NuFACT08 Summer School

Muon Capture, Phase-Energy Rotation and Cooling

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Fermilab

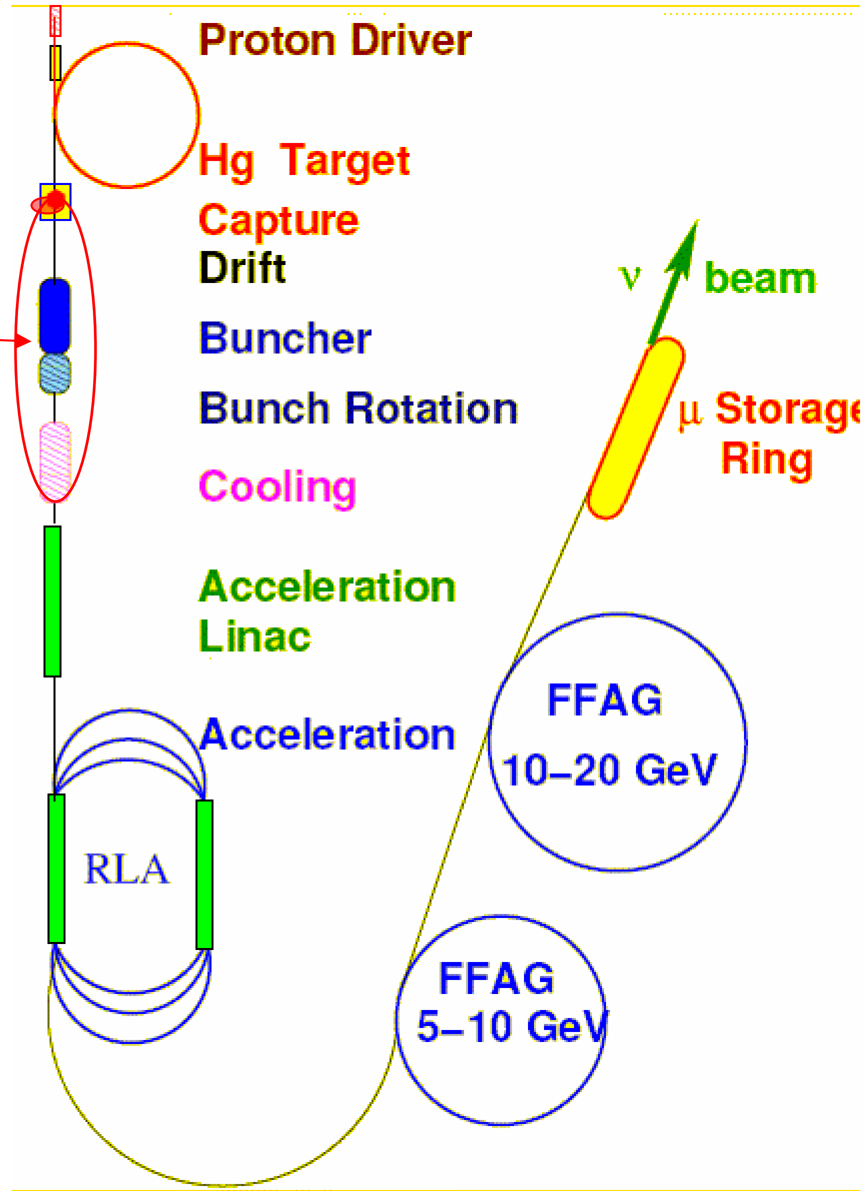
Outline

- Lecture I –Introduction– Optics and Capture
  - General introduction
    - ν–Factory; μ^+–μ^- Collider
    - Optics review
  - Muon capture for cooling
    - ν–Factory and μ^+–μ^- Collider
  - variations

- Lecture II – Cooling
  - Ionization cooling concepts
  - Cooling for a neutrino factory
  - Muon collider —cooling
  - Experiments
  - Ion cooling

Neutrino Factory – Study 2A

- Proton driver
  - Produces proton bunches
  - ~10 GeV, ~10^{15}p/s, ~50Hz bunches
- Target and drift
  - $\pi \rightarrow \mu$ (> 0.2 μ/p)
- **Buncher, bunch rotation, cool**
- Accelerate μ to 20 GeV or more
  - Linac, RLA and FFAGs
- **Store** at 20 GeV (0.4ms)
- $\mu \rightarrow e + \nu_\mu + \nu_e^*$
- Long baseline ν Detector
- $>10^{20}$ ν/year

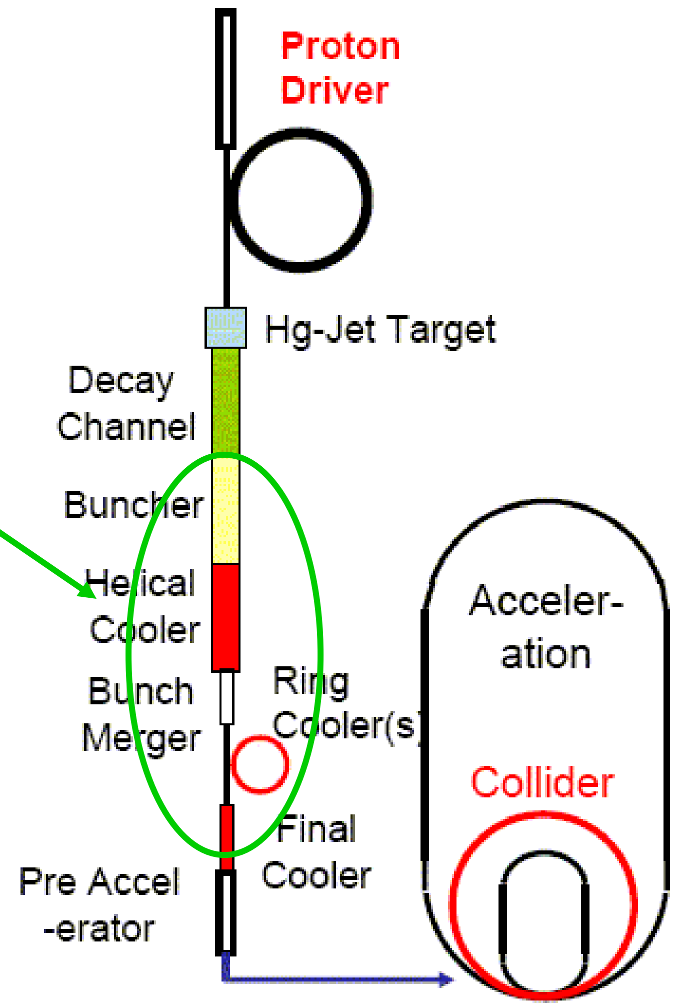


Overview of μ^+–μ^- Collider

- **Proton Driver** (1-4 MW) – proton bunches on target produce π′s
- **Front-end:** π decay $\rightarrow \mu + \nu$ collect and cool μ′s: (phase rotation + **ionization cooling**)
- **Accelerator** - to full energy (linac + RLAs to TeV)
- **μ - Collider Ring**

  Store μ**’s** until decay (~300 B turns)
- **$\mu^+ - \mu^- \rightarrow$ X**

  high-energy collisions

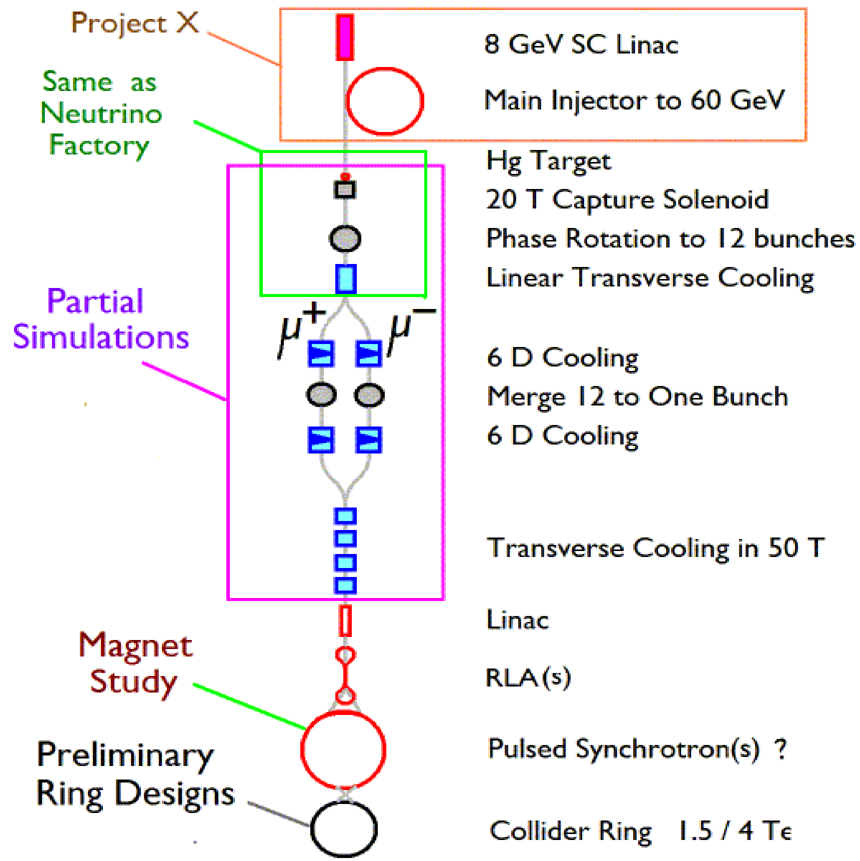


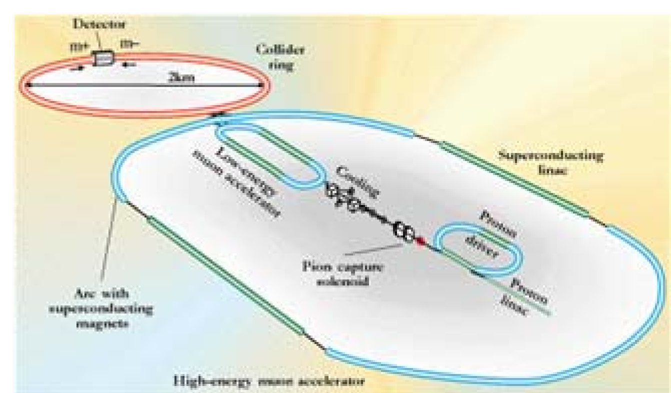
$\mu^+ - \mu^-$ Collider

Collider Parameters

C of m Energy	1.5	4	TeV
Luminosity	1	4	10^{34} cm^{2}sec^{-1}
Beam-beam Tune Shift	0.1	0.1	0.
Muons/bunch	2	2	10^{12}
Ring <bending field>	5.2	5.2	T
Ring circumference	3	8.1	km
Beta at IP $= \sigma_z$	10	3	mm
rms momentum spread	0.1	0.12	%
Muon Beam Power	7.5	9	MW
Required depth for ν rad (1)	13	135	m
Muon survival (2)	0.07	0.07	
Repetition Rate	12	6	Hz
Proton Driver power	≈4	≈ 1.8	MW
Trans Emittance	25	25	pi mm mrad
Long Emittance	72,000	72,000	pi mm mrad

$$\sigma_{IR} = 6.0 \text{ to } 2.0 \ \mu$$





Primary limitation: μ–Lifetime

- Muons decay in time:
  - $\gamma\,\tau_0 = \gamma$ 2.2 μs
- Or in a distance:
  - $\beta\gamma\ c\tau_0 = \beta\gamma$ 660 m

$$\frac{dN}{ds} = -\frac{1}{\beta\gamma\, c\tau_0} N$$

$$N = N_0 e^{-\frac{z}{\beta\gamma c \tau_0}}$$

- Need acceleration, and cooling, much faster than decay:

$$V'_{rf} \gg \frac{m_\mu c^2}{e\, c\tau_0} \cong 0.16\, MV/m$$

$$E = E_0 + eV'_{rf}\, s$$

- Need > ~10MV/m
  - Must bunch, cool, and accelerate

$$\frac{N}{N_0} = \left[\frac{E_0}{E}\right]^{\frac{m_\mu c^2}{c\tau_0 e V'_{rf}}}$$

