



NuFact08 Summer School- Ionization Cooling

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Fermilab

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Outline

- **v-Factory and $\mu^+ - \mu^-$ Collider:**
- **Ionization Cooling**
 - Cooling description
 - Heating – Longitudinal Cooling
 - Emittance Exchange - Partition Numbers
 - Solenoidal focusing
 - Helical Cooler-PIC-REMEX
 - Low-Energy Cooling
- **Cooling Scenarios**
- **Other Applications**
 - Nuclear physics, stopped μ 's
- **Experimental Studies**
 - Mice
 - Mucool
 - Muons, Inc....
 - MCTF



References

- A. N. Skrinsky and V.V. Parkhomchuk, Sov. J. Nucl. Phys. **12**, 3(1981).
- D. Neuffer, Particle Accelerators **14**, 75 (1983)
- D. Neuffer, “ $\mu^+\mu^-$ Colliders”, CERN report 99-12 (1999).
- **D. Neuffer, “Introduction to Muon Cooling”, NIM A532, p. 26 (2004).**
- Y. Derbenev and R. Johnson, Phys. Rev. ST A. B. **8**, E041002 (2005);
- **Simulation tools**
 - R. Fernow, **ICOOL**
<http://pubweb.bnl.gov/users/fernnow/www/icool/readme.html>
 - T. Roberts, G4BeamLine (Muons, Inc.)
<http://www.muonsinc.com/>
- **Collaboration Efforts**
 - Muon Collaboration: http://www.cap.bnl.gov/mumu/mu_home_page.html
 - Muon Collider Task Force: <https://mctf.fnal.gov/>
 - **MICE** Collaboration: <http://hep04.phys.iit.edu/cooldemo/>
 - UKNF group (RAL)



Cooling Requirements

- Beam from target has

$$\varepsilon_{\perp,\text{rms}} \approx 2 \times 10^{-2} \text{ m-rad}; \varepsilon_{\parallel,\text{rms}} \approx 1 \text{ m}$$

- $\Delta x \approx 0.1 \text{ m} \times 20 \text{ MeV/c}; \Delta z \approx 1 \text{ m} \times \delta E = 100 \text{ MeV};$

- **μ -Storage Ring ν -Factory**

- Goal is to collect maximum number of μ^+ and/or μ^- that fit within accelerator / storage ring acceptances
- Transverse cooling by $\sim 10 \times$ is sufficient
- $\varepsilon_{\perp,\text{rms}} \approx 0.006$ to $0.002 \text{ m-rad}; \varepsilon_{\parallel,\text{rms}} \approx 0.06 \text{ m-rad/bunch}$

- **$\mu^+ - \mu^-$ Collider**

- Goal is maximal cooling of maximum number of **both** μ^+ AND μ^- ; high luminosity needed.
- Cooling by $> \sim 100 \times$ in each of $\varepsilon_x, \varepsilon_y, \varepsilon_z$ is required
- $\varepsilon_{\perp,\text{rms}} \approx 0.5$ to $0.025 \times 10^{-4} \text{ m-rad}; \varepsilon_{\parallel,\text{rms}} \approx 0.04 \text{ m-rad}$



Ionization Cooling-general principle



Transverse cooling:

- Particle loses momentum $P(\perp$ and \parallel) in material

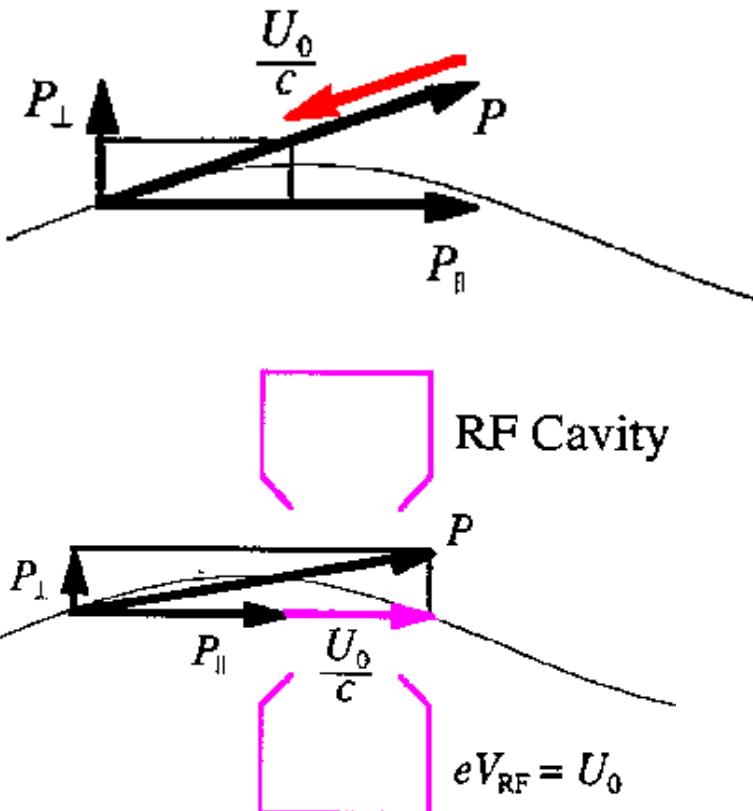
$$P_x \rightarrow P_x \left[1 - \frac{dP}{ds} \frac{\Delta s}{P} \right]$$

- Particle regains P_{\parallel} (only) in RF

$$\varepsilon_{\perp} \rightarrow \varepsilon_{\perp} \left[1 - \frac{dP}{ds} \frac{\Delta s}{P} \right]$$

- Multiple Scattering in material increases rms emittance:

$$\Delta\varepsilon_{\perp,N} = \beta\gamma \frac{\langle x^2 \rangle}{2\varepsilon_{\perp}} (\Delta \langle \theta_x^2 \rangle) = \beta\gamma \frac{\beta_{\perp}}{2} \left(\frac{E_s^2}{(\beta cp)^2 L_R} \right) \Delta z$$



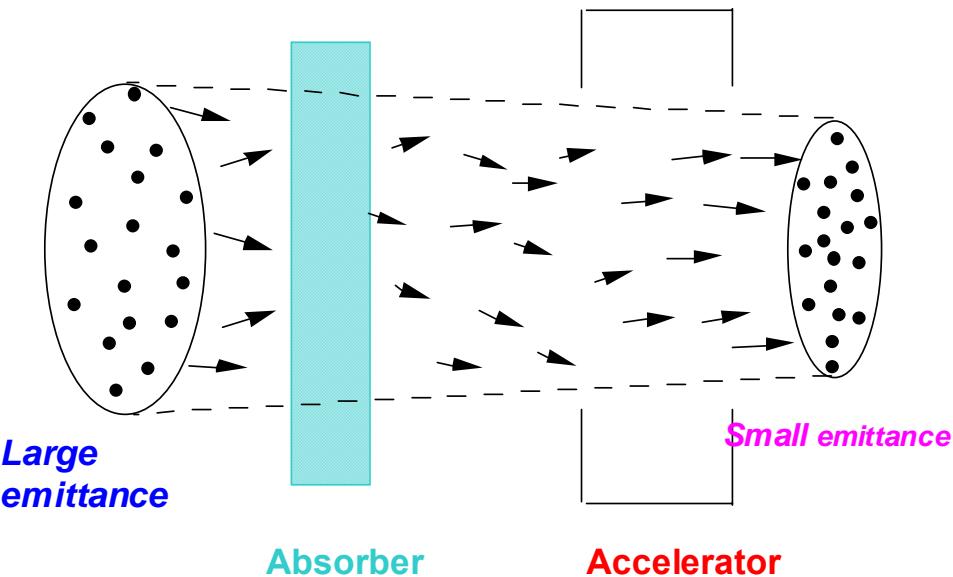


Ionization Cooling Principle

Loss of transverse momentum
in absorber:

$$p_x \rightarrow p_x \left[1 - \frac{dp}{ds} \frac{\Delta s}{p} \right]$$

$$\varepsilon_{\perp,N} \rightarrow \varepsilon_{\perp,N} \left[1 - \frac{dp}{ds} \frac{\Delta s}{p} \right]$$



$$\frac{d\varepsilon_N}{ds} = -\frac{1}{P_\mu} \frac{dP_\mu}{ds} \varepsilon_N + \frac{\beta\gamma\beta_\perp}{2} \frac{d\langle \theta_{rms}^2 \rangle}{ds}$$

Momentum loss is
opposite to motion,
 p , p_x , p_y , ΔE decrease

$$= -\frac{1}{\beta^2 E_\mu} \frac{dE_\mu}{ds} \varepsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E}$$

Heating by
multiple scattering

Momentum gain
is purely longitudinal



Combining Cooling and Heating:



$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E} \frac{dE}{ds} \varepsilon_N + \frac{\beta\gamma \beta_\perp}{2} \frac{d\langle \theta_{rms}^2 \rangle}{ds}$$

- Low-Z absorbers (H_2 , Li, Be, ...) to reduce multiple scattering
- High Gradient RF
 - To cool before μ -decay ($2.2\gamma \mu s$)
 - To keep beam bunched
- Strong-Focusing at absorbers
 - To keep multiple scattering less than beam divergence ...
⇒ Quad focusing ?
⇒ Li lens focusing ?
⇒ Solenoid focusing ?

$$\frac{d\langle \theta_{rms}^2 \rangle}{ds} = \frac{z^2 E_s^2}{\beta^2 c^2 p_\mu^2 L_R}$$



Transverse cooling limits

- Transverse Cooling – equilibrium emittance

equilibrium scattering angle

$$\epsilon_{N,eq} = \frac{\beta_\perp E_s^2}{2\beta m_\mu c^2 L_R \frac{dE_\mu}{ds}}$$

$$\sigma_\theta = \sqrt{\frac{E_s^2}{2\beta^2 \gamma m_\mu c^2 L_R \frac{dE_\mu}{ds}}}$$

Material Properties for Ionization Cooling

Material	Symbol	Z, A	Density gm/cm ³	dE/ds (min.) MeV/cm	L _R Cm	L _R dE/ds MeV	$\sigma_\theta \cdot \beta \gamma^{1/2}$	$g_x \beta \epsilon_N / \beta_\perp$ mm-mrad/cm
Hydrogen	H ₂	1, 1	0.071	0.292	865	252.6	0.061	37
Lithium	Li	3, 7	0.534	0.848	155	130.8	0.084	71
Lith. H	LiH	3+, 7+	0.9	1.34	102	137	0.0824	68
Beryllium	Be	4, 9	1.848	2.98	35.3	105.2	0.094	88
Carbon	C	6, 12	2.265	4.032	18.8	75.8	0.110	122
Aluminum	Al	13, 27	2.70	4.37	8.9	38.9	0.154	238
Copper	Cu	29, 63.5	8.96	12.90	1.43	18.45	0.224	503
Tungsten	W	74, 184	19.3	22.1	0.35	7.73	0.346	1200

- Want materials with small multiple scattering (large L_R), but relatively large dE/ds, density (ρ)
- Want small β_\perp at absorbers => strong focusing
- - equilibrium emittances ($/\beta_\perp$) smallest for low-Z materials



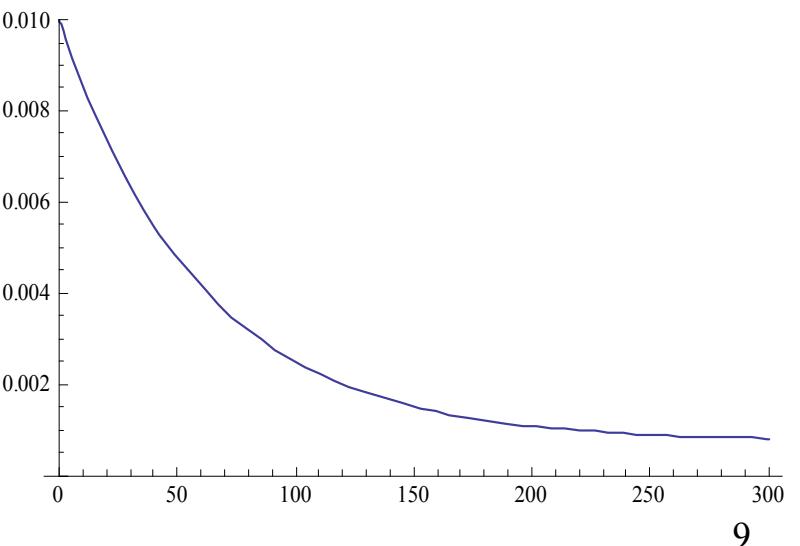
Problem



- Plug in some sample values for the cooling equations; solve for ε_t equilibrium and $\varepsilon_t(s)$
 - try $dE/ds = 5 \text{ MeV/m}$, $p = 300 \text{ MeV/c}$,
 - $\beta_\perp = 0.1 \text{ m}$, Li absorbers

$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E} \frac{dE}{ds} \varepsilon_N + \frac{\beta\gamma}{2} \frac{\beta_\perp}{ds} \frac{d\langle \theta_{rms}^2 \rangle}{ds}$$

$$\frac{d\langle \theta_{rms}^2 \rangle}{ds} = \frac{f E_s^2}{\beta^2 c^2 p_\mu^2 L_R}$$

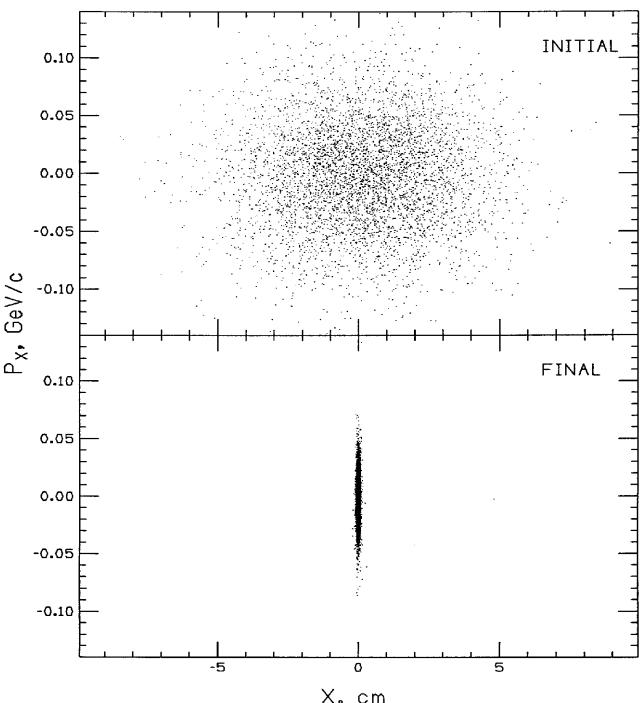




Ionization Cooling difficulties



- Must focus to very small β_{\perp}
 - $\beta_{\perp} : 1\text{m} \rightarrow \sim 1\text{mm}$
- Intrinsic scattering of beam is large
 - $\theta_{\text{rms}} > \sim 0.1$ radians
- Intrinsic momentum spread is large
 - $\sigma_P/P > \sim 0.03$
- Cooling must occur within muon lifetime
 - $\tau_{\mu} = 2.2\gamma \text{ } \mu\text{s}$ or $L_{\mu} = 660 \beta\gamma \text{ m pathlength}$
- Does not (directly) cool longitudinally



Longitudinal “Cooling”

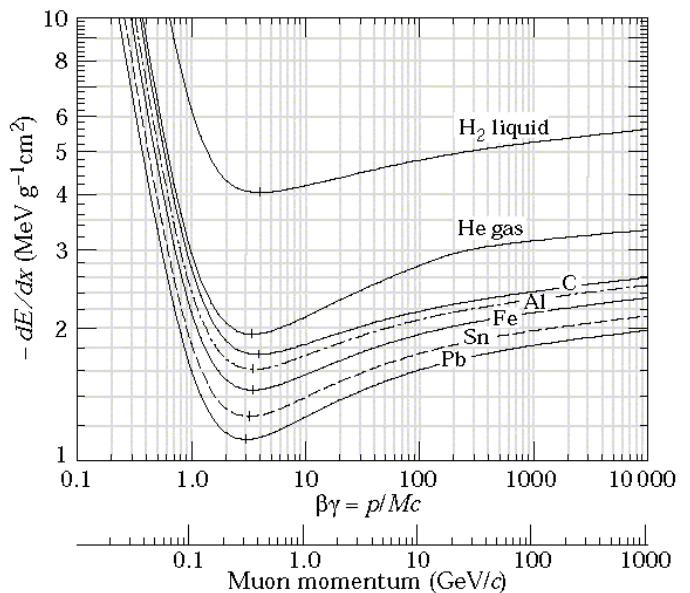
$$\frac{dE}{ds} = 4\pi N_A \rho r_e^2 m_e c^2 \frac{Z}{A} \left[\frac{1}{\beta^2} \ln \left(\frac{2m_e c^2 \gamma^2 \beta^2}{I(Z)} \right) - 1 - \frac{\delta}{2\beta^2} \right]$$

- Energy cooling occurs if the derivative :

$$\partial(dE/ds)/\partial E = g_L (dp/ds)/p > 0$$

$$g_L = -\frac{2}{\gamma^2} + 2 \frac{\left(1 - \frac{\beta^2}{\gamma^2}\right)}{\left(\ln \left[\frac{2m_e c^2 \beta^2 \gamma^2}{I(Z)} \right] - \beta^2\right)}$$

