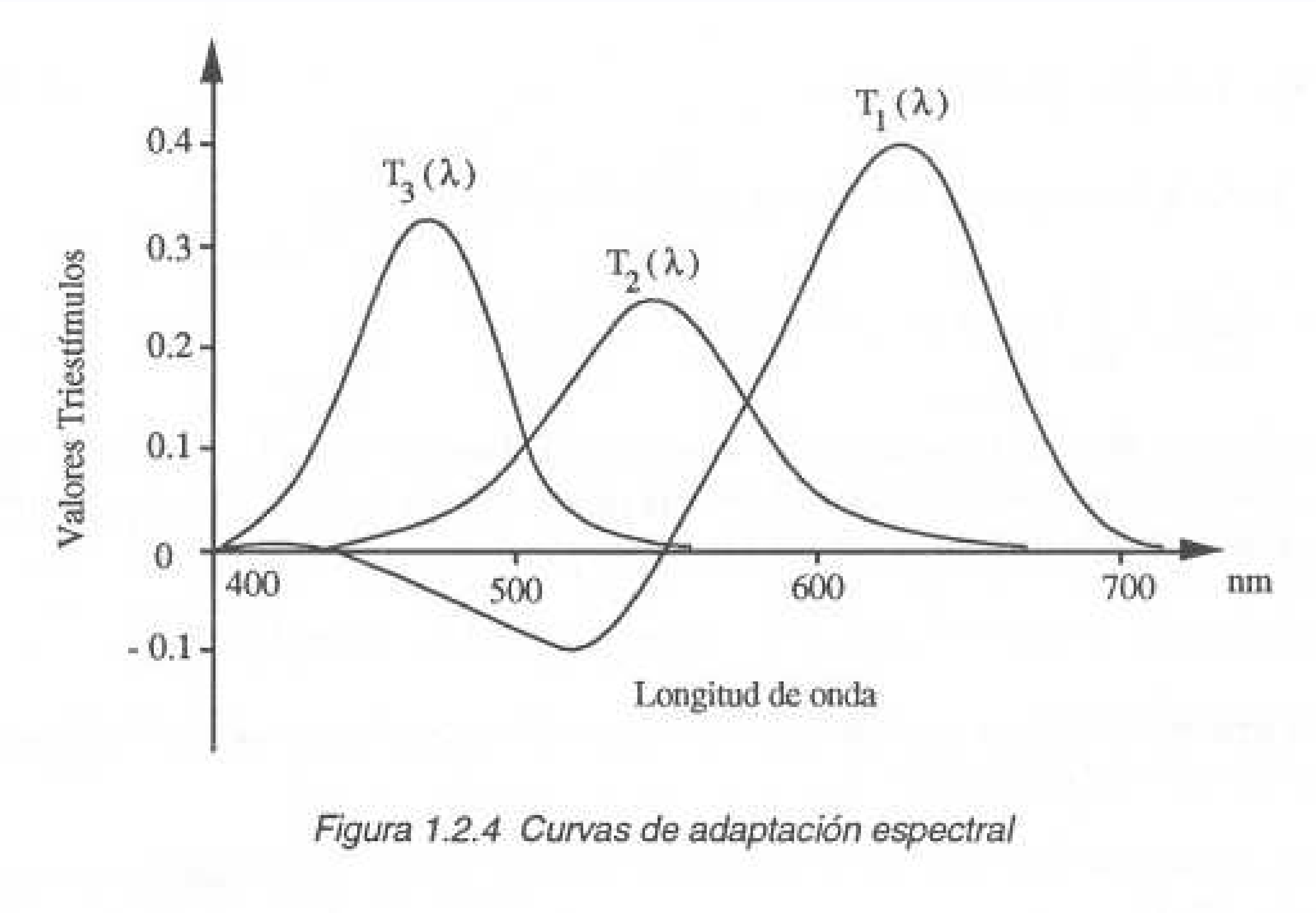


Figure 7: Triestimulus functions.

