Baryon Chiral Perturbation Theory

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Outline (1)

Part I: Basics

- construction of the meson-baryon Lagrangian
- power counting and its failure
- heavy-baryon ChPT and infrared regularisation

Part II: πN σ-term and strangeness in the nucleon (1)

- quark mass dependence of the nucleon mass
- sigma term and πN scattering

Outline (2)

Part III: Strangeness in the nucleon (2)

- Strangeness form factors and parity-violating $e^- p$ scattering
- Chiral perturbation theory: a failure

Part IV: Isospin-violating form factors

- Why? isospin violation and strangeness
- Chiral perturbation theory: a success

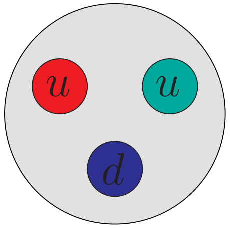
Part V: Crimes and omissions

- The role of the $\Delta(1232)$ resonance
- Two- and more-nucleon systems

Part III: Strangeness in the nucleon (2)

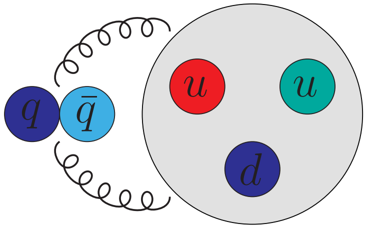
Strangeness in the nucleon (1)

- naive quark model of the proton:



Strangeness in the nucleon (1)

- virtual quark–antiquark pairs:



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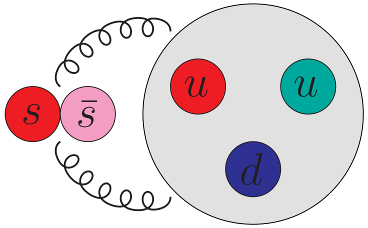
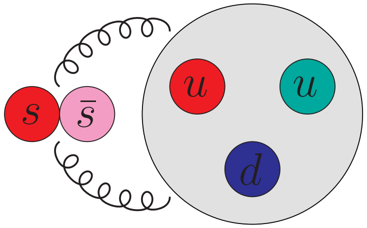


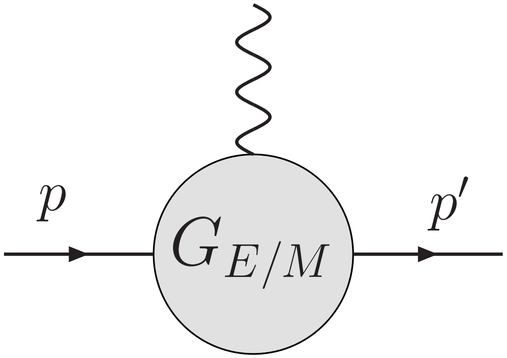
Strangeness in the nucleon (1)



- virtual strange quark–antiquark pairs:
- How “strange” is the nucleon?

$\langle N\|\bar{s}s\|N\rangle$	contribution to mass $\Rightarrow$ σ-term
$\langle N\|\bar{s}\gamma_\mu\gamma_5 s\|N\rangle$	contribution to spin
$\langle N\|\bar{s}\gamma_\mu s\|N\rangle$	contribution to magnetic moment

- vector form factors:



- electric + magnetic form factors
- contribution of the three lightest quarks

$u,\ d,\ s$: $\boxed{G^{u,d,s}_{E/M}}$

Reminder: nucleon vector form factors

- definition of nucleon vector form factors:

$$\langle N(p')|\bar{q}\gamma_\mu q|N(p)\rangle \;=\; \bar{u}(p')\Big\{\underbrace{F_1^q(t)}_{\text{Dirac}}\gamma_\mu + \frac{i}{2m_N}\sigma_{\mu\nu}(p'-p)^\nu\,\underbrace{F_2^q(t)}_{\text{Pauli}}\Big\}u(p)$$

momentum transfer $t = (p'-p)^2 = -Q^2 < 0$

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- Sachs form factors:

$$G_E^q(t) = F_1^q(t) + \frac{t}{4m_N^2}F_2^q(t) = Q^q\left\{1 + \frac{1}{6}\langle r_E^2\rangle\, t + \mathcal{O}(t^2)\right\}$$

$$G_M^q(t) = F_1^q(t) + F_2^q(t) = \mu^q\left\{1 + \frac{1}{6}\langle r_M^2\rangle\, t + \mathcal{O}(t^2)\right\}$$

Q^q: charge, μ^q: magnetic moment

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Q^q: charge, μ^q: magnetic moment

- Fourier transform G_E, G_M: charge/magnetisation distributions
  interpretation of $\sqrt{\langle r_{E/M}^2\rangle}$ as charge/magnetisation radii

Strangeness in the nucleon (2)

- wanted: flavour decomposition of the vector current $\Rightarrow \; G^{s}_{E/M}$
- electromagnetic current: $J_\mu^{\text{EM}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s \quad \Rightarrow$

$$G^{\gamma,p}_{E/M} = \frac{2}{3}G^{u}_{E/M} - \frac{1}{3}\left(G^{d}_{E/M} + G^{s}_{E/M}\right)$$

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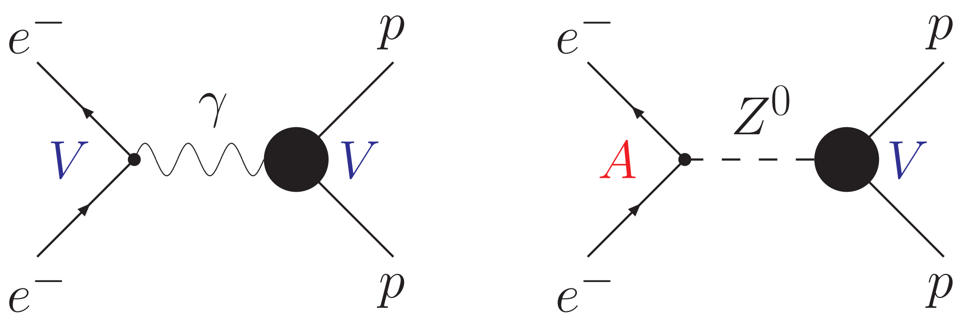
$$G^{\gamma,p}_{E/M} = \frac{2}{3}G^{u}_{E/M} - \frac{1}{3}\left(G^{d}_{E/M} + G^{s}_{E/M}\right)$$

