Blockade and superdrag in exciton-condensate Josephson junctions

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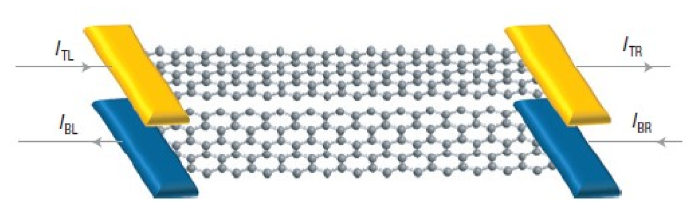
Abstract

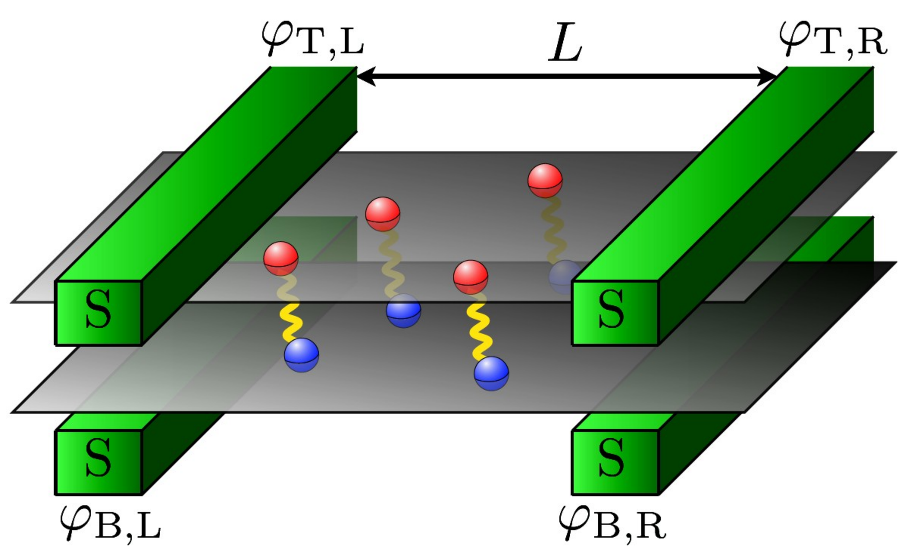
Boson and Fermion-pair condensation are the most remarkable phenomena in statistical physics because they promote quantum behaviour from the microscopic to the macroscopic world. Here we examine the conversion of charged supercurrents of superconductors (Cooper pair condensates) into neutral supercurrents of electron-hole pair (exciton) condensates. We show that perfect conversion is possible via a new pair Andreev-like scattering mechanism. Because neutral currents are protected from electrical noise, this property could have important applications in the design of coherent electronic devices.

Exciton condensate in bilayers 1

- Exciton condensates (ECs) are ordered states of a solid in which macroscopic phase coherence is established between electrons and holes in different bands[1,2].
- Spontaneous coherence between separate two-dimensional electron layers has been reported in quantum Hall bilayers[3,4].
- When the two layers are contacted separately[3-6], bilayer ECs can exhibit transport anomalies associated with counterflow supercurrents[7].



EC Josephson junctions 2



$$L \gg \frac{\hbar v_F}{|\Gamma|}$$

- The system is a Superconductor-EC-Superconductor (S-EC-S) structure: two closely-spaced layers, assumed to host an EC, independently contacted to four superconducting electrodes.

Order parameters:

superconductor $\quad \Delta \propto \langle \Psi_{T/B\uparrow} \Psi_{T/B\downarrow} \rangle$

exciton $\quad \Gamma \propto \langle \Psi^\dagger_{B\sigma} \Psi_{T\sigma} \rangle$

- Independent phase bias is applied to the top and bottom contacts. In the presence of these biases, Josephson currents flow through the double layer. Below we show that, since the EC is gapped, when L is long only dissipationless condensate current can contribute to the Josephson current. The EC and the dissipationless nature of its counterflow supercurrent can therefore be revealed by a purely coherent equilibrium measurement when contacted by superconducting electrodes.