Color representation : three dimensional vectors

Perceptual properties of color are described in terms of three parameters:

- Luminance: perceived intensity or brightness (energy), depends on the response of the eye
- Hue : attribute associated to the predominant wavelength (usually, what we refer as color in ordinary language)
- Saturation : Amount of achromatic light in the perceived color

Comments on representation of the perceptual attributes of color

The chromaticity diagram

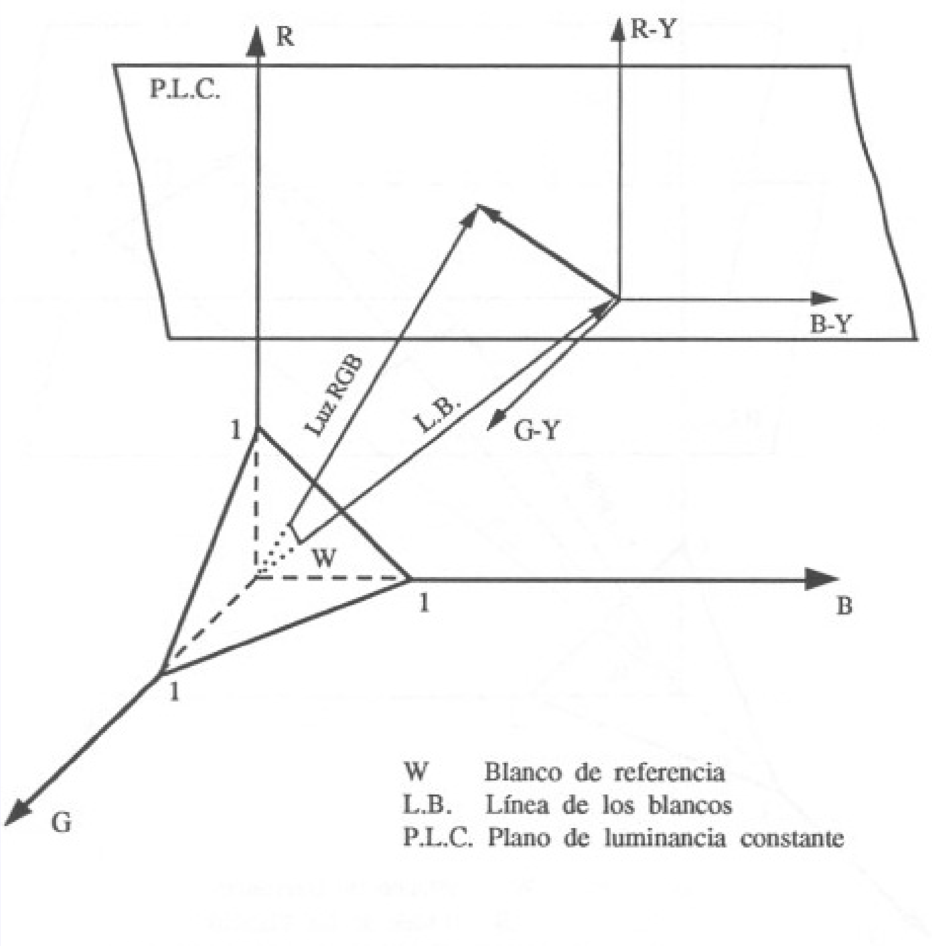


Figura 1.2.11 Sistema tridimensional RGB

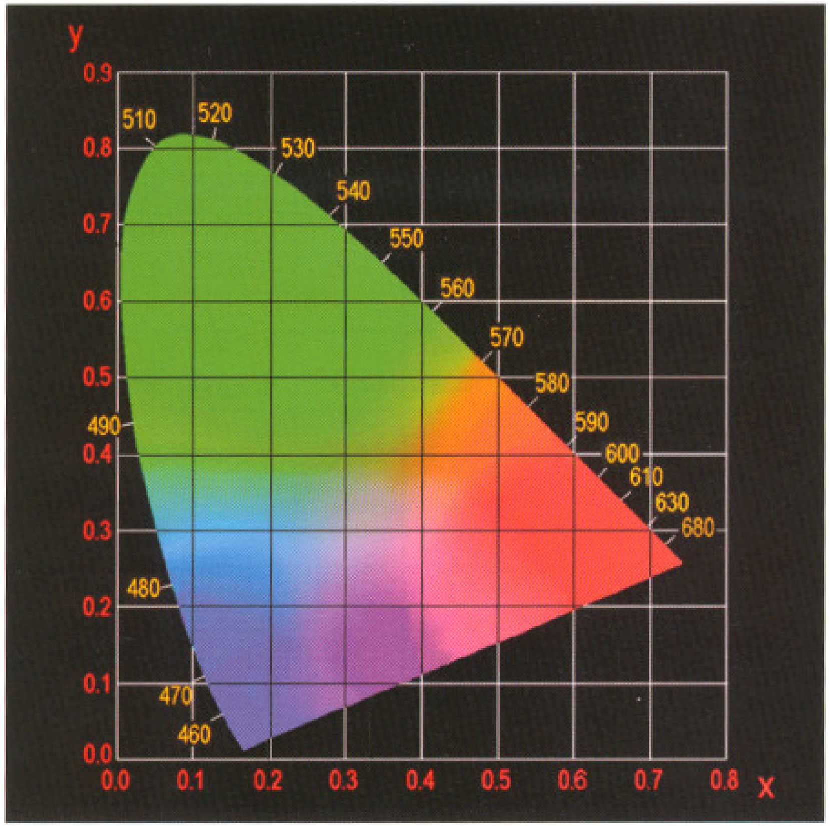


Figure 4.7 Color chromaticity $x - y$ diagram for a 2-deg field (CIE 1931).

Figure 8: The chromaticity diagram.

The chromaticity diagram

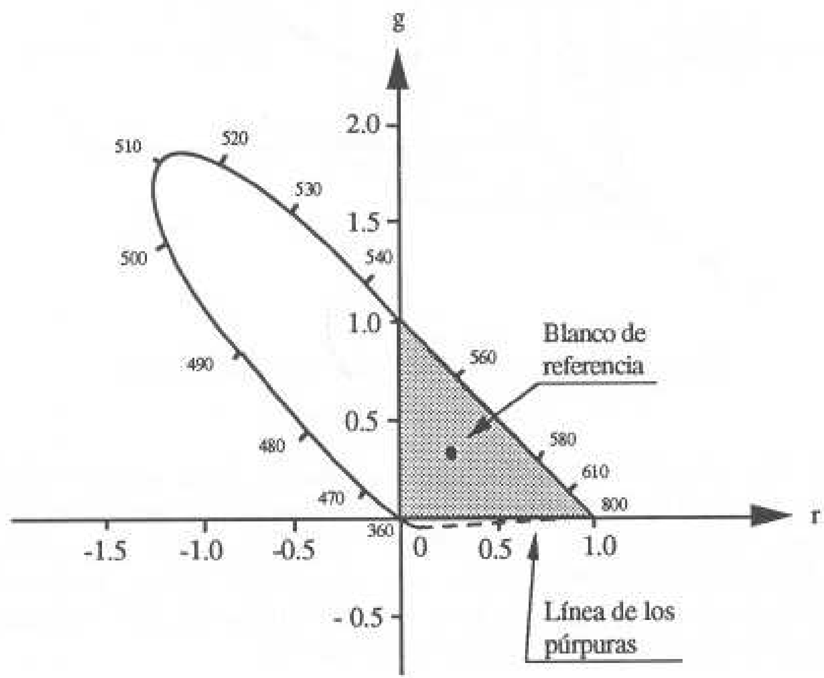


Figura 1.2.6 Diagrama de cromaticidad en el sistema CIE

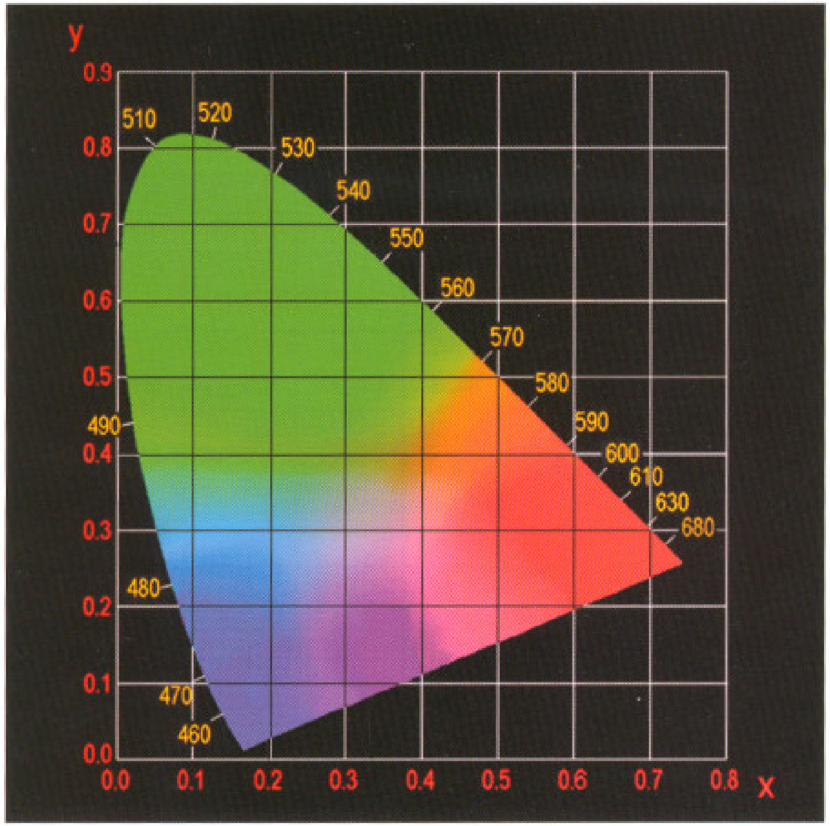


Figure 4.7 Color chromaticity $x - y$ diagram for a 2-deg field (CIE 1931).