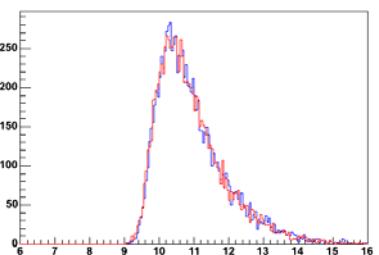
- $g_L(E)$ is negative for $E < \sim 0.2$ GeV and only weakly positive for $E > \sim 0.2$ GeV



Energy straggling increases energy spread

$$\frac{d\langle \Delta E_{rms}^2 \rangle}{ds} \cong 4\pi (r_e m_e c^2)^2 n_e \gamma^2 \left(1 - \frac{\beta^2}{2}\right)$$

⇒ Ionization cooling does not effectively cool longitudinally





“Emittance exchange” enables longitudinal cooling:

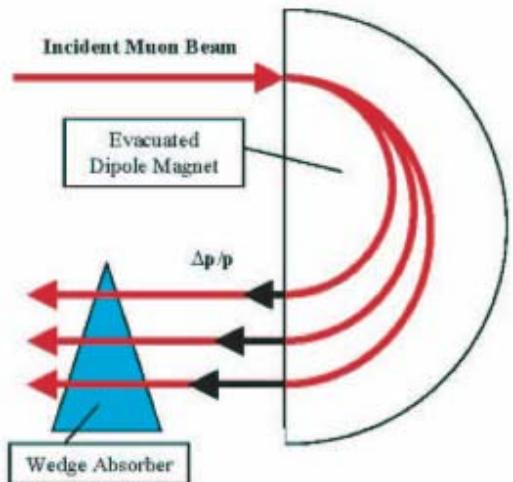


Figure 1. Use of a Wedge Absorber for Emittance Exchange

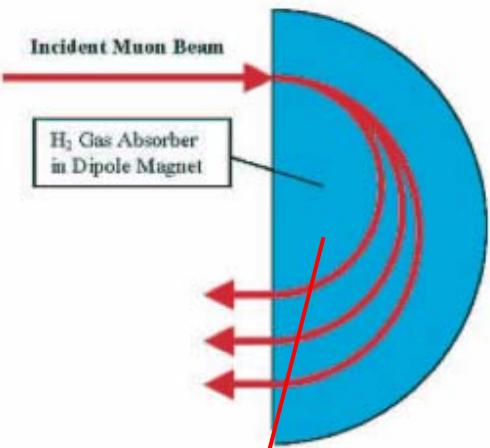
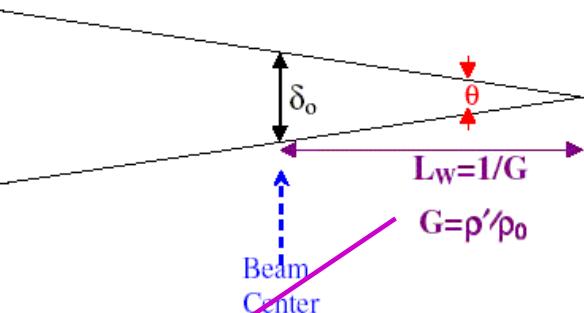


Figure 2. Use of Continuous Gaseous Absorber for Emittance Exchange

Geometry of wedge absorber



- Cooling derivative is changed by use of dispersion + wedge
(Dependence of energy loss on energy can be increased)

$$\frac{\partial \frac{dE}{ds}}{\partial E} \Rightarrow \left. \frac{\partial \frac{dE}{ds}}{\partial E} \right|_0 + \frac{1}{\beta c p} \frac{dE}{ds} \frac{\eta \rho'}{\rho_0} = g_L \frac{dp/ds}{p}$$

(if due to path length)

$$\Delta g_L = \frac{1}{\gamma_T^2} = \frac{(ka)^2}{1+(ka)^2} \left(\frac{a}{p} \frac{dp}{da} \right)^{-1}$$



Partition Numbers, δE - δt cooling

With emittance exchange the longitudinal partition number g_L changes:

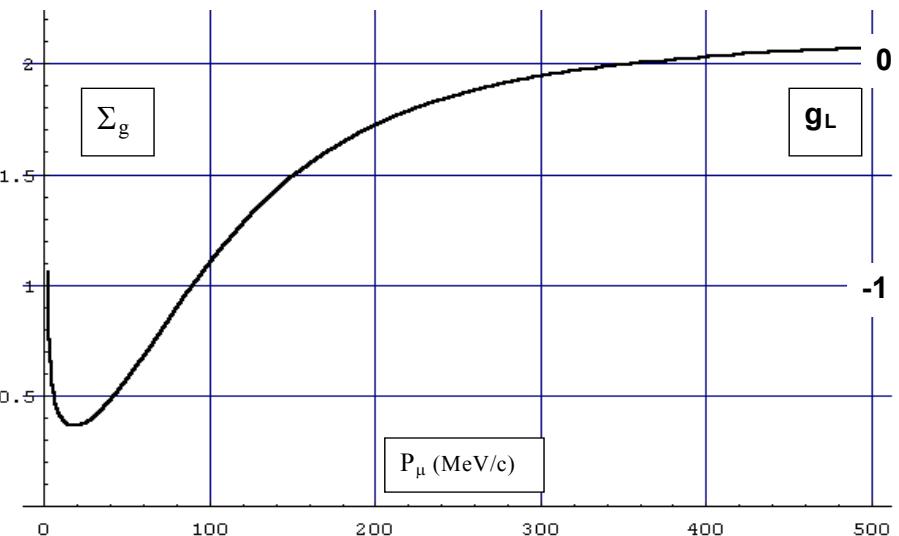
$$g_L \Rightarrow g_{L,0} + \frac{\eta \rho'}{\rho_0}$$

But the transverse cooling partition number decreases:

$$g_x \Rightarrow 1 - \frac{\eta \rho'}{\rho_0}$$

The sum of the cooling partition numbers (at $P = P_\mu$) remains constant:

$$\Sigma_g(P_\mu) = g_x + g_y + g_L = 2 + g_{L,0}$$



$$\Sigma_g = 2\beta^2 + 2 \frac{\left(1 - \frac{\beta^2}{\gamma^2}\right)}{\left(\ln\left[\frac{2m_e c^2 \beta^2 \gamma^2}{I(Z)}\right] - \beta^2\right)}$$

$$\Sigma_g > 0$$



Cooling + Energy straggling ...

Energy spread (σ_E) cooling equation:

$$\frac{d\sigma_E^2}{ds} = -2 \frac{g_L \frac{dE}{ds}}{\beta^2 E} \sigma_E^2 + 4\pi (r_e m_e c^2)^2 n_e \gamma^2 \left(1 - \frac{\beta^2}{2} \right)$$

Equilibrium σ_p :

$$\frac{\sigma_p}{p} = \sqrt{\frac{m_e \gamma}{2g_L m_\mu} \times \frac{1 - \frac{\beta^2}{2}}{\log[] - \beta^2}}$$

Longitudinal Emittance Cooling equation:

$$\frac{d\varepsilon_L}{ds} = -\frac{g_L}{p_\mu} \frac{dp_\mu}{ds} \varepsilon_L + \frac{\beta_{ct}}{2} \frac{d\langle \Delta E_{rms}^2 \rangle}{ds}$$

$$\beta_{ct} = \sqrt{\frac{1}{\beta^3 \gamma eV' \cos\varphi_s} \frac{\lambda_{RF}}{2\pi} \frac{\alpha_p}{mc^2}}$$

Longitudinal Cooling requires:

- Positive g_L (η , “wedge”), Strong bunching (β_{ct} small)
- Large V_{rf} , small λ_{rf}

Energy loss/recovery

Before decay requires:

$$V'_{rf} \gg \frac{\Delta p_\mu}{L_\mu} \gg \frac{(m_\mu \beta \gamma)}{L_0 \beta \gamma} = \frac{105.66 \text{ MeV}}{660 \text{ m}} = 0.16 \text{ MeV/m}$$



μ Cooling Regimes

Neutrino Factory
 π^+
Muon Collider

- Efficient cooling requires:

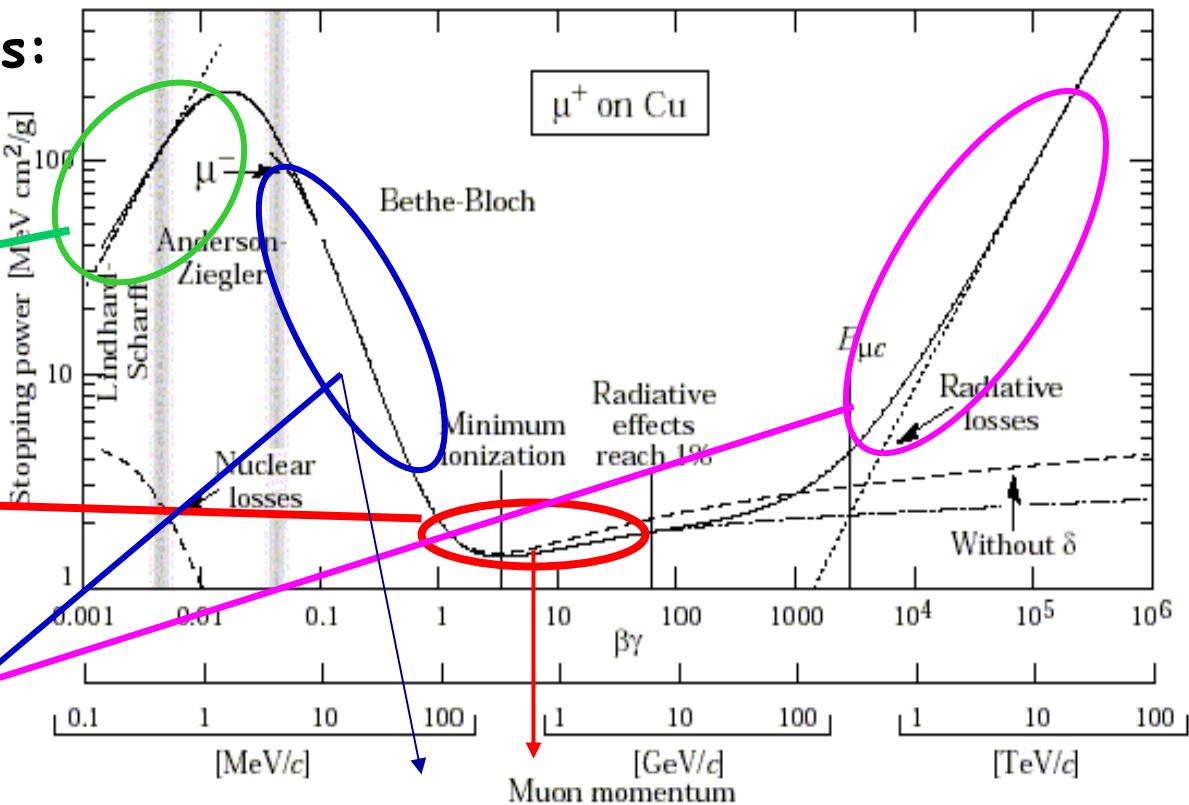
$$\frac{\partial \frac{dE}{dx}}{\partial E} > \sim 0$$

- Frictional Cooling ($< 1 \text{ MeV}/c$) $\Sigma_g = \sim 3$

- Ionization Cooling ($\sim 0.3 \text{ GeV}/c$) $\Sigma_g = \sim 2$

- Radiative Cooling ($> 1 \text{ TeV}/c$) $\Sigma_g = \sim 4$

- Low- ϵ_t cooling $\Sigma_g = \sim 2\beta^2$
(longitudinal heating)



$$\frac{dE}{ds} = 4\pi N_A \rho r_e^2 m_e c^2 \frac{Z}{A} \left[\frac{1}{\beta^2} \ln \left(\frac{2m_e c^2 \gamma^2 \beta^2}{I(Z)} \right) - 1 - \frac{\delta}{2\beta^2} \right]$$

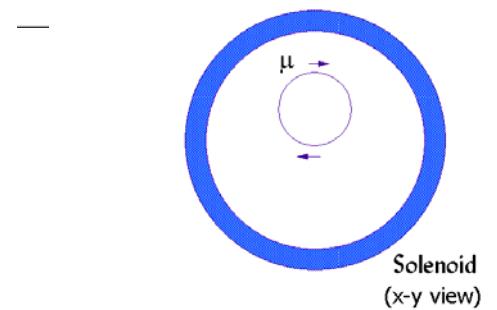
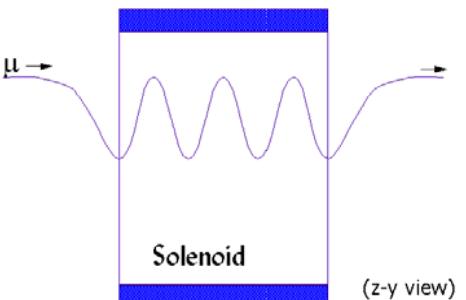


Focusing for Cooling

- Strong focussing needed – magnetic quads, solenoids, Li lens ?
- Solenoids have been used in most (recent) studies
 - Focus horizontally and vertically
 - Focus both μ^+ and μ^-

$$r'' = - \left(\frac{B}{2B\rho} \right)^2 r \quad \beta_{\perp, \text{equil.}} = \frac{2B\rho}{B}$$

- Strong focussing possible:
 - $\beta_{\perp} = 0.13m$ for $B=10T$, $p_{\mu} = 200 \text{ MeV}/c$
 - $\beta_{\perp} = 0.0027m$ for $B=50T$, $p_{\mu} = 20 \text{ MeV}/c$
- But:
 - Solenoid introduces angular motion
 - L damped by cooling + field flips
 - B within rf cavities ?



$$\theta' = B/(2 B\rho)$$



Solenoidal focusing for cooling

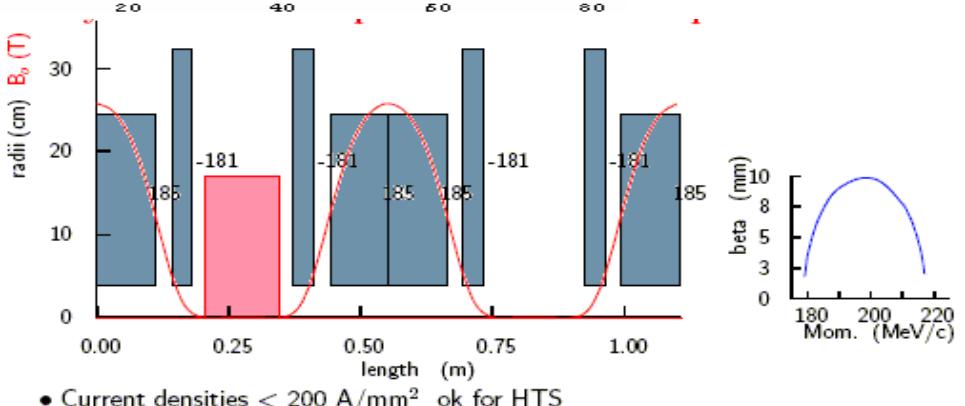
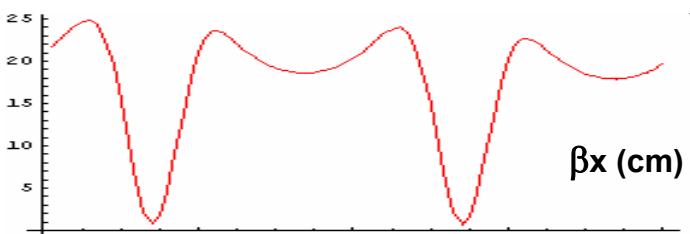
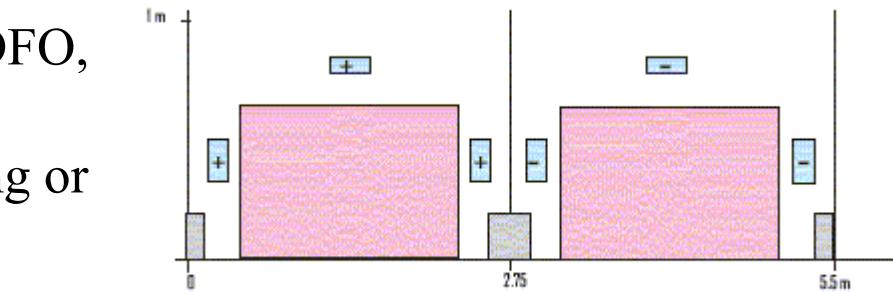
Fernow, Palmer PRSTAB 10, 064001 (2007)



- Lattices are sequences of solenoids and drifts (rf interlaced) (+,-)
 - FOFO, ASOL, RFOFO, SFOFO, DODO, SOSO ...
- Can have nearly constant focusing or focusing to small β^*
- Large $\delta p/p$ acceptance possible
 - Need $>\pm 10\% \delta p/p$
- Low β^* can be much less than:

$$\beta_{\perp, \text{equil.}} = \frac{2B\rho}{B}$$

- $>5\times$ smaller



- Current densities $< 200 \text{ A/mm}^2$ ok for HTS



Cooling with \perp - exchange and solenoids

Wang and Kim, NIM A532, 260 (2004)



Example: rms Cooling equations with dispersion and wedges (at $\eta'=\alpha=0$) in x-plane:

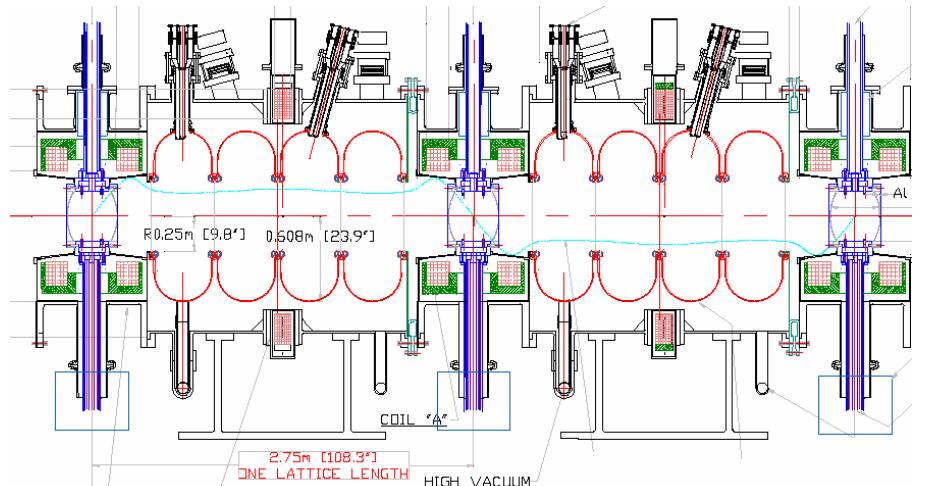
$$\boxed{\begin{aligned}\frac{d\epsilon_x}{ds} &= -g_x \frac{dP}{ds} \frac{\epsilon_x}{P} + \beta_{\perp} \frac{d\theta_{rms}^2}{ds} + \frac{1}{2} \frac{dP}{ds} \beta_{\perp} \theta'_L L + \frac{1}{2} H_x \frac{d\delta_{rms}^2}{ds} & H_x = \frac{\eta^2}{\beta_x} \\ \frac{d\epsilon_y}{ds} &= -\frac{dP}{ds} \frac{\epsilon_x}{P} + \beta_{\perp} \frac{d\theta_{rms}^2}{ds} + \frac{1}{2} \frac{dP}{ds} \beta_{\perp} \theta'_L L & \theta'_L = B/(2B\rho) \\ \frac{d\epsilon_z}{ds} &= -g_L \frac{dP}{ds} \frac{\epsilon_z}{P} + \frac{1}{2} \beta_z \frac{d\delta_{rms}^2}{ds} + \frac{1}{2} \frac{\eta^2}{\beta_z} \frac{d\theta_{rms}^2}{ds} & L = x'y' - yx'\end{aligned}}$$

$$\frac{dL}{ds} = -(1 - \frac{\delta g}{2}) \frac{dP}{ds} L + \frac{1}{2} \frac{dP}{ds} \beta_{\perp} \theta'_L (\epsilon_x + \epsilon_y) + \frac{\eta^2}{\beta_z} \frac{d\delta_{rms}^2}{ds}$$

The additional correlation and heating terms are "small" in "well-designed" systems.



Study 2 Cooling Channel (\approx MICE)

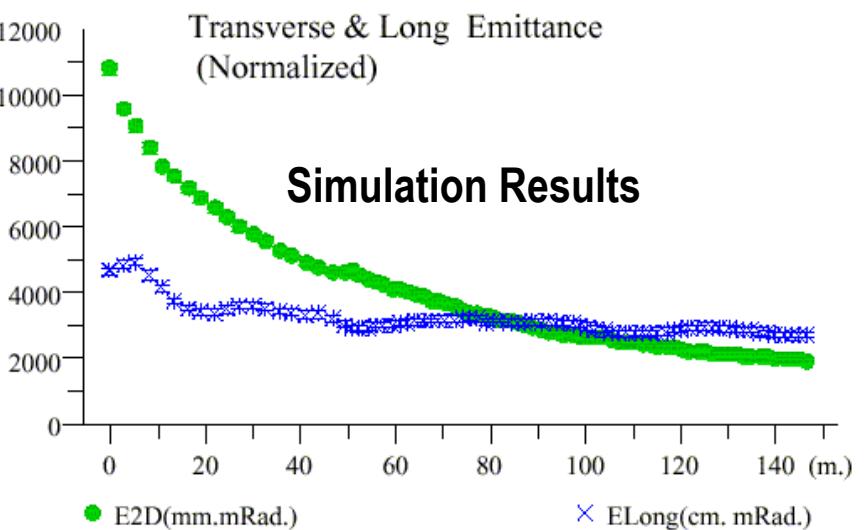


sFOFO 2.75m cells

- Cell contains
 - Rf for acceleration/bunching
 - H₂ absorbers
 - Solenoidal magnets

108 m cooling channel consists of:
16 2.75m cells + 40 1.65m cells

Focusing increases along channel:
 B_{\max} increases from 3 T to 5.5 T

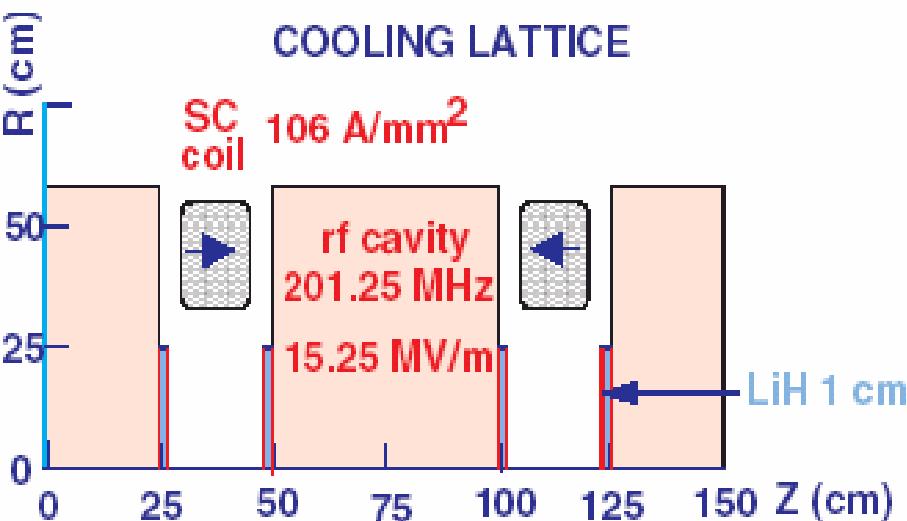




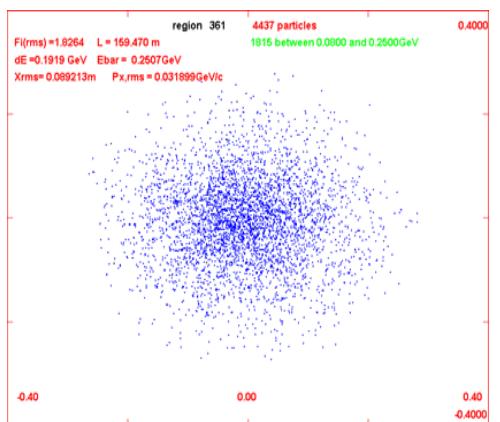
v-Factory Study 2A cooling channel



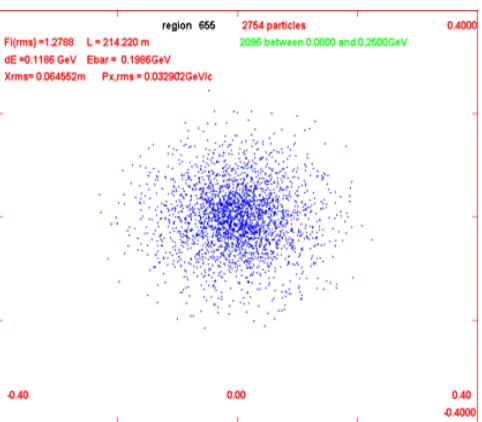
- Lattice is weak-focusing
 - $B_{\max} = 2.5\text{T}$, solenoidal
 - $\beta_{\perp} \cong 0.8\text{m}$
- Cools transversely
 - ε_{\perp} from ~ 0.018 to $\sim 0.007\text{m}$
 - in $\sim 70\text{m}$



Before



After cooling



Problem: Check with rms cooling equations; How is answer changed if H_2 is used?

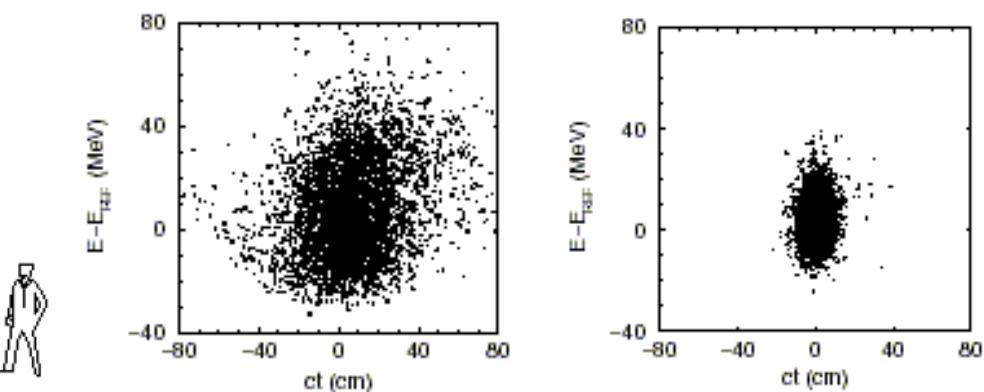
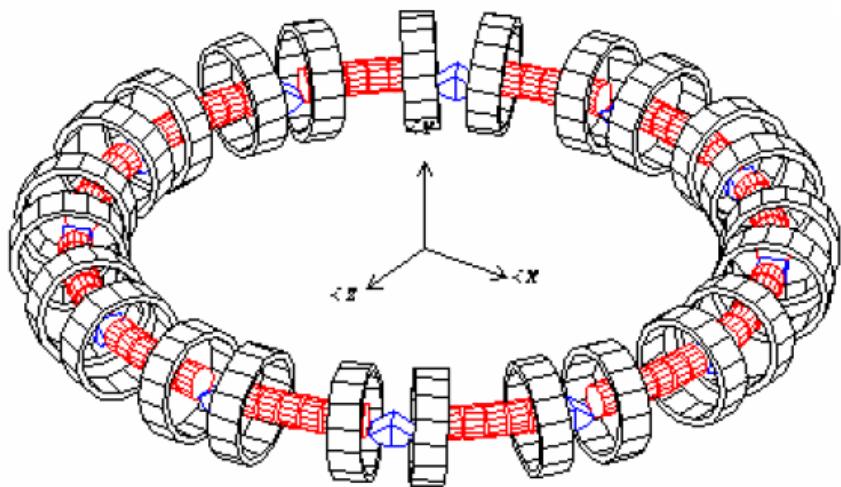
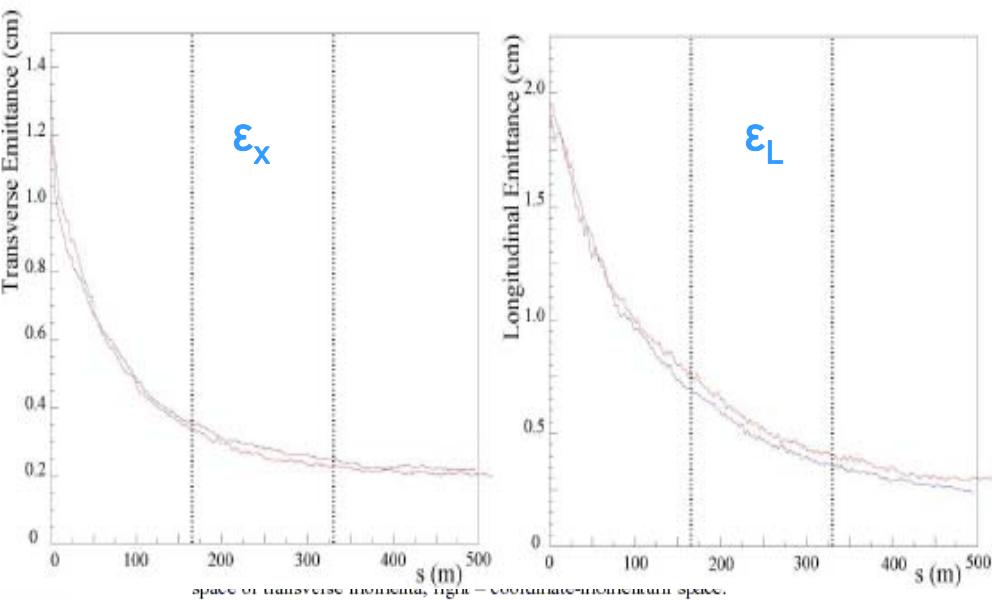
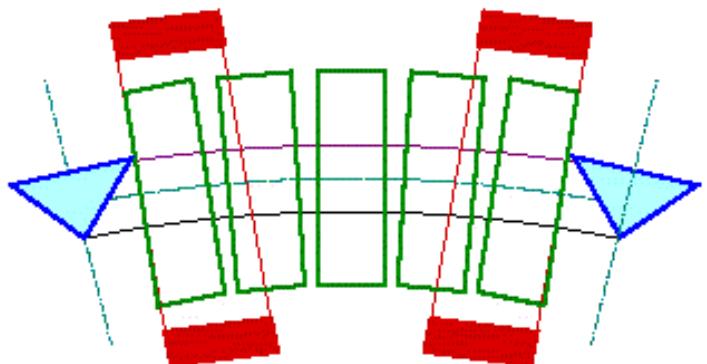


RFOFO Ring Cooler performance

R. Palmer et al., PR STAB 8, 061003 (2005)



- Cools longitudinally and transversely



E-ct before and after



$\mu^+ - \mu^-$ Collider Cooling Scenarios

Palmer et al.



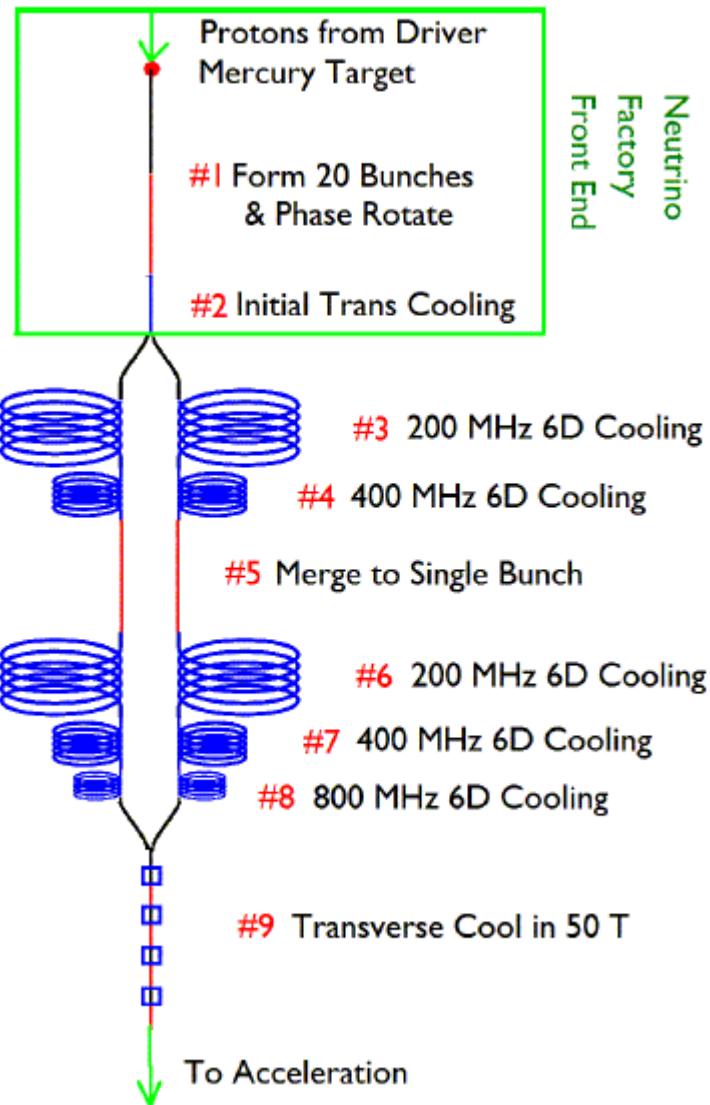
- requires **energy cooling** and **emittance exchange** (and anti-exchange) to obtain small $\varepsilon_L, \varepsilon_x, \varepsilon_y$

- Start with large beam from target, compress and cool, going to stronger focussing and bunching the beam gets smaller ...

