Massive Dirac fermions in single-layer graphene

D. V. Khveshchenko

UNC-Chapel Hill

Benasque, 29/07/09

Outline

- Effects of Coulomb and e-ph interactions on Dirac fermions

PRL 87, 246802 (2001);
NPB 642, 515 (2004);
PRB 73, 115104 (2006) ;
PRB 74, 161402(R) (2006);
J. Phys.: Condens. Matter, 21, 075303 (2009).

- Effects of magnetic fields on (interacting) Dirac fermions

PRL 87, 206401 (2001);
PRB 75, 153405 (2007)

- Effects of (long-range-correlated) disorder

PRL 96, 027004 (2006);
PRB 75, 241406 (R) (2007);
EuroPhys.Lett. 82, 57008 (2008)

Motivation

•Does graphene remain a (chiral) Fermi liquid down to zero doping…or else?

•What are the effects of genuine long-range (unscreened) Coulomb interactions?

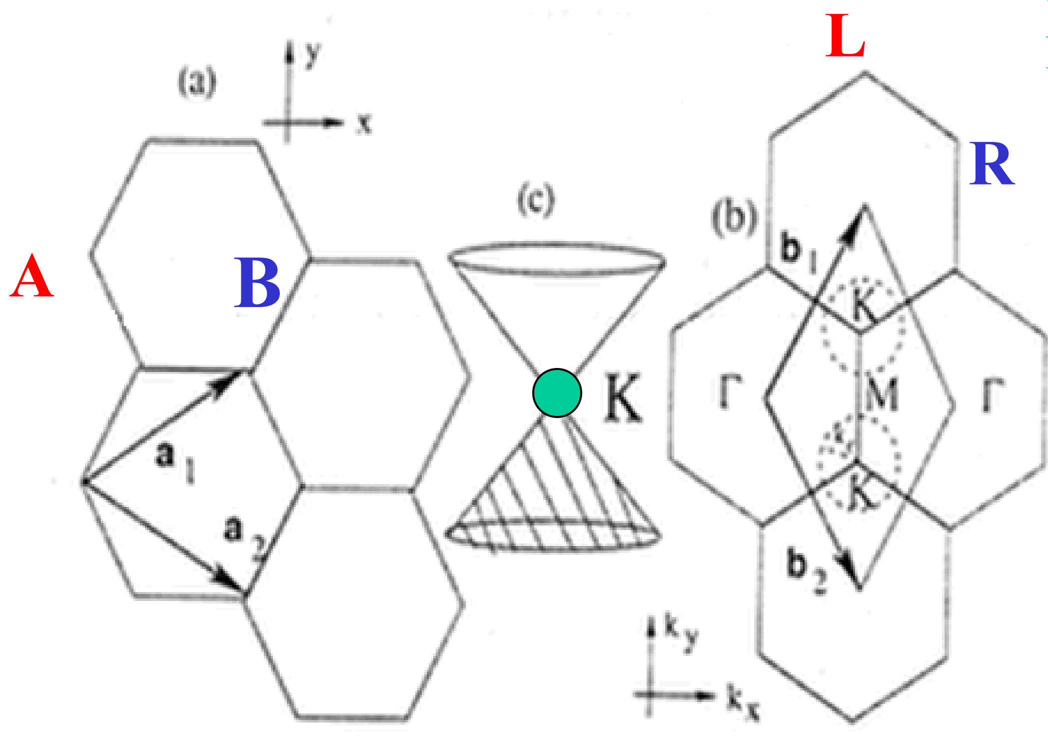
•What is the effect of magnetic field in the presence of long-range Coulomb interactions?

•What are the effects of physically relevant (long-range-correlated) disorder?

Massless Dirac fermions in graphene

•Nodal quasiparticle excitations

P.R. Wallace, ’47
G. W. Semenoff, ’84;
E. Fradkin, ’86



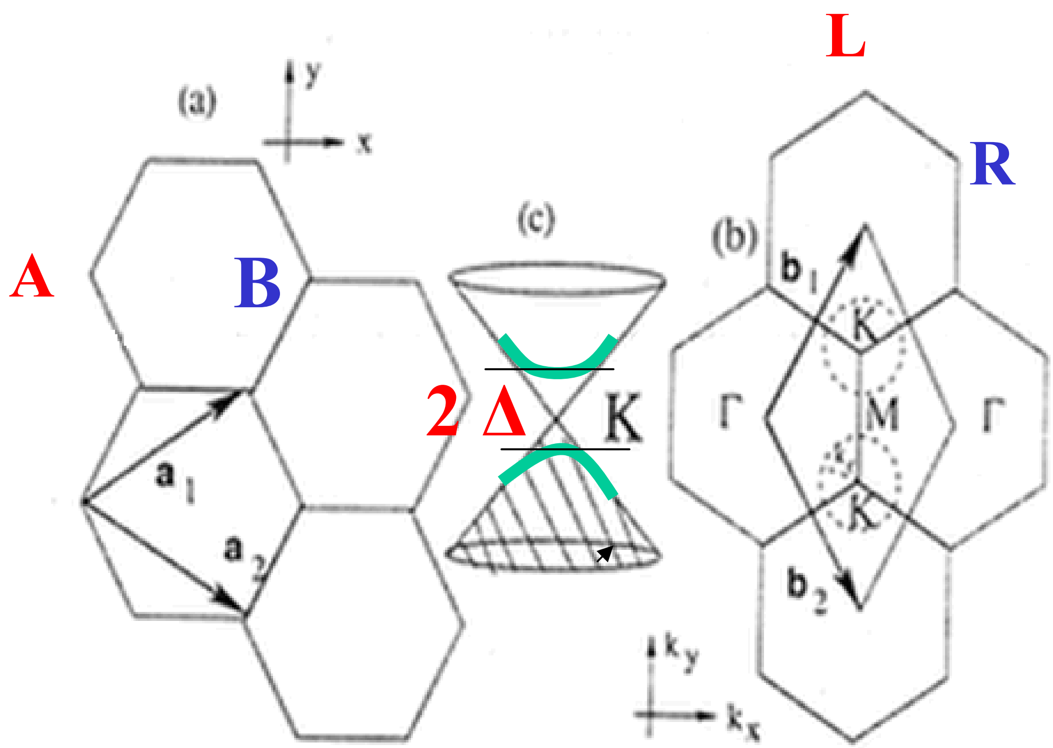
•Dirac (bi-) spinors: $\Psi = (\psi^{\sigma}_{LA}, \psi^{\sigma}_{LB}, \psi^{\sigma}_{RA}, \psi^{\sigma}_{RB})$