- different linear combination: weak vector current

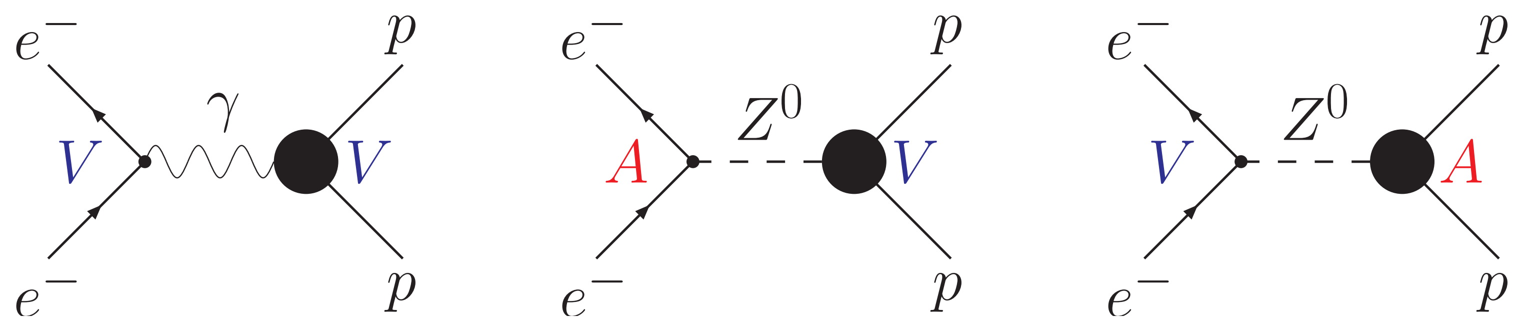
$$G^{Z,p}_{E/M} = \left(1 - \frac{8}{3}\sin^2\theta_W\right)G^{u}_{E/M} - \left(1 - \frac{4}{3}\sin^2\theta_W\right)\left(G^{d}_{E/M} + G^{s}_{E/M}\right)$$



parity violating electron scattering!

SAMPLE@MIT-Bates, HAPPEX@JLab, A4@MAMI, G0@JLab

Parity-violating electron scattering



- Measure the helicity-dependent interference; asymmetry:

$$A = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = -\frac{G_F\, Q^2}{4\sqrt{2}\pi\alpha}\, \frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - (1-4\sin^2\theta_W)\epsilon' G_M^\gamma G_A}{\epsilon (G_E^\gamma)^2 + \tau (G_M^\gamma)^2}$$

$$\tau = \frac{Q^2}{4m_N^2}\,, \quad \epsilon = \left(1 + 2(1+\tau)\tan^2\tfrac{\theta}{2}\right)^{-1}\,, \quad \epsilon' = \sqrt{\tau(1+\tau)(1-\epsilon^2)}$$

- also depends on axial form factor of the nucleon:

$$\langle N(p')|\bar{q}\gamma_\mu\gamma_5 q|N(p)\rangle \;=\; \bar{u}(p')\left\{G_A(t)\gamma_\mu\gamma_5 + \ldots\right\}u(p)$$

$G_A(0) = 1.26$ (from neutron decay, neutrino scattering)

Parity-violating electron scattering

$$A = -\frac{G_F\, Q^2}{4\sqrt{2}\pi\alpha} \frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - (1 - 4\sin^2\theta_W)\epsilon' G_M^\gamma G_A}{\epsilon (G_E^\gamma)^2 + \tau (G_M^\gamma)^2}$$

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- order of magnitude:

$$A \propto \frac{Q^2}{M_Z^2} \propto \mathcal{O}(10^{-4}) \qquad \text{for} \qquad Q^2 \approx 1\ \text{GeV}^2$$

Parity-violating electron scattering

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Parity-violating electron scattering

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- backward direction: $\theta = \pi \;\Rightarrow\; \epsilon = 0$, no contribution of G_E^Z
- forward direction: $\theta = 0 \;\Rightarrow\; \epsilon' = 0$, no contribution of G_A
- assume $G_{E/M}^\gamma$, G_A as known

  $\Rightarrow$ extract $G_{E/M}^Z$ from measured asymmetry!

Strangeness in the nucleon (3)

- two linear combinations

$$G^{\gamma,p}_{E/M}\,, \quad G^{Z,p}_{E/M}$$

  depend on three flavour form factors $G^{u,d,s}_{E/M}$

- need third linear combination: isospin-(charge-)symmetry!

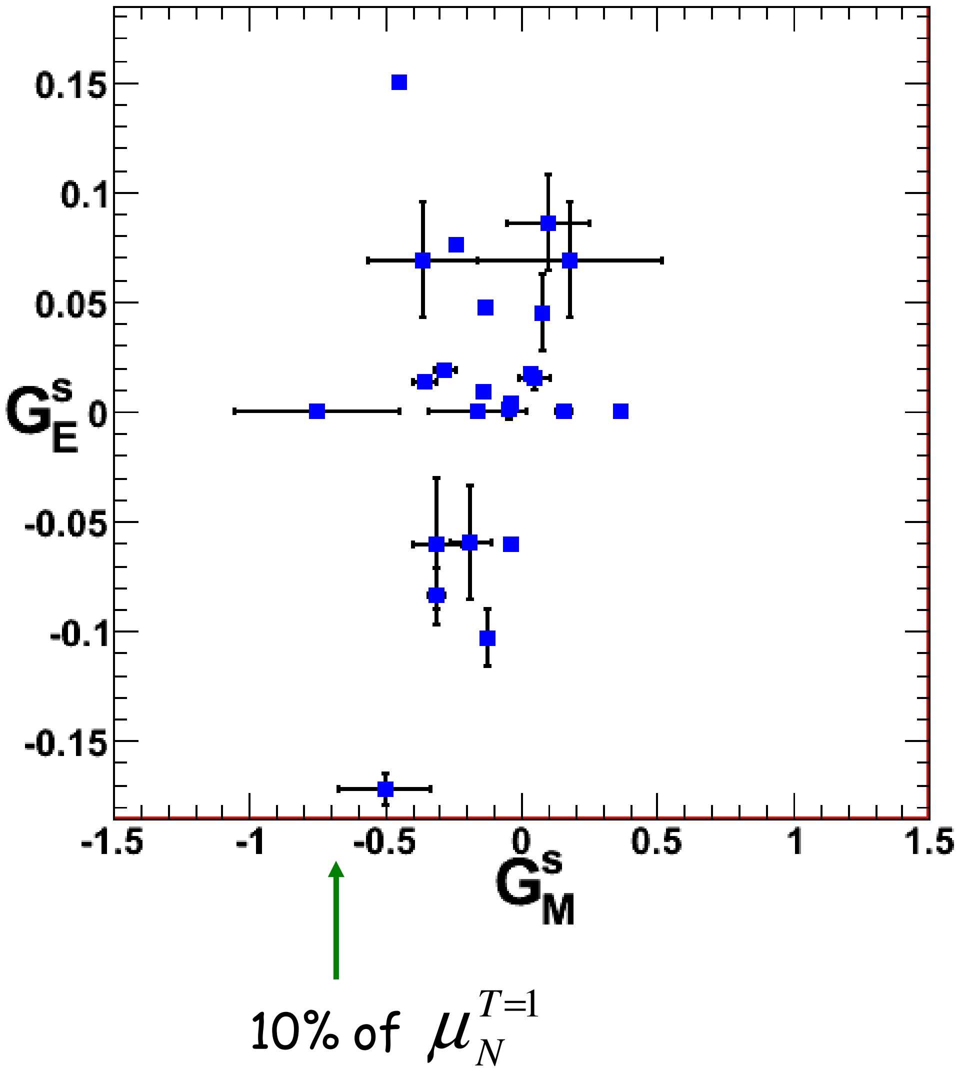
$$u \leftrightarrow d \;+\; p \leftrightarrow n\,, \qquad \text{i.e.} \quad G^{u,n}_{E/M} = G^{d,p}_{E/M} \quad \text{etc.}$$

