CLASSICAL AND QUANTUM ASPECTS OF TOMOGRAPHY

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In this course I will consider some aspects of tomographic maps. In the first lecture I will introduce the Radon transform, which is the key mathematical tool for reconstructing the tomographic map of both the Wigner quasidistribution of a quantum state and the probability distribu- tion on the phase space of a classical particle.

The original transform was introduced by Radon, who proved that a differentiable func- tion on the 3-dimensional Euclidean space can be determined explicitly by means of its integral over the planes. I will prove the orig- inal Radon inversion formula and its general- ization to n-dimensional spaces.

In the second lecture I will consider a broader framework and look at the more general prob- lem of expressing a function on a manifold in terms of its integrals over certain submani- folds. This has become an important topic in integral geometry with many applications ranging from partial differential equations, group representations, and X-ray technology.

The focus will be on invariant (or equivari- ant) transformations under some simmetry groups from the space of functions on one ge- ometrical space to the space of functions on another geometrical space.

In the last lecture I will show some possi- ble generalizations of the above picture to the quantum case. A straightforward generaliza- tion derives from the phase-space description of quantum mechanics through Wigner qua- sidistribution functions. We will see how this map can be considered as a specific tomo- graphic version of the star-product quantiza- tion.

References

[1] S. Helgason, The Radon Transform,Birkhauser, Boston, 1999

[2] M.G.A. Paris and J. Rehacek, Quantum State Estimation, Lecture Notes in Physics Vol. 649 (Springer, Berlin, 2004

[3] J. Bertrand and P. Bertrand, Found. Phys. 17 397 (1987)

[4] G.M. DAriano, S. Mancini, V.I. Manko and P. Tombesi, Quantum Semiclass. Opt. 8 1017 (1996)

[5] O.V. Manko, V.I. Manko and G. Marmo, J. Phys. A: Math. Gen. 35 699 (2002)