• Massless Dirac Hamiltonian: $H = \psi^{+} i\, v_F \gamma \partial\, \psi$

Massive Dirac fermions in graphene

- Nodal quasiparticle excitations

P. R. Wallace, ’47
G. W. Semenoff, ’84;
E. Fradkin, ’86;



- Dirac (bi-) spinors: $\Psi = (\psi^{\sigma}_{LA}, \psi^{\sigma}_{LB}, \psi^{\sigma}_{RA}, \psi^{\sigma}_{RB})$

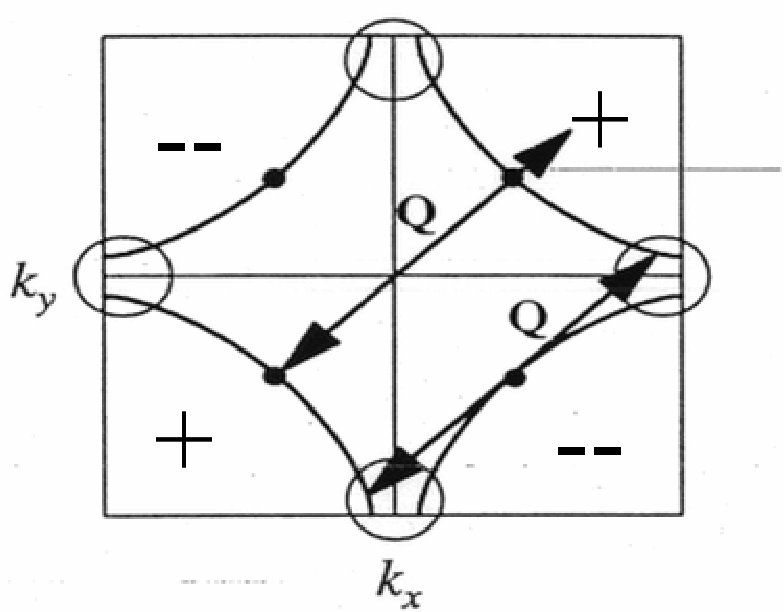
- Massive Dirac Hamiltonian: $H = \psi^{+}(i\, v_F \gamma \partial + \Delta)\, \psi$

Massive Dirac fermions: Higgs-Yukawa model

- **d-wave** superconductors ($\mu=0$)

Emergent fermion mass = second superconducting pairing:

d → d +is (id) S. Sachdev et al '99; DVK and J. Paaske, '00



$\Delta_d \sim (\cos k_x - \cos k_y)$

$\varepsilon \sim (\cos k_x + \cos k_y)$

$E(k) = (\varepsilon^2 + \Delta^2_d + \Delta^2_{is/id})^{1/2} \sim$

$\sim (v_x^2 k_x^2 + v_y^2 k_y^2 + \Delta^2_{is/id})^{1/2}$

•Dichalcogenides (2D **f-CDW ?**);
•He3-A, Bismuth (3D)…

Critical coupling: $g > g_c$

Massive Dirac fermions: Lorentz-invariant QED_3

- CSB in QED_3 T.Appelquist et al, '88

$$L = i \sum^{N}_{f=1} \bar{\psi}_f \gamma(\partial + A)\psi_f + F^2/2g \qquad F = \partial \times A$$

- Chiral rotation symmetry for **massless** fermions

$$\psi_f^{L,R} = (1 \pm \gamma_5)/2\, \psi_f^{L,R} \rightarrow \exp(i\gamma_5\varphi)\, \psi_f^{L,R}$$

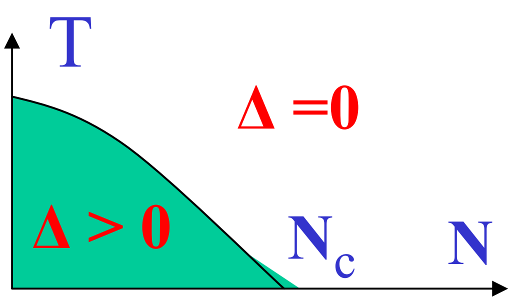
- CSB order parameter, $U(2N) \rightarrow U(N) \times U(N)$

$$\Delta \sim \sum^{N}_{f=1} \langle \bar{\psi}_f \psi_f \rangle$$

CSB phase transition

$\Delta \neq 0$, $N < N_c$

$\Delta = 0$, $N > N_c$ (for arbitrary g)



Critical number of species Nc

Different Dirac fermion masses

- 4-spinor wave functions:

$$\Psi(p) = (\psi_{C,n,\alpha}(p), \tau_2^{nm} s_2^{\alpha\beta} \psi^\dagger_{C,m,\beta}(-p))$$

$$\psi = (\frac{1+\tau_3}{2} + i\frac{1-\tau_3}{2} \otimes \sigma_2)(A_1, B_1, A_2, B_2)^T$$

$$\rho_n \otimes \sigma_a \otimes \tau_i \otimes s_\alpha$$

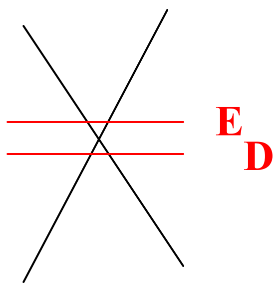
- Dirac mass terms (p-h):

$$\psi^\dagger \sigma_3 \otimes \tau_1 \otimes s_0 \psi = A^\dagger_{L\alpha} B_{R\alpha} + B^\dagger_{L\alpha} A_{R\alpha} + h.c.$$

$$\psi^\dagger \sigma_3 \otimes \tau_3 \otimes s_0 \psi = \sum_{i=L,R} (A^\dagger_{i\alpha} A_{i\alpha} - B^\dagger_{i\alpha} B_{i\alpha})$$

$$\psi^\dagger \sigma_3 \otimes \tau_0 \otimes s_0 \psi = \sum_{i=L,R} sgni(A^\dagger_{i\alpha} A_{i\alpha} - B^\dagger_{i\alpha} B_{i\alpha})$$

$$\psi^\dagger \sigma_3 \otimes \tau_2 \otimes s_0 \psi = iA^\dagger_{L\alpha} B_{R\alpha} - iB^\dagger_{L\alpha} A_{R\alpha} + h.c.$$