  $\Rightarrow$ use the neutron form factor as third input

$$\boxed{G^{Z,p}_{E/M} = \left(1 - 4\sin^2\theta_W\right)G^{\gamma,p}_{E/M} - G^{\gamma,n}_{E/M} - G^{s}_{E/M}}$$

- measured $G^{Z,p}_{E/M}$ allow to extract $G^{s}_{E/M}$

Model predictions for strangeness form factors



- models all over the place; sizeable!
- G_M^s tends to be negative

Armstrong, Talk at MENU 07

ChPT: Generic power counting for nucleon form factors

- Sachs form factors:

$$G_E(t) = F_1(t) + \frac{t}{4m_N^2} F_2(t) = \underbrace{Q}_{\mathcal{O}(p)} + \underbrace{\frac{1}{6}\langle r_E^2 \rangle\, t}_{\mathcal{O}(p^3)} + \ldots$$

$$G_M(t) = F_1(t) + F_2(t) = \underbrace{\mu}_{\mathcal{O}(p^2)} + \underbrace{\frac{1}{6}\langle r_M^2 \rangle\, t}_{\mathcal{O}(p^4)} + \ldots$$

- polynomial contributions (i.e. counterterms) to the electric/magnetic radii appear at leading/subleading one-loop order

Why ChPT cannot predict μ^s and $\langle (r_E^s)^2 \rangle$

Musolf, Ito 1997

- Three diagonal vector currents in SU(3):

$$
\begin{aligned}
J_\mu^{(3)} &= \bar{q}\frac{\lambda^3}{2}\gamma_\mu q \propto \text{isovector el.magn. current}, \quad \lambda^3 = \text{diag}(1,-1,0) \\
J_\mu^{(8)} &= \bar{q}\frac{\lambda^8}{2}\gamma_\mu q \propto \text{isoscalar el.magn. current}, \quad \lambda^8 = \frac{1}{\sqrt{3}}\,\text{diag}(1,1,-2) \\
J_\mu^{(0)} &= \bar{q}\frac{\lambda^0}{2}\gamma_\mu q \propto \text{baryon number current}, \quad \lambda^0 = \sqrt{\frac{2}{3}}\,\text{diag}(1,1,1)
\end{aligned}
$$

- The "physical" currents are

$$
J_\mu^{\text{EM}} = J_\mu^{(3)} + \frac{1}{\sqrt{3}} J_\mu^{(8)} \qquad\qquad J_\mu^{\text{s}} = \sqrt{\frac{2}{3}} J_\mu^{(0)} - \frac{2}{\sqrt{3}} J_\mu^{(8)}
$$

Why ChPT cannot predict μ^s and $\langle (r_E^s)^2 \rangle$

- Consequence: wherever the electromagnetic current matrix elements have low-energy constants, there is a new one for the strange current matrix elements!

- Example: magnetic moments

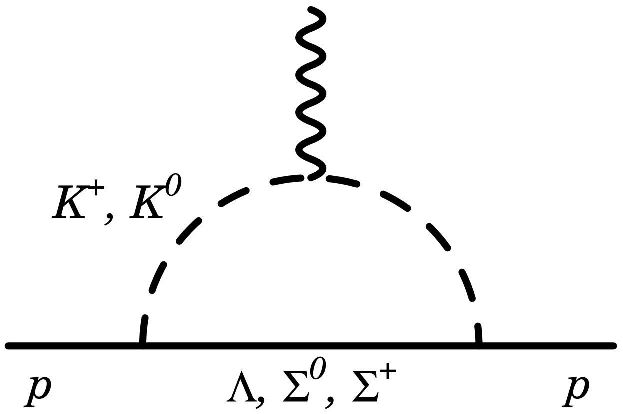
$$\mathcal{L}^{(2)} = \frac{b_6^{D/F}}{8m_N} \langle \bar{B} \sigma^{\mu\nu} \left[F_{\mu\nu}^{+}, B \right]_{\pm} \rangle + \frac{b_6^{0}}{8m_N} \langle \bar{B} \sigma^{\mu\nu} B \rangle \langle F_{\mu\nu}^{+} \rangle$$

- $b_6^{D/F}$ can be fitted to μ_p, μ_n or the octet magnetic moments, but b_6^0 appears only in the strange magnetic moment!

- Same pattern for all other low-energy constants
  $\Rightarrow$ need to fit these to experimental results

A low-energy theorem for $\langle (r_M^s)^2 \rangle$

- How can there possibly be a low-energy theorem??
- Answer: leading non-analytic loop effects!