- $\delta p/p \sim 10\%, \sigma_\theta \sim 0.1$

- Bunching rf frequency increases

- In final cooling stages longitudinal emittance increases while transverse emittance decreases

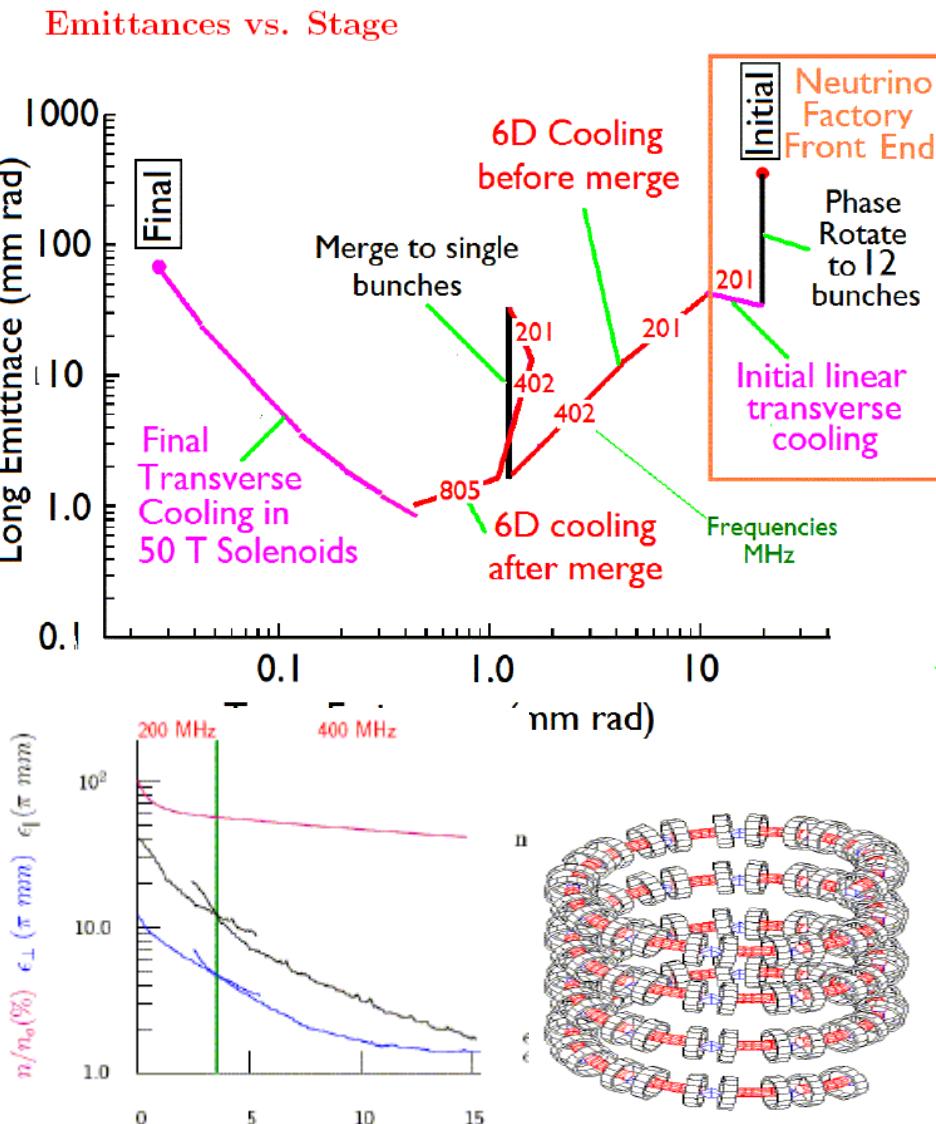




Cooling Scenario for Collider



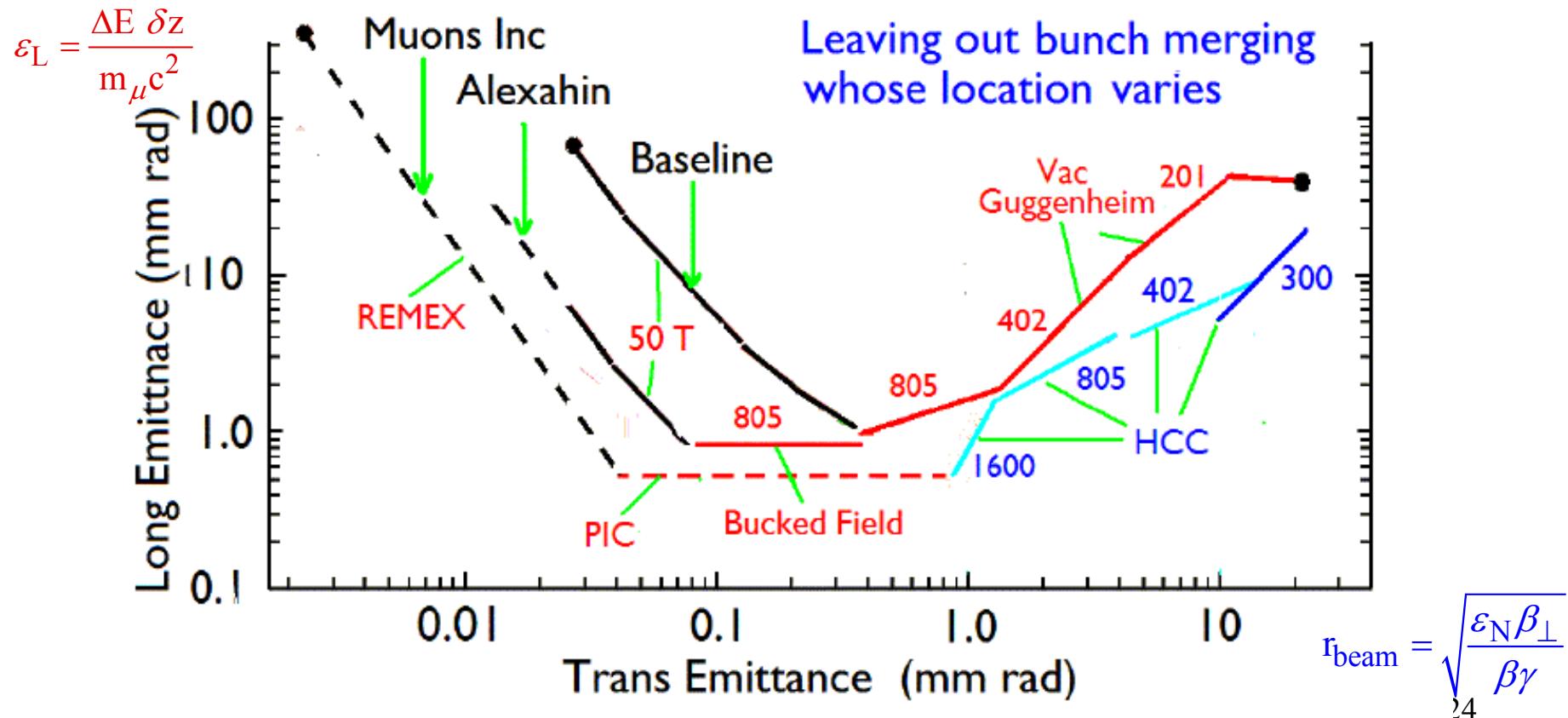
- **Steps 1,2:** Bunching, phase rotation, cooling (=ν factory)
 - σ_μ : **10cm → 6cm**
- **3,4:** 6-D cooling with 200, 400 MHz "Ring Coolers"
 - σ_μ : **6cm → 2.4cm → 1.0cm**
- **5:** compress to 1 bunch
- **6, 7:** 6-D 200, 400 MHz Coolers
 - σ_μ : **3cm → 1.0cm**
- **8:** 800 MHz "Ring Cooler"
 - σ_μ : **1.0cm → 0.3cm**
- **9:** up to 50T coolers (H_2 , solenoids)
 - σ_μ : **0.4cm → 0.08cm**
- **Total length of system ~0.8km**





Other Cooling scenarios

- HCC- Helical Cooling Channel
- PIC-Parametric-resonance Ionization Cooling
 - Use resonance beam dynamics to intensify focusing
- REMEX, low-energy emittance exchange
- Very low energy cooling





Helical 6-D Cooler

Y. Derbenev and R. Johnson, Phys. Rev. STAB. 8, E041002 (2005)



- Magnetic field is solenoid B_0 + dipole + quad + ...
- System is filled with H_2 gas, includes rf cavities
- Cools 6-D (large E means longer path length)

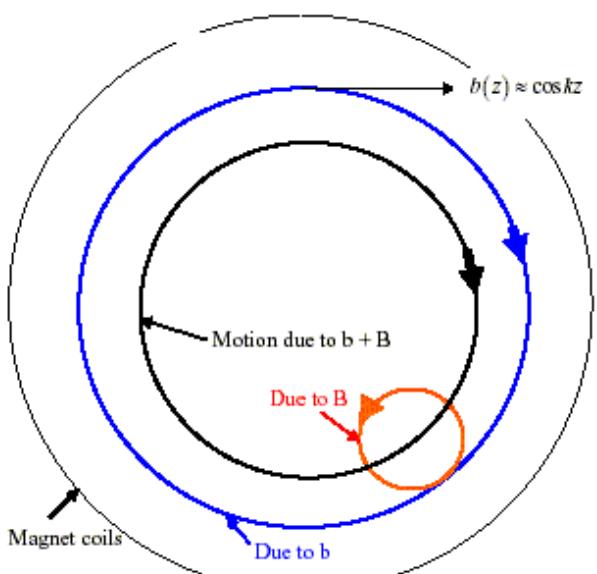
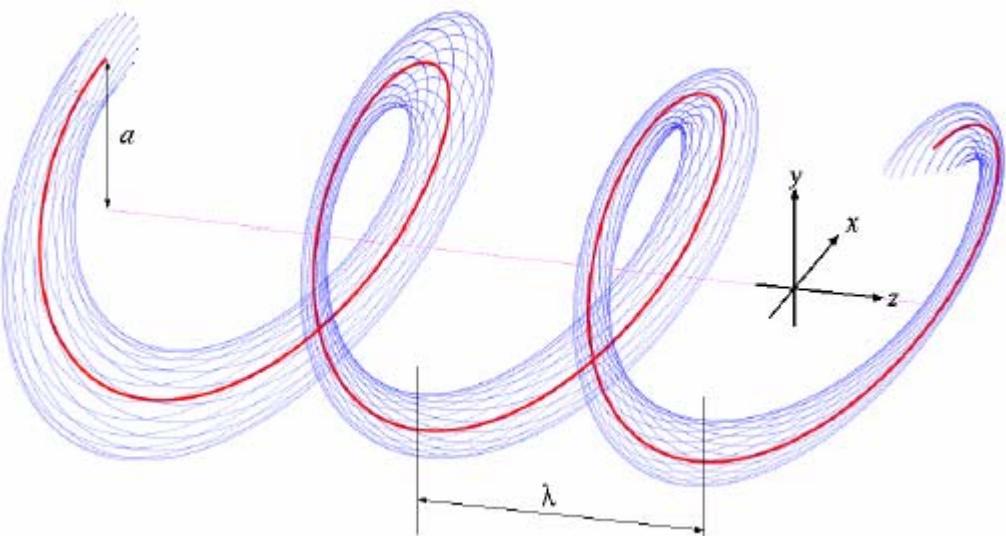
$$\Delta g_L = \frac{1}{\gamma_T^2} = \frac{(ka)^2}{1+(ka)^2} \left(\frac{a dp}{p da} \right)^{-1}$$

Key parameters:

$a, k=2\pi/\lambda$, solenoid field B_0 , transverse field $b(a)$

$$p(a) = \frac{\sqrt{1+(ka)^2}}{k} \left[B - \frac{1+(ka)^2}{ka} b(a) \right]$$

$$\begin{aligned} F_{h-dipole} &= p_z \times B_\perp; \quad b = B_\perp \\ F_{solenoid} &= -p_\perp \times B_z; \quad B = B_z \end{aligned}$$

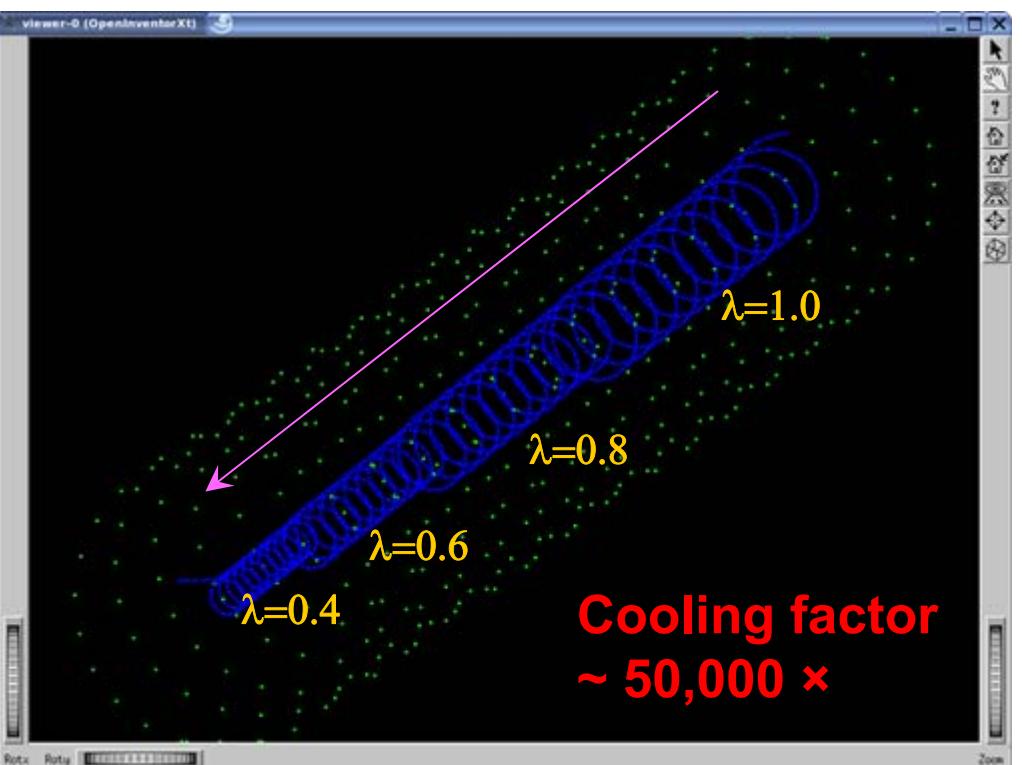




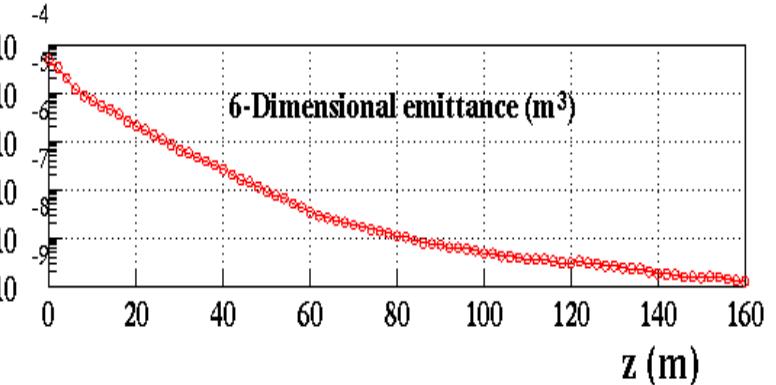
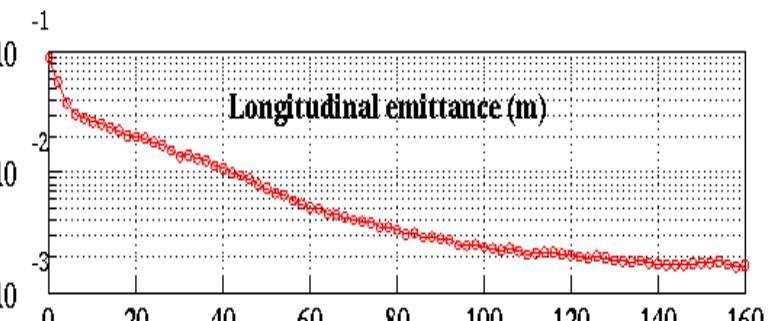
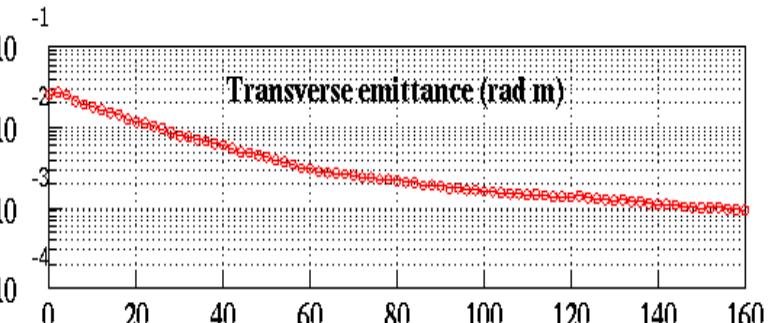
Helical Wiggler 3-D Cooling ($P_{\mu}=250\text{MeV}/c$)

Series HCCs

		Segment				
		1st	2nd	3rd	4th	
L	Length	m	50	40	30	40
λ	Helix period	m	1.0	0.80	0.60	0.40
a	Reference orbit radius	m	0.16	0.13	0.095	0.064
κ	Helix pitch		1.0	1.0	1.0	1.0
B	Solenoidal component	T	-6.95	-8.68	-11.6	-17.4
b_d	Helix dipole coefficient	T	1.81	2.27	3.02	4.53
b_q	Helix quadrupole coefficient	T/m	-0.35	-0.44	-0.59	-0.88
b_1	Helix sextupole coefficient	T/m ²	0.031	0.039	0.051	0.077



