XVIII International Fall Workshop on Geometry and Physics

Benasque, September 6–11, 2009

Mini Courses

Paolo Facchi
Universitá di Bari

Classical and quantum aspects of tomography

In this course I will consider some aspects of tomographic maps.

In the first lecture I will introduce the Radon transform, which is the key mathematical tool for reconstructing the tomographic map of both the Wigner quasidistribution of a quantum state and the probability distribution on the phase space of a classical particle. The original transform was introduced by Radon, who proved that a differentiable function on the 3-dimensional Euclidean space can be determined explicitly by means of its integral over the planes. I will prove the original Radon inversion formula and its generalization to n-dimensional spaces.

In the second lecture I will consider a broader framework and look at the more general problem of expressing a function on a manifold in terms of its integrals over certain submanifolds. This has become an important topic in integral geometry with many applications ranging from partial differential equations, group representations, and X-ray technology. The focus will be on invariant (or equivariant) transformations under some simmetry groups from the space of functions on one geometrical space to the space of functions on another geometrical space.

In the last lecture I will show some possible generalizations of the above picture to the quantum case. A straightforward generalization derives from the phase-space description of quantum mechanics through Wigner quasidistribution functions. We will see how this map can be considered as a specific tomographic version of the star-product quantization.

References:

[1] S. Helgason, *The Radon Transform* (Birkhauser, Boston, 1999)
[2] M.G.A. Paris and J. Rehacek, *Quantum State Estimation*, Lecture Notes in Physics Vol. 649 (Springer, Berlin, 2004)
[3] J. Bertrand and P. Bertrand, Found. Phys. **17** 397 (1987)
[4] G.M. D'Ariano, S. Mancini, V.I. Man'ko and P. Tombesi, Quantum Semiclass. Opt. **8** 1017 (1996)
[5] O.V. Man'ko, V.I. Man'ko and G. Marmo, J. Phys. A: Math. Gen. **35** 699 (2002)

Recommended lectures: As a very agreeable introduction to the problem I suggest the classical book by one of the chief contributors to the modern theory of the Radon transform, Ref. [1].

Peter J. Olver
University of Minnesota

Moving frames, Lie pseudo-groups, and their applications

The course will center on the theory and applications of the new, equivariant approach to the classical method of moving frames. The lecture will begin with the case of finite-dimensional Lie group actions, and then move on to more recent developments for infinite-dimensional pseudo-groups. A variety of applications, including structure theory for pseudo-groups, explicit construction and classification of differential invariants, differential invariant algebras, symmetry groups of differential equations, invariant variational problems, and equivalence problems arising in geometry, computer vision and elsewhere will be presented.

Prerequisites are basic knowledge of differential geometry and Lie groups. The required aspects of jets, contact structures, prolongation, the variational bicomplex and Lie pseudo-groups will be developed during the course of the lectures.

References:

[1] Fels, M., and Olver, P.J., *Moving coframes. II. Regularization and theoretical foundations*, Acta Appl. Math. 55 (1999) 127-208.
[2] Olver, P.J., *A survey of moving frames, in: Computer Algebra and Geometric Algebra with Applications*, Li, H., Olver, P.J., Sommer, G., eds., Lecture Notes in Computer Science, vol. 3519, Springer-Verlag, New York, 2005, pp. 105-138.
[3] Olver, P.J., and Pohjanpelto, J., *Maurer-Cartan forms and the structure of Lie pseudo-groups*, Selecta Math. 11 (2005) 99-126.
[4] Olver, P.J., and Pohjanpelto, J., *Moving frames for Lie pseudo-groups*, Canadian J. Math. 60 (2008) 1336-1386.
[5] Olver, P.J., and Pohjanpelto, J., *Differential invariant algebras of Lie pseudo-groups*, Adv. in Math., to appear.