- diagram of $\mathcal{O}(p^3)$ generates

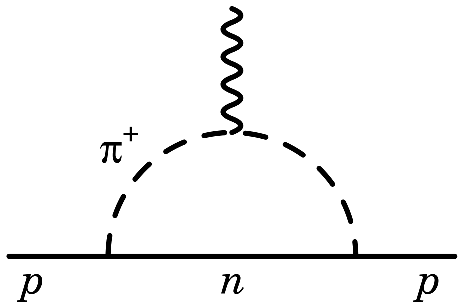
$$\langle (r_M^s)^2 \rangle = -\frac{5D^2 - 6DF + 9F^2}{48\pi F_K^2} \frac{m_N}{M_K}$$

- remember: low-energy constant only at $\mathcal{O}(p^4) \Rightarrow$ "suppressed"

Hemmert, Meißner, Steininger 1998

A low-energy theorem for $\langle (r_M^s)^2 \rangle$

- Known in the isovector magnetic radius since long:



$$\Rightarrow \quad \langle r_{M,v}^2 \rangle = \frac{g_A^2}{8\pi F_\pi^2 \mu_v} \frac{m_N}{M_\pi}$$

Bég, Zapeda 1972

- Physical picture:
  pion-/kaon-cloud becomes infinite-ranged in the chiral limit $M_\pi,\ M_K \to 0$.
- masses, coupling constants known $\Rightarrow$ parameter-free prediction

$$\boxed{\langle (r_M^s)^2 \rangle = -0.115\ \text{fm}^2}$$

- use this to extrapolate measurement of $G_M^s(Q^2)$ at finite $Q^2 = -t$ to the strange magnetic moment μ_s

A low-energy theorem for $\langle (r_M^s)^2 \rangle$

How stable is the low-energy theorem for $\langle (r_M^s)^2 \rangle$?

$\Rightarrow$ Next-to-leading order corrections ($\mathcal{O}(p^4)$):

Instead of

$$\langle (r_M^s)^2 \rangle^{(3)} = -0.115 \text{ fm}^2$$

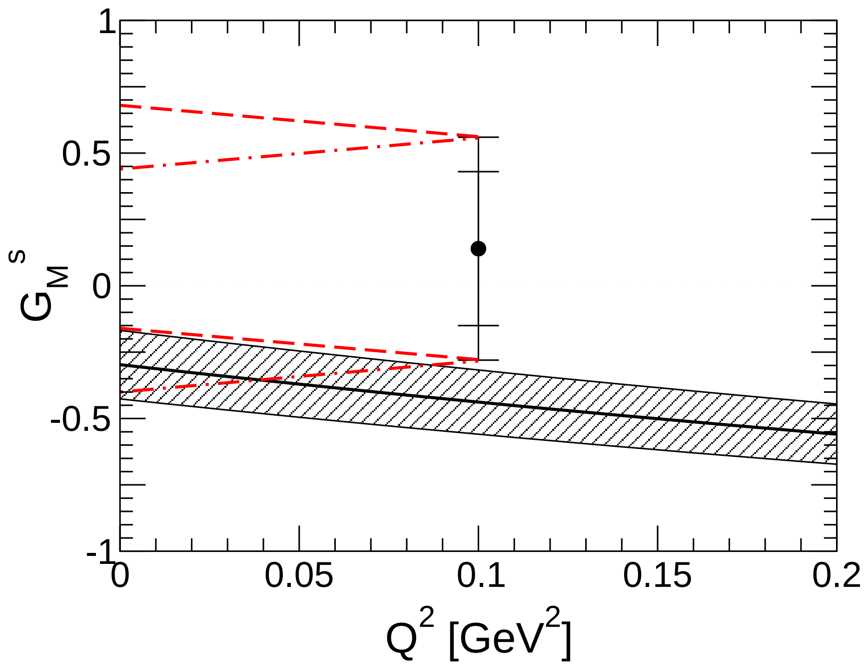
find

$$\boxed{\langle (r_M^s)^2 \rangle^{(4)} = -\left(0.04 + 0.3\, b_s^r\right) \text{ fm}^2}$$