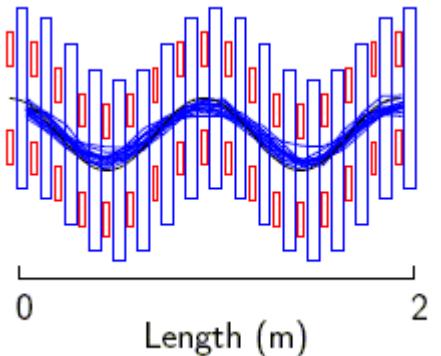
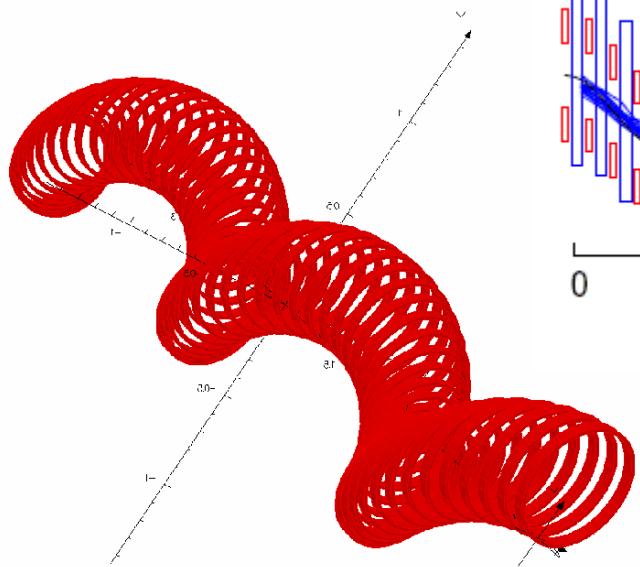
$\lambda = 1.0 \text{ m}$ $\lambda = 0.8 \text{ m}$ $\lambda = 0.6 \text{ m}$ $\lambda = 0.4 \text{ m}$



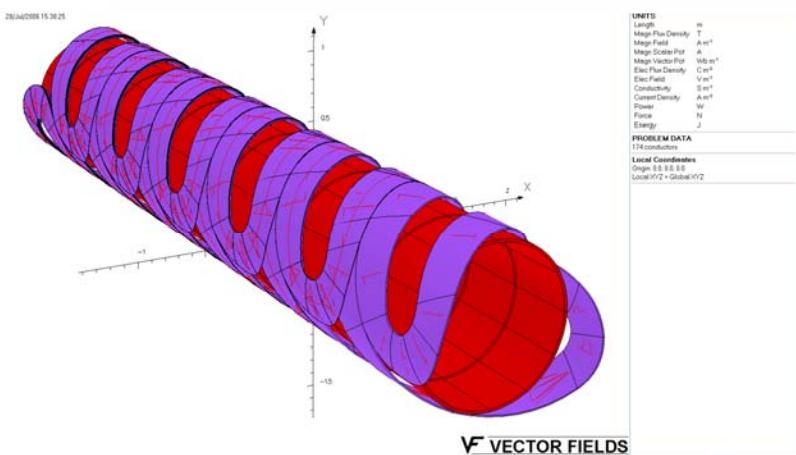


Comments on Helical Cooling channel

- Requires fitting magnets + rf into very tight geometry



Reference Example

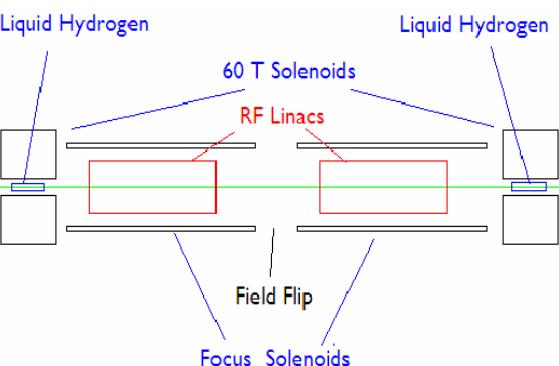
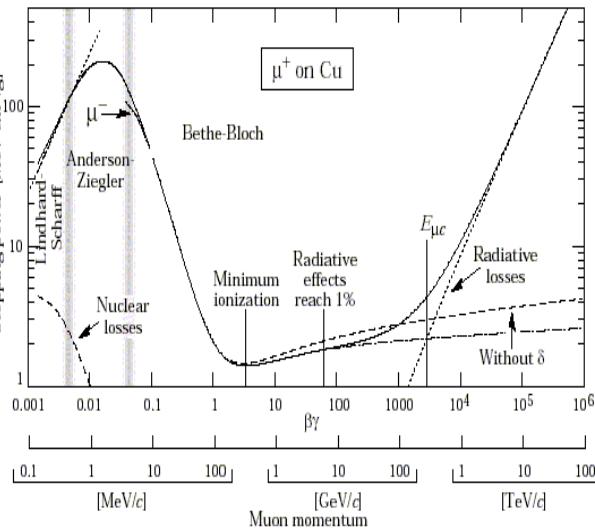


P_μ	200 MeV/c (0.67T-m)
λ	1.0m
a	0.16m
$\kappa = ka = P_\perp / P_z$	1
B	5.5T (Bp/B=0.12m)
$b(a)$	1.28T (Bp/b=0.52m)
$b'(a)$	-0.46T/m
$D(a)$ - dispersion	0.29m
$\Delta_g = 1/\gamma_T^2$	0.9



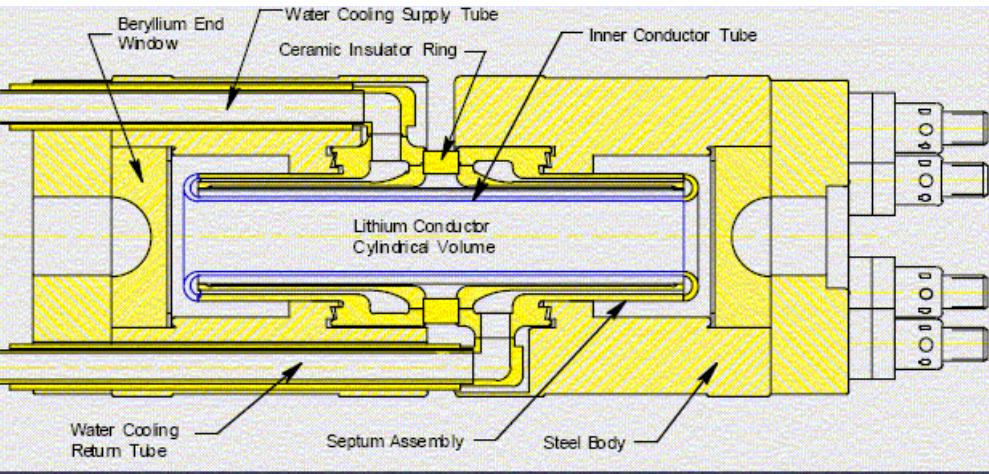
Low-Energy “cooling”=emittance exchange

- dP_μ/ds varies as $\sim 1/\beta^3$
- “Cooling” distance becomes very short
- Focusing can get quite strong:
 - Solenoid: $\beta_\perp \approx \frac{2B\rho}{B} = \frac{2P_\mu}{0.3B}$
 - $\beta_\perp = 0.0013\text{m}$ at $50\text{T}, 10\text{MeV}/c$
- $\varepsilon_{N,\text{eq}} = 1 \times 10^{-6} \text{ m}$ at $10\text{MeV}/c$
 - Small enough for “low-emittance” collider
- But Beam is heated longitudinally
 - ($\varepsilon_{6-\text{D}}$ is \sim constant)



Li-lens cooling

- **Lithium Lens** provides strong-focusing and low-Z absorber in same device
- **Liquid Li-lens** may be needed for highest-field, high rep. rate lens
- **BINP (Silvestrov)** started prototype liquid Li lens for FNAL, but not completed



$$\beta_{\perp} \approx \sqrt{\frac{B\rho}{B'}} = \sqrt{\frac{P_{\mu}}{0.3B'}}$$

μ-Cooling Li lens parameters

B(T)	B' (T/m)	radius(cm)	Length(m)	I (MA)	$\tau(\delta=0.7r)$
10	1000	1	1	0.50	0.25ms
15	3000	0.5	1	0.375	64μs
20	8000	0.25	1	0.25	16μs

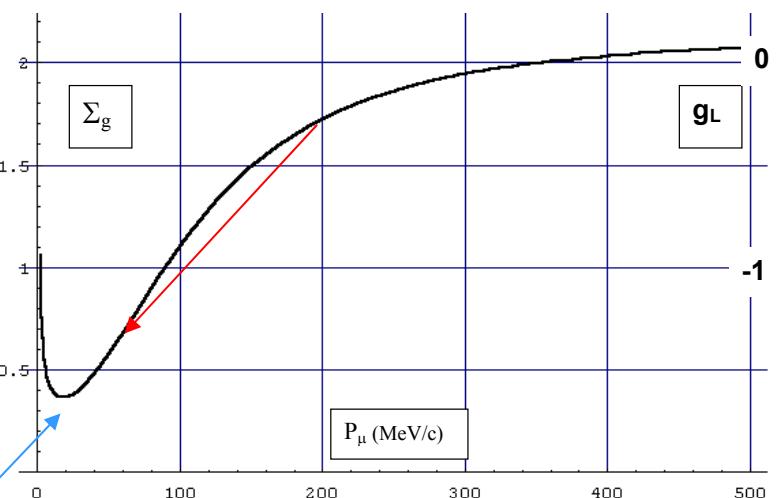
$\beta_{\perp} = 0.026m$ (200 MeV/c, 1000 T/m)

$\beta_{\perp} = 0.004m$ (40 MeV/c, 8000 T/m)



Other applications- not just muons!

- Stopping μ beam
 - (for μ^2e conversion experiment)
 - C. Ankenbrandt et al.,
Muons, Inc.
- For BCNT neutron source
 - Y. Mori - KURRI
- For beta-beam source
 - C. Rubbia et al
 - Nucl. Inst. and Meth. A 568,
475 (2006).
- ...

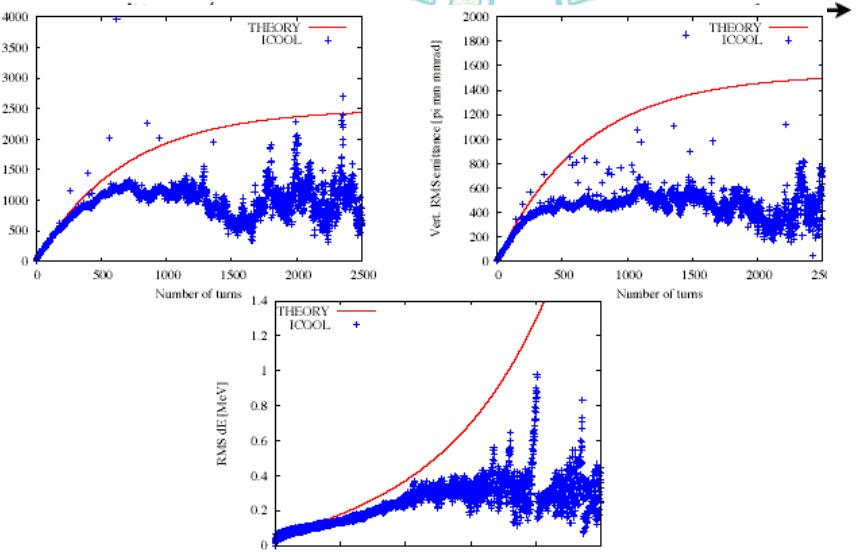
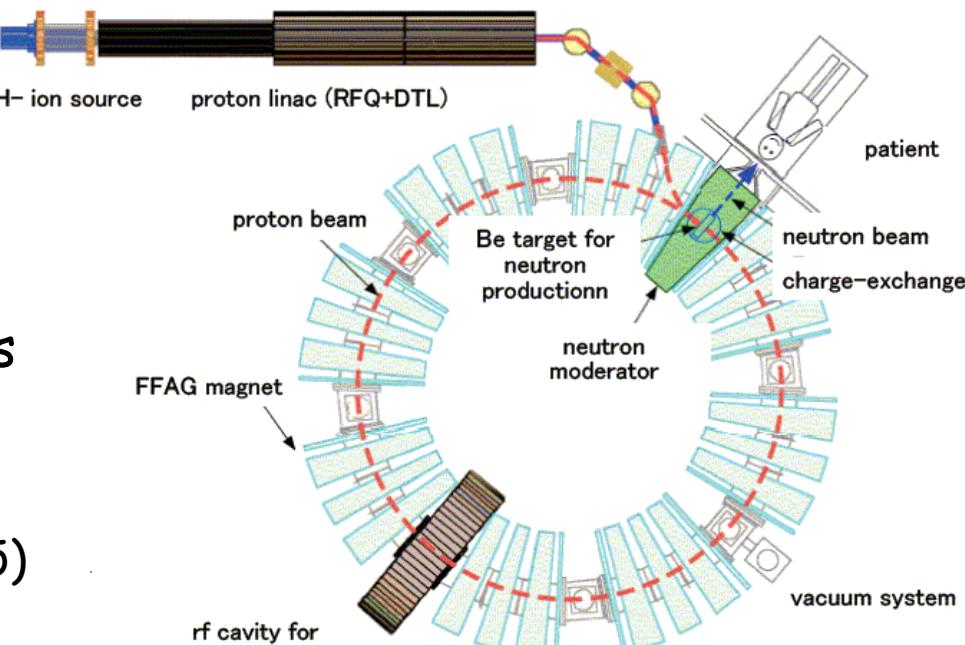




FFAG-ERIT neutron source (Mori, KURRI)



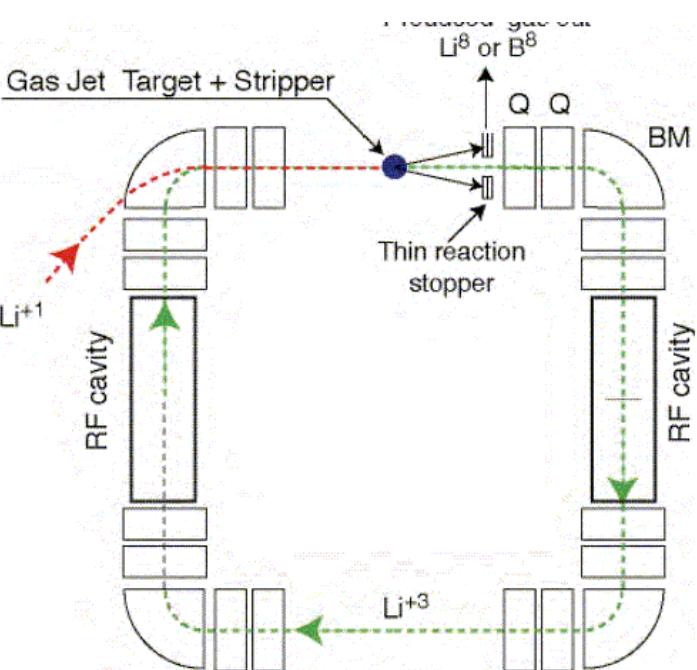
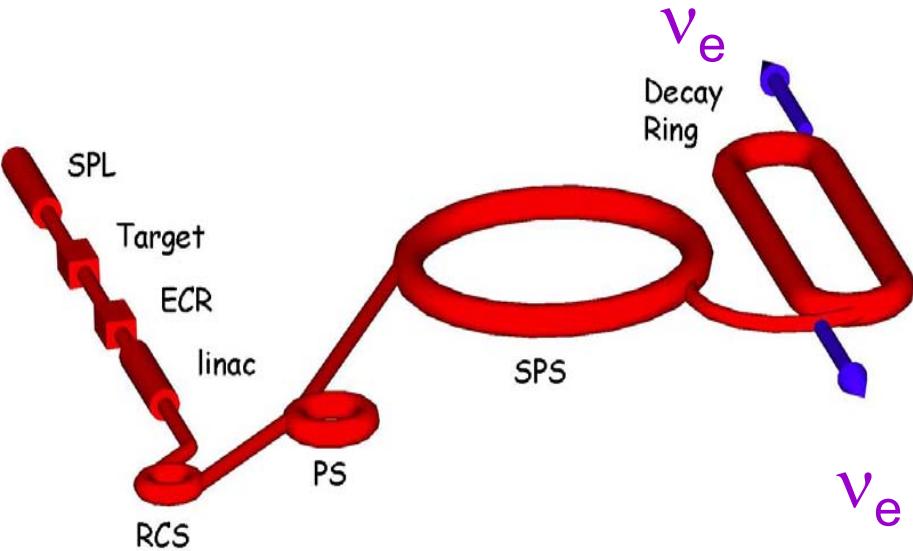
- Ionization cooling of protons/ ions is unattractive because nuclear reaction rate \approx energy-loss cooling rate
- **But** can work if the goal is beam storage to obtain nuclear reactions
 - Absorber is beam target, add rf
- ERIT-P-storage ring to obtain neutron beam (Mori-Okabe, FFAG05)
- **10 MeV protons** ($\beta = v/c = 0.145$)
 - ^{10}Be target for neutrons
 - $5\mu\text{m}$ Be absorber, wedge (possible)
 - $\delta E_p \approx 36 \text{ keV/turn}$
- Increases beam lifetime to ~ 1000 turns (not actually cooling ...)





β -beam Scenario (Rubbia et al.)