Figure 9: The chromaticity diagram.

Retinex theory

Fundamental observation: color constancy principle. We are able to "see" the colors of objects (Objects retain their color identity) under a great variety of lighting conditions.

$\Longrightarrow$ The quantities related to the perception of color cannot be the responses of the three types of cones in the retina. What are the processes involved in the retina and brain ?

Retinex is a theory proposed by Edwin Land in 1959 which tried to explain the process of human color vision. He proposed a series of experiments trying to elucidate the type of computations done by the human vision system to be able to identify and see the familiar colors of objects in a varying environment (discount the illuminant).

As a consequence he was able to normalize the color presentation obtained in photography and had immediate applications to obtain visual realism in photos (it is implemented in cameras). There are many variants of the algorithm he proposed (in particular, one which is NASA Retinex algorithm).

Retinex theory

Figure 10: Example of NASA Retinex

Retinex theory

Figure 11: Example of NASA Retinex.

Retinex theory

From Edward Adelson: " The amount of light coming to the eye from an object depends on the amount of light striking the surface, and on the proportion of light that is reflected. If a visual system only made a single measurement of luminance, acting as a photometer, then there would be no way to distinguish a white surface in dim light from a black surface in bright light. Yet humans can usually do so, and this skill is known as lightness constancy.
The constancies are central to perception. An organism needs to know about meaningful world-properties, such as color, size, shape, etc. These properties are not explicitly available in the retinal image, and must be extracted by visual processing."

Lightness: Relative amount of light reflected.

Example: A black ball remains the same either indoors or in sunlight. The absolute amount of light reflected changes, the relative amount of light reflected remains the same. Lightness remains the same, brightness changes.

Retinex theory

Land and McCann experiments tried to show:

- What we compute: lightness (independently of flux arriving to the eye). They used Mondrians.

- How we compute it (the response of the cones being flux measures): describe the biological mechanisms that convert flux into a pattern of lightness. Moreover: lightness is related to object's reflectance.

- There are three independent processes computing the lightness in the short, middle, and long wave bands and the comparison of these three quantities leads to the perception of color.