where $|b_s^r| \leq 1$

$\Rightarrow$ corrections large

$\Rightarrow$ b_s^r theoretical uncertainty

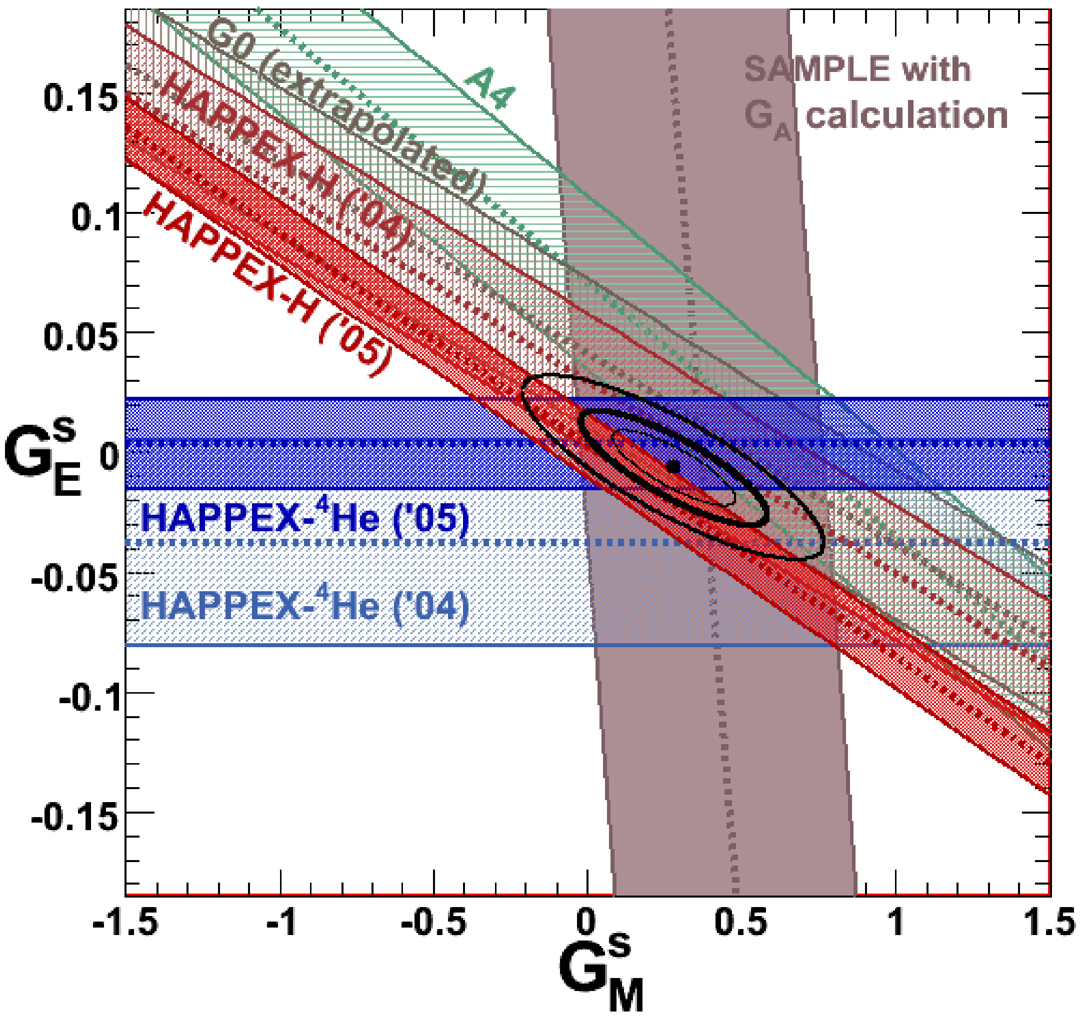


Hammer, Puglia, Ramsey-Musolf, Zhu 2003

BK 2002, 2005

Conclusion on strangeness:

. . . let the experimenters do their job!



Armstrong, Talk at MENU 07

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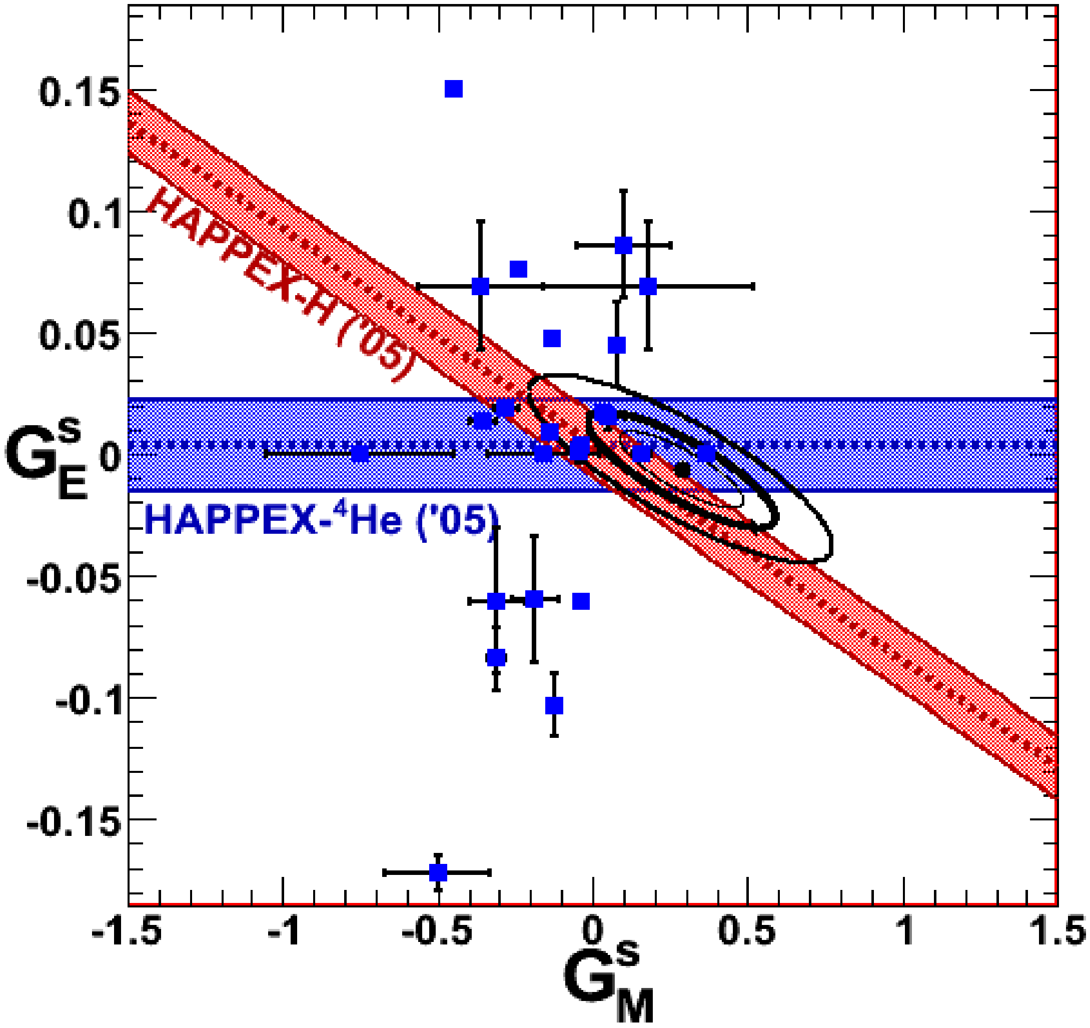

Armstrong, Talk at MENU 07

Part IV: Isospin-violating form factors

Strangeness and isospin violation

- Remember: third linear combination: isospin symmetry!

$$u \leftrightarrow d \; + \; p \leftrightarrow n \,, \qquad \text{i.e.} \quad G_{E/M}^{u,n} = G_{E/M}^{d,p} \quad \text{etc.}$$

$\Rightarrow$ use the neutron form factor as third input

$$\boxed{G_{E/M}^{Z,p} = \left(1 - 4\sin^2\theta_W\right) G_{E/M}^{\gamma,p} - G_{E/M}^{\gamma,n} - G_{E/M}^{s}}$$

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- without isospin conservation:

$$G^{u,d}_{E/M} = \frac{2}{3}\left(G^{d,p}_{E/M} - G^{u,n}_{E/M}\right) - \frac{1}{3}\left(G^{u,p}_{E/M} - G^{d,n}_{E/M}\right)$$

$\Rightarrow$ isospin violation generates “pseudo-strangeness”!