- **β -beam** – another ν_e source
 - Produce accelerate, and store unstable nuclei for β -decay
 - Example: ${}^8\text{B} \rightarrow {}^8\text{Be} + \text{e}^+ + \nu$ or ${}^8\text{Li} \rightarrow {}^8\text{Be} + \text{e}^- + \nu^*$
- Source production can use ionization cooling
 - Produce Li and inject at 25 MeV
 - nuclear interaction at gas jet target produces ${}^8\text{Li}$ or ${}^8\text{B}$
 - ${}^6\text{Li} + {}^3\text{He} \rightarrow {}^8\text{B} + \text{p}$
 - Multiturn storage with ionization “cooling” maximizes ion production
 - ${}^8\text{Li}$ or ${}^8\text{B}$ is caught, is ion source for β -beam accelerator
 - Concept needs development





β -beams example: ${}^6\text{Li} + {}^3\text{He} \rightarrow {}^8\text{B} + \text{n}$

- Beam: **25MeV ${}^6\text{Li}^{+++}$**
 - $P_{\text{Li}} = 530 \text{ MeV}/c$ $B\beta = 0.6 \text{ T-m}$; $v/c = 0.094$ $\mathbf{J}_{z,0} = -1.6$
- Absorber: **${}^3\text{He}$**
 - $Z=2, A=3, I=31\text{eV}, z=3, a=6$
 - $dE/ds = 1180 \text{ MeV/gm/cm}^2, L_R = 70.9 \text{ gm/cm}^2$
- If $g_{x,y,z} = 0.13$ ($\Sigma_g = 0.4$), $\beta_\perp = 0.3\text{m}$ at absorber
 - **Must mix both x and y with z**
 - $\varepsilon_{N,\text{eq}} = \sim 0.000046 \text{ m-rad}$,
 - $\sigma_{x,\text{rms}} = \sim 2 \text{ cm}$ at $\beta_\perp = 1\text{m}$
- $\sigma_{E,\text{eq}}$ is $\sim 0.4 \text{ MeV}$
- Promising but many problems ...
- **Better with ${}^3\text{He beam}$, ${}^6\text{Li target}$**
 - D. Neuffer, NIM A 583, p.109 (2008).

$$\varepsilon_{N,\text{eq}} \cong \frac{z^2 \beta_\perp E_s^2}{2 J_x \beta a m_p c^2 L_R \frac{dE_{z,a}}{ds}}$$

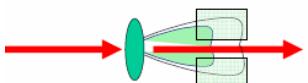
$$\sigma_{E,\text{eq}}^2 = \frac{(m_e c^2)(a m_p c^2) \beta^4 \gamma^3}{2 J_L \ln[] \left(1 - \frac{\beta^2}{2}\right)}$$



Beta beam source

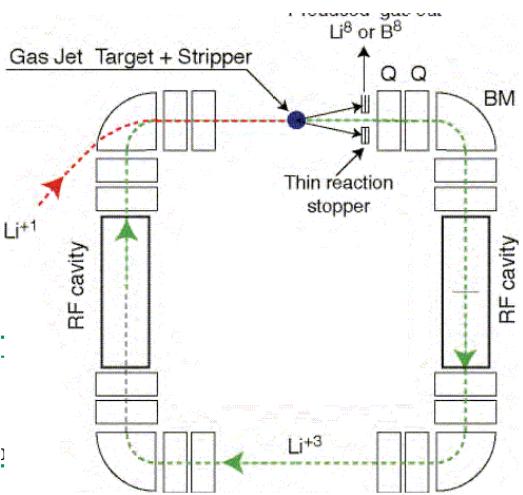
- Key Difficulties

- Gas jet target
- Separation of created from circulating ions
- MW cooling power



- Easier with He-3 beam, Li-6 target

- Liquid Li “waterfall” target
- B-8 more separated from He-3
- 0.5 MW

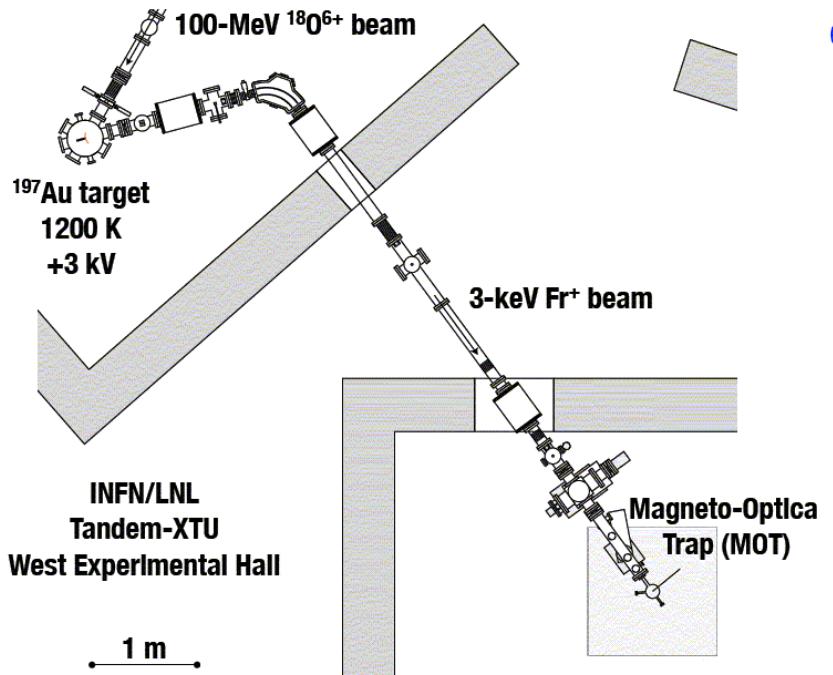


Low energy Ion cooling for B-8 production

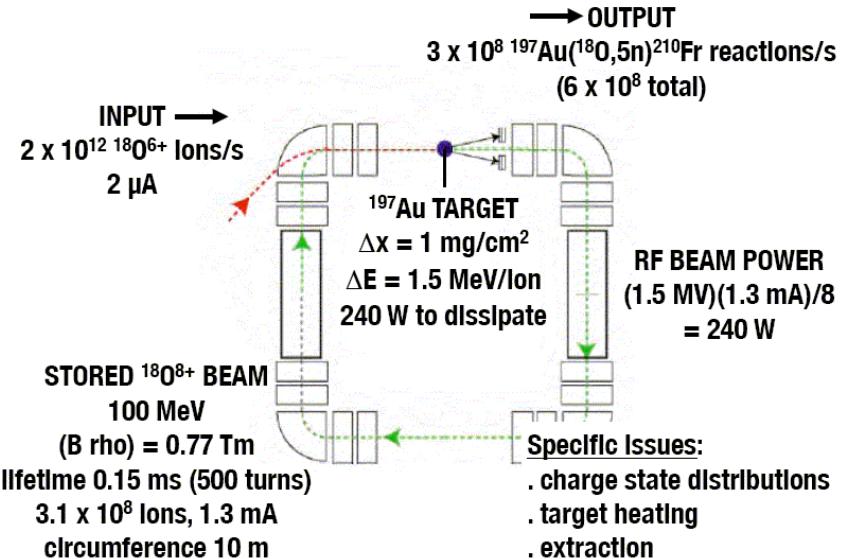
Parameter	Symbol	Reverse Dynamics	Direct Scenario
Beam	${}^3\text{Li}$	${}^3\text{He}$	
Absorber		${}^3\text{He}$	${}^6\text{Li}$
Momentum	P	530 MeV/c	265 MeV/c
Kinetic energy	T_a	25	12.5 MeV
Speed	$\beta = v/c$	0.094	0.094
Absorber density	ρ_{ref} (liquid or solid)	0.09375	0.46
Energy loss	dE/ds	110.6 MeV/cm	170.4 MeV/cm
Radiation Length		756cm	155cm
Betatron functions	β_{\perp}, η	0.3m, 0.3m	0.3m, 0.3m
Rms angle	$\delta\theta_{\text{rms}} (\beta_t = 0.3\text{m})$	$2.25 K_s {}^\circ$	$3.8 K_s {}^\circ$
Rms beam size	σ_t (at $\beta_t = 1\text{m}$)	$2.15 K_s \text{ cm}$	$3.6 K_s \text{ cm}$
Absorber thickness ()	L_{abs} (liquid)	0.018cm	0.00725cm
Characteristic Cooling Length	$(dP/ds/P)^{-1}$	0.45cm	0.147cm
Multiple scattering	$d(\theta^2)/ds$	$8.84 \times 10^{-4} K_s^2/\text{cm}$	$0.0078 K_s^2/\text{cm}$
Energy straggling	$d(\delta E^2)/ds$	0.0886 MeV ² /cm	0.143
Sum of partition numbers.	ΣJ_i	0.4	0.4
Eq. transverse emittance	$\epsilon_{T,N,\text{rms}}$	$4.35 \times 10^{-5} K_s^2 \text{ m}$	$0.000123 K_s^2 \text{ m}$
Equilibrium $\delta P/P$ ($J_z=0.13$)	δ_{rms}	0.0078	
Maximum production angle	θ_{max}	$14.5 {}^\circ$	$30 {}^\circ$



Another possible application: Fr atoms



Could this scheme work for francium?

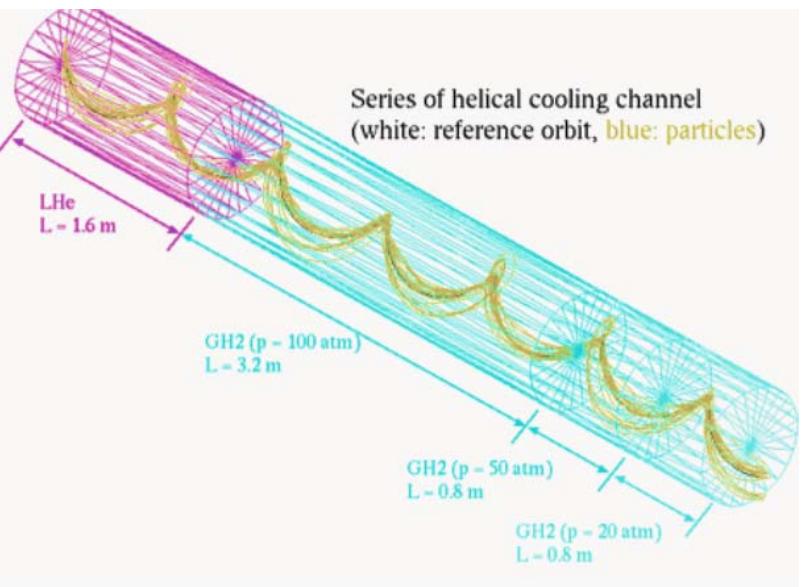
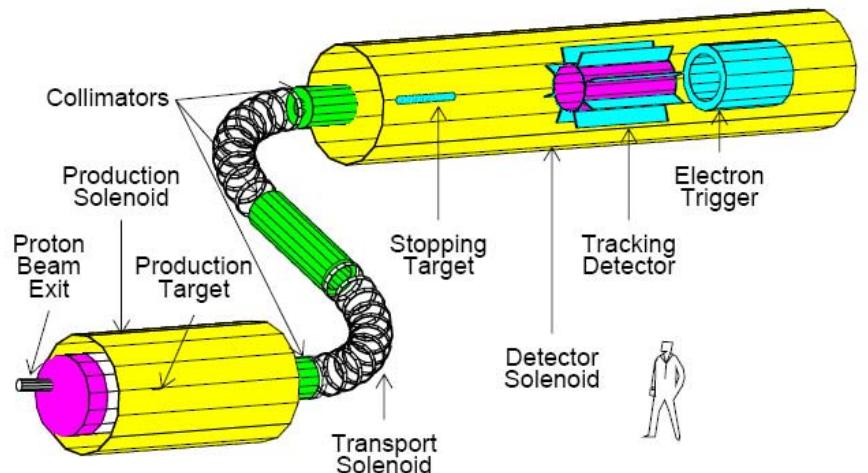


- Francium desirable for atomic parity violation measurements and Electric dipole moment.
- Scantari et al., INFN



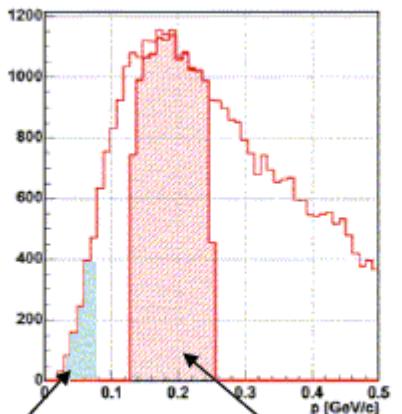
μ 2e experiment ~MECO

MECO

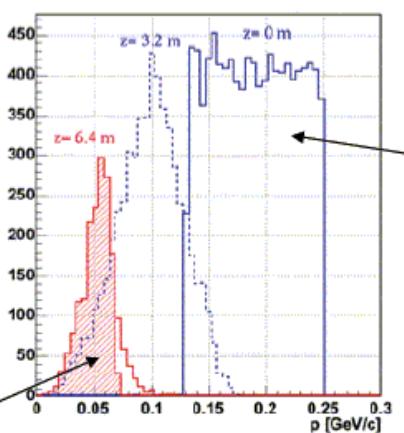


- Mu-E COnversion Experiment
 - $\mu^- + Z \rightarrow e^- + Z$
- Stopped μ^- beam
- Helical energy-loss cooling channel can greatly increase μ intensity
 - C. Ankenbrandt et al., Muons Inc./FNAL

Pi / Mu Production from Target



~ Acceptance of MECO design



Acceptance of HCC



Ionization Cooling

Experimental R&D Program



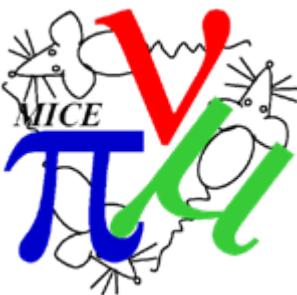
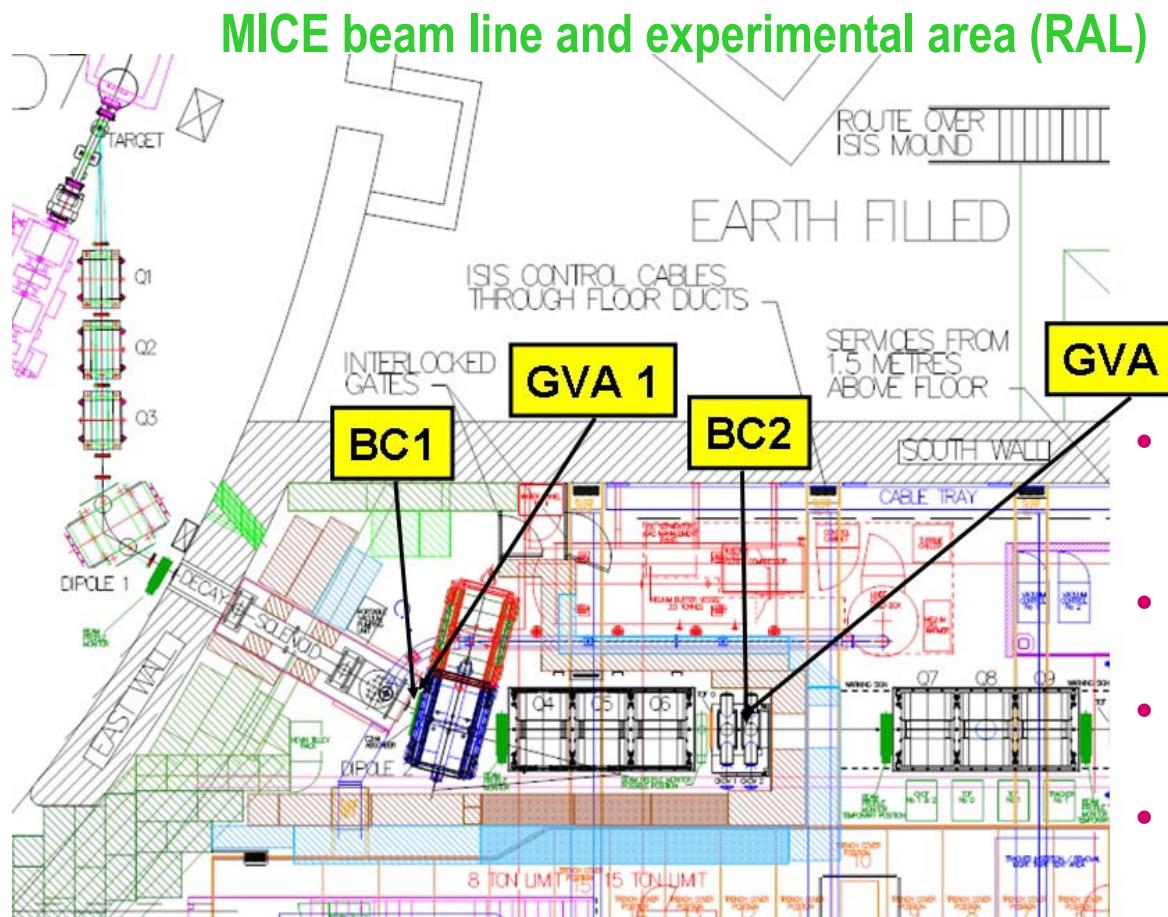
- **MICE** –International Muon Ionization Cooling Experiment
 - μ -beam at RAL ISIS
 - Systems test of complete cooling system
- **MuCOOL** Program
 - Rf, absorber, magnet R&D-supports MICE
 - MuCOOL test area (Fermilab)
 - Muon Collider Task Force
- **MUONS, Inc.** (R. Johnson, et al.)
 - High-pressure rf cavities
 - Helical cooler, Parametric resonance cooler



MICE beam line (ISIS, RAL)