Retinex theory

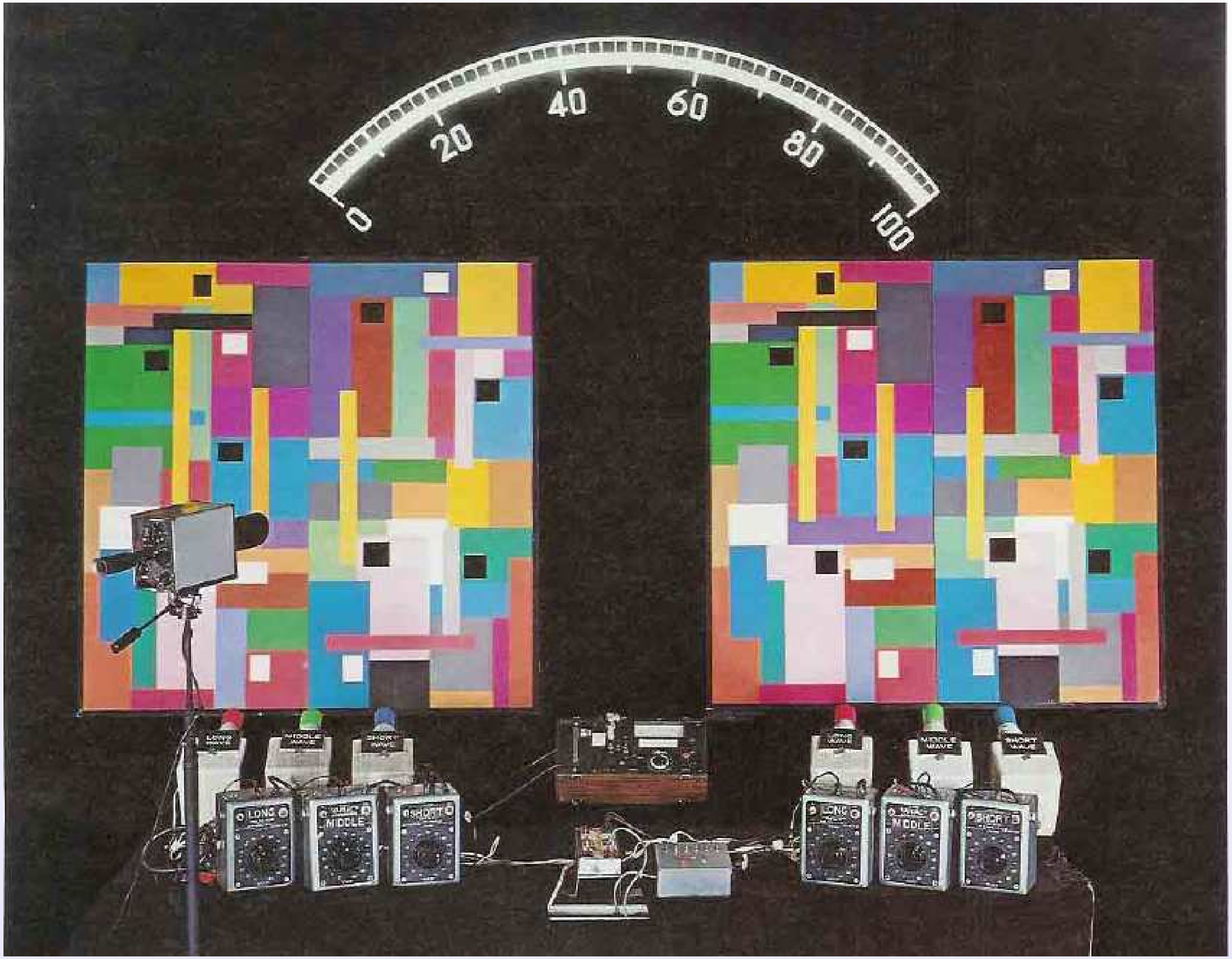
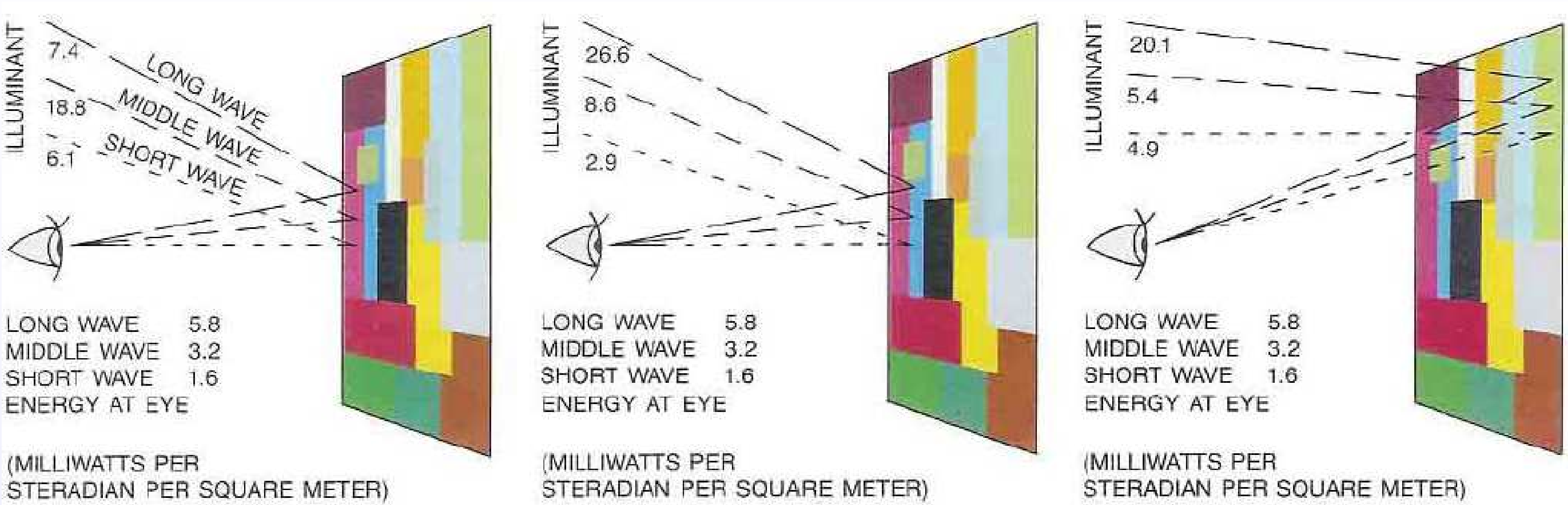


Figure 12: The experimental set up of Land's experiment.