Invited Talks

Ilka Agricola
Philipps-Universität Marburg

Connections and Dirac operators with torsion

In this survey talk, I will give a general introduction to special geometries carrying a characteristic connection with torsion - for example, contact manifolds, almost Hermitian manifolds, G_2 and Spin(7) manifolds. I will show how the geometry of the manifold is visible in the torsion 3-form of the connection and that this connection, a natural generalization of the Levi-Civita connection, defines an interesting class of Dirac operators that includes Kostant's cubic Dirac operator or the Dolbeault operator. These Dirac operators are in turn the right instrument for studying spinorial field equations on these manifolds, as they occur in superstring theories.

References:

[1] I. Agricola, *The Srní lectures on non-integrable geometries with torsion.* Arch. Math. **42** (2006) 5-84.

Gil Ramos Cavalcanti
Utrech University

Local forms for branes in generalized complex 4 manifolds

Branes in a generic generalized complex 4-manifold can be either points in the complex locus, Lagrangians intersecting the complex locus at a few points and the complex locus itself. A neighborhood of each such brane is determined by data on the brane. Some of these neighborhoods resemble a neighborhood of a Lagrangian manifold in a symplectic manifold while other are related to complex submanifolds and holomorphic Poisson structures. We will present the neighborhood theorems and, if time allows, will give applications of such theorems.

Marius Crainic
Utrech University

Stability of symplectic leaves and normal forms around them

I will start by reminding some of the classical results on stability of leaves of foliations (Reeb-Thurston stability), the slice theorem, the stability of orbits of group actions (Stowe-Hirsch).
Then I will explain the analogous result in Poisson geometry (joint work with R. L. Fernandes) and a normal form theorem around symplectic leaves (generalizing Conn's linearization theorem). If there will be time, I will explain how all these results (in foliation theory, equivariant geometry, Poisson geometry) can be unified using Lie algebroids.

Fernando Falceto
Universidad de Zaragoza

Pre-Poisson branes and Poisson sigma models

We present classical and quantum aspects of Poisson sigma models in manifolds with boundary. We introduce the notion of pre-Poisson submanifold (the analog of pre-symplectic for Poisson geometry) and discuss some of its properties. We show that pre-Poisson submanifolds are the most general boundary conditions for the Poisson sigma model compatible with the symmetries. We also comment on several applications of the model for integration and quantization of Poisson brackets.

Ximo Gual-Arnau
Universitat Jaume I, Castelló

Rotational integral formulae in space forms

Integral formulas obtained from the measure of totally geodesic submanifolds ('affine spaces or planes') intersecting a compact submanifold in a space form, and applications to the theory of geometric probability, are available from appropriate representations of the motion invariant density (measure) of these linear spaces [6]. Prompted by advances in microscopic sampling and measurement techniques, a new branch of stereology, *local stereology*, has been developed during the last decade [1]. On the other hand, there is a remarkable interest in geometric tomography to obtain a correspondence between results concerning projections and those concerning sections through a fixed point *(rotational formulae)*, [2].
Here we derive rotational formulas for the integral of mean curvature of a closed hypersurface in a space form. Particular cases of interest in stereology are considered for $\mathbb{R}^3$, [3, 4, 5].

References:

[1] A. Baddeley and E.B.V. Jensen. *Stereology for Statisticians.* Chapman & Hall/ CRC, 2004.
[2] R.J. Gardner. *Geometric Tomography.* Cambridge University Press, Cambridge, 1995.
[3] X. Gual-Arnau and L.M. Cruz-Orive. *A new expression for the density of totally geodesic submanifolds in space forms, with stereological applications.* Diff. Geom. Appl. 27 (2009), 124-128.
[4] X. Gual-Arnau, L.M. Cruz-Orive and J. J. Nuño-Ballesteros. *A new rotational integral formula for intrinsic volumes in space forms.* Preprint.
[5] E.B.V. Jensen and J. Rataj. *A rotational integral formula for intrinsic volumes.* Adv. Appl. Math. 41 (2008), 530-560.
[6] L. A. Santaló. *Integral Geometry and Geometrical Probability.* Addison-Wesley Reading, Massachusetts, 1976.