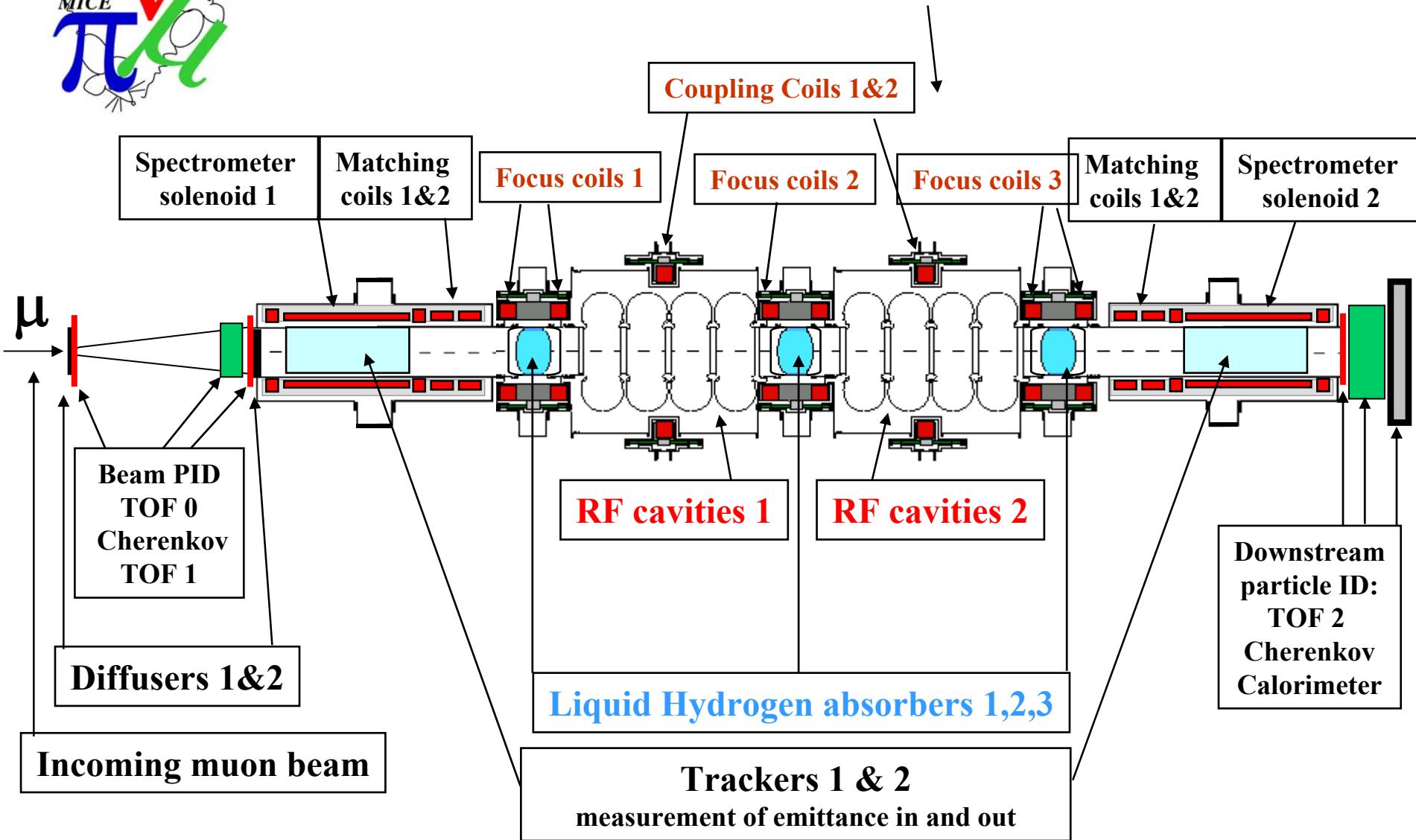
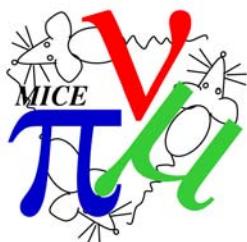


- MICE (International Muon Ionization Cooling Experiment)
- To verify ionization cooling (for a neutrino factory)
with a test of a complete cooling module in a muon beam



- **Target** – Titanium paddle inserted into ISIS beam
- **GVA 1,2** – Scintillating paddles
- **BC1, 2** – Scintillating fiber arrays
- **CKOV 1,2** – Cherenkov threshold detectors to identify pions

10% cooling of 200 MeV muons requires ~ 20 MV of RF
single particle measurements $\Rightarrow \Delta (\varepsilon_{\text{out}} / \varepsilon_{\text{in}}) = 10^{-3}$

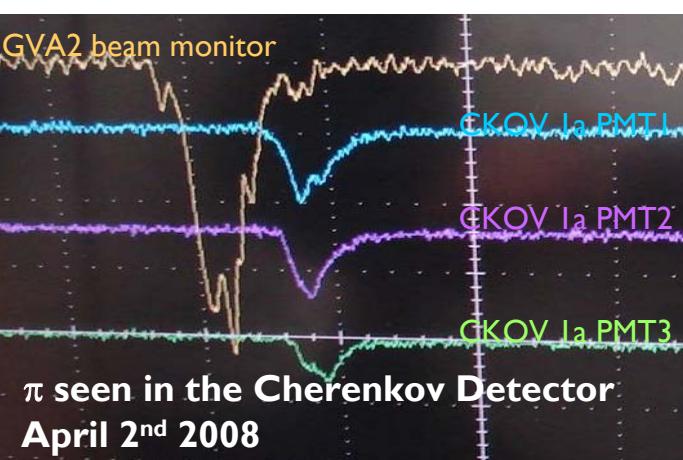
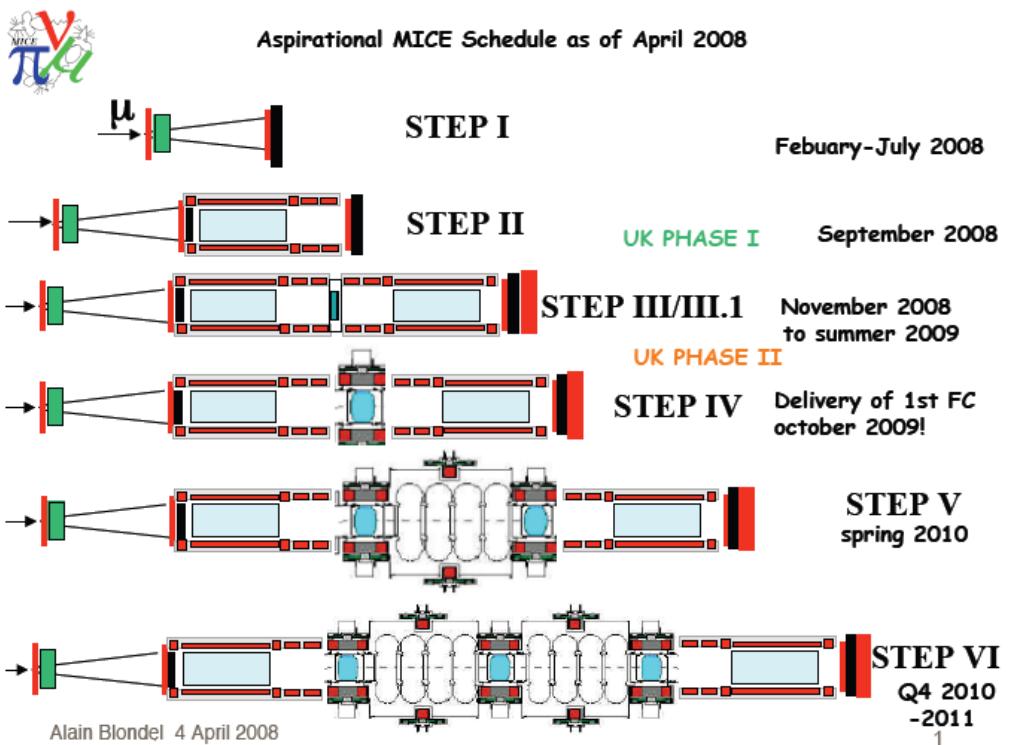
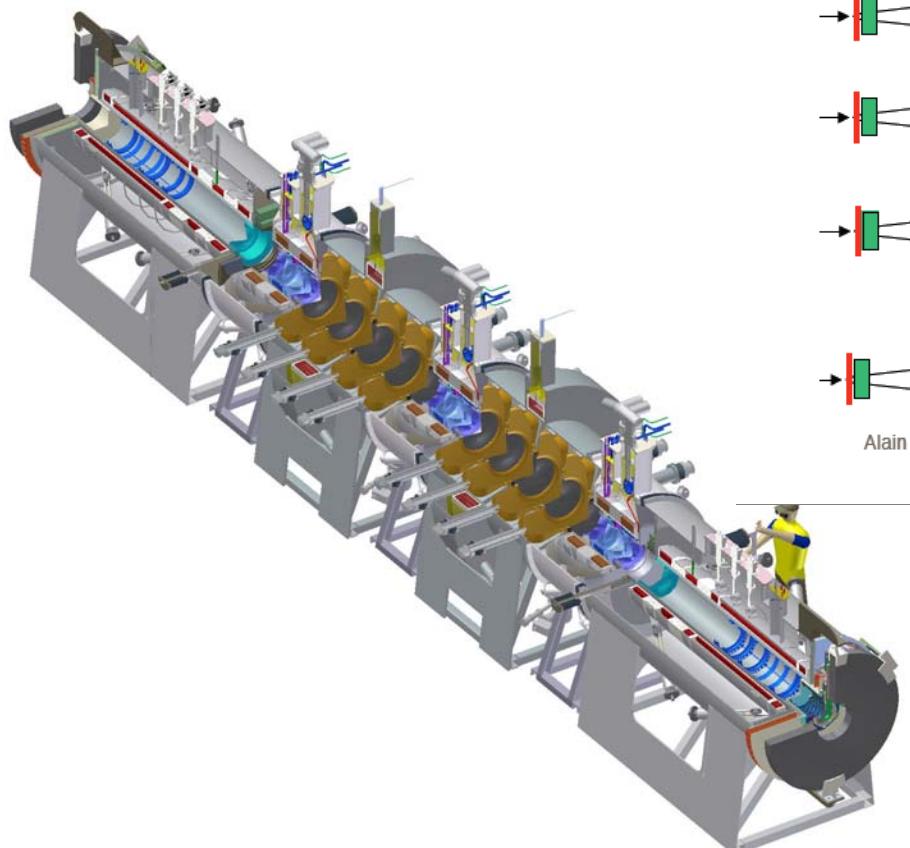




Muon Ionization Cooling Experiment (MICE)



MICE
Measurement of Muon Cooling
Emitance Measurement @ 10^{-3}
First Beam February 2008





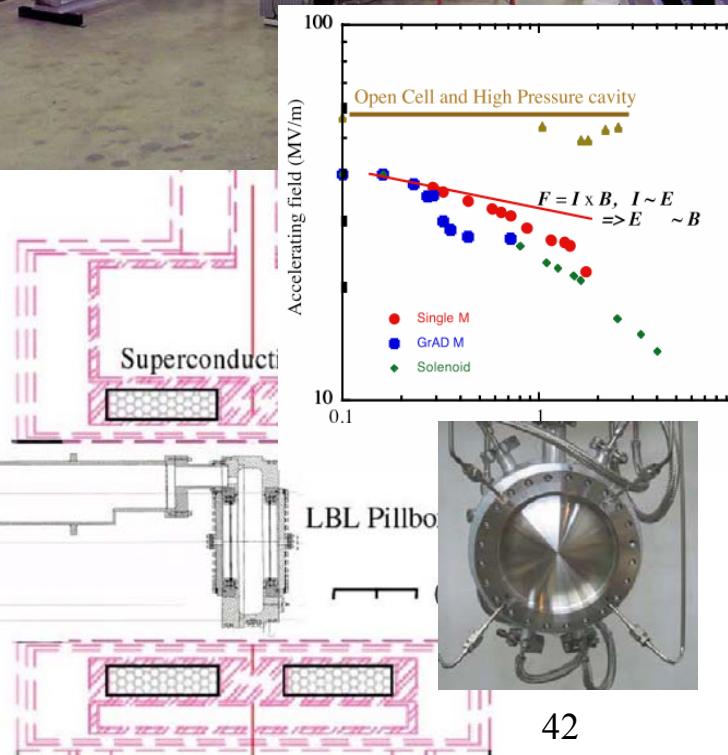
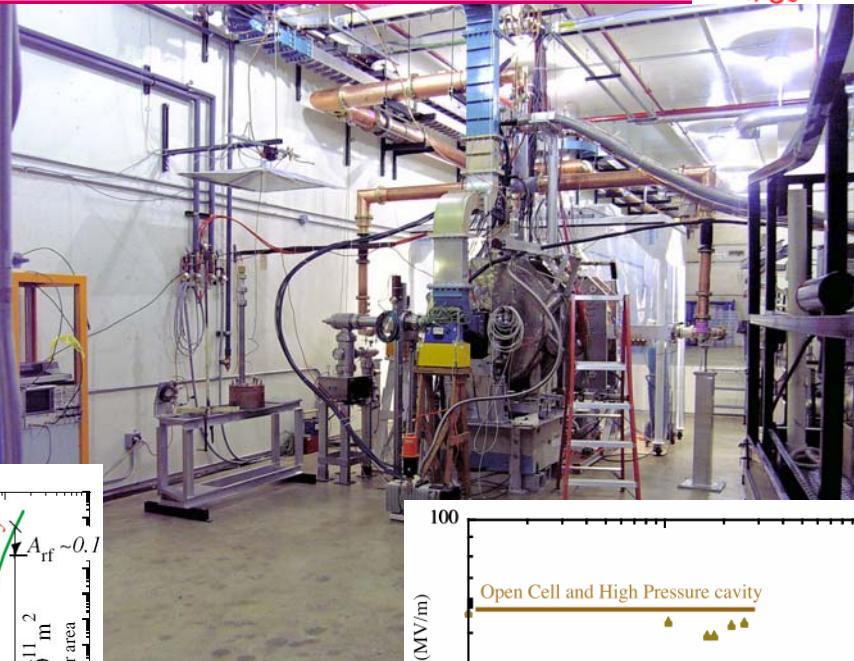
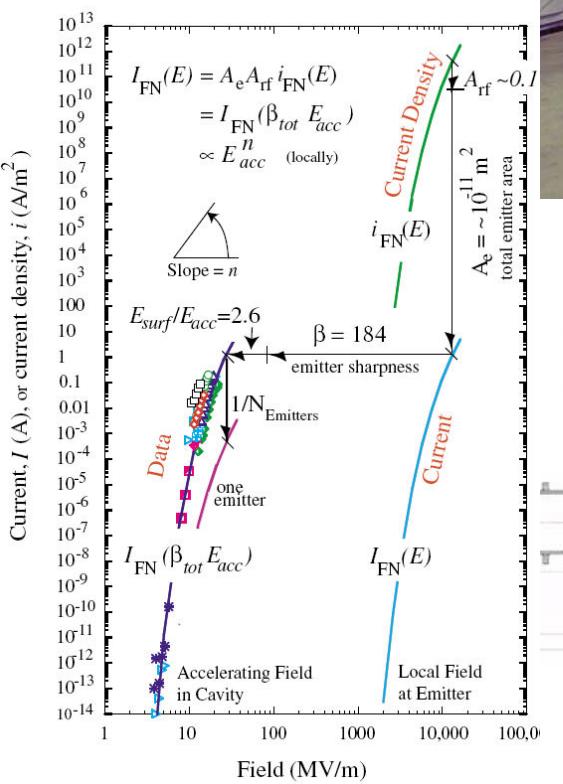
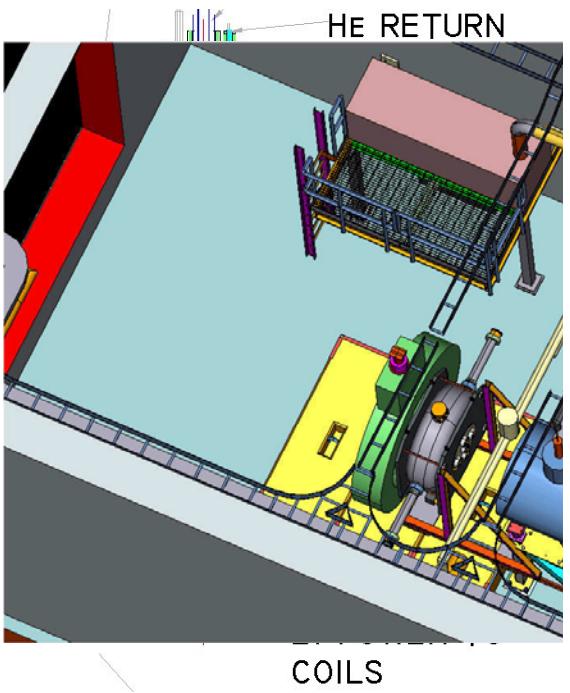
MICE components





MuCOOL-MTA experimental program

- Rf: 805, 201 MHz, gas-filled
 - **201MHz reached 16 MV/m**
 - 805 MHz 3T, gas-cavity test
- H₂ absorbers
- Solenoids