Beam Optics: Basic Equations

- Maxwell's equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla \times \frac{\vec{B}}{\mu} - \frac{\partial \varepsilon \vec{E}}{\partial t} = \vec{J}$$

- Equation of motion:

$$\frac{d(m\gamma\vec{v})}{dt} = q\vec{E} + q(\vec{v} \times \vec{B})$$

- Transport/focusing uses magnetic fields
  - Dipoles, quads, solenoids, …
  - Horns, Li lens, …

- Acceleration uses electric fields
  - rf cavities
  - Induction modules

Magnetic motion–Bending

$$\frac{d(m\gamma\vec{v})}{dt} = q(\vec{v} \times \vec{B})$$

Bending radius:

$$R = \frac{m\gamma v}{qB} = \frac{B\rho}{B}$$

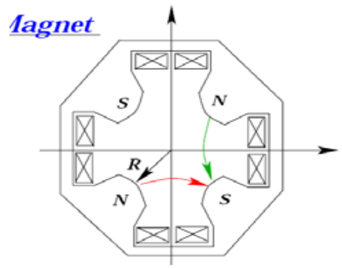
$$B\rho \ \ (\mathrm{T-m}) = \frac{\mathrm{P\ (GeV/c)}}{0.3}$$

- Force is perpendicular to motion
- γ, β=v/c, P remain constant
- If **v, B** are perpendicular, motion is circular
- mγv/q = BR =P/q is called the magnetic rigidity or **B**$\boldsymbol{\rho}$
- 1 GeV/c particle has Bρ=10^9/c= 3.33 T-m
- Bending angle in magnet with field B, length L is θ=BL/ Bρ

Betatron Functions–focusing

- Assume focusing is linear around the orbit:

$$\frac{d^2x}{ds^2} + k_x(s)\,x = 0$$



- ~Harmonic oscillator (k(s))
- Solution:

$$x = A\sqrt{\beta_x(s)}\; e^{i\phi(s)}$$

- where:

$$\left(\sqrt{\beta_x(s)}\right)'' + k_x(s)\sqrt{\beta_x(s)} - \frac{1}{\left(\sqrt{\beta_x(s)}\right)^3} = 0$$

$$\phi_x(s) = \int \frac{ds}{\beta_x(s)}$$

- Focussing forces:
  - Quads: $k_x(s) = \pm B'(s)/B\rho$

    $k_y(s) = -\,k_x(s)$
  - Solenoids: $k(s) = (B/2B\rho)^2$
    - With x–y rotation
- Need starting amplitude and derivative to specify motion
  - $\beta_x(0)$, $\beta_x'(0)$
- In periodic structure these are set by requiring that $\beta_x(s)$ be periodic.

$$A = \sqrt{\varepsilon_{amplitude}}$$

At equilibrium $\beta_x = (1/k)^{1/2}$

$$M = \begin{pmatrix} \cos\phi + \alpha\sin\phi & \beta\sin\phi \\ -\frac{1+\alpha^2}{\beta}\sin\phi & -\cos\phi + \alpha\sin\phi \end{pmatrix}$$

Exercise: show that these equations are consistent

Solenoidal Focussing

- Magnetic field
  - Cylindrical symmetry

$$\vec{B} = -\frac{r}{2}\frac{\partial B_z(z)}{\partial z}\vec{e}_r + B_z(z)\vec{e}_z$$

$$x'' - \frac{B_z}{B\rho}y' - \frac{B_z'}{2B\rho}y = 0$$

$$y'' + \frac{B_z}{B\rho}x' + \frac{B_z'}{2B\rho}x = 0$$

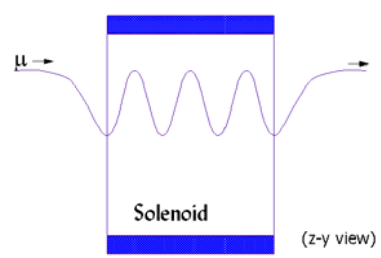
- Change to R–ϕ notation

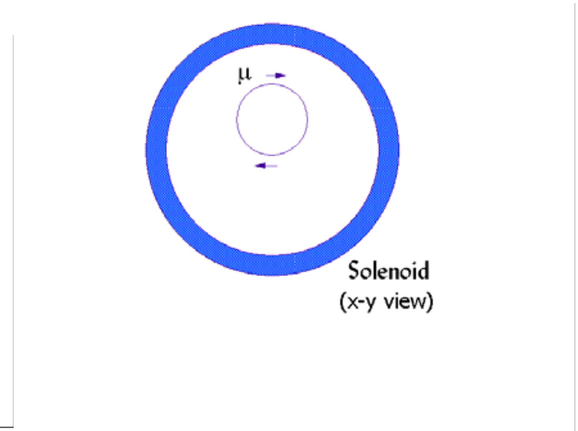
$$x + iy = R\,e^{i\phi}$$

- Solution is rotation in ϕ, focussing in R:

$$\phi' = -\tfrac{1}{2}\frac{B_z}{B\rho}$$

$$R'' = -\tfrac{1}{4}\left(\frac{B_z}{B\rho}\right)^2 R$$





Comparison of focussing

- Quadrupole:
  - Focuses x or y
  - Proportional to 1/P

$$x'' = -\frac{B'}{B\rho}x$$

- Solenoid
  - Focuses both x and y
  - Focuses both μ^+ and μ^-
  - Proportional to $1/P^2$

$$x'' = -\left(\frac{B}{2B\rho}\right)^2 x$$

- Solenoid better for low–energy
  - For $B\rho < \sim B_o\, a$ (~1 T-m ?)

Betatron function discussion

- Beam optics typically described in terms of betatron functions
- Use "rms" Emittance (beam phase space area)

$$\sigma_x^2 = \beta_{\perp,x}\varepsilon_{x,geom.} = \frac{\beta_{\perp,x}(s)\varepsilon_{x,N}}{\beta\gamma}$$

- Emittance also defined as:

$$\varepsilon_{x,geom}^2 = \langle x^2\rangle\langle x'^2\rangle - \langle x\,x'\rangle^2 \qquad (m_o c)^2\,\varepsilon_{x,N}^2 = \langle x^2\rangle\langle p_x{}^2\rangle - \langle x\,p_x\rangle^2$$

- References are often unclear on whether normalized or geometric emittance is used; also
  - Fermilab convention: $\varepsilon_f = 6\pi\varepsilon_{rms}$
  - CERN convention: $\varepsilon_{CERN} = 4\pi\varepsilon_{rms}$

Adiabatic damping and normalized emittance

- Under acceleration:
  - $\delta x\ \delta p_x$ remain constant (normalized emittance)
  - $\delta x\ \delta x' = \delta x\ \delta p_x/p$ is proportional to $(1/p)$
  - geometric emittance decreases (beam size decreases)

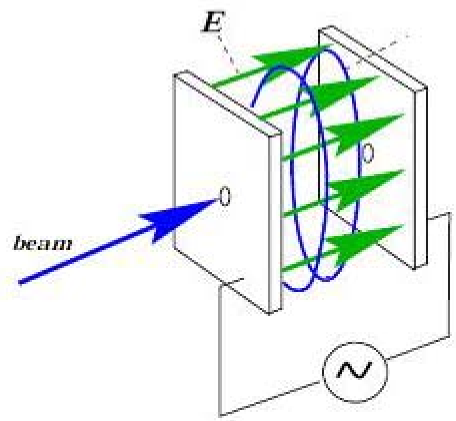
$$\sigma_x = \sqrt{\frac{\beta_{\perp,x}\varepsilon_{N,x}}{\beta\gamma}}$$

- Similarly,
  - $\delta ct\ \delta E$ remain constant
  - $\delta z\ \delta p/p$ decreases as $1/p$ (acceptances set by $\delta p/p$)

- Acceptances improve as beam is accelerated
  - Use higher-frequency rf, weaker transverse focusing

Longitudinal motion

- Choose variables for longitudinal motion: $(\delta z, \delta P_z) \rightarrow (\delta\Phi, \delta E)$



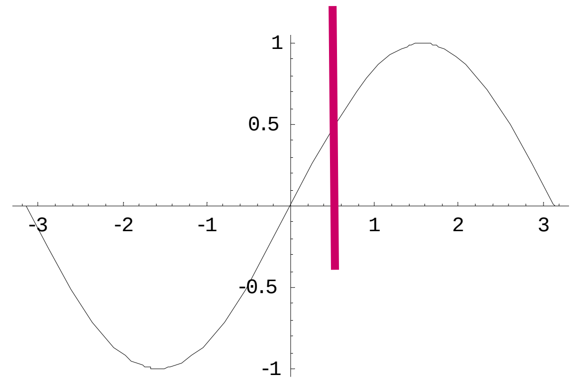
$$\delta z = \frac{\lambda_0}{2\pi}\delta\phi$$

$$\frac{d\Delta E}{ds} = eV'(\cos(\phi+\phi_s) - \cos\phi_s) \cong -eV'\sin\phi_s\,\phi$$

$$\frac{d\phi}{ds} = \left(\frac{1}{\beta_0} - \frac{1}{\beta}\right)\frac{2\pi}{\lambda_0} \cong \frac{1}{\beta^3\gamma^3}\frac{2\pi}{\lambda_0}\frac{\Delta E}{mc^2}$$

$$\mathrm{E_z}(\phi) = \mathrm{V'_{rf}}\cos\left(\frac{2\pi}{\lambda_{\mathrm{rf}}}(\mathrm{z}-\mathrm{z_o})\right) = \mathrm{V'_{rf}}\cos(\phi-\phi_{\mathrm{o}})$$

$$\left[\alpha_p = \frac{1}{\gamma^2} \quad \text{in linac}\right] \qquad \left[\alpha_{\mathrm{p}} = \frac{1}{\gamma^2} - \frac{1}{\gamma_{\mathrm{t}}^2}\right] \text{in ring}$$



$$\frac{d^2\phi}{ds^2} \cong -\frac{\alpha_p}{\beta^3\gamma}\frac{2\pi}{\lambda_0}\frac{eV'\sin\phi_s}{mc^2}\phi$$

Longitudinal motion stuff

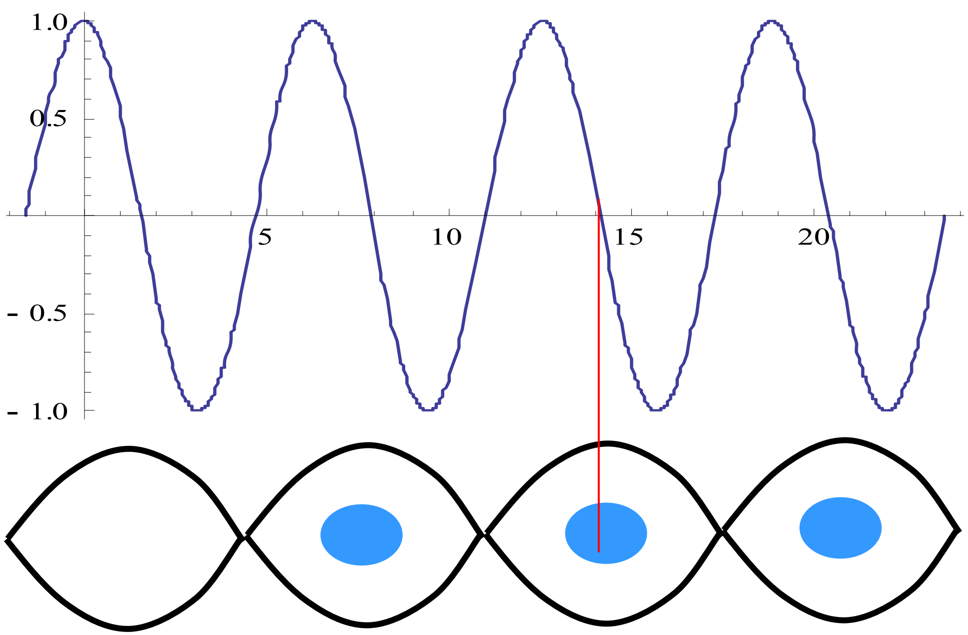
- Distance for oscillation

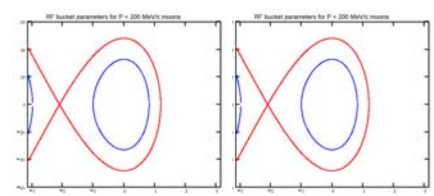
$$\lambda_{osc} = 2\pi\sqrt{\frac{\beta^3\gamma\lambda_0 mc^2}{2\pi\alpha_p eV'\sin\phi_s}}$$