Carlos Herdeiro
Universidade do Porto

Minimal Five dimensional supergravities and complex geometries

Five dimensional minimal supergravities have played an important role in recent advances in string theory. In this lecture we shall discuss the relation between timelike solutions of such theories admitting Killing spinors and four dimensional complex geometries. In the ungauged theory (vanishing cosmological constant $\Lambda = 0$) the solutions are determined in terms of a hyper-Kahler base space; in the gauged theory ($\Lambda < 0$) the complex geometry is Kahler; in the de Sitter theory ($\Lambda > 0$) the complex geometry is hyper-Kahler with torsion (HKT). In the latter case some details of the derivation are given, using spinorial geometry techniques. The method for constructing explicit solutions is presented in each case, and some solutions of physical importance briefly discussed. We shall then turn to null solutions in the de Sitter theory. These are defined in terms of a one parameter family of 3-dimensional constrained Einstein-Weyl spaces called Gauduchon-Tod structures. They admit a geodesic, expansion-free, twist-free and shear-free null vector field and therefore are a particular type of Kundt geometry. When the Gauduchon-Tod structure reduces to the 3-sphere, the null vector becomes recurrent, and therefore the holonomy is contained in Sim(3), the maximal proper subgroup of the Lorentz group SO(4, 1). For these geometries, all scalar invariants built from the curvature are constant. Explicit examples will again be presented.

References:

[1] Jai Grover, Jan B. Gutowski, Carlos A.R. Herdeiro, Patrick Meessen, Alberto Palomo-Lozano, Wafic A. Sabra, *Gauduchon-Tod structures, Sim holonomy and De Sitter supergravity.* Accepted in JHEP, `arXiv:0905.3047 [hep-th]`

[2] Jai Grover, Jan B. Gutowski, Carlos A.R. Herdeiro, Wafic Sabra, *Five Dimensional Minimal Supergravities and Four Dimensional Complex Geometries.* AIP Conf.Proc.1122: 129-136, 2009, `arXiv:0901.4066 [hep-th]`

[3] Jai Grover, Jan B. Gutowski, Carlos A.R. Herdeiro, Wafic Sabra, *HKT Geometry and de Sitter Supergravity.* Nucl.Phys.B809: 406-425, 2009, `arXiv:0806.2626 [hep-th]`

[4] Pau Figueras, Carlos A.R. Herdeiro, F. Paccetti Correia, *On a class of 4D Kahler bases and AdS(5) supersymmetric Black Holes.* JHEP 0611:036, 2006, hep-th/0608201

[5] Carlos A.R. Herdeiro, *Spinning deformations of the D1 - D5 system and a geometric resolution of closed timelike curves.* Nucl.Phys.B665: 189-210,2003.

Melvin Leok
University of California, San Diego

Discrete Analogues of the Hamilton-Pontryagin Principle, Dirac Structures, and Dirac Mechanics

Variational integrators, which are obtained by a discrete Hamilton's principle, provide a rich class of geometric structure-preserving numerical methods, and can be viewed as a discrete analogue of Lagrangian mechanics, with an associated discrete symplectic structure and a discrete Noether's theorem. These methods, together with discrete differential geometry, serve as the basis of a systematic program to develop computational geometric mechanics and geometric control theory.

We will describe how discrete Hamiltonian mechanics can be obtained from a corresponding discretization of Hamilton's phase space principle that is valid in the absence of hyperregularity of the Hamiltonian, and further demonstrate that both discrete Lagrangian and Hamiltonian mechanics can be derived from the more general Hamilton- Pontryagin variational principle.

In addition to discussing the variational formulation of Dirac mechanics, we will also describe the discrete Dirac structure that is associated with the flow of Hamilton-Pontryagin integrators. Dirac structures can be viewed as generalizations of symplectic and Poisson structures, and the discrete analogues are obtained by considering the geometry of symplectic maps and their associated generating functions, in a manner analogous to the construction of continuous Dirac structures in terms of the geometry of symplectic vector fields and their associated Hamiltonians.