Isospin violation and chiral perturbation theory (1)

- isospin violation: $m_u \neq m_d$
- calculate pion mass difference:

$$M_{\pi^0}^2 = M_{\pi^+}^2 \left\{ 1 - \frac{(m_d - m_u)^2}{8\hat{m}(m_s - \hat{m})} + \ldots \right\}$$

plug in quark mass ratios ...

$$\begin{aligned} M_{\pi^+} - M_{\pi^0} &\approx 0.1 \text{ MeV} \\ \text{vs.} \quad (M_{\pi^+} - M_{\pi^0})_{\text{exp}} &\approx 4.6 \text{ MeV} \end{aligned}$$

- second source of isospin violation: $q_u \neq q_d$!
- inclusion of photon effects via minimal substitution insufficient
  introduce quark charge matrix $Q = e \, \text{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right) = \mathcal{O}(p)$
- one single term at $\mathcal{O}(e^2) = \mathcal{O}(p^2)$:

$$\mathcal{L}_{\text{em}}^{(2)} = C \langle Q U Q U^\dagger \rangle$$

generates $\left(M_{\pi^+}^2 - M_{\pi^0}^2\right)_{\text{em}} = \left(M_{K^+}^2 - M_{K^0}^2\right)_{\text{em}} = 2Ce^2/F^2$

Isospin violation and chiral perturbation theory (2)

- ChPT: treat

$$m_u \neq m_d \qquad \text{and} \qquad q_u \neq q_d$$

  simultaneously and consistently
- no fixed hierarchy between both effects:

$$\begin{aligned} M_{\pi^+} - M_{\pi^0} &\simeq 4.5\ \text{MeV}_{\text{em}} + 0.1\ \text{MeV}_{m_u \neq m_d} \\ m_n - m_p &\simeq -0.8\ \text{MeV}_{\text{em}} + 2.1\ \text{MeV}_{m_u \neq m_d} \\ \epsilon_{\pi^0\eta} &\simeq \left(\epsilon_{\pi^0\eta}\right)_{m_u \neq m_d} \qquad (\text{e.g. in } \eta \to 3\pi) \end{aligned}$$

- much smaller effect than SU(3) breaking
- no m_s $\Rightarrow$ better convergence behaviour expected

Generic power counting revisited

- “polynomial” isospin breaking suppressed by

$$m_d - m_u = \mathcal{O}(p^2) \qquad \text{or} \qquad e^2 = \mathcal{O}(p^2)$$

- therefore leading moments:

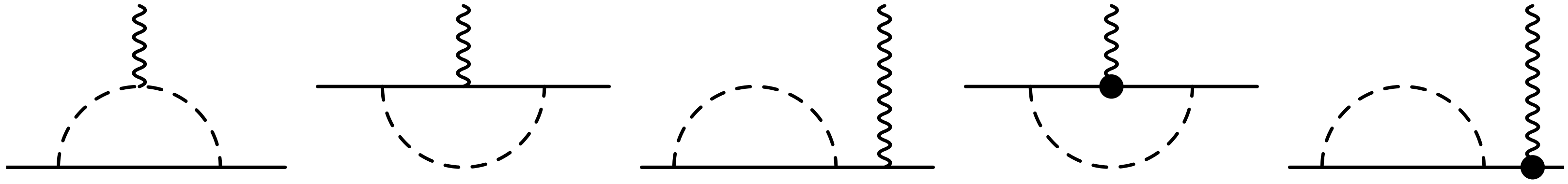
$$G_E^{u,d}(t) = \underbrace{\rho_E^{u,d}\, t}_{\mathcal{O}(p^5)} + \mathcal{O}(t^2)$$

$$G_M^{u,d}(t) = \underbrace{\kappa^{u,d}}_{\mathcal{O}(p^4)} + \underbrace{\rho_M^{u,d}\, t}_{\mathcal{O}(p^6)} + \mathcal{O}(t^2)$$

- calculate $G_E^{u,d}$ up to $\mathcal{O}(p^4)$, $G_M^{u,d}$ up to $\mathcal{O}(p^5)$

Isospin violating form factors in ChPT

- (non-trivial) diagrams:



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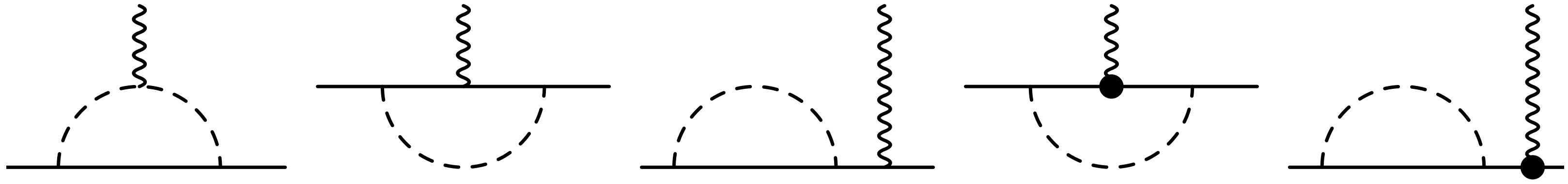


- radii: can be expressed in terms of $\Delta m = m_n - m_p$

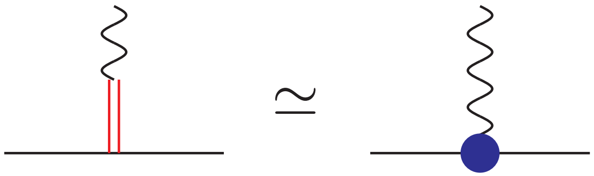
$$\rho_E^{u,d} = \frac{5\pi C}{6M_\pi m_N}\,, \quad \rho_M^{u,d} = \frac{2C}{3M_\pi^2}\left\{1 - \frac{7\pi}{4}\frac{M_\pi}{m_N}\right\}\,, \quad C = \frac{g_A^2 m_N \Delta m}{16\pi^2 F_\pi^2}$$

BK, Lewis 2006

- missing: unknown low-energy constant in $\kappa^{u,d}$
  $\Rightarrow$ resonance saturation

Resonance saturation

- low-energy constants parameterise effects of heavy (non-Goldstone boson) states:

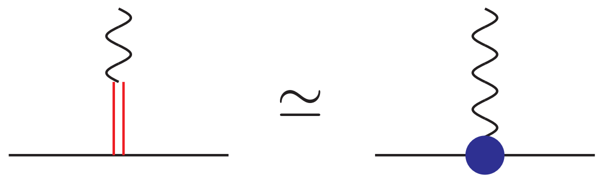


$$\frac{1}{M_{\text{res}}^2 - t} \simeq \frac{1}{M_{\text{res}}^2}\left\{1 + \frac{t}{M_{\text{res}}^2} + \dots\right\}$$

- modern form of “vector meson dominance”