Retinex theory



IDENTICAL ENERGY FLUXES AT THE EYE provide different color sensations in the Mondrian experiments. In this example, with the illuminants from the long-wave, middle-wave and short-wave illuminators adjusted as indicated, an area that looks red continues to look red (*left*), an area that looks blue continues to look blue (*middle*) and an area that looks green continues to look green (*right*), even though all three are sending to the eye the same triplet of long-, middle- and short-wave energies. The same triplet can be made to come from any other area: if the area is white, it remains white; if the area is gray, it remains gray; if it is yellow, it remains yellow, and so on.

Figure 13: Land's experiment.

Retinex theory: from brightness to lightness

Let $I(i)$ denote the intensity and $L(i)$ denote the lightness of pixel i. Assume that $I(i) \in (0, 1]$ (normalized range).

The three ingredients: Paths, Ratios, Reset.

• Paths: it permits to compare the relative reflectances of two parts of the image (permits to compare colors of adjacent areas of the image) Select a pixel i and a set of paths γ_k, $k = 1, \ldots, N$, ending at i starting at some pixel. The path γ_k has n_k pixels.

• Ratios: it permits to normalize the illumination.

Let x_{t_k} and $x_{t_{k+1}}$ be two consecutive pixels in path γ_k and let

$$R_{t_k} = \frac{I(x_{t_{k+1}})}{I(x_{t_k})}$$

• Reset: find the area of highest reflectance (and start afresh from it) and the lightness of any part of the image is a fraction of it.

Retinex theory: from brightness to lightness

- Reset: find the area of highest reflectance (and start afresh from it) and the lightness of any part of the image is a fraction of it.

Let $L_k(i)$ be the contribution to the lightness from path γ_k. Then

$$L(i) = \frac{1}{N}\sum_{k=1}^{N} L_k(i)$$

Formula for $L_k(i)$:

$$L_k(i) = \Pi_{t_k=1}^{n_k-1}\delta_k(R_{t_k})$$

where $\delta_k(R_{t_k}) = R_{t_k}$ unless $R_{t_k}\Pi_{t_k=1}^{t_k-1}\delta_k(R_{t_k}) >> 1$ in which case we reset $\Pi_{t_k=1}^{t_k}\delta_k(R_{t_k}) = 1$ since we have found a local reference for white.

Remark: $L(i) \geq I(i)$. This simplified formulation is taken from [Provenzi et al.]

Variants of Retinex algorithm

- Brownian retinex: use random sprays instead of paths
- Horn's approach (Marr version): Recall $I = Il \times R$. Then

$$\log\, I = \log\, Il + \log\, R.$$

Notice that, being Il almost uniform, $\log\, Il$ is almost constant while $\log\, R$ has sharp jumps at edges. Thus we eliminate the slow variations taking

$$d = \begin{cases} \Delta \log\, I & \text{if } |\Delta \log\, I| > \tau \\ \\ 0 & \text{otherwise} \end{cases}$$

Then compute lightness by solving

$$\Delta L = d$$

The manifold of color parameters [Resnikoff]

Let $\mathcal{P}$ be the set of perceived lights.

Operations: Linear combinations mean mixing. Equality means color matching equality.

- Cone Axiom : If $x \in \mathcal{P}$ and $\alpha > 0$, then $\alpha x \in \mathcal{P}$.
- No absence from superposition: If $x \in \mathcal{P}$, then there is no $y \in \mathcal{P}$ such that $x + y = 0$.
- Convex combination (Grassmann 1853, Helmholtz 1866): If $x, y \in \mathcal{P}$ and $\alpha \in [0, 1]$, then $\alpha x + (1 - \alpha)y \in \mathcal{P}$.
- Any four perceived lights are linearly dependent (Grassmann 1853)

Thus $\mathcal{P}$ is a cone in a vector space $\mathcal{V}$ of dimension ≤ 3.

Let $GL(\mathcal{P})$ be the group of orientation preserving linear maps in $\mathcal{V}$ which preserve the cone $\mathcal{P}$. Its elements represent changes of background illumination.

- $\mathcal{P}$ is globally homogeneous with respect to changes of background illumination.