References:

[1] M. Leok and T. Oshawa, *Discrete Dirac Structures and Variational Discrete Dirac Mechanics*, arXiv:0810.0740v1 [math.SG]

Pablo Mira
Universidad Politécnica de Cartagena

Conformal metrics of constant curvature on planar domains

A conformal Riemannian metric $ds^2 = e^u|dz|^2$ on a planar complex domain $\Omega \subset \mathbb{R}^2 \equiv \mathbb{C}$ has constant curvature $K \in \mathbb{R}$ if, and only if, u verifies the elliptic PDE

$$\Delta u + 2K\, e^u = 0.$$

In addition, the boundary $\partial\Omega$ has constant geodesic curvature $\kappa \in \mathbb{R}$ with respect to the Riemannian metric ds^2 if and only if u satisfies the Neumann condition

$$\frac{\partial u}{\partial \nu} = -2\kappa e^{u/2}$$

along $\partial\Omega$, where here ν stands for the unit conormal of $\partial\Omega$.

These conformal metrics of constant curvature have been studied from a global perspective for many years, and their study has been applied to understand the behaviour of complete surfaces of constant mean curvature in Riemannian and Lorentzian spaces of constant curvature.

In this conference we will classify the Riemannian metrics of constant curvature with constant geodesic curvature on the boundary, in the case that the domain Ω is a disk, a half-plane or a punctured disk. This generalizes many previous results on the same problem. For that we will use, rather than the usual sophisticated techniques of elliptic theory, an alternative approach via complex analysis and differential geometry. These results have been obtained jointly with Jose A. Galvez (see [1])

References:
[1] J.A. Gálvez and P. Mira. *The Liouville equation in a half-plane.* J. Diff. Equations, 2009.

Contributed Talks

Stanislaw Ewert-Krzemieniewski
West Pomeranian University of Technology at Szczecin

Bundle-like metrics on tangent bundle

Let $(M, g, \mathcal{F})$ be a semi-Riemannian manifold with metric g and non-degenerated foliation $\mathcal{F}$: Let

$$TM = \mathcal{D} \bigoplus \mathcal{D}^{\perp}$$

and $D^{\perp}$ be the intrinsic connection on $\mathcal{D}^{\perp}$: A metric g on M is said to be bundle-like for the non-degenerated foliation $\mathcal{F}$ if the induced semi-Riemannian metric on $\mathcal{D}^{\perp}$ is parallel with respect to the intrinsic connection $\mathcal{D}^{\perp}$ (see [1]).

Supposing that a Riemannian metric g on M is bundle-like we shall give a necessary and sufficient condition for a g-natural metric G on TM to be bundle-like. We shall also indicate another bundle-like metrics on TM.

References:
[1] Bejancu, Aurel; Farran, Hani Reda *Foliations and geometric structures.* Mathematics and Its Applications (Springer), 580. Springer, Dordrecht, 2006. x+300 pp. ISBN: 978-1-4020-3719-1; 1-4020-3719-8
[2] Kowalski, Oldrich; Sekizawa, Masami *Natural transformations of Riemannian metrics on manifolds to metrics on tangent bundles. A classification.* Bull. Tokyo Gakugei Univ. (4) 40 (1988), 1–29.
[3] Reinhart, Bruce L. *Foliated manifolds with bundle-like metrics.* Ann. of Math. (2) 69 1959 119–132.

Anna Fino and Adriano Tomassini
Università di Parma

On the Cohomology of Almost Complex Manifolds

Cohomological properties of manifolds has been recently investigated and studied by many authors. A very special situation is represented by *Kähler manifolds*, in view of the Hodge decomposition for the cohomology ring and the consequent theory of topological obstructions. Another special class is given by *Symplectic manifolds* (M^{2n}, ω) satisfying the *Hard Lefschetz Condition* (see e.g. [1]).