- Adiabatic (if L > ~λ_{osc})

- Rf Bucket energy width

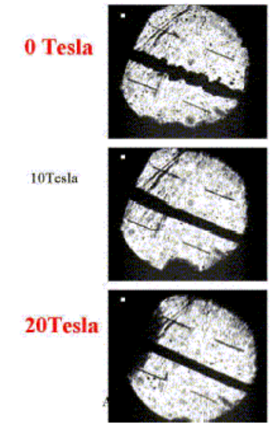
$$\Delta E = \pm\sqrt{\frac{eV'\lambda_0 mc^2\beta^3\gamma}{\pi\alpha_p}}\sqrt{2\phi_s\sin\phi_s - \phi_s\cos\phi_s}$$



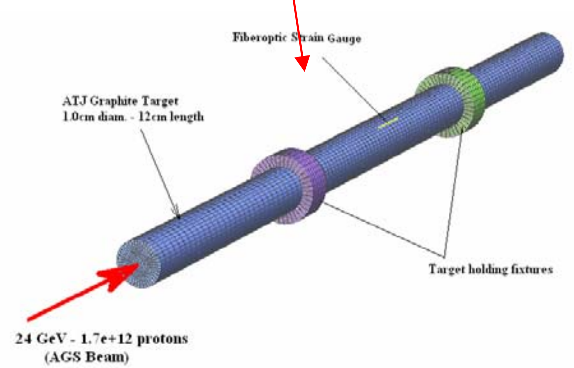


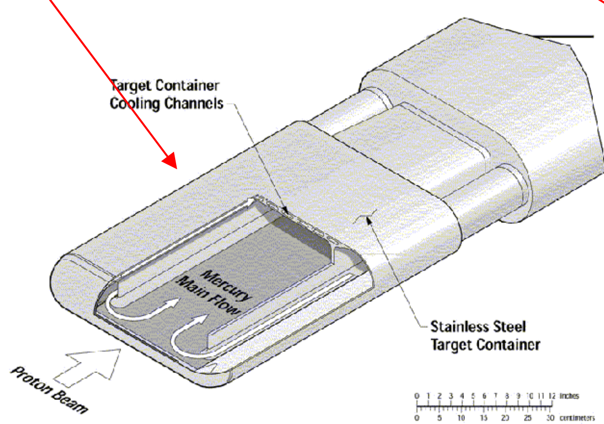
Target for π production

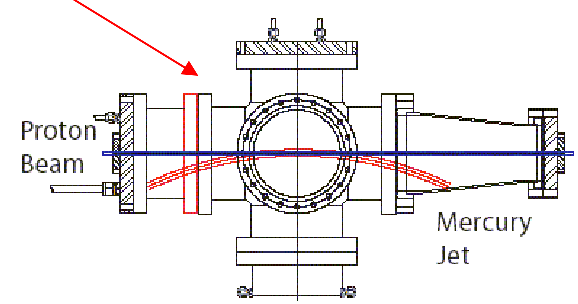
- Typical beam: 10 GeV protons up to 4 MW
  - 1m long bunches up to 4×10^{13}/bunch, 60Hz
- Options:
  - Solid targets
    - C (graphite targets) (NUMI)
    - Solid metal (p-source) – rotating Cu-Ni target
  - Liquid Metal targets
    - SNS – type (confined flow)
    - MERIT test – Hg jet in free space
    - November 2007– experiment
      - Best for 4MW ??



Hg jets

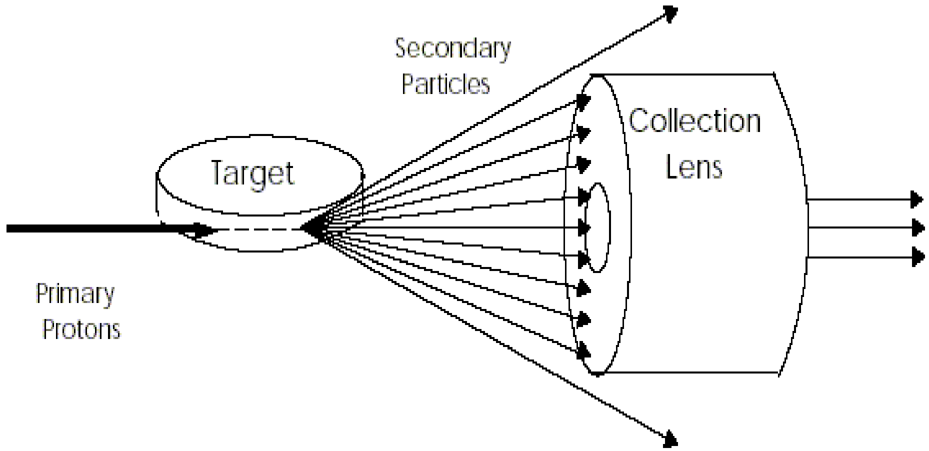




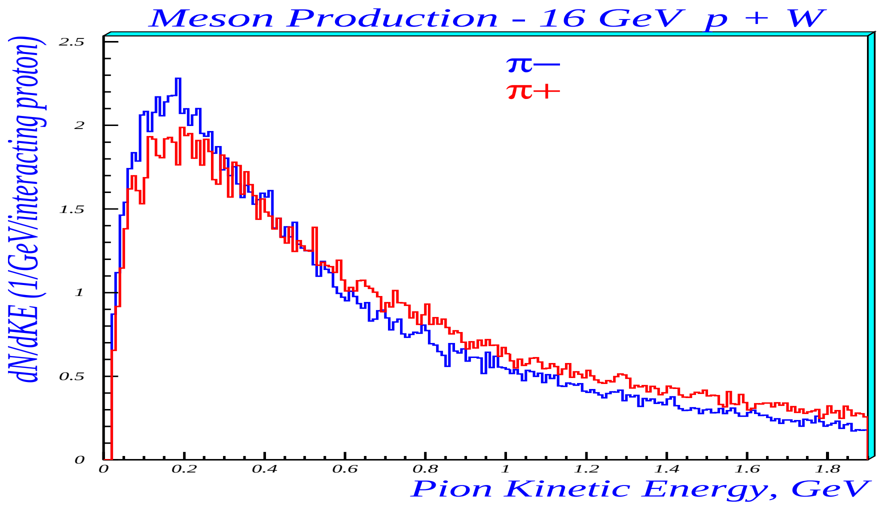


π capture from target

- Protons on target produce large number of π's
  - Broad energy range (0 to 10+GeV)
  - More at lower energies
  - Transverse momentum (up to ~0.3GeV/c)



- Capture beam from target
- Options:
  - Li lens
  - Magnetic horn
  - Magnetic Solenoid

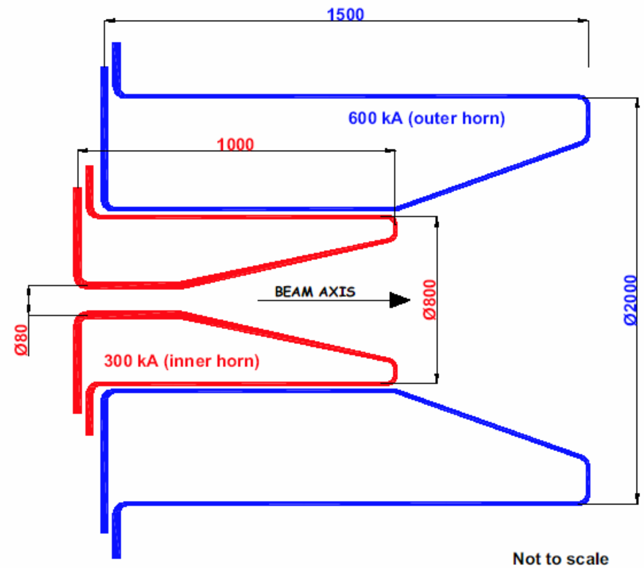


Magnetic Horn after target

- Baseline capture for superbeams/NUMI
- Magnetic field from I on wall

$$B_\theta(r) = 0 \text{ inside conductor}$$

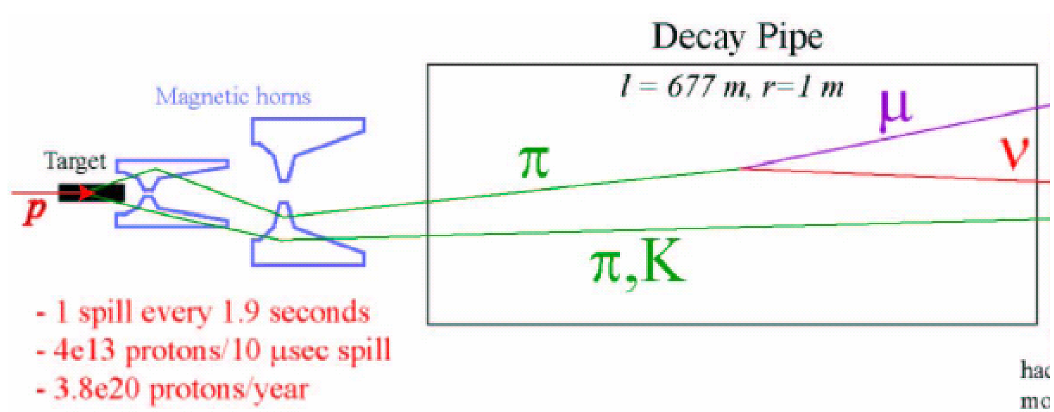
$$B_\theta(r) = \mu_o \frac{I_{total}}{2\pi r} \qquad \Delta\theta_{focus} = \frac{B_\theta L_{path}}{3.33 P_{\pi/\mu}}$$



- Lenses can be tuned to obtain narrow band or broad-band acceptance

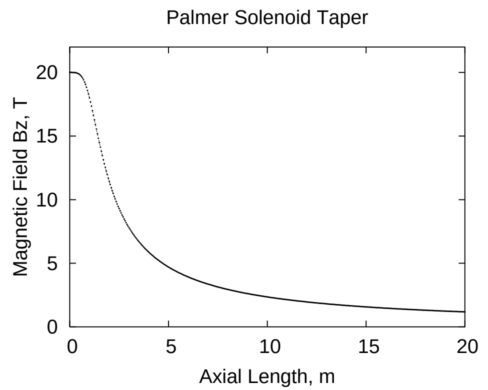
- Pulsed current, thin conductors
  - Breakage over many pulses
  - Beam lost on material
- focuses + or – particles

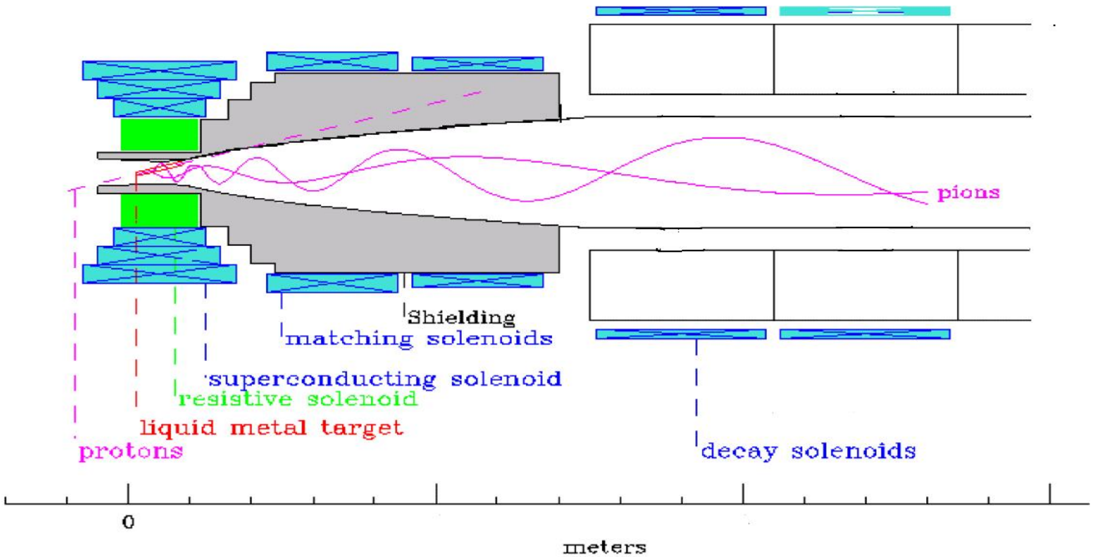
NUMI beam line

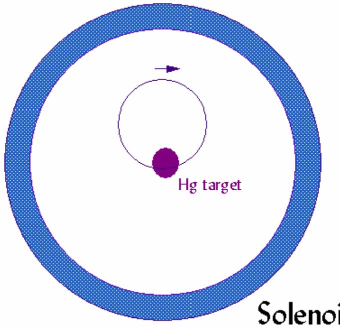


Solenoid lens capture

- Target is immersed in high field solenoid
- Particles are trapped in Larmor orbits
  - Produced with $p = p_{\|}, p_{\perp}$
  - Spiral with radius $r = p_{\perp}/(0.3\ B_{sol}) = B\rho_{\perp}/B$
  - Particles with $p_{\perp} < 0.3\ B_{sol}R_{sol}/2$ are trapped
  - $p_{\perp,max} < 0.225$ GeV/c for $B=20T$, $R_{sol} = 0.075m$
  - Focuses both + and – particles