The manifold of color parameters

Then $\mathcal{P}$ is an homogeneous space isomorphic to $GL(\mathcal{P})/K$ where K is a subgroup of $GL(\mathcal{P})$ which leaves some point of $\mathcal{P}$ fixed.

• Observation: $GL(\mathcal{P})/K = \mathbb{R}^+ \times SL(\mathcal{P})/K$ where SL are the matrices of determinant 1.

• Theorem: Either $\mathcal{P}$ is equivalent to $\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$ or to $\mathbb{R}^+ \times SL(2, \mathbb{R})/SO(2)$.

The metric structure of $\mathcal{P}$.

- Axiom: Perceptual distances in $\mathcal{P}$ are invariant under changes in background illumination, that is

$$d(x,y) = d(gx,gy) \qquad \forall x,y \in \mathcal{P},\ \forall g \in GL(\mathcal{P})$$

In case that $\mathcal{P} \equiv \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$ then

$$ds^2 = \alpha_1 \left(\frac{dx_1}{x_1}\right)^2 + \alpha_2 \left(\frac{dx_2}{x_2}\right)^2 + \alpha_3 \left(\frac{dx_3}{x_3}\right)^2 \quad \alpha_i > 0.$$

If $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and $u_k = \log x_k$, then the metric is the euclidean metric.

Consequence If $C_1 = (R_1, G_1, B_1)$, $C_2 = (R_2, G_2, B_2)$ are the coordinates of two colors, then its distance is

$$d^2(C_1, C_2) = (\log \frac{R_1}{R_2})^2 + (\log \frac{G_1}{G_2})^2 + (\log \frac{B_1}{B_2})^2.$$

Enhancement of Color Images: basic principles

The general structure of the energy functional

- Let $I_0, I : \Omega \to (0,1]$ be images, I_0 given, $\Omega = [0,1]^2$.
- Visual adaptation to the average intensity level of the scene:

$$D_1(I) = \int_\Omega d_1(I(x), \frac{1}{2})\, dx \tag{1}$$

- Attachment to data $I_0(x)$

$$D_2(I) = \int_\Omega d_2(I(x), I_0(x))\, dx \tag{2}$$

- Enhance contrast: we define a contrast measure as a function $\overline{c} : (0,+\infty) \times (0,+\infty) \to \mathbb{R}$ which is continuous, symmetric in (a,b), i.e., $\overline{c}(a,b) = \overline{c}(b,a)$, increasing when $\min(a,b)$ decreases or $\max(a,b)$ increases.

Examples : $\overline{c} = |a-b|$, $\overline{c}(a,b) = \frac{\max(a,b)}{\min(a,b)}$.

Inverse contrast : decreasing when $\min(a,b)$ decreases or $\max(a,b)$ increases.

Enhancement of Color Images: basic principles

The general structure of the energy functional

Let $w : \Omega \times \Omega \to \mathbb{R}^+$ be a positive symmetric kernel such that $w(x, y)$ measures the mutual influence of both pixels x, y. We assume that the kernel is normalized

$$\int_\Omega w(x, y)\, dy = 1 \qquad \forall x \in \Omega. \tag{3}$$

We define the contrast energy term by

$$C_w(I) = \int_\Omega \int_\Omega w(x, y)\, c(I(x), I(y))\, dxdy. \tag{4}$$

We define a perceptually inspired color correction energy functional by

$$E_w(I) = D(I) + C_w(I), \tag{5}$$

Enhancement of Color Images: basic principles

In order to remove the color cast, we assume that $c(a,b)$ is an homogeneous function (of degree zero), that is,

$$c(\lambda a, \lambda b) = c(a,b) \quad \forall \lambda, a, b \in (0,\infty), \tag{6}$$

Then $c(a,b)$ is a function of a/b. Thus $c(I(x), I(y))$ can be written as a monotone nondecreasing function of $\frac{\min(I(x),I(y))}{\max(I(x),I(y))}$.

Further support: Weber-Fechner's law states that in the experiment of minimal perceptible difference the *Weber-Fechner ratio* $\mathcal{R}_{WF} \equiv \frac{|I_1 - I_0|}{I_0} = |I_1/I_0 - 1|$ remains constant: the perceived contrast is a function of I_1/I_0.

Problem: It would be interesting to know if given an inverse contrast function c, there are increasing functions $g, h : (0,\infty) \to (0,\infty)$ such that $g(c(h(a), h(b))) = \frac{a}{b}$ for any $0 < a < b$. Probably some other assumptions on c are required.