In this talk we will study some cohomological properties of almost complex manifolds (see [3]). More precisely, let J be an almost complex structure on a $2n$-dimensional manifold M. Then J acts in a natural way on the bundle of 2-forms $\Omega^2(M)$ so that

$$\Omega^2(M) = \Omega_J^{1,1}(M)_{\mathbb{R}} \oplus (\Omega_J^{2,0}(M) \oplus \Omega_J^{0,2}(M))_{\mathbb{R}}\,.$$

Then, it is natural to consider the cohomology groups $H_J^{1,1}(M)_{\mathbb{R}}$, $H_J^{(2,0),(0,2)}(M)_{\mathbb{R}}$ as the subspaces of $H^2(M,\mathbb{R})$ whose elements are cohomology classes represented by real differential forms of type $(1,1)$, $(2,0)+(0,2)$ respectively.
Following Li and Zhang [5], an almost complex structure J on M is said to be $\mathcal{C}^\infty$*-pure* if $H_J^{1,1}(M)_{\mathbb{R}} \cap H_J^{(2,0),(0,2)}(M)_{\mathbb{R}} = \{0\}$,
$\mathcal{C}^\infty$*-full* if $H^2(M,\mathbb{R}) = H_J^{1,1}(M)_{\mathbb{R}} + H_J^{(2,0),(0,2)}(M)_{\mathbb{R}}$.

A similar definition can be given in terms of currents (see [5]). In general, there is no relation between the two notions.
One motivation in the study of the cohomology spaces $H_J^{1,1}(M)_{\mathbb{R}}$ and $H_J^{(2,0),(0,2)}(M)_{\mathbb{R}}$ on a symplectic manifold (M,ω) is related to the description of the *J-tamed symplectic cone* $\mathcal{K}_J^t(M)$, namely

$$\mathcal{K}_J^t(M) = \{[\omega] \in H^2(M,\mathbb{R}) \mid \omega \text{ is tamed by } J\}$$

In [2], it is showed that if M is a compact 4-dimensional manifold, then any almost complex structure J on M is $\mathcal{C}^\infty$-pure and full.
This is a special situation which just occurs in dimension 4; indeed a 6-dimensional example in [3] provides a compact 6-dimensional manifold endowed with an almost complex structure which is not $\mathcal{C}^\infty$-pure. Furthermore, the relationship between the concepts of $\mathcal{C}^\infty$-pure and full and pure and full almost complex structures is investigated, showing that on a compact symplectic manifold, which satisfies the Hard Lefschetz condition, a $\mathcal{C}^\infty$-pure and full almost complex structure is pure and full. Finally, some families of $\mathcal{C}^\infty$-pure and full almost complex structures are carefully described.

References:

[1] G.R. Cavalcanti, M. Fernández, V. Muñoz, *Symplectic resolutions, Lefschetz property and formality*. Adv. Math. **218** (2008) 576–599.
[2] T. Draghici, T.J. Li, W. Zhang, *Symplectic forms and cohomology decomposition of almost complex 4-manifolds*. arXiv:0812.3680.
[3] A. Fino, A. Tomassini, *On some cohomological properties of almost complex manifolds*. arXiv:0811.4675v2, to appear in J. of Geom. Anal..
[4] A. Fino, A. Tomassini, in preparation.
[5] T.J. Li, W. Zhang, *Comparing tamed and compatible symplectic cones and cohomological properties of almost complex manifolds*. arXiv:0708.2520.

Magdalena Caballero
Universidad de Córdoba

Willmore surfaces in Generalized Robertson-Walker spacetimes and static spacetimes

It is well known the existence of a closed link between Willmore surfaces and elastic curves. Many examples of this relation are:

- Willmore surfaces of revolution in R^3 are obtained from clamped free elastic curves in the hyperbolic plane (M. Barros).
- Willmore surfaces of revolution in L^3 with null axis are generated by clamped free elastic curves in the anti de Sitter plane (M. Barros and M. Ortega).
- Willmore tori in S^5 foliated by fibers of the standard Hopf map are generated by closed elastic curves in CP^2 (4), (M. Barros, O. Garay and D. Singer).

Let (M,g) be a semi-Riemannian manifold and let γ be a nondegenerate curve immersed in (M,g). S^1 is a Willmore surface in $(S^1 \times M, \epsilon dt^2 + f^2 g)$ if and only if γ is a free elastic curve in (M,g) (M. Barros).
The Willmore surfaces appearing in all the previous examples, except for the second one, are invariant under the action of a compact group of transformations. This fact is the key of the proof of those results. We focus on the technique used to obtain Willmore surfaces of revolution in L^3 with null axis. We modify it to obtain a result for Willmore surfaces in 3-dimensional Lorentzian product spaces, from which we get two characterizations of Willmore surfaces invariant under a (not necessarily compact) group of transformations, in terms of elastic curves. The first one in Generalized Robertson-Walker spacetimes of dimension 3 (an extension in dimension 3 of the result given in the last item). The second one is a result for Willmore surfaces in 3-dimensional static spacetimes.