Muons, Inc. Experimental Program

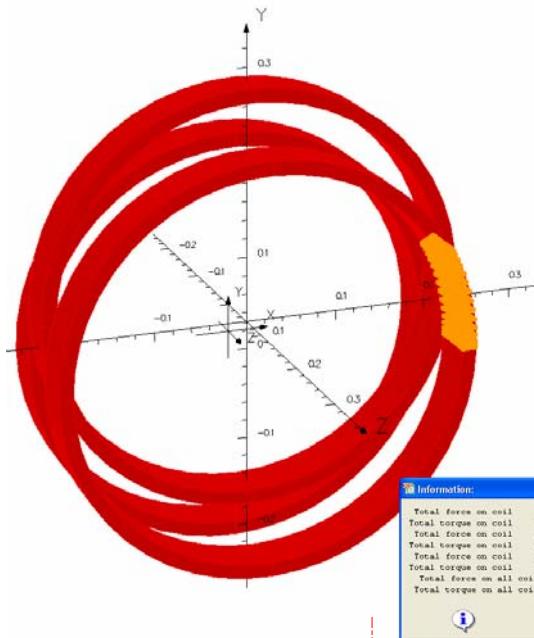
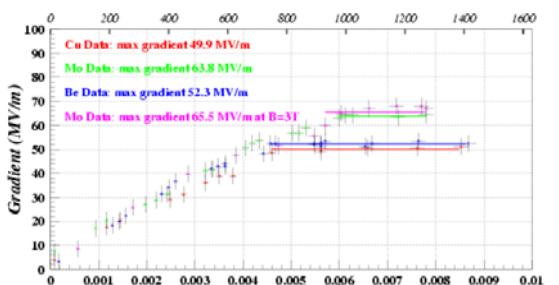


• High Pressure Gas Cavities

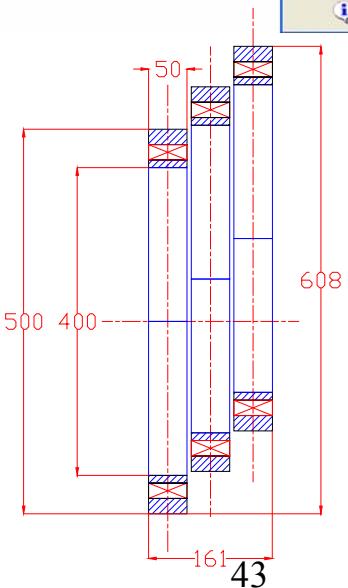
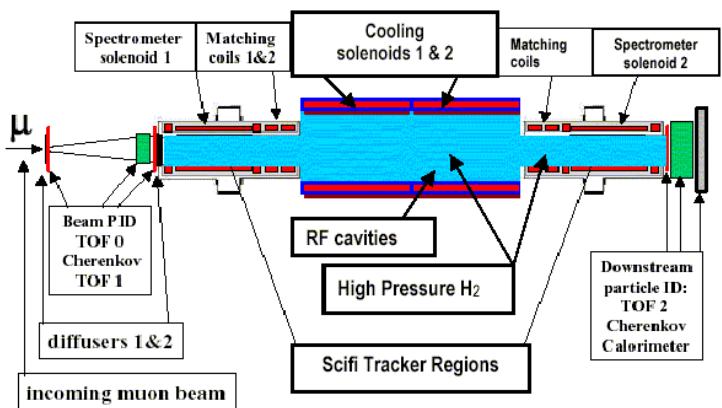
- Absorber and energy recovery in
- Gas limits breakdown, may permit **higher gradients**
- Can operate in magnetic fields?

• Helical Wiggler 6-D cooling

- 3-coil magnet prototype



MANX





Summary

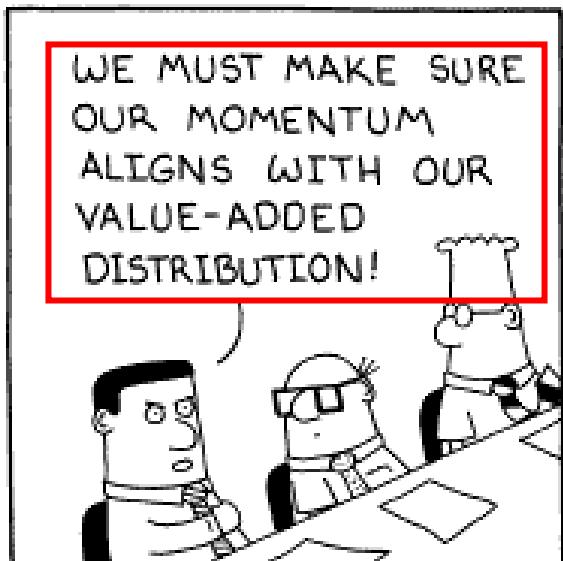
- Cooling for **Neutrino Factory** is practical
 - Components are being built & tested
- **Collider** cooling scenario has made great progress but needs development
 - **Longitudinal cooling** by large factors ...
 - **Transverse cooling** by very large factors
 - **Final beam compression** with emittance exchange

Other Ionization Cooling applications are appearing

Cooling Summary:



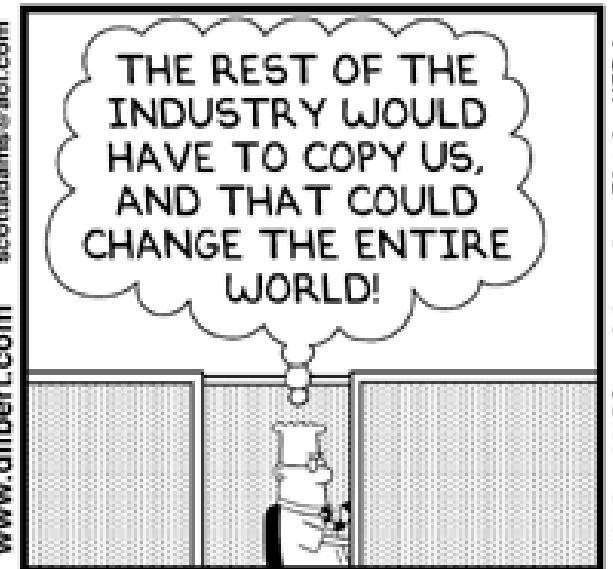
www.dilbert.com



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Summary



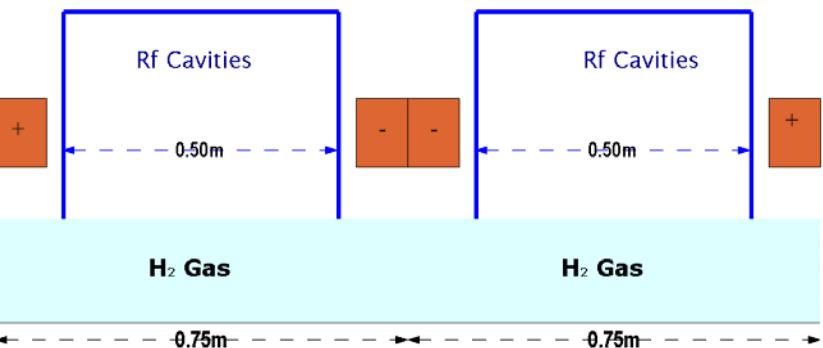
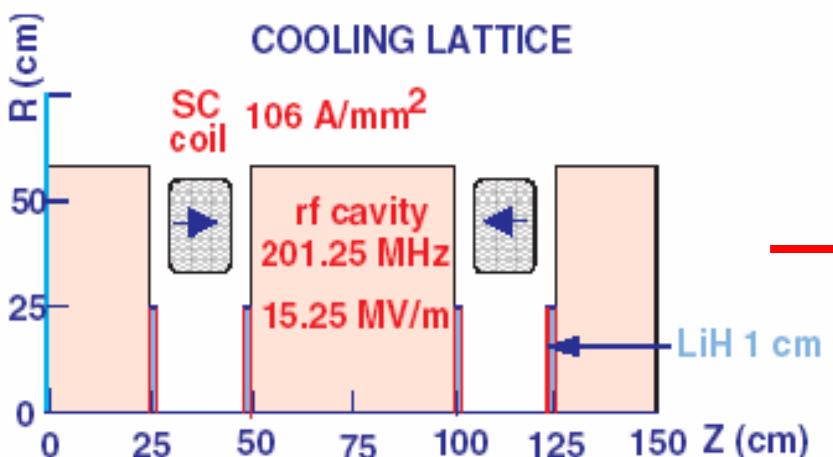
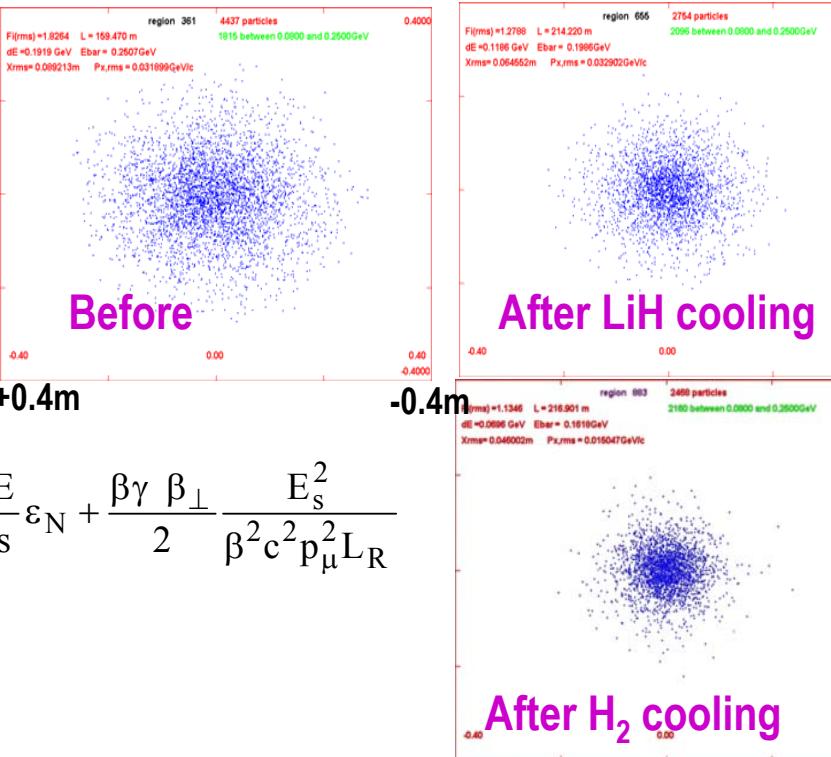
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Variation: ν-Factory Cooling Channel

- Cooling is limited:
 - LiH absorber, $\beta_{\perp} \approx 0.8m$
 - ε_{\perp} from ~0.018 to ~0.0076m in ~80m
 - $\varepsilon_{eq} \approx 0.0056m$
- Could be improved
 - H₂ Absorber (120A) or smaller β_{\perp}
 - $\varepsilon_{\perp} \rightarrow \sim 0.0055$
 - $\varepsilon_{eq} \approx 0.003m$
 - ~20% more in acceptance

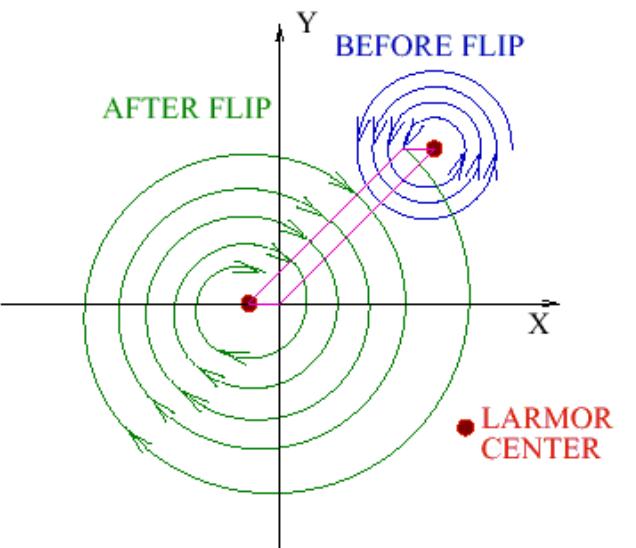
$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta c p_{\mu}} \frac{dE}{ds} \varepsilon_N + \frac{\beta \gamma \beta_{\perp}}{2} \frac{E_s^2}{\beta^2 c^2 p_{\mu}^2 L_R}$$





Solenoidal Focusing and Angular Momentum

- Angular motion with focusing complicates cooling
 - Energy loss in absorbers reduces P_{\perp} , including P_{θ}
- Orbits cool to **Larmor centers**, not $r = 0$



Solution:

Flip magnetic fields; new Larmor center is near $r=0$



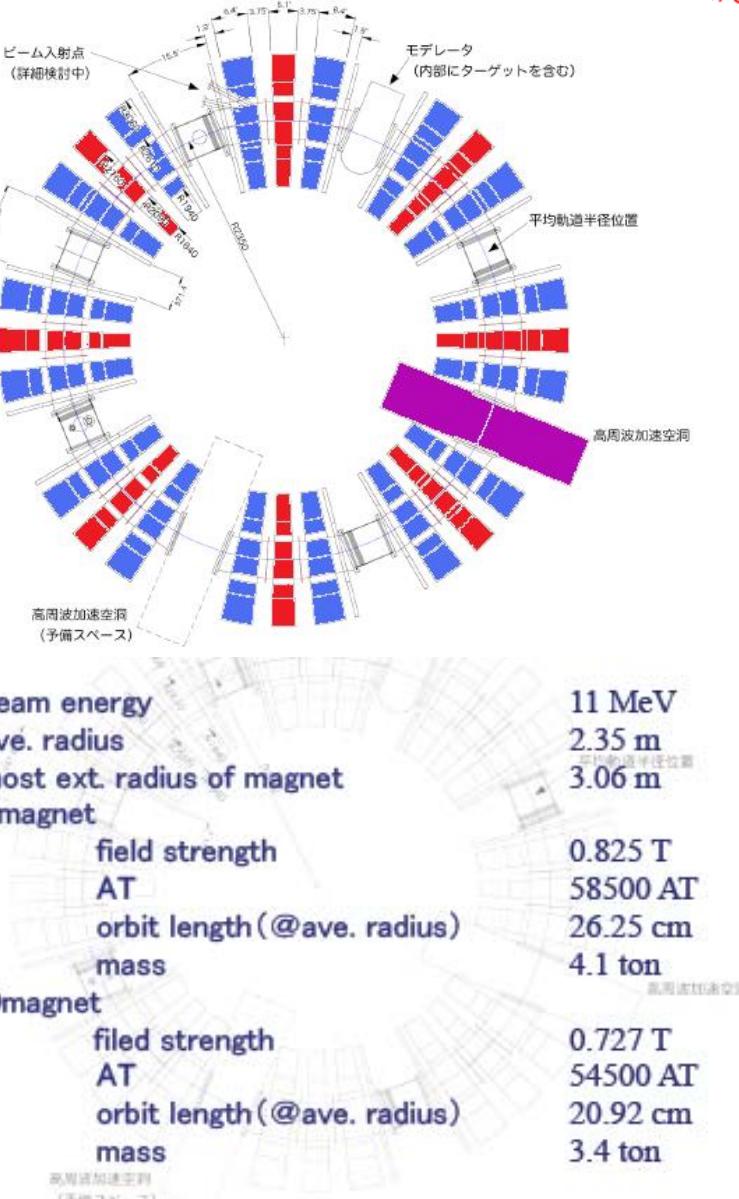
FFAG-ERIT – under construction



With baseline parameters, cannot cool both x and δE

$$\sum J_i \approx 0.36; J_z = -1.6$$

- Optimal x - E exchange could increases storage time from ~ 1000 to 3000 turns
- With x - y - E coupling, can cool 3-D with $g_i = 0.12$
 - Cooling time would be ~ 5000 turns
 - With $\beta_{\perp} = 0.2m$, $\delta E_{rms} = 0.4\text{MeV}$
 $\varepsilon_{\perp,N} = 0.0004\text{m}$ ($x_{rms} = 2.3\text{cm}$)
 - $x_{rms} = 7.3\text{cm}$ at $\beta_{\perp} = 2\text{m}$
- **Construction complete:**
 - **November 2007**
 - First “cooling” demonstration

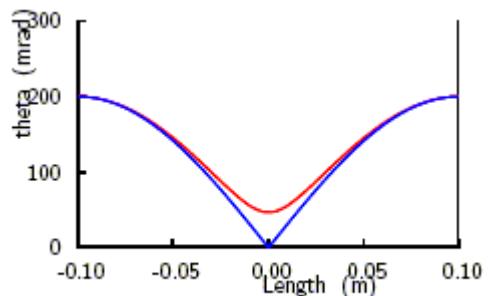
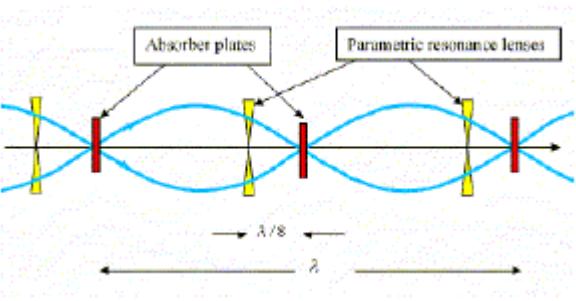


PIC-Parametric-resonance Ionization Cooling

(Y. Derbenev) (also Balbekov, 1997)

Excite $\frac{1}{2}$ integer parametric resonance (in Linac or ring)

- Similar to vertical rigid pendulum or $\frac{1}{2}$ -integer extraction
- Elliptical phase space motion becomes hyperbolic
- Use $xx' = \text{const}$ to reduce x , increase x'
- Use Ionization Cooling to reduce x'



Then:

$$\begin{pmatrix} e^{-\Lambda} & 0 \\ 0 & e^{+\Lambda} \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & e^{-\lambda} \end{pmatrix} \xrightarrow{\text{red arrow}} \begin{pmatrix} e^{-\Lambda} & 0 \\ 0 & 1 \end{pmatrix}$$