P & T

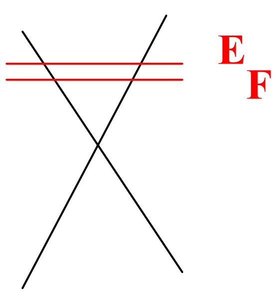
- Majorana mass terms (p-p/h-h):

$$\psi \sigma_0 \otimes \tau_1 \otimes s_0 \psi = \sum_{i=L,R} (A_{i\alpha} A_{i\beta} + B_{i\alpha} B_{i\beta}) s_2^{\alpha\beta}$$

$$\psi \sigma_0 \otimes \tau_2 \otimes s_0 \psi = \sum_{i=L,R} sgni(A_{i\alpha} A_{i\beta} + B_{i\alpha} B_{i\beta}) s_2^{\alpha\beta}$$

$$\psi \sigma_0 \otimes \tau_0 \otimes s_2 \psi = (A_{L\alpha} B_{R\alpha} - B_{L\alpha} A_{R\alpha})$$

$$\psi \sigma_0 \otimes \tau_3 \otimes s_0 \psi = (A_{L\alpha} B_{R\alpha} - B_{L\alpha} A_{R\alpha}) s_2^{\alpha\beta}$$



Coulomb interacting Dirac fermions

- Non-Lorentz-invariant Hamiltonian of graphene (no disorder):

$$H = iv_F \sum_{\alpha=1,2} \int_{\mathbf{r}} \Psi_\alpha^\dagger [\hat{\sigma}_x \nabla_x + (-1)^\alpha \hat{\sigma}_y \nabla_y] \Psi_\alpha$$

$$+ \frac{v_F}{4\pi} \sum_{\alpha,\beta=1,2} \int_{\mathbf{r}} \int_{\mathbf{r}'} \Psi^\dagger{}_\alpha(\mathbf{r}') \Psi_\alpha(\mathbf{r}') \frac{g}{|\mathbf{r} - \mathbf{r}'|} \Psi^\dagger{}_\beta(\mathbf{r}) \Psi_\beta(\mathbf{r})$$

- Dirac fermion propagator:

$$\hat{G}(\epsilon, \mathbf{p}) = [\epsilon - \hat{\rho}_3 \otimes (v \vec{\sigma}_{\|} \vec{p} - \mu + \vec{s}\vec{B}) + \hat{\Sigma}(p)]^{-1}$$

$$\omega = \mu + \sigma B \pm E(p)$$

$$E(p) = \sqrt{v^2(p)p^2 + \Delta^2(p)}$$

- Effective interaction:

$$V_C(\omega, q) = \frac{2\pi g v}{q + 2\pi g v \Pi(\omega, q)} \qquad g = \frac{e^2}{\epsilon v} \approx \frac{2.16}{\epsilon}$$

$$\Pi(\omega, q) = \frac{N q^2}{16\sqrt{v^2(q) q^2 - \omega^2}}$$

NOT **V(q)=const**

NOT **weak**

Excitonic pairing between Dirac fermions: gap equation

- Dirac fermion self-energy: $$\hat{\Sigma}(p) = \sum_{\mathbf{q}} \int \frac{d\omega}{2\pi} V(\mathbf{p}-\mathbf{q}, \epsilon - \omega) \frac{\omega + v\vec{\sigma}_{\|}\vec{q} + \hat{\Sigma}(p)}{\omega^2 - E^2(p) + i0}$$

- Velocity renormalization (diagonal term):

I.L.Aleiner et al, '07

D.T.Son, '07

$$\frac{v(p)}{v} = \frac{g}{g(p)} = (\frac{\Lambda}{p})^{\eta} \qquad \eta = \frac{8}{\pi^2 N}$$

- Gap equation (off-diagonal term): $$\Delta(p) = \sum_{\mathbf{q}} \frac{2\pi g v}{|\mathbf{p}-\mathbf{q}|} \frac{\Delta(q)}{2E(q)} \tanh \frac{E(q)}{2T}$$

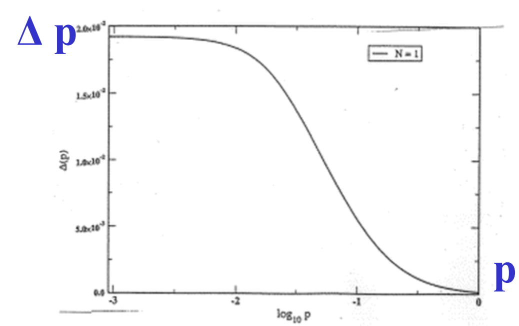
- Differential form: $$\frac{d^2\Delta(p)}{dp^2} + \frac{2+\eta_N}{p}\frac{d\Delta(p)}{dp} + \frac{g_N(1+\eta_N)}{2p^{2-\delta\eta}}\frac{\Delta(p)}{\Lambda^{\delta\eta}} = 0$$

- Boundary conditions: $$\frac{d\Delta(p)}{dp}|_{p=\Delta/v} = 0 \qquad [(1+\eta_N)\Delta(p) + p\frac{d\Delta(p)}{dp}]|_{p=\Lambda/v} = 0$$