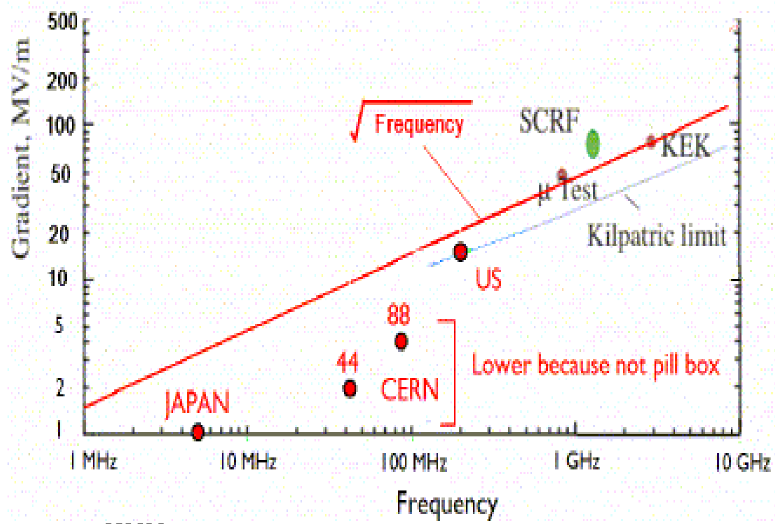




Solenoid
(x-y view)

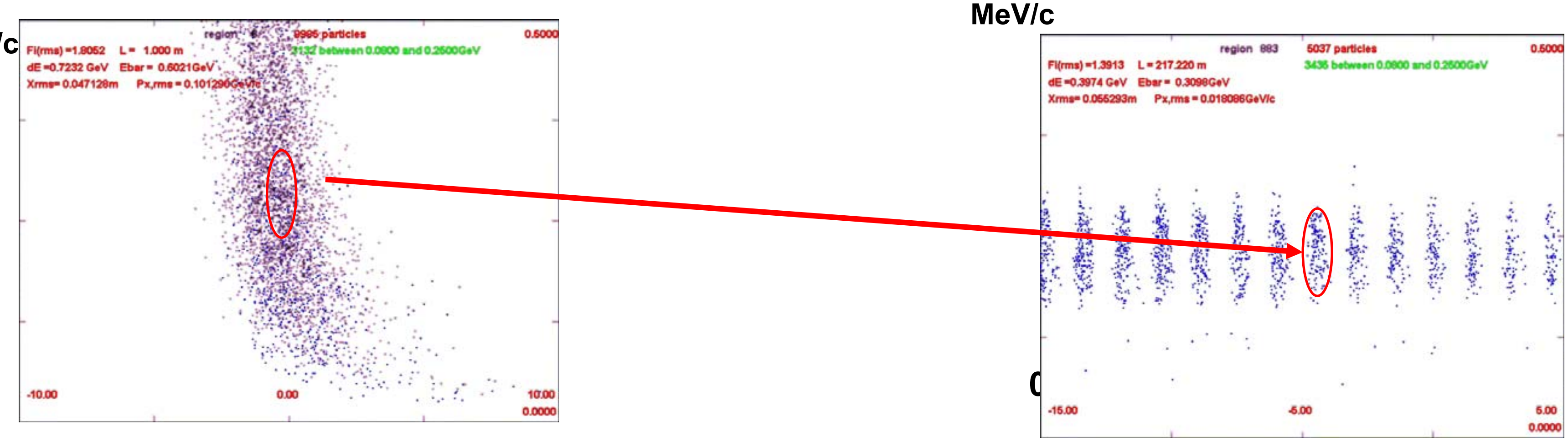
Target to Cooling channel match

- **Transverse match:** 20T to ~2T solenoid
  - **P_t = 0.225→ 0.07GeV/c**
  - **R= 25cm: $\sigma_x \approx$ 0.1m; $\theta_x \approx$ 0.1**
- **Longitudinal match:**
  - **rf ~200 MHz (λ=1.5m) V' >10 MV/m**
  - **Optimum cooling is:**
    - **P =~200MeV/c, δP/P ~10%**
  - **Want both signs (μ+, μ-)**



500 MeV/c

500 MeV/c



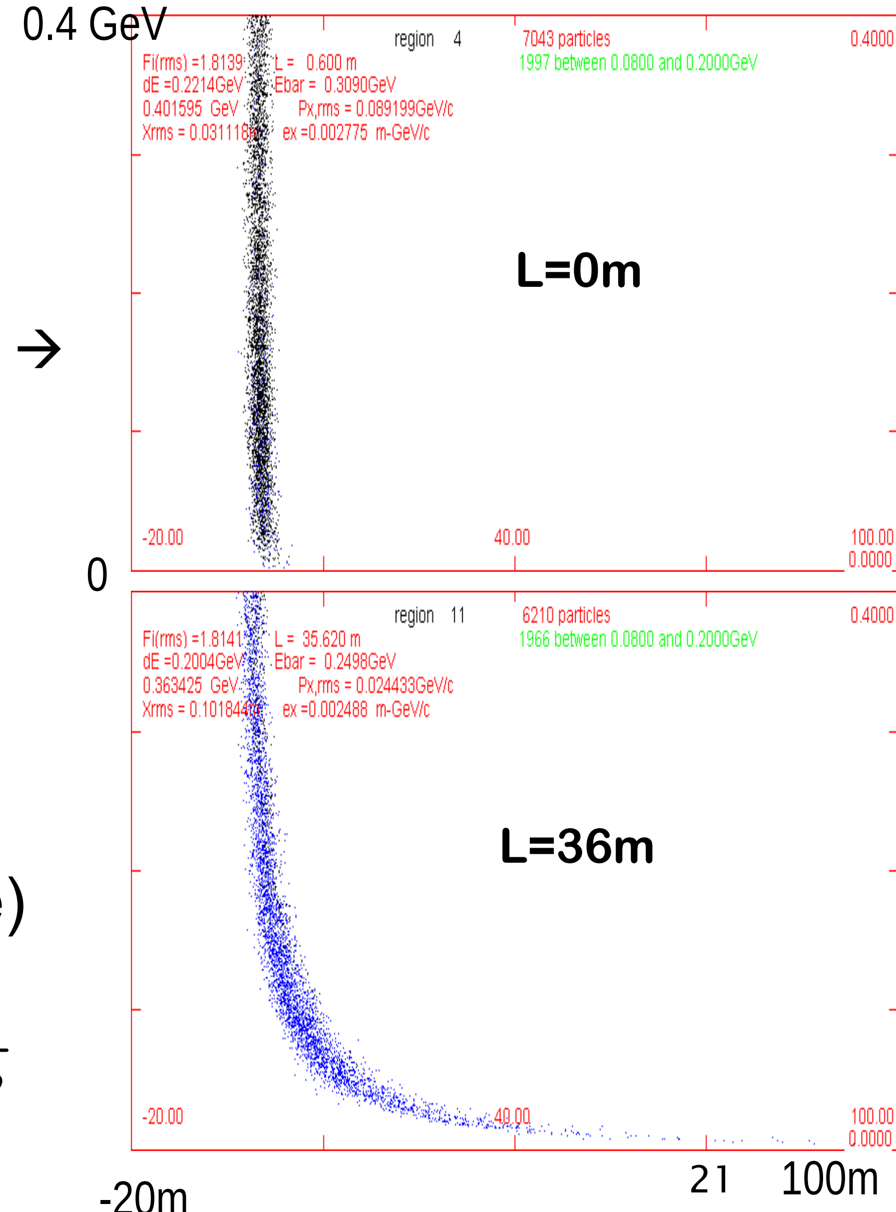
-10m **10m**

π→μν decay in transport

- π-lifetime is $2.60\times10^{-8}\gamma$ s
  - L = 7.8 βγ m
- For π → μ+ν,

  $\langle P_{T,rms}\rangle$ is 23.4 MeV/c, E_μ=0.6 to 1.0E_π
- Capture relatively low-energy π → μ
  - 100 – 300 MeV/c
- Beam is initially short in length
  - Bunch on target is 1 to 3 ns rms length
- As Beam drifts down beam transport, energy-position (time) correlation develops:

$$c\tau_{arrival} = \frac{L}{\beta}$$

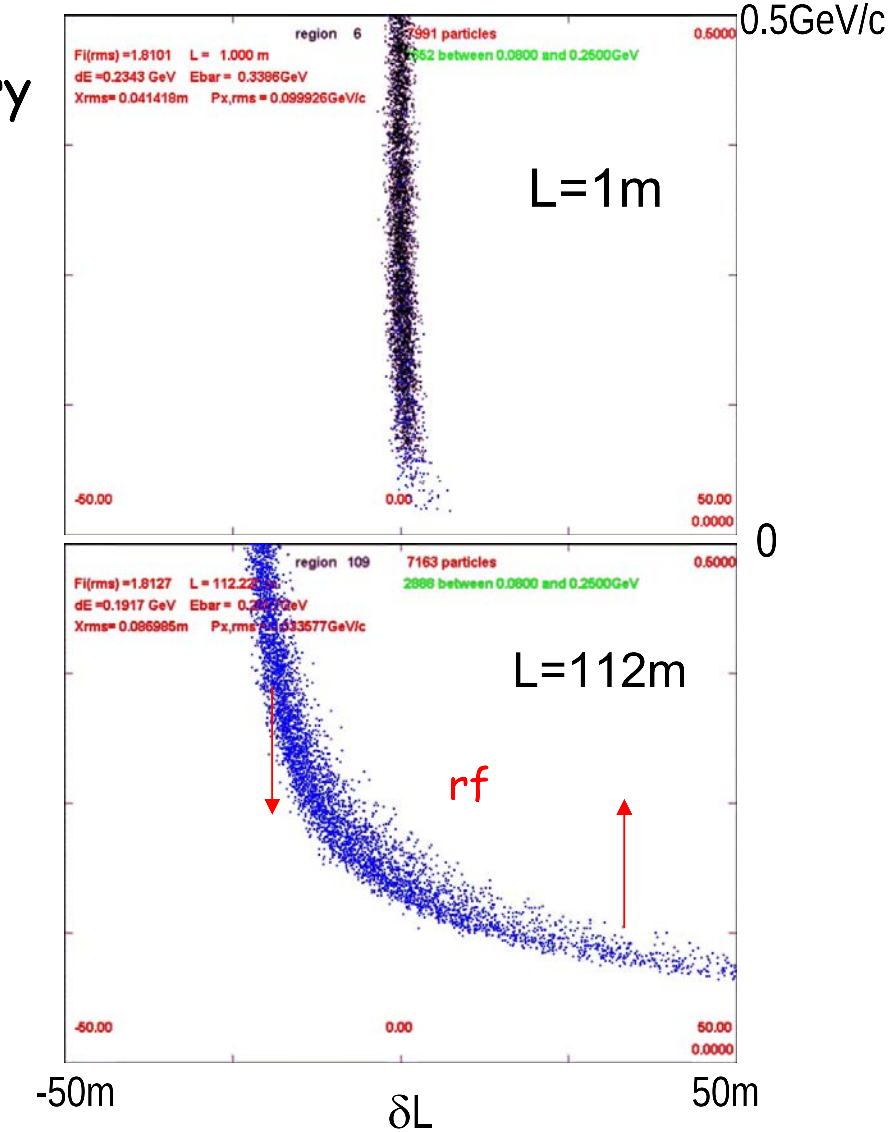


Phase-energy rotation

- To maximize number of ~monoenergetic μ's, neutrino factory designs use phase-energy rotation