References:

[1] M. Barros, M. Caballero y M. Ortega. Rotational Surfaces in L3 and Solitons in the Nonlinear Sigma Model. Communications in Mathematical Physics. Accepted
[2] M. Caballero, Willmore surfaces in Generalized Robertson-Walker spacetimes and static spacetimes. Preprint

I. Struchiner
University of Utrecht

The Classifying Lie Algebroid of a Geometric Structure

For classes of geometric structures which can be characterized by coframes, we will show how to construct its classifying Lie algebroid, which has the following properties:

1. To each point on the base there corresponds a germ of the geometric structure belonging to the class.
2. Two germs of geometric structures in the class are equivalent if and only if they correspond to the same point on the base of the Lie algebroid.
3. The isotropy Lie algebra at a point is isomorphic to the symmetry Lie algebra of the corresponding geometry.

We will also show that if the classifying Lie algebroid is integrable, then its Lie groupoid can be used to find explicit examples of geometries in the class, and also to obtain global results about their symmetries and moduli space. This is joint work with Prof. Rui Loja Fernandes (Instituto Superior Técnico, Lisboa).

J.C. Marrero, A. de Nicola and E. Padrón
Universidad de La Laguna

Reduction of Poisson-Nijenhuis Lie Algebroids

We will present our results on the reduction of Poisson-Nijenhuis Lie algebroids to Symplectic-Nijenhuis Lie algebroids with nondegenerate Nijenhuis tensor. We generalize the work of Magri and Morosi on the reduction of Poisson-Nijenhuis manifolds.

Antonio Cañete
Universidad de Sevilla

The isoperimetric problem in the plane with a piecewise constant density

In general, the isoperimetric problem in a given surface M seeks for the least-perimeter set in M enclosing a prescribed quantity of area. In the last years this problem has been studied in a density setting, that is, considering a positive function which is used to weight the perimeter and the area in M. We will focus our talk in the case of R^2 endowed with a piecewise constant density, where a remarkable fact occurs: the boundary of an isoperimetric solution may not be a smooth curve, since when this boundary passes through regions with different values of density, a corner is formed according to a certain rule, which is analogous to the Snell refraction rule of Optics and Physics. We will study this problem in some particular cases (densities taking different values inside and outside a ball, or a strip, or with different values in the two half-planes), describing the corresponding isoperimetric solutions. This is part of a joint work with Michele Miranda and Davide Vittone.

Antonio Alarcón and José Antonio Gálvez
Universidad de Murcia

A proper harmonic map from the unit disk to the complex plane

Picard proved that any analytic function from the complex plane $\mathbb{C}$ to $\mathbb{C} - \{z, w\}$ must be constant. This follows from the fact that there is no nonconstant holomorphic functions from $\mathbb{C}$ to the unit disk $\mathbf{D}$. More generally, it is natural to ask wether $\mathbf{D}$ and $\mathbb{C}$ are equivalent under harmonic diffeomorphisms. On the one hand, Heinz proved that there is no harmonic diffeomorphisms from $\mathbf{D}$ to $\mathbb{C}$ with flat metrics. This result was a key step in his proof of the Bernstein Theorem for minimal surfaces in $\mathbb{R}^3$. On the other hand, Collin and Rosenberg constructed harmonic diffeomorphisms from $\mathbb{C}$ to the hyperbolic disk. It is conjectured by Schoen and Yau that there is no proper harmonic maps from $\mathbf{D}$ to $\mathbb{C}$ with flat metrics. This conjecture is related with some problems on minimal surfaces theory. We show a counterexample to this conjecture.