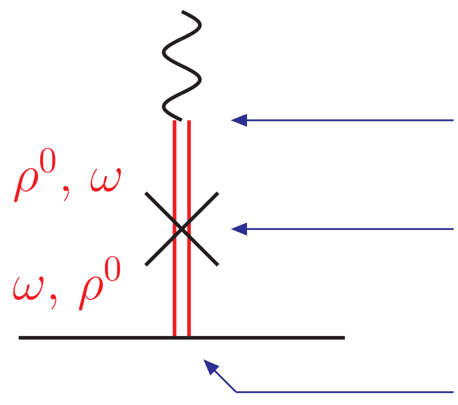
Resonance saturation

- low-energy constants parameterise effects of heavy (non-Goldstone boson) states:



$$\frac{1}{M_{\text{res}}^2 - t} \simeq \frac{1}{M_{\text{res}}^2}\left\{1 + \frac{t}{M_{\text{res}}^2} + \ldots\right\}$$

- modern form of “vector meson dominance”
- here: $\rho - \omega$ mixing



$F_\rho,\ F_\omega$ (decay constants) from $\rho^0,\ \omega \to e^+e^-$

$\Theta_{\rho\omega}$ (mixing angle) from pion form factor

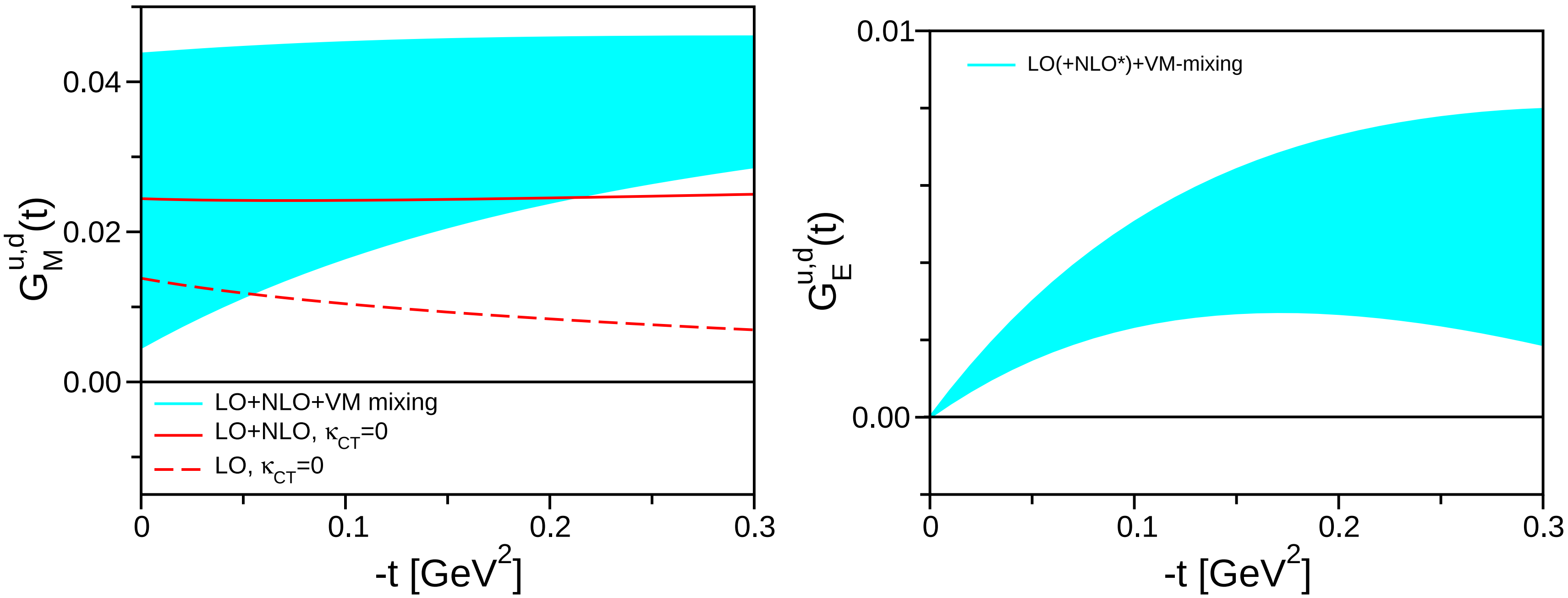
$g_{\rho,\omega},\ \kappa_{\rho,\omega}$ from form factor dispersion analysis

Belushkin, Hammer, Meißner 2007

- higher-order low-energy constants (in $\rho^{u,d}_{E/M}$) “for free”

Results (1)

- uncertainties in various couplings generate error bands:



- isospin breaking on the percent level
- t-dependence moderate
- unlike in some quark models, $G_M^{u,d}(0) \neq 0$!

Results (2)

- compare isospin breaking to strangeness at $Q^2 = 0.1$GeV^2:

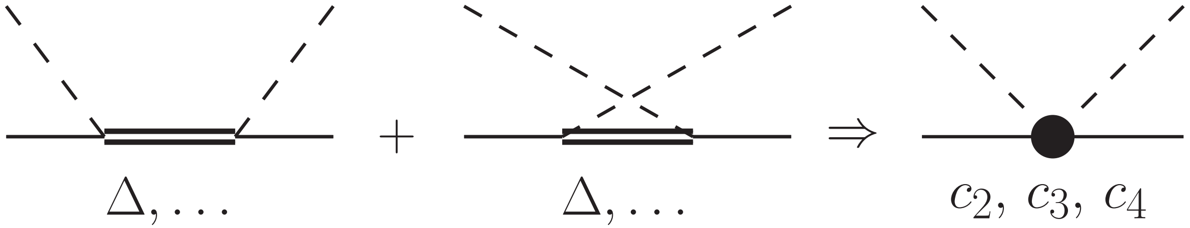
experiment	electric/magnetic	G^s (measured)	$G^{u,d}$ (calculated)
SAMPLE	G_M	$0.37 \pm 0.20 \pm 0.26 \pm 0.07$	$0.02 \ldots 0.05$
A4	$G_E + 0.106\, G_M$	0.071 ± 0.036	$0.004 \ldots 0.010$
HAPPEX	$G_E + 0.080\, G_M$	$0.030 \pm 0.025 \pm 0.006 \pm 0.012$	$0.004 \ldots 0.009$

- conclusion: isospin violation for now smaller than other (experimental) uncertainties
- necessary correction for future precision determinations of strange matrix elements

Part V: Crimes and omissions

The role of the $\Delta(1232)$ resonance (1)

- discussed the contribution of vector mesons in LECs; there are baryon resonances, too!