Enhancement of Color Images: basic principles

Examples:

$$C_w^{\mathrm{id}}(I) := \frac{1}{4}\int_\Omega\int_\Omega w(x,y)\,\frac{\min(I(x),I(y))}{\max(I(x),I(y))}\,dxdy, \tag{7}$$

$$C_w^{\log}(I) := \frac{1}{4}\int_\Omega\int_\Omega w(x,y)\,\log\left(\frac{\min(I(x),I(y))}{\max(I(x),I(y))}\right)\,dxdy, \tag{8}$$

$$C_w^{-\mathcal{M}}(I) := -\frac{1}{4}\int_\Omega\int_\Omega w(x,y)\,\mathcal{M}\left(\frac{\min(I(x),I(y))}{\max(I(x),I(y))}\right)\,dxdy, \tag{9}$$

where

$$\mathcal{M}\left(\frac{\min(I(x),I(y))}{\max(I(x),I(y))}\right) := \frac{1-\frac{\min(I(x),I(y))}{\max(I(x),I(y))}}{1+\frac{\min(I(x),I(y))}{\max(I(x),I(y))}} \equiv \frac{|I(x)-I(y)|}{I(x)+I(y)}, \tag{10}$$

Enhancement of Color Images: basic principles

The entropy dispersion term:

$$\begin{aligned} D^{\mathcal{E}}_{\alpha,\beta}(I) &:= \alpha \int_\Omega \left(\frac{1}{2} \log \frac{1}{2I(x)} - \left(\frac{1}{2} - I(x) \right) \right) dx \\ &+ \beta \int_\Omega \left(I_0(x) \log \frac{I_0(x)}{I(x)} - (I_0(x) - I(x)) \right) dx, \end{aligned}$$

where $\alpha, \beta > 0$.

The final energy is:

$$E^{\mathrm{f}}_{w,\alpha,\beta}(I) = D^{\mathcal{E}}_{\alpha,\beta}(I) + C^{\mathrm{f}}_{w}(I).$$

where $f = \mathrm{id}, \log, -\mathcal{M}$.

Perceptual Color Correction using Variational Techniques

We minimize them with a variant of a gradient descent approach:

$$\text{Minimize} \quad \int_\Omega \left(I^{k+1}(x) \log \frac{I^{k+1}(x)}{I^k(x)} - \left(I^{k+1}(x) - I^k(x)\right) \right) dx + E_w(I^{k+1})$$

Formally, this leads to

$$I_t = -I \nabla_I E_w(I)$$

Important: This process is applied separately to each RGB channel (Retinex assumption).

Perceptual Color Correction using Variational Techniques

The numerical approach requires some care to have an algorithm of complexity $O(N \log N)$, N being the number of pixels of the image:

$$I^{k+1}(x) = \frac{I^k(x) + \Delta t \left(\frac{\alpha}{2} + \beta I_0(x) + \frac{1}{2} R^{\mathrm{id}}_{\epsilon, I^k}(x) \right)}{1 + \Delta t(\alpha + \beta)}, \tag{11}$$

where

$$R^{\mathrm{id}}_{\epsilon, I^k}(x) := \int_\Omega w(x,y) \frac{\min_\epsilon(I^k(x), I^k(y))}{\max_\epsilon(I^k(x), I^k(y))} s_\epsilon(I^k(x) - I^k(y))\, dy. \tag{12}$$

The algorithm is based on a Taylor expansion of the integrand and writing in a way that can be solved using the FFT:

$$R^{(n)}(x) = \sum_{j=0}^{n} f_j(I(x)) \int_\Omega w(x,y)\, g_j(I(y))\, dy, \tag{13}$$

Results: Color Correction

Figure 14: Original, min/max

Results: Color Correction

Figure 15: Original, Michelson

Results: Color Correction

Figure 16: Original, min/max

Results: Color Correction

Figure 17: Original, Michelson

Results: Color Correction

Figure 18: Original, min/max

Results: Color Correction

Figure 19: Original, Michelson

Results: Color Correction

Figure 20: Original, log

Results: Color Correction

Figure 21: Original, Michelson

Results: Color Correction

Figure 22: Original, Michelson

Results: Color Correction

Figure 23: Random Spray Retinex, Michelson

Results: Color Correction

Figure 24: Original, Michelson with gamma correction

Results: Color Correction

Figure 25: Original, Michelson with gamma correction

Results: Color Correction

Figure 26: Original, Michelson with gamma correction

Results: Color Correction

Figure 27: Original, min/max, min/max with noise control