Isabel Fernández
Universidad de Sevilla

Entire maximal graphs with singularities in the Lorentz-Minkowski space $\mathbb{L}^3$

It is well-known that the unique entire maximal (i.e., zero mean curvature) graphs in $\mathbb{L}^3$ are the spacelike planes. Therefore, it is natural to allow the existence of singularities in theses surfaces. In this talk we will deal with entire maximal graphs whose singular set is the *smallest* possible, that is, a finite number of points. Our main goal is to show that the number of maximal graphs on a given circular domain (up to biholomorphisms) is always 2^n, where $n + 1$ is the number of singularities.

A special Talk

Ilka Agricola
Philipps-Universität Marburg

Old and new on the exceptional Lie group G_2

Moreover, we hereby obtain a direct definition of our 14-dimensional simple group which is as elegant as one can wish for. With these words, Friedrich Engel summarized his research on the exceptional Lie group G_2 at a talk on June 11, 1900 at the Royal Saxonian Academy of Science. Indeed, the description initiated by Friedrich Engel and accomplished by his doctoral student Walter Reichel in 1907 is one of remarkable scientific insight. It is at the foundation of many surprising results and developments in modern differential geometry and leads directly to the exceptional role that G_2 is playing in current superstring models. In my talk, I will retrace the discovery of the exceptional group G_2, the history of the completely forgotten mathematician Walter Reichel, and give an introduction to modern developments.

References:
[1] I. Agricola, *Old and New on the exceptional Lie group G_2*- Notices of the AMS **55** (2008), 922–929.

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Posters

Rui Loja Fernandes and Pedro Frejlich
Instituto Superior Tecnico

h-principles in Poisson Geometry

We formulate a sufficient cohomological criterion for a regular almost-Poisson structure on an open manifold to be homotopic to a regular Poisson one.

Ricardo Gallego Torrome
University of Lancaster

There are no Strictly Landsberg Surfaces

We show that there are not pure $\mathcal{C}^5$ regular y-global Landsberg surfaced. The proof is based on the averaged connection associated with the linear Chern's connection and the classification of irreducibles holonomies of torsion-free affine connections. It consists on exausting all the possible cases and showing that in dimension 2 Landsberg condition implies Berwald condition.

Leonardo Colombo
ICMAT-CSIC and Universidad Nacional de La Plata

A Survey of Optimal Control for Underactuated Mechanical Systems in Quasivelocities

Quasivelocities are the components of a mechanical system?s velocity relative to the set of vector fields that span the fibres of the tangent bundle of the configuration space. These vector fields need not be associated with (local) configuration coordinates.

A good example of quasivelocities is the set of components of the body angular velocity of a rigid body rotating about a fixed point. One of the reason for using quasivelocities is that the Euler-Lagrange equations written in generalized coordinates are not effective for analyzing the dynamics of a mechanical systems of interest.

In this poster we use the quasivelocities for the analyzed of the underactuated mechanical control systems.

Ricardo Gallego Torrome
University of Lancaster

Averaged Lorentz Dynamics and Comparison with Standard Lorentz Dynamics

A geometric formulation of the Lorentz law as the differential equations for auto-parallel integral curves of a linear connection ${}^L\nabla$ is presented. We define an associated covariant derivative also denoted by ${}^L\nabla$. Then, we use an averaging procedure to obtain the *averaged connection* and the associated *averaged dynamics*. This *averaged dynamics* is simpler than the original one. We proof also that in the ultra-relativistic limit and for concentrated 1-particle probability distributions, the auto-parallel curves of the averaged connection remain close to the auto-parallel curves of ${}^L\nabla$.

A. Hurtado and C. Rosales
Universidad de Granada

Stability of constant mean curvature surfaces in the sub-Riemannian three-sphere

In this work, we study first and second order minima of the area under volume-constraint in the sub-Riemannian three-sphere.

Rafael M. Rubio Ruiz
Universidad de Córdoba

Uniqueness of constant mean curvature surfaces inmersed in a slab in certain 3-dimensional Robertson-Walker spacetimes and Calabi-Bernstein type problem

Several uniqueness results for the spacelike slices in certain Robertson-Walker spacetimes are proved under boundeness assumptions either on the mean curvature function of the spacelike surface or on the restriction of the time coordinate on the surface when the mean curvature is constant. In the non-parametric case, a uniqueness result and a non-existence one are proved for bounded entire solutions of some constant mean curvature spacelike differential equations.