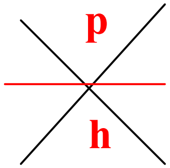
- WKB solution:

$$\Delta^{\pm}(p) = \frac{C_{\pm}}{p^{1-\delta\eta/2}P(p)^{1/2}} \exp(\pm i \int_{\kappa}^{p} P(p')dp')$$

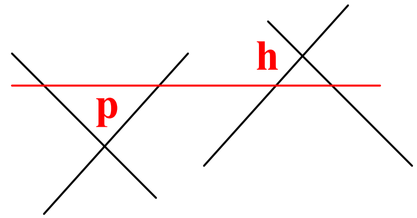
$$P^2(p) = \frac{1}{p^2}[g\frac{1+\eta_N}{2}(\frac{p}{\Lambda})^{\delta\eta} - \frac{(1+\eta_N)^2}{4}]$$



Excitonic pairing: undoped case



NOT

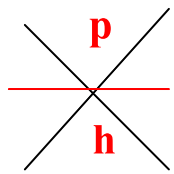


-finite in-plane field
-biased bi-layer

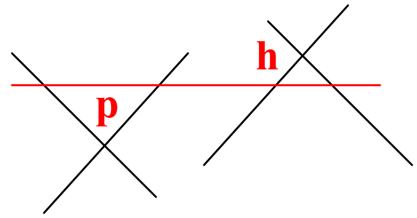
•Critical coupling: $\tilde{g} = \frac{g}{1 + \pi N g / 8\sqrt{2}} = 1/2, \;\; \eta = 0$ DVK,'01

Cf. Atomic collapse in the single-particle problem of a charged impurity in graphene (g=1/2).

Excitonic pairing: undoped case



NOT



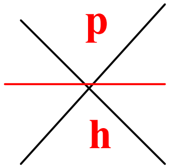
-finite in-plane field
-biased bi-layer

•Critical coupling: $\tilde{g} = \dfrac{g}{1 + \pi N g / 8\sqrt{2}} = 1/2$, $\eta = 0$

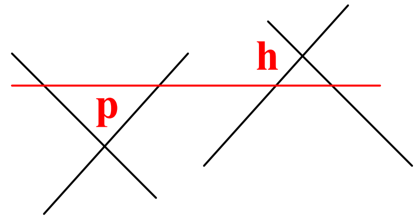
$$g_c = \frac{1}{2}\left(1 + \eta_N + (3\pi\delta\eta)^{2/3} + \ldots\right), \quad \eta > 0$$

DVK,'08

Excitonic pairing: undoped case



NOT



-finite in-plane field
-biased bi-layer

•Critical coupling: $\tilde{g} = \frac{g}{1 + \pi N g / 8\sqrt{2}}$ $= 1/2$, $\eta = 0$

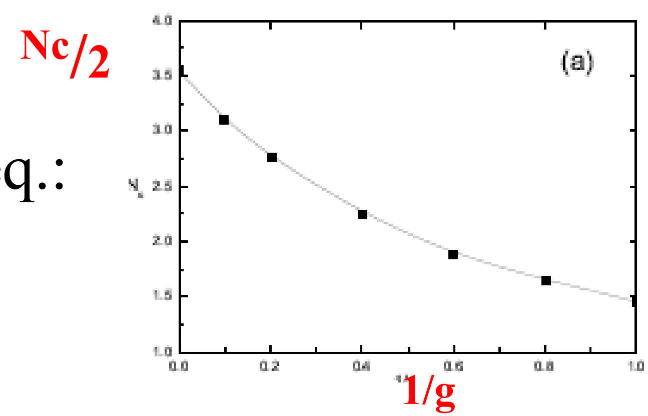
$$g_c = \frac{1}{2}\left(1 + \eta_N + (3\pi\delta\eta)^{2/3} + \ldots\right), \quad \eta > 0$$

•Maximum gap: $\Delta = v\Lambda \exp\left(-\frac{2\pi - 4\delta}{\sqrt{g - g_c}}\right)$, $\eta = 0$

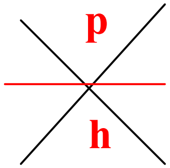
$$\Delta \sim v\Lambda(g - g_c)^{2/\eta} \quad , \eta > 0$$

•Numerical solution of the gap eq.:

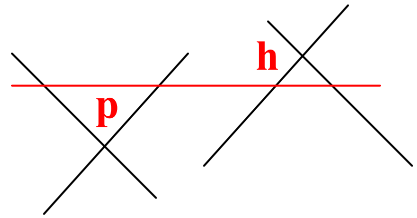
G.Liu et al, Phys. Rev. B 79, 205429 (2009)



Excitonic pairing: undoped case



NOT



-finite in-plane field
-biased bi-layer

•Critical coupling: $\tilde{g} = \frac{g}{1 + \pi N g / 8\sqrt{2}} = 1/2$, η=0

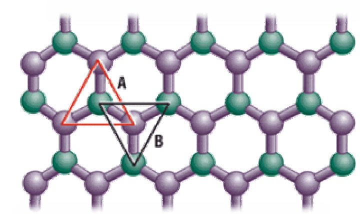
$g_c = \frac{1}{2}(1 + \eta_N + (3\pi\delta\eta)^{2/3} + \ldots)$, η>0

•Maximum gap: $\Delta = v\Lambda \exp(-\frac{2\pi - 4\delta}{\sqrt{g - g_c}})$, η=0

$\mathbf{\Delta \sim v\Lambda(g - g_c)^{2/\eta}}$, η>0

•Lifting of the sublattice (A/B) degeneracy:

CDW **$\Delta \sim \rho_A - \rho_B$** Q=(√3/2,1/2,(1))π



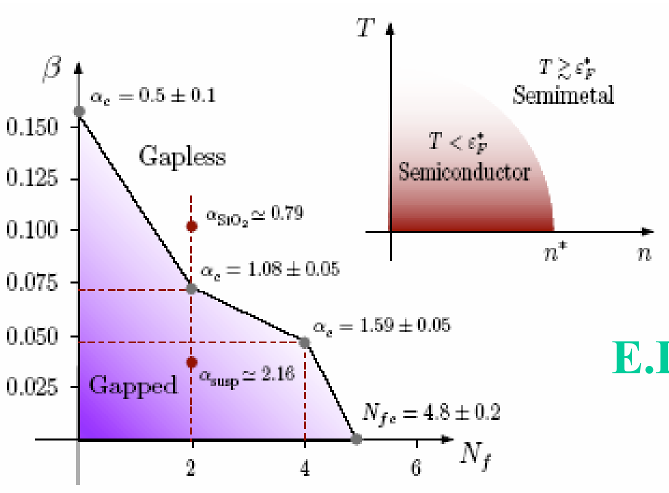
Excitonic insulator transition in undoped graphene

- QED_3 : Gap equation: $N_c = 3.2$
  MC simulations: $N_c < 1.5$ (>1.0 ?)