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Model 3

Mean field Hamiltonian

$$\hat{\mathcal{H}} = \int_{-\infty}^{\infty} dx\ \hat{\Psi}^\dagger(x)\mathcal{H}(x)\hat{\Psi}(x)$$

$$\hat{\Psi} = (\Psi_{T\uparrow}, \Psi_{B\uparrow}, \Psi^\dagger_{T\downarrow}, \Psi^\dagger_{B\downarrow})$$

$$\mathcal{H}(x) = \begin{pmatrix} -\frac{\hbar^2\partial_x^2}{2m} & \Gamma(x) & \Delta_T(x) & 0 \\ \Gamma^*(x) & \frac{\hbar^2\partial_x^2}{2m} & 0 & \Delta_B(x) \\ \Delta_T^*(x) & 0 & \frac{\hbar^2\partial_x^2}{2m} & -\Gamma^*(x) \\ 0 & \Delta_B^*(x) & -\Gamma(x) & -\frac{\hbar^2\partial_x^2}{2m} \end{pmatrix}$$

In the limit of infinite Δ, the presence of the superconductors can be accounted for by boundary conditions:

$$\Psi_{T\downarrow(\uparrow)+}(0) = \pm i\ e^{i\varphi_{T,L}} \Psi^\dagger_{T\uparrow(\uparrow)-}(0)$$

$$\Psi_{B\downarrow(\uparrow)+}(0) = \mp i\ e^{i\varphi_{B,L}} \Psi^\dagger_{B\downarrow(\uparrow)-}(0)$$

- Current conservation implies linear phase variation for Γ

$$\Gamma(x) = |\Gamma| e^{i\gamma_0 + 2iqx}$$

Supercurrent 4

When phase biases are applied to the four electrodes, supercurrents flow in both layers. The EC weak-link supports two contributions to the Josephson current:

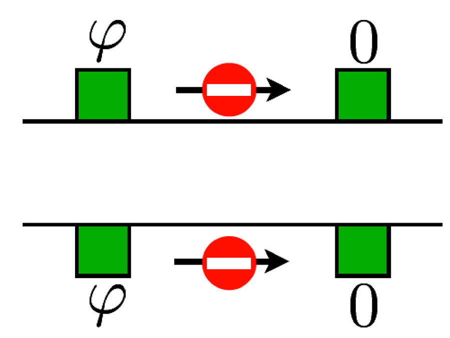
- The quasiparticle channel contribution, in which Cooper pairs propagate by the virtual quasiparticles in the double layer. This is present in ordinary weak links. If, however, L is large such a contribution is exponentially suppressed because of the gap in the quasiparticle excitation spectrum of the EC.
- Conversion of supercurrent into superfluid excitonic current. It can be visualized as a correlated Andreev reflection in which an electron and hole (in different layers) enter the EC and propagate without dissipation to the other end of the double layer. There a similar process occurs to convert the exciton current back into a Cooper-pair current. This process survives also in the long-junction limit.

For $|\Delta| \gg |\Gamma| \gg \hbar v_F/L$
at zero temperature, the supercurrents in top and bottom layers flow in opposite directions and exhibit a sawtooth form:

$$I^{(0)}_{T/B} = \pm \frac{e v_F}{2\pi L}(\varphi_T - \varphi_B)$$

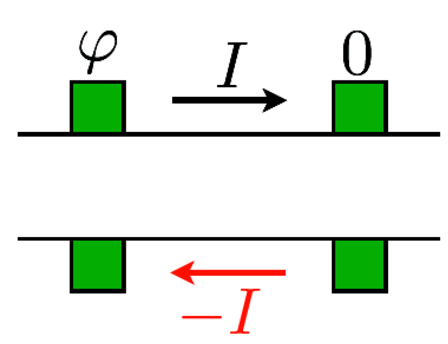
where $\varphi_\alpha \equiv \varphi_{\alpha,L} - \varphi_{\alpha,R}$ is defined modulo 2π