- Requires:
  - "short" initial p-bunch
  - Drift space
  - Acceleration (induction linac or rf)
    - at least ±100 MV
- Goal:
  - Accelerate "low-energy tail"
  - Decelerate "high-energy head:
  - Obtain long bunch
    - with smaller energy spread

$$\delta L = \delta \frac{L}{\beta(p_\mu)}$$



Phase Energy rotation options

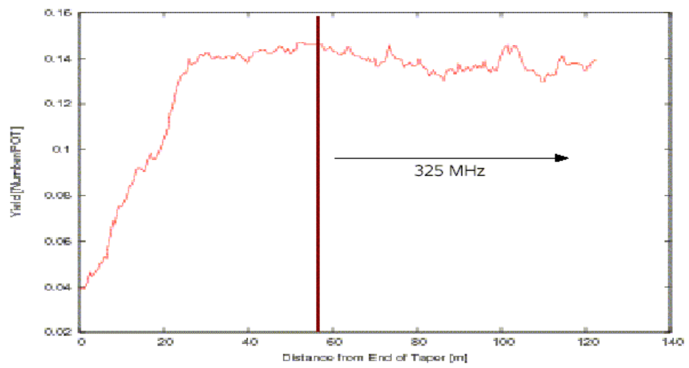
- Single bunch capture
  - Low-frequency rf (~30MHz)
  - Best for collider (?) (but ~ only μ^+ or μ^-)

- Induction Linac
  - Nondistortion capture possible
  - Very expensive technology, low gradient
  - Captures only μ^+ or μ^-

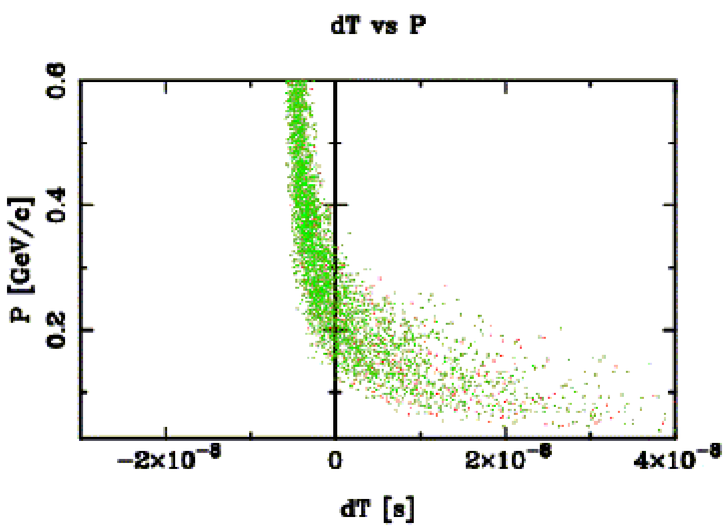
- "High Frequency" buncher and phase rotation
  - Captures into string of bunches (~200MHz)
  - Captures both μ^+ and μ^-

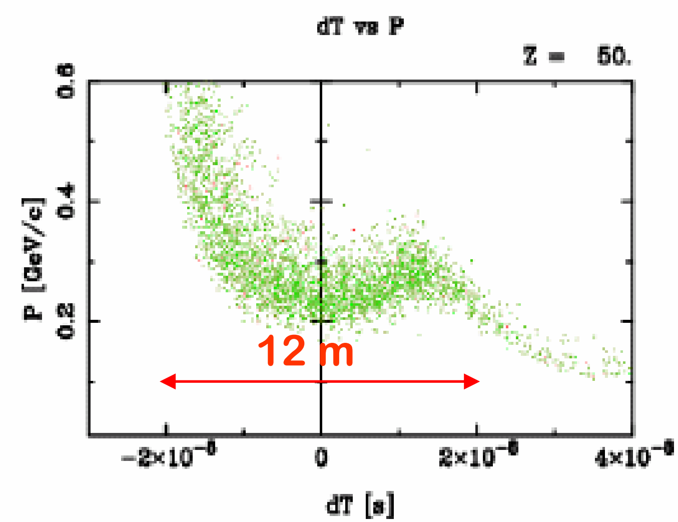
Phase/energy rotation

- Low-frequency rf; capture into single long bunch
  - 25MHz – 3MV/m
  - +25% 50MHz
  - 10m from target to 50m
- But:
  - Low-frequency rf is very expensive
  - Continuation into cooling and acceleration a problem (200MHz?)



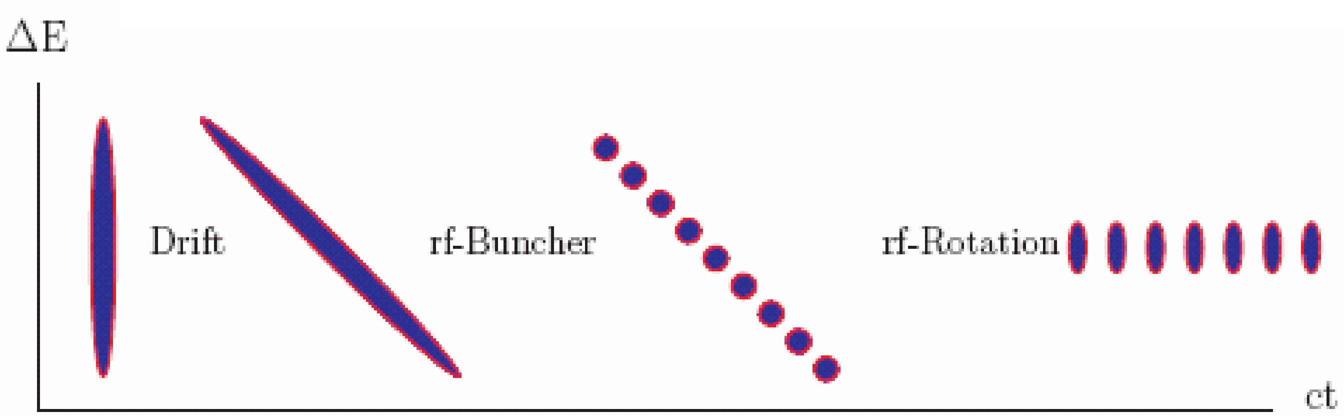
Only captures one sign …





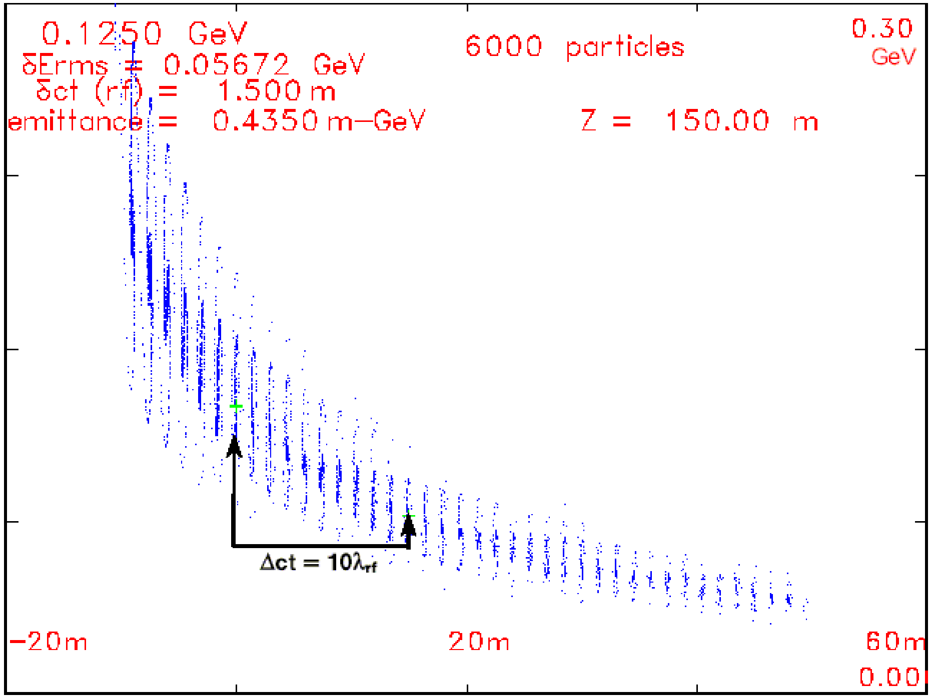
High-Frequency bunch and ϕ-E rotation

- Base Line Neutrino factory Solution:
  - Uses only f > 200MHz rf
  - Captures both signs (μ^+, μ^-)



Adiabatic Buncher overview

- Want **rf phase to be zero** for reference energies as beam travels down buncher
- Spacing must be N λ_{rf}
  $\Rightarrow \lambda_{rf}$ increases (rf frequency decreases)
- **Match to λ_{rf}= ~1.5m** at end:
- **Gradually increase rf gradient** (linear or quadratic ramp)



Example: λ_{rf} : 0.90→1.5m
For s =90 to 150m

$$\delta L = \delta \frac{L}{\beta(p_\mu)}$$

Adiabatic Buncher example

- Adiabatic buncher (z=90→150m)
- Set T_0, $\delta(1/\beta)$:
  - 125 MeV/c, 0.01
- In buncher:

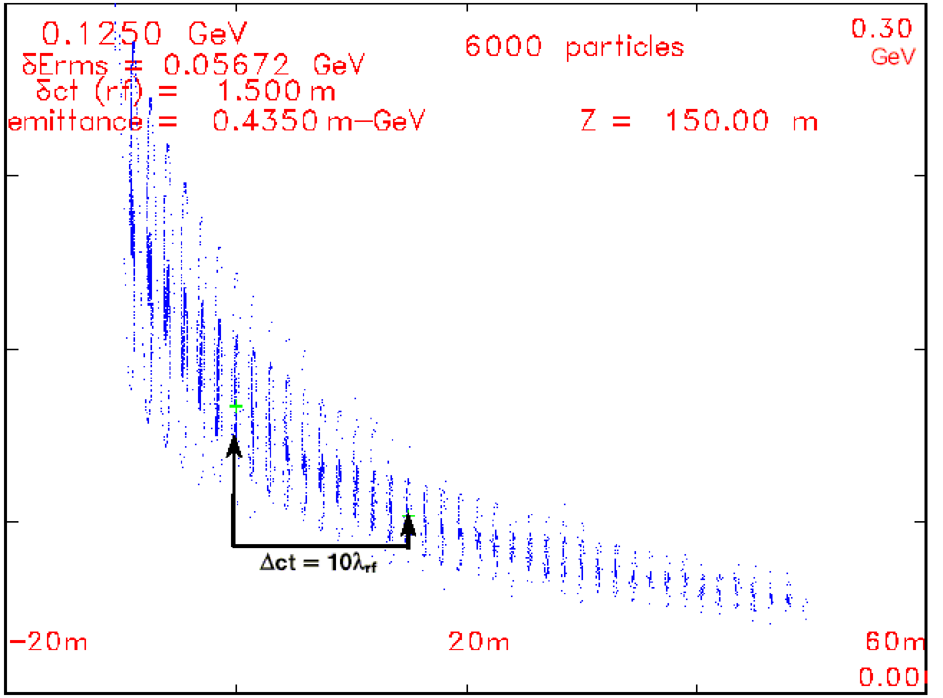
$$\lambda_{rf}(z) = z\,\delta(1/\beta)$$

- Match to λ_{rf}=1.5m at end:

$$L_{tot}\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)=L_{tot}\,\delta\left(\frac{1}{\beta}\right)=\lambda_{rf}=1.5\text{m}$$

- zero-phase with $1/\beta$ at integer intervals of $\delta(1/\beta)$:

$$\frac{1}{\beta_n}=\frac{1}{\beta_0}+n\,\delta\left(\frac{1}{\beta}\right)$$



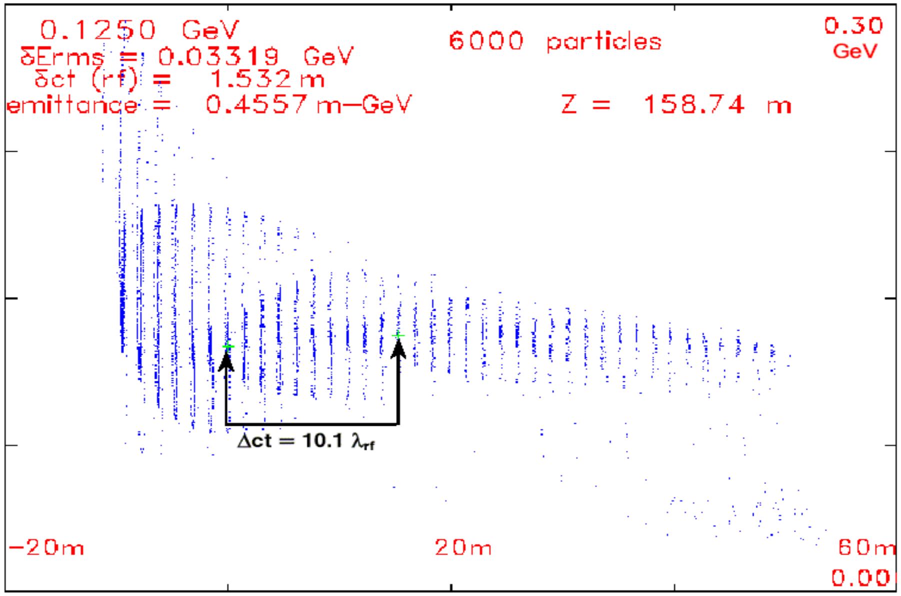
λ_{rf} : 0.90→1.5m

- Adiabatically increase rf gradient:

$$E_{rf}(z)=2\frac{(z-z_D)}{(L_{tot}-z_D)}+6\frac{(z-z_D)^2}{(L_{tot}-z_D)^2}\ MV/m$$

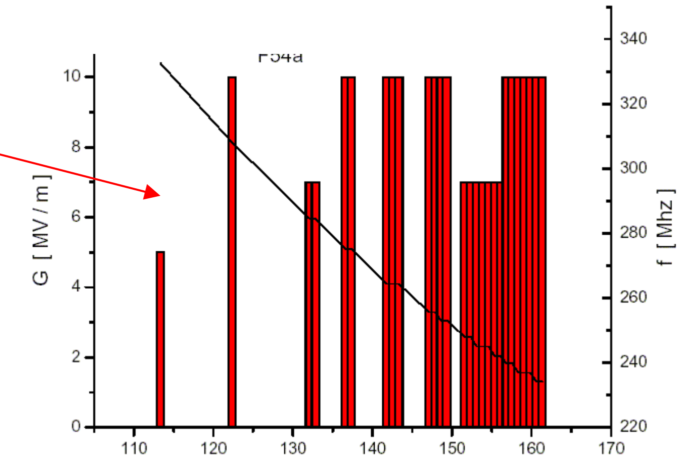
$\phi-\delta E$ Rotation

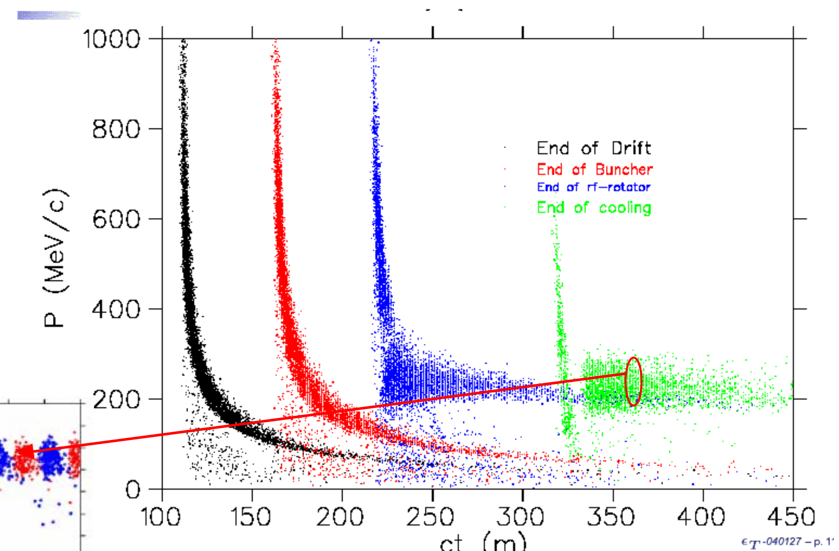
- At end of buncher, change rf to decelerate high-energy bunches, accelerate low energy bunches
- Reference bunch at zero phase, set λ_{rf} less than bunch spacing (increase rf frequency)
- Place low/high energy bunches at accelerating/decelerating phases
- Can use fixed frequency (requires fast rotation) or
- Change frequency along channel to maintain phasing

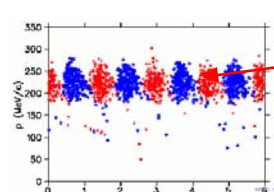


Study2A June 2004 scenario

- Drift –110.7m
- Bunch –51m
  - $V\delta(1/\beta)$ =0.0079
  - 12 rf freq., 110MV
  - 330 MHz → 230MHz
- ϕ–E Rotate – 54m – (416MV total)
  - 15 rf freq. 230→ 202 MHz
  - P_1=280 , P_2=154 δN_V = 18.032
- Match and cool (80m)
  - 0.75 m cells, 0.02m LiH
- "Realistic" fields, components
- Captures both μ^+ and μ^-
- ~0.23 μ/p within reference acceptance at end
- Rms emittance cooled from $\epsilon_\perp$ = 0.0185 to $\epsilon_\perp$ = ~0.008m





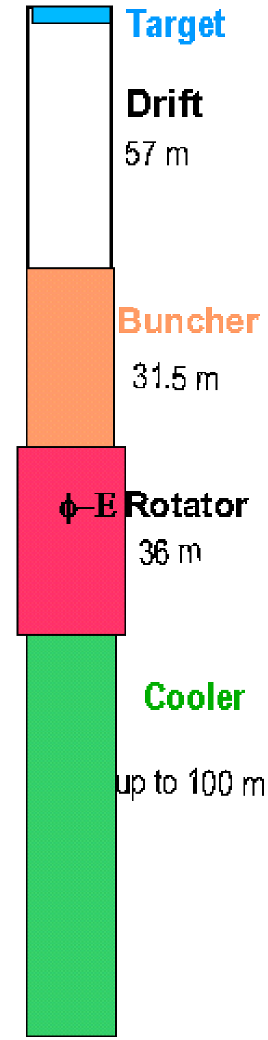


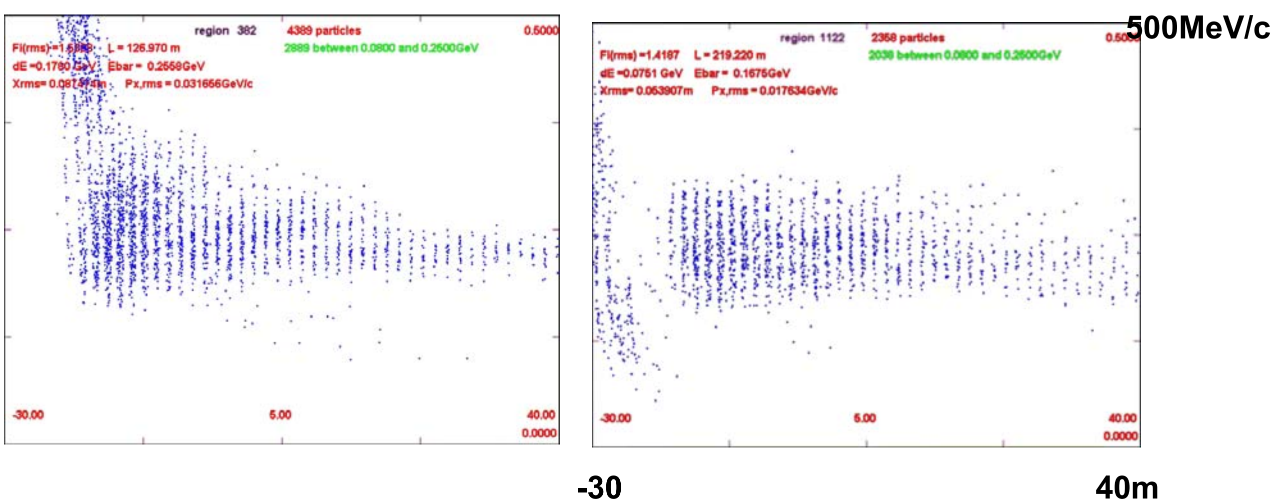
Features/Flaws of Study 2A Front End

- Fairly **long** section – ~300m long
- Produces long bunch trains of ~**200 MHz** bunches
  - ~80m long (~50 bunches)
- Transverse cooling is ~2½ in x and y
  - No cooling or more cooling ?
- Requires rf within magnetic fields
  - in current lattice, rf design
  - 12 MV/m at B = 1.75T

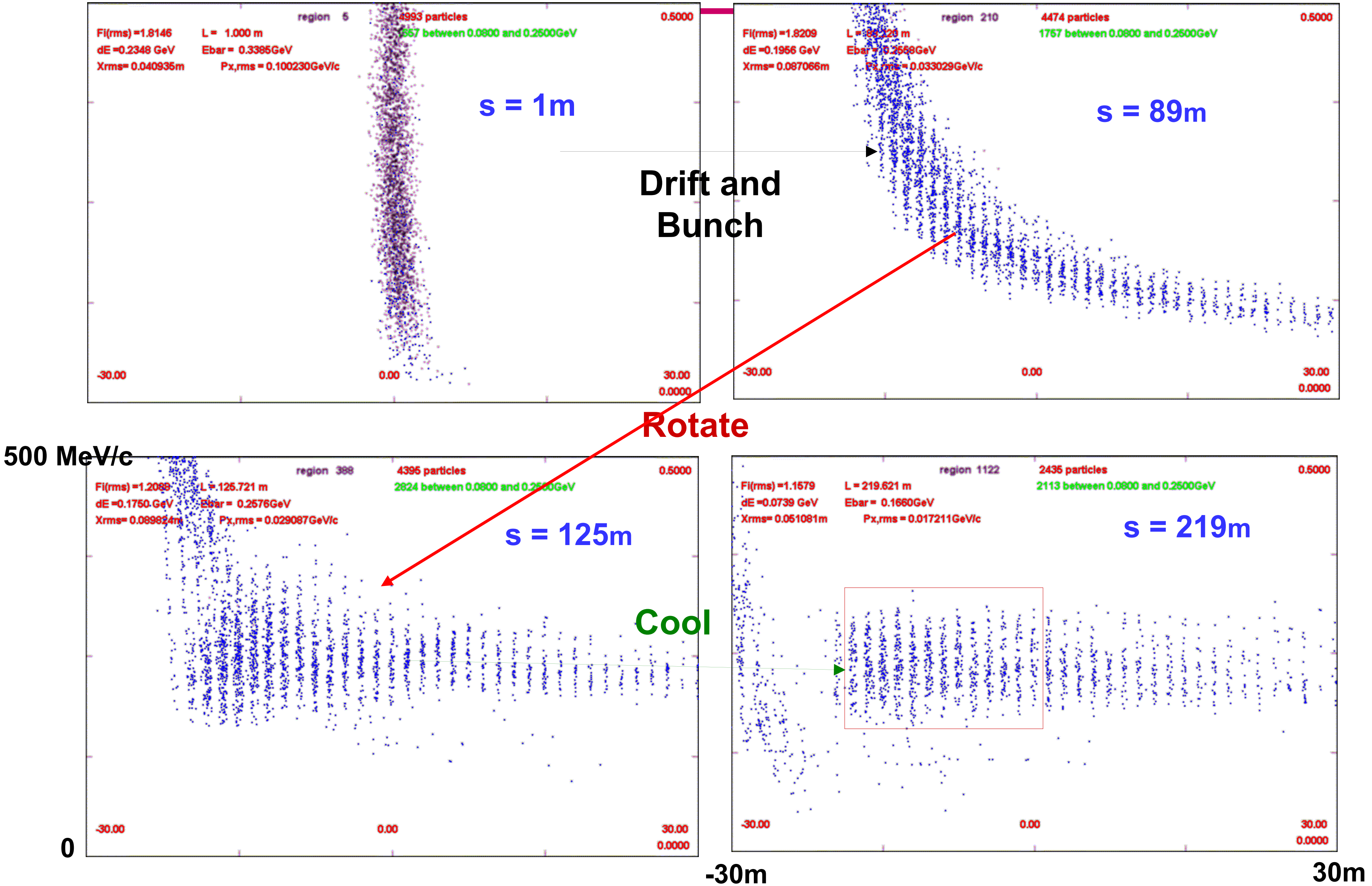
Shorter Bunch train example

- Reduce drift, buncher, rotator to get shorter bunch train:
  - 217m ⇒ 125m
  - 57m drift, 31m buncher, 36m rotator
  - Rf voltages up to 15MV/m (×2/3)
- Obtains ~0.25 μ/p_{24} in ref. acceptance
  - Slightly better ?
    - ~0.23 μ/p for Study 2B baseline
- 80+ m bunchtrain reduced to < 50m
  - Δn: 18 -> 10

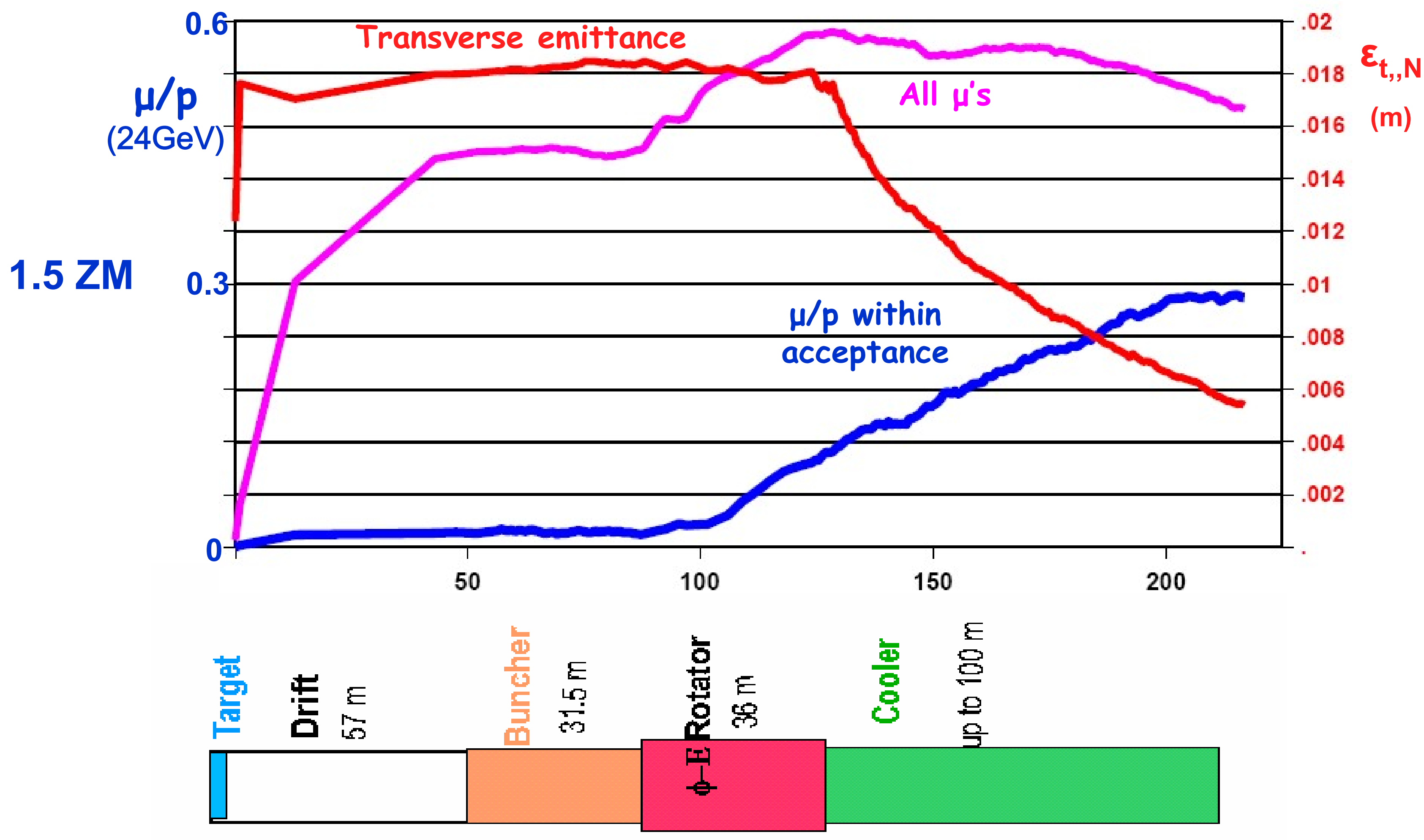




Simulations (N_B=10)



Example: $N_B = 10$, H_2 cooling



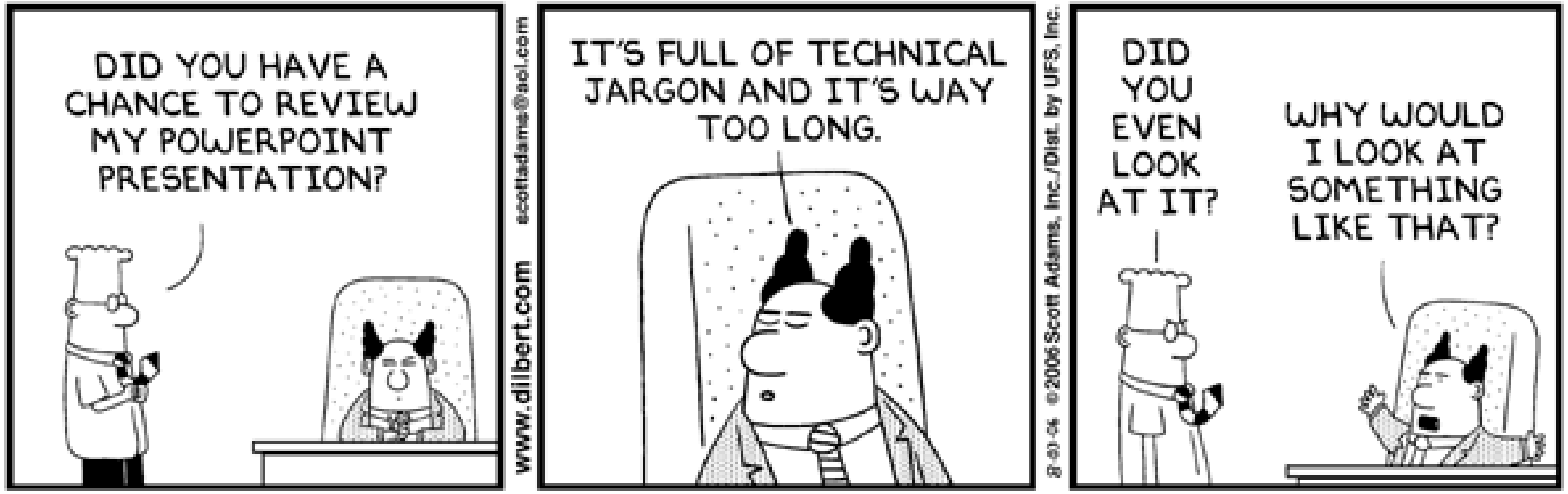
Summary

- Front end for ν-Factory
  - Target
- Initial $\phi-\delta E$ Rotation for neutrino factory discussed
  - High-frequency (multibunch)
- Matches into cooling and acceleration for neutrino factory and collider
- Captures both μ^+ and μ^- bunches

References

- “Cost-effective design for a neutrino factory”, J. Berg et al., PRSTAB 9, 011001(2006)
- “Recent progress in neutrino factory …”, M. Alsharo’a et al., PRSTAB 6, 081001 (2003)
- “Beams for European Neutrino Experiments (BENE) CERN-2006-005
- S. Ozaki et al., Feasibility Study 2, BNL-52623(2001).
- N. Holtkamp and D. Finley, eds., Study 1, Fermilab-Pub-00/108-E (2000).
- The Study of a European neutrino factory complex, CERN/PS/2002-080.
- R. Palmer- NuFACT07 Summer School lecture notes

Summary slide

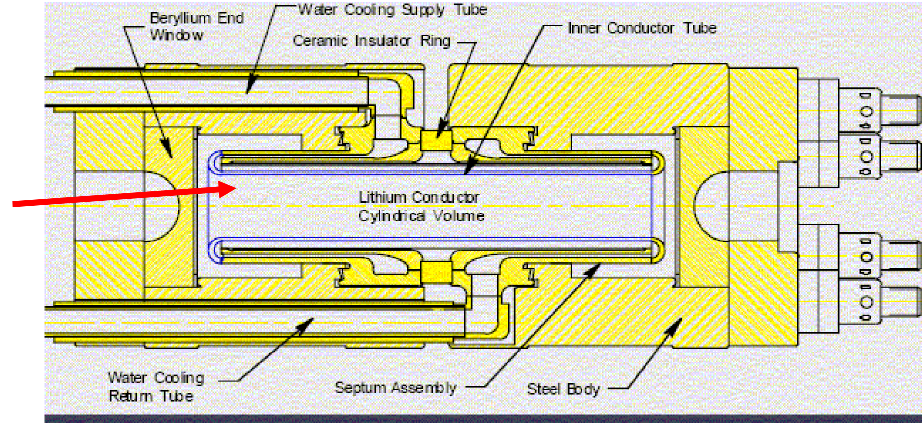


Extra Slides

Li Lens properties

- Current-carrying conducting cylinder
- Focusing Field:

$$B_\theta(r) = \mu_0 I_{total} \frac{r}{2\pi R_0^2}$$

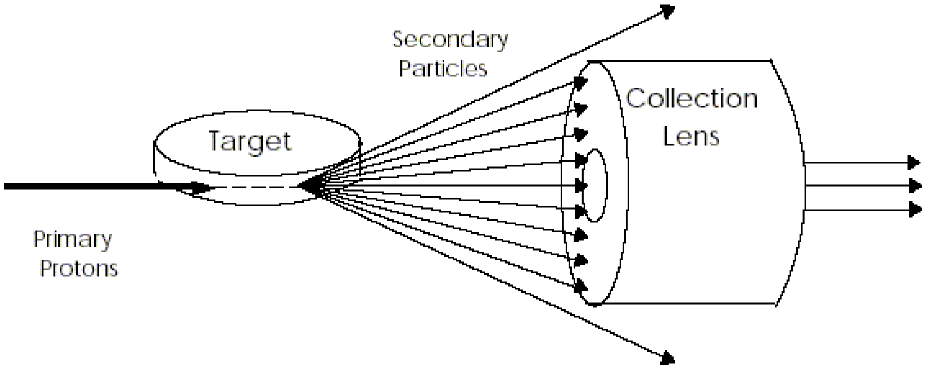


- Fermilab values:
  - R_0=1cm, I=0.5MA, L=15cm, B(R_0)=10T
  - Focuses 9GeV/c p with $p_\perp < 0.45$ GeV/c

Focusing angle:

Θ=(0.3B(r) L)/P

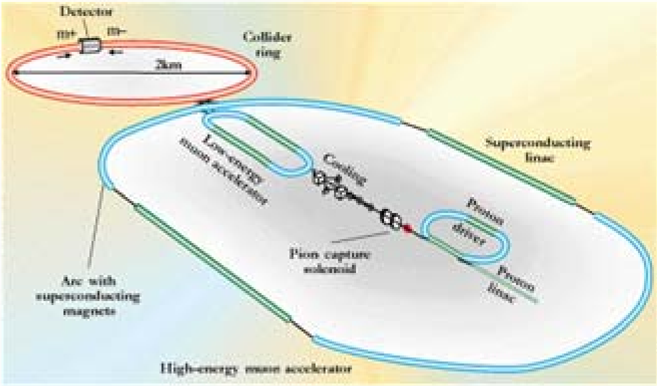
- Problems
  - Pulsed at <1Hz, need liquid for 10^+ H
  - Absorbs particles (π,p-bar)
  - Forward capture
  - Captures only one sign