- lowest-lying baryon resonance: $\Delta(1232)$

$$m_\Delta - m_N \approx 2M_\pi$$

- moreover, the Δ couples strongly to the πN system resonance saturation:

$$c_2^\Delta \approx 3.8 \qquad c_3^\Delta \approx -3.8 \qquad c_4^\Delta \approx 1.9$$

Bernard, Kaiser, Meißner 1997

- N and Δ become degenerate in the large-N_c limit

The role of the $\Delta(1232)$ resonance (2)

- phenomenological extension of chiral perturbation theory: include the Δ/the spin-$\frac{3}{2}$ decuplet as explicit degrees of freedom
- potentially improved convergence in some observables

  Jenkins, Manohar 1991

  e.g. (obviously!) P_{33} partial wave in πN scattering

  Fettes, Meißner 2000
- formal development: ϵ-expansion

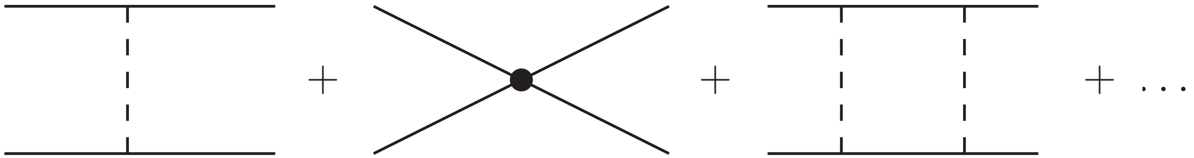
$$p = \mathcal{O}(\epsilon) \qquad M_\pi = \mathcal{O}(\epsilon) \qquad m_\Delta - m_N = \mathcal{O}(\epsilon)$$

Hemmert, Holstein, Kambor 1998

- considerable difficulties in formulating a consistent covariant theory for spin-$\frac{3}{2}$ fields Pascalutsa

Two- and more-nucleon systems

- extended ChPT from $\pi\pi$ to πN systems — why not NN systems, too?



- NN systems has bound states: the deuteron!
  $\Rightarrow$ non-perturbative effect
  $\Rightarrow$ cannot calculate NN amplitudes perturbatively
- the S-wave scattering lengths are unnaturally large:

$$a(^1S_0) \approx -23.8\,\text{fm}\ ,\ a(^3S_1) \approx 5.4\,\text{fm}\ \gg\ 1/M_\pi \approx 1.4\,\text{fm}$$

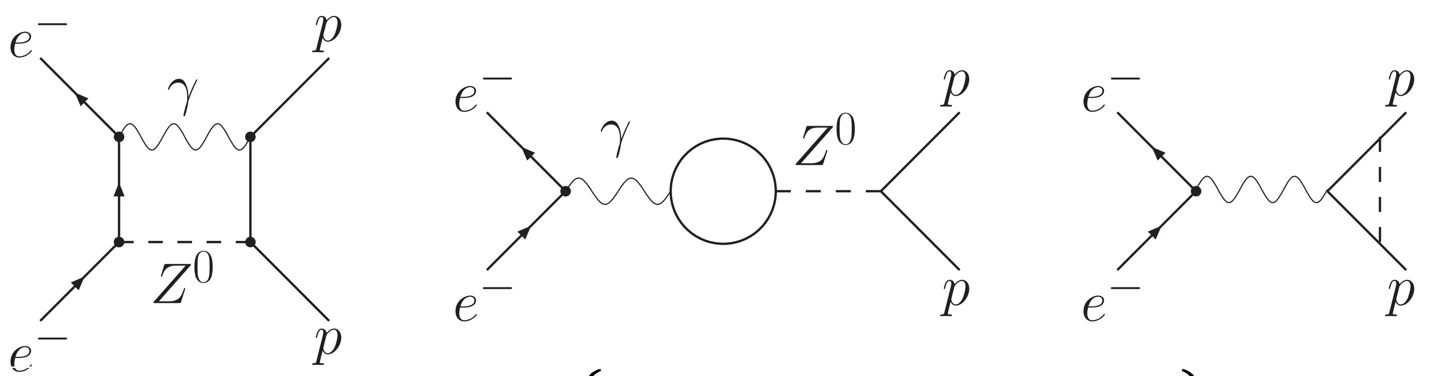
- (one) solution: solve Lippmann-Schwinger equation *exactly*
  calculate the potential perturbatively in the chiral expansion

see E. Braaten's lectures

Spares

Radiative corrections to the axial form factor

- G_A suppressed by $1 - 4\sin^2\theta_W \approx 0.08$
- radiative corrections: generate anapole moment



$$\langle N(p')|\bar{q}\gamma_\mu q|N(p)\rangle \;=\; \bar{u}(p')\Big\{\ldots + \big(t\gamma_\mu - \not{q}\, q_\mu\big)\gamma_5 F_A(t)\Big\}u(p) \quad [q_\mu = p'_\mu - p_\mu]$$

- relative importance: anapole form factor F_A “suppressed” by

$$\eta = \frac{8\pi\sqrt{2}\alpha}{1 - 4\sin^2\theta_W} \approx 3.45$$

- conclusion: maybe G_A not quite as well known…

Why isospin-violating ffs. are not quite *that* difficult

- claim: calculate $G_E^{u,d}$ up to $\mathcal{O}(p^4)$, $G_M^{u,d}$ up to $\mathcal{O}(p^5)$
- task: calculate all possible one- and two-loop diagrams with virtual pions and photons including all possible isospin breaking

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(1) no contribution from pion mass difference $M_{\pi^+}^2 - M_{\pi^0}^2 \;\propto\; e^2$:

charge symmetry: $u \leftrightarrow d$, $p \leftrightarrow n$

charge independence: general rotations in isospin space

$M_{\pi^+}^2 - M_{\pi^0}^2$ only breaks charge independence:

$$u \leftrightarrow d \;\;\Rightarrow\;\; \pi^+ \leftrightarrow \pi^- \;,\;\; \pi^0 \leftrightarrow \pi^0$$

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$$u \leftrightarrow d \;\; \Rightarrow \;\; \pi^+ \leftrightarrow \pi^- \; , \;\; \pi^0 \leftrightarrow \pi^0$$

(2) no two-loop diagrams contribute (to $G_M^{u,d}$):