- The magnitude depends only on the difference ($\varphi_T - \varphi_B$)
- Parallel flow ($\varphi_T = \varphi_B$): "exciton blockade" of Josephson current



- Counterflow ($\varphi_T = -\varphi_B$): maximal Josephson current, equal to the one for a ballistic SNS junction

- Superdrag ($\varphi_T = \varphi$, $\varphi_B = 0$): when current flows in one layer due to a phase bias in that layer, a current equal in magnitude but opposite in direction flows in the other layer. This is a consequence of perfect conversion of exciton current into supercurrent.



Finite temperature 5

In the regime: $\begin{cases} \hbar v_F/L \ll k_B T \ll |\Gamma| \\ \hbar v_F q \ll |\Gamma| \end{cases}$

The existence of a dissipationless (counterflow) channel also has a spectacular impact on the temperature dependence of the critical current:

$$I_{T/B} = \pm \frac{2 e v_F}{\pi}\ q \left[1 - \sqrt{2\pi\beta|\Gamma|}\ \frac{\sinh(q L_{th})}{q L_{th}}\ e^{-\beta|\Gamma|}\right]$$

- Thermal fluctuations are dominated by the excitonic gap, the ground-state current is essentially unaffected by thermal fluctuations
- This occurs even when

$$L_{th} \ll L \qquad L_{th} = \hbar v_F/(k_B T)$$

- For decoupled layers the critical current is exponentially suppressed

Andreev processes coherently occurring in the two layers transform Cooper pairs into electron-hole pairs of the EC, which are protected from thermal decoherence by the excitonic gap.

The EC counterflow channel is responsible for an *exponential enhancement* of the critical current.

Discussion 6

The sawtooth dependence on ($\varphi_T - \varphi_B$) is general, valid beyond 1D, as long as L is much larger than the EC coherence length:

- the supercurrent in the bulk is purely carried by the EC and is proportional to q
- by applying the gauge transformation, the superconducting phase biases can be gauged away:

$$\Psi_{\alpha\sigma}(r) = e^{i[\varphi_{\alpha,L} + (\varphi_{\alpha,R} - \varphi_{\alpha,L})x/L]/2}\ \widetilde{\Psi}_{\alpha\sigma}(r)$$

As a result, an effective vector potential appears in each intra-layer Hamiltonian and the EC order parameter transforms into

$$\widetilde{\Gamma}(x) = |\Gamma| \exp\left[2i\left(q - \frac{\varphi_T - \varphi_B}{4L}\right)x\right]$$

Energy minimization implies that the quantity in round brackets in the argument of the exponent vanishes, since in the new gauge the system is effectively phase unbiased. This fixes

$$q = \frac{\varphi_T - \varphi_B - 2\pi J}{4L}$$

The current-phase relationship is always of the sawtooth form, independently of the details of the experimental setup.

For a 2D setup the expression for the current must simply be multiplied by the number of open transverse channels.

Conclusions

- We have proposed a new mechanism for dissipationless transport associated to the conversion of the EC neutral supercurrent into Cooper-pair supercurrent.
- The setup to observe this effect is a double-layer EC, each layer being separately connected to two superconducting electrodes.
- The supercurrent depends only on the difference between the superconducting phase biases in the two layers.

Graphene double layers seem to be a promising candidate for the realization of the phenomena described in view of recent observations:

- Josephson effect with graphene-sheet weak links[8-12]
- electrically isolated double-layers graphene sheets[13]
- on-going efforts to achieve EC in double-layer graphene[14-16] (expected due to: higher carrier density, weaker dielectric screening, graphene bands being nearly perfectly particle-hole symmetric)