$\mu^+ - \mu^-$ Collider Parameters

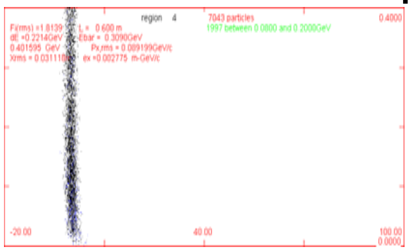
Muon Collaboration

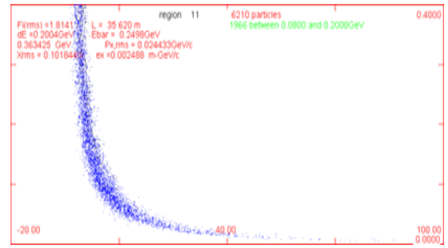
Parameter	4TeV(2000)	4TeV low emittance
Collision Energy($2E_\mu$)	4000	4000 GeV
Energy per beam(E_μ)	2000	2000 GeV
Luminosity($L=f_0 n_s n_b N_\mu^2/4\pi\sigma^2$)	**10^{35}**	**10^{35} $cm^{-2}s^{-1}$**
Source Parameters	**3.8 MW**	**1.0 MW p-beam**
Proton energy(E_p)	16	8 GeV
Protons/pulse(N_p)	$4\times2.5\times10^{13}$	$2\times2\times10^{13}$
Pulse rate(f_0)	15	20 Hz
Collider Parameters		
Mean radius(R)	1200	1000 m
μ /bunch($N_{\mu\pm}$)	1.25×10^{12}	2×10^{11}
Number of bunches(n_B)	4	2
Storage turns($2n_s$)	1500	2000
Norm. emittance(ε_N)	**0.6×10^{-2}**	**2.5×10^{-4}** cm-rad
Geom.. emittance($\varepsilon_t=\varepsilon_N/\gamma\beta$)	**3×10^{-7}**	1.3×10^{-8} cm-rad
IR Beam size $\sigma=(\varepsilon\beta_0)^{1/2}$	3.1	0.36 μm



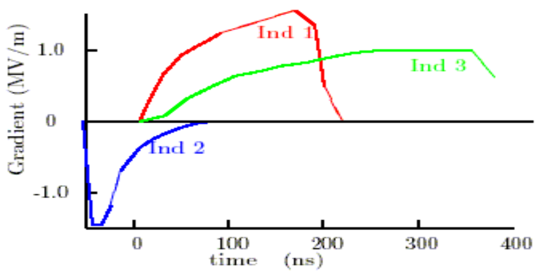
Study 2: Induction Linacs

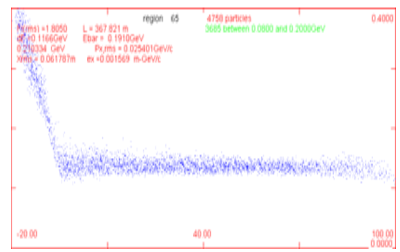
- **Drift** to develop Energy- phase correlation



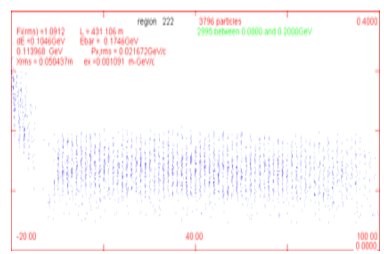


- Accelerate tail; decelerate head of beam, non-distortion (280m induction linacs (!))

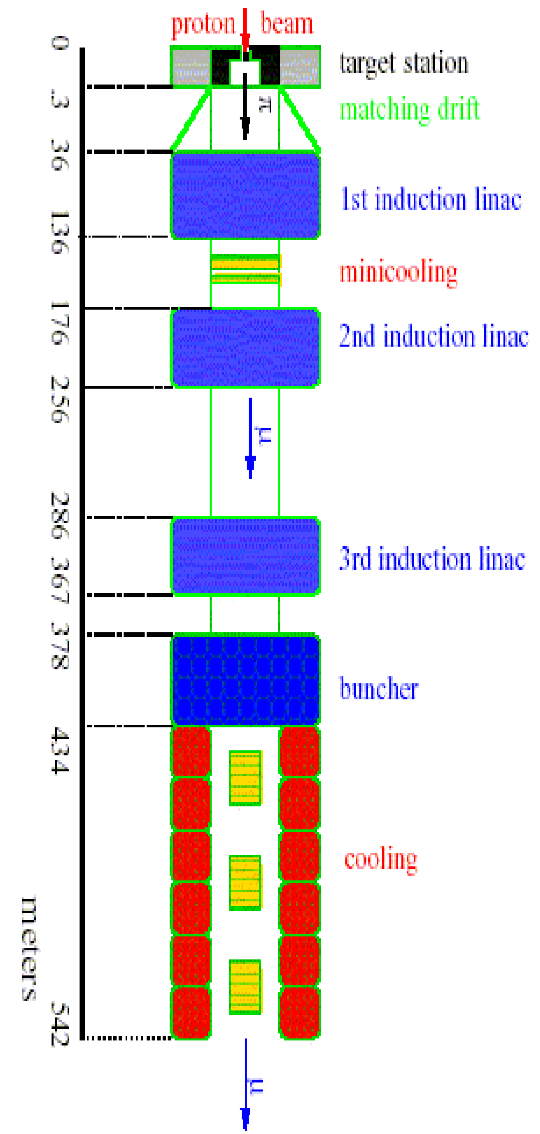




- **Bunch** at 200 MHz
  - ~0.2 μ/p

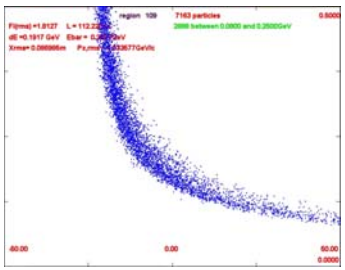


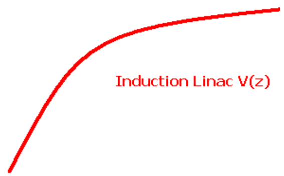
- Inject into 200 MHz cooling system
  - Cools transversely (to ε_t= ~0.002m

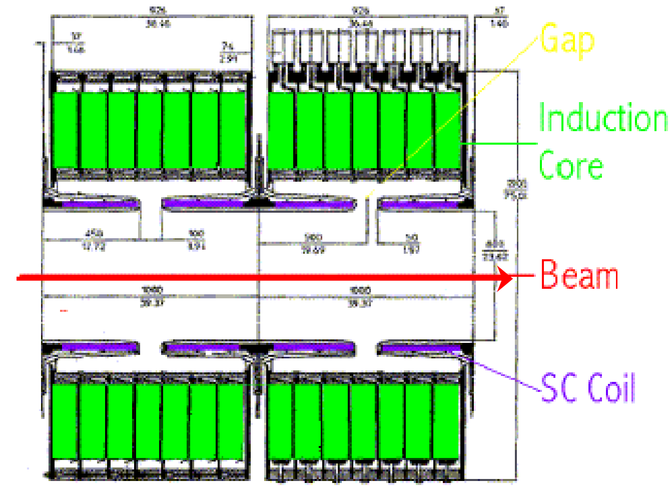


Induction Linac for φ−E Rotation

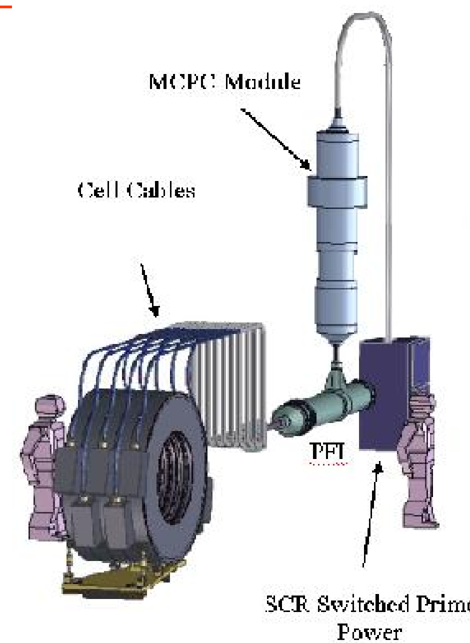
- Induction Linac can provide long pulse for φ-E rotation
- Arbitrary voltage waveform possible





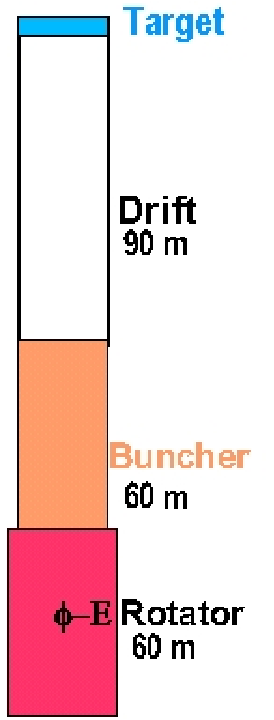


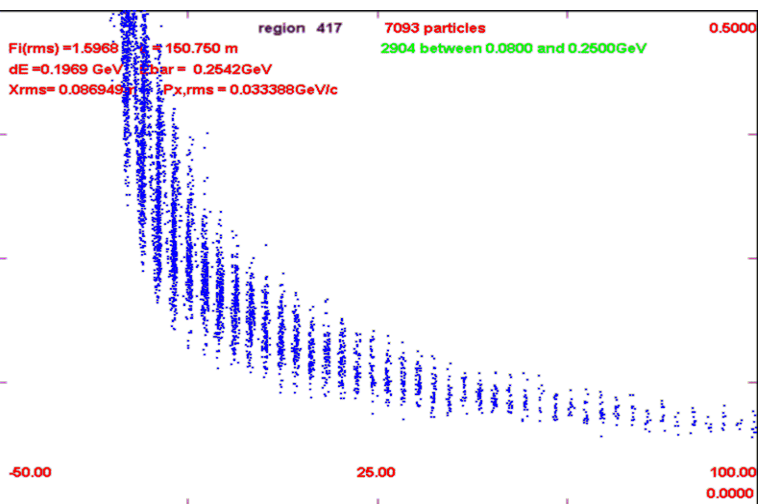
- Limited to < ~1MV/m
  - need > ~200MV, > 200m
- Very expensive, large power requirements
- Only captures one sign

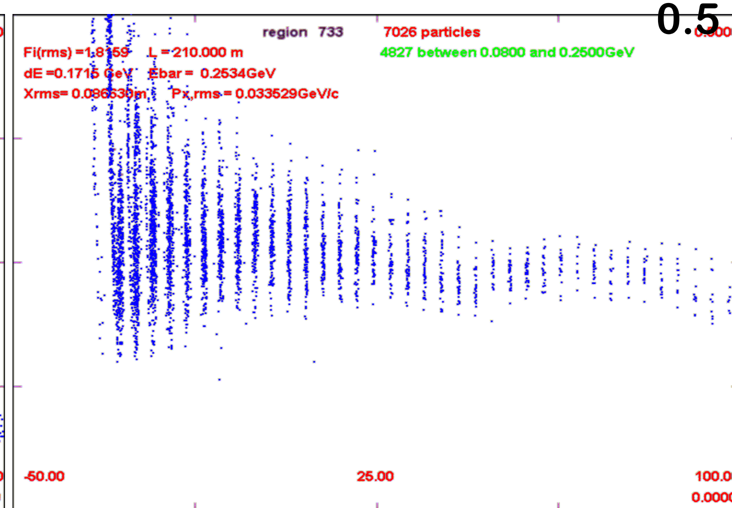


Another example: ~88 MHz

- Drift – 90m
- Buncher–60m
  - Rf gradient 0 to 4 MV/m
  - Rf frequency: 166→100 MHz
  - Total rf voltage 120MV
- Rotator–60m
  - Rf gradient 7 MV/m – 100→87 MHz
  - 420MV total
- Acceptance ~ study 2A (but no cooling yet)
  - Less adiabatic







0.5 GeV/c

0 GeV/c