S. Vilariño, J.C. Marrero, N. Román-Roy and M. Salgado
Uiversidad de Santiago de Compostela

Symmetries, Noether's theorem and reduction in k-cosymplectic field theories

The k-cosymplectic formulation provides us with a very appropriate and simple geometric framework for working with certain kinds of field theories, both in the Lagrangian and the Hamiltonian formalisms. Using this framework, we introduce a class of symmetries for Hamiltonian k-cosymplectic fiels theories, and study their associated conservation laws by means of a suitable generalization of Noether's theorem. For these symmetries, we propose also a geometric reduction procedure which is based on the Marsden-Weinstein reduction theorem.

Antonio Otal and Luis Ugarte
Universidad de Zaragoza

Special metrics and hypo contact structures on product Lie groups

We classify product Lie groups admitting hypo structures with compatible contact form. We evolve such structures according to Conti-Salamon evolution equations to construct Riemannian metrics with holonomy equal to $SU(3)$.

Rafael Ramirez and Natalia Shadovskaia
Universitat Rovira i Virgili

Cartesian approach to constrained mechanical systems with three degree of freedom

In the history of mechanics, there have been two points of view for studying mechanical systems: The Newtonian and the Cartesian. According the Descartes point of view, the motion of mechanical systems is described by the first-order differential equations in the N dimensional configuration space Q. In this paper we develop the Cartesian approach for mechanical systems with three degrees of freedom and with constraint which are linear with respect to velocity. The obtained results we apply to discuss the integrability of the geodesic flows on the surface in the three dimensional Euclidian space and to analyze the integrability of a heavy rigid body in the Suslov and the Veselov cases.

Alexander Shermenev
Russian Academy of Sciences

Wave equation in cylinder coordinates

An interaction of two acoustical waves in a cylinder is studied within quadratic approximation. When the cylinder coordinates are used, the usual perturbation techniques in separation of variables method inevitably lead to a series of overdetermined systems of linear algebraic equations for the unknown coefficients (in contrast with the Cartesian coordinates). However, if we formally introduce a new function satisfying the first system of this series, all these overdetermined systems become compatible (remaining overdetermined) for the special case of the nonlinear acoustical wave equation. Using the new function and quadratic polynomials of the Bessel functions of radius, we explicitly express the coefficients of the resulting harmonics. It gives solutions describing two-waves interaction which are found with the same accuracy as the nonlinear acoustical equation is derived. As a consequence, a general boundary problem can be explicitly solved in these terms.

References:

[1] Shermenev, A. *Nonlinear acoustic waves in tubes* Acta Acustica, vol. 89 (2003) 426–429.
[2] Shermenev, A. *Separation of variables for the nonlinear wave equation in cylindrical coordinates* Physica D: Nonlinear Phenomena, 212:3-4 (2005) pp 205-215.

Cédric Martínez Campos
ICMAT - CSIC
Higher-Order Classical Field Theory
We propose a differential-geometric setting for the dynamics of a higher-order field theory, based on the Skinner and Rusk formalism for mechanics. This approach incorporates aspects of both, the Lagrangian and the Hamiltonian description, since the field equations are formulated using the Lagrangian on a higher-order jet bundle and the canonical multisymplectic form on its affine dual. The result is that we obtain a unique and global intrinsic description of the dynamics.

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Discrete Hamilton-Pontryagin mechanics and generating functions on Lie groupoids
We present a discrete analog of the recently introduced Hamilton- Pontryagin variational principle in Lagrangian mechanics. This unifies two, previously disparate approaches to discrete Lagrangian mechanics: either using the discrete Lagrangian to define a finite version of Hamilton's action principle, or treating it as a symplectic generating function. This is demonstrated for a discrete Lagrangian defined on an arbitrary Lie groupoid; the often encountered special case of the pair groupoid (or Cartesian square) is also given as a worked example.