J.Kogut et al, 0808.2720

- Graphene: Gap equation: $g_c =$ 1.13 DVK,0807.0676
  Nc = 7.2 for g→Infty
  Actual values: g = 2.16 (free standing)
  = 0.8 (SiO_2)
  N=2
  Later MC simulations: gc = 1.1 for N=2 E.Drut et al, 0807.0834
  Nc = 4.8 for g→Infty

T.Hands et al, 0808.2714



E.Drut and T.Lahde, 0905.1320

Excitonic insulator transition in undoped graphene

- QED_3 : Gap equation: $N_c = 3.2$

  MC simulations: $N_c < 1.5$ (>1.0 ?)

  J.Kogut et al, 0808.2720

- Graphene: Gap equation: $g_c = 1.13$ DVK,0807.0676

  $Nc = 7.2$ for $g \rightarrow$ Infty

  Actual values: $g = 2.16$ (free standing)

  $= 0.8$ (SiO_2)

  N=2

  Later MC simulations: $gc = 1.1$ for N=2 E.Drut et al, 0807.0834

  $Nc = 4.8$ for $g \rightarrow$ Infty

  T.Hands et al, 0808.2714

- Free-standing graphene: $\Delta \sim$ 5-10 meV

Excitonic insulator transition in undoped graphene

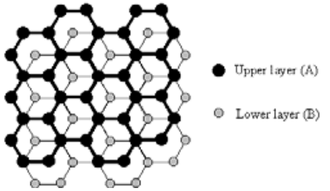
- QED_3 : Gap equation: $N_c = 3.2$
  MC simulations: $N_c < 1.5$ (>1.0 ?)

J.Kogut et al, 0808.2720

- Graphene: Gap equation: $g_c = 1.13$ DVK,0807.0676
  Nc = 7.2 for g→Infty
  Actual values: g = 2.16 (free standing)
  = 0.8 (SiO_2)
  N=2
  Later MC simulations: gc = 1.1 for N=2 E.Drut et al, 0807.0834
  Nc = 4.8 for g→Infty

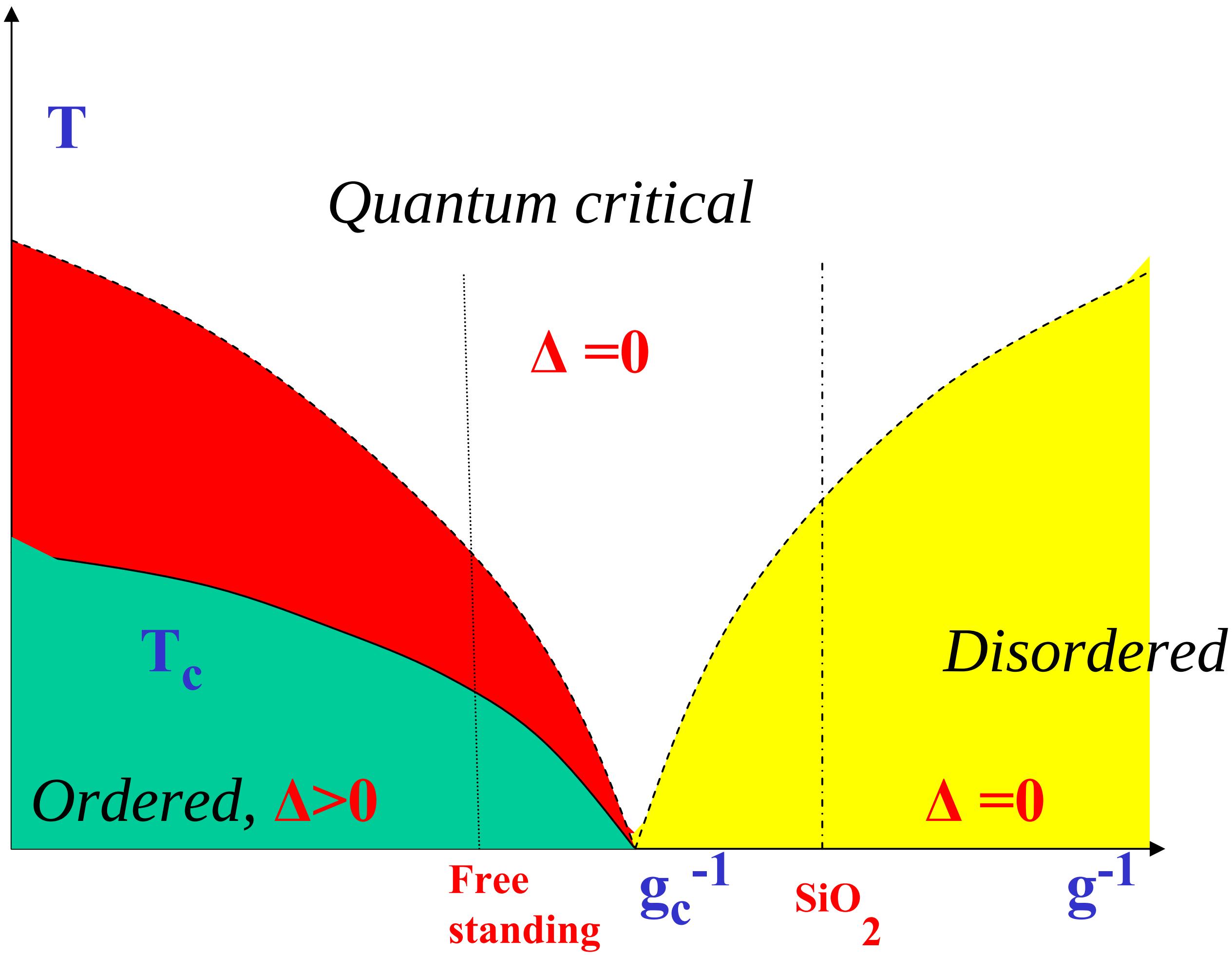
T.Hands et al, 0808.2714

- HOPG: EI is further stabilized by inter-layer Coulomb repulsion



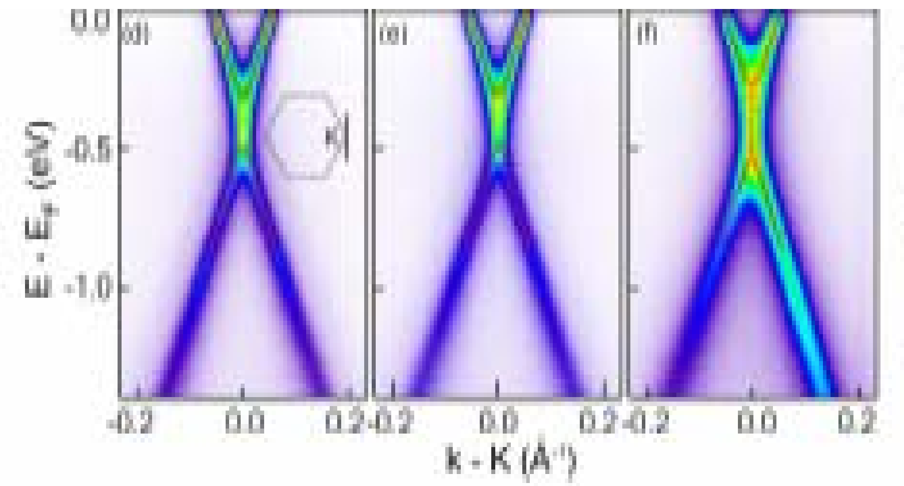
Quantum-critical behavior in undoped graphene?

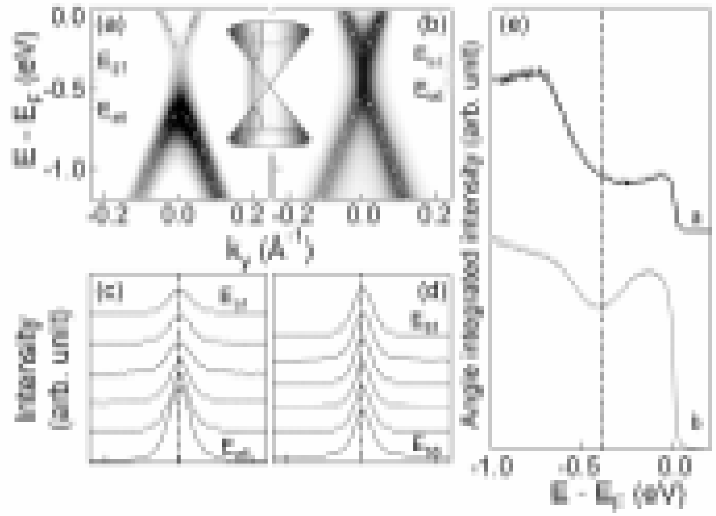
Coulomb coupling:
$g = e^2/\varepsilon v_F$



T=0 quantum critical point at **$g=g_c$**

Dirac fermion mass in epitaxial graphene? (ARPES)

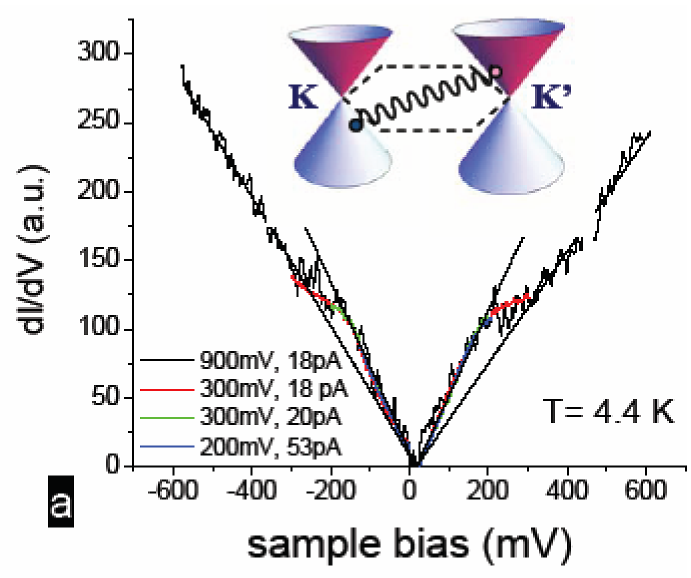


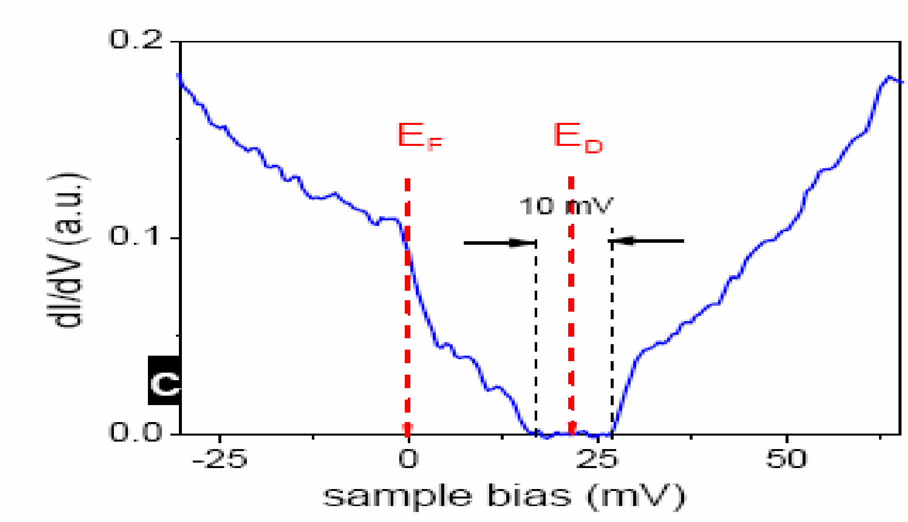


A. Lanzara et al '07

Strong substrate-related effects:
Large gap/mass ~130meV

Dirac fermion mass in suspended graphene? (STM)





E. Andrei et al '08

No substrate:

Small gap/mass~10meV

Moderately strong Coulomb interactions: photoemission

• Electron spectral function:
undoped, quantum-critical regime, **g<gc**

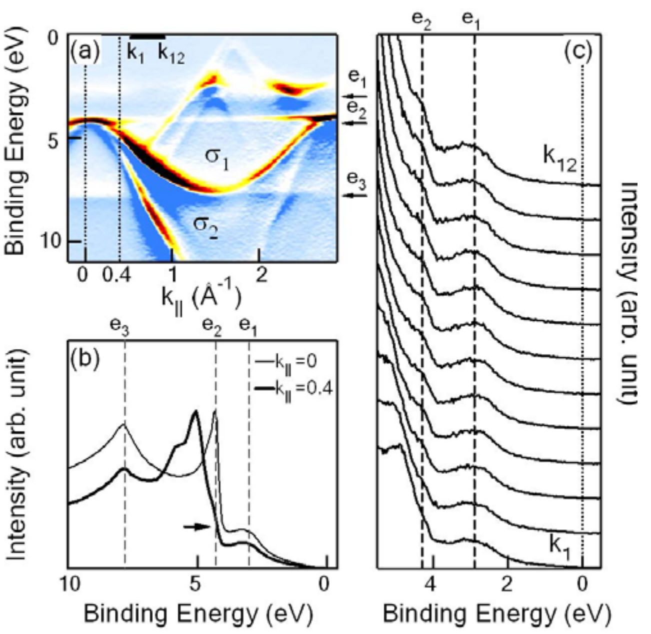
$$\Gamma(\epsilon, \mathbf{p}) \propto \theta(p_\mu^2) \frac{p_\mu^2}{max[\epsilon, v_F p]} \ln g, \quad max[\epsilon, v_F p] > T$$
$$\propto \theta(p_\mu^2) (\frac{p_\mu^2 T}{max[\epsilon, v_F p]})^{1/2}, \quad max[\epsilon, v_F p] < T$$

$p^2{}_\mu = E^2 - v^2 \mathbf{p}^2$

• **NOT** just "~ E"

•Formally related problem:
normal quasiparticles in d-wave cuprates

J. Paaske and DVK, '00;
A.Chubukov and A.Tsvelik, '05



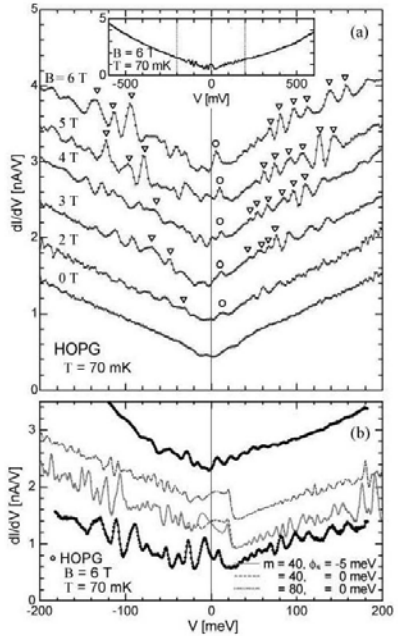
A. Lanzara et al, '05

Moderately strong Coulomb interactions: tunneling

- Tunneling DOS:

$$\nu(\epsilon) \approx -\frac{1}{\pi} ImTr \int_{-\infty}^{\infty} \hat{G}_0^R(0,t) e^{-S(t)+i\epsilon t} dt$$

$$S(t) = \int_0^{\Lambda} \frac{d\omega}{4\pi} \sum_{\mathbf{q}} ImU(\omega, \mathbf{q}) \coth\frac{\omega}{2T} \int_0^t dt_1 \int_0^t dt_2 e^{-i\omega(t_1 - t_2)} \left\langle e^{i\mathbf{q}(\mathbf{r}(t_1)-\mathbf{r}(t_2))} \right\rangle$$

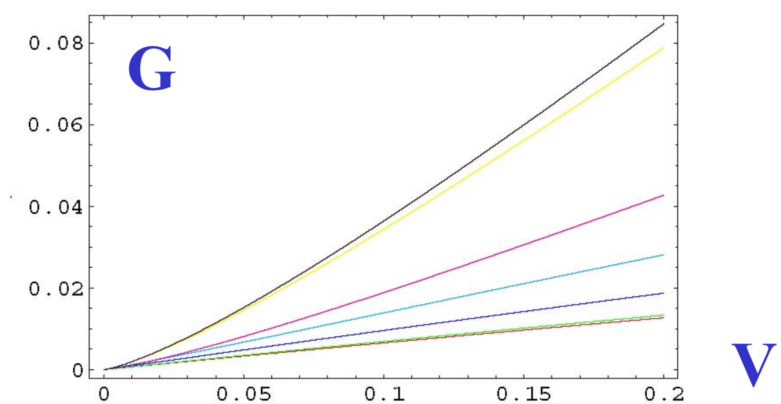


- Tunneling conductance:

$$G(V) \propto \frac{d}{dV} \int_0^{\infty} \mathcal{G}^R(\mathbf{0}, t) \mathcal{G}_0^R(\mathbf{0}, t) e^{iVt} dt$$

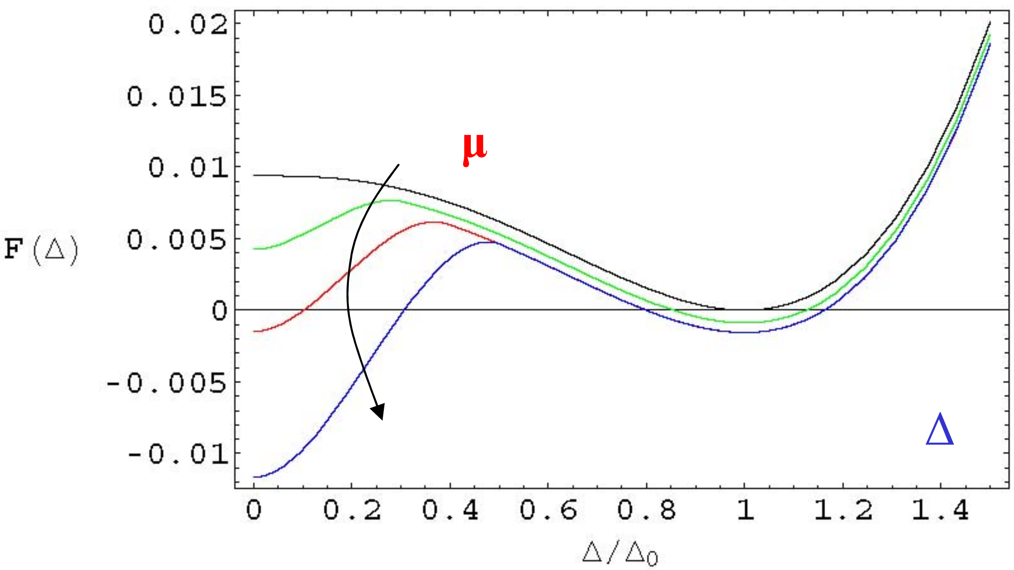
T.Matsui et al, '05

$G(V,T) \sim \max[V, T]^{1+\eta(g,V)}$

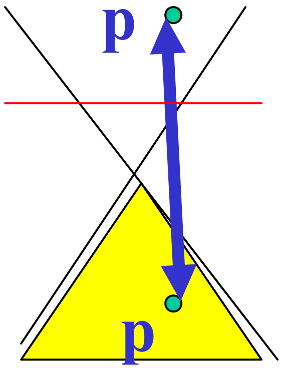


Excitonic pairing: finite doping

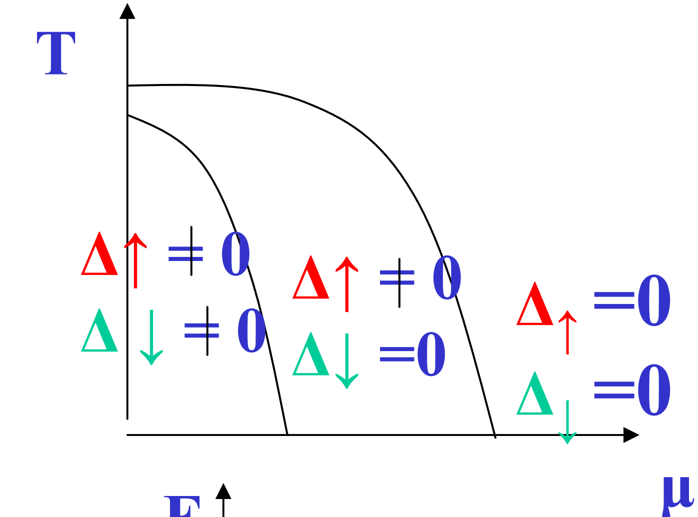
- Electron density dependence: first order transition

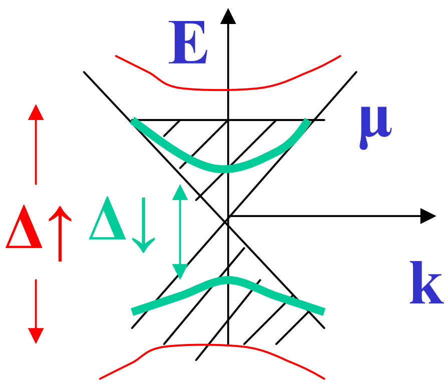


- Degeneracy between singlet and triplet pairing is lifted



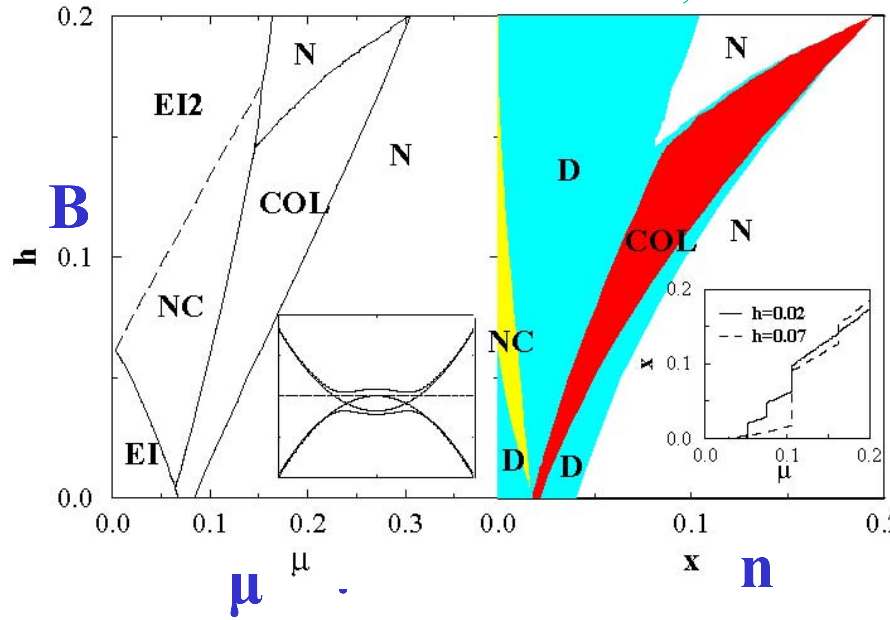
Excitonic (weak?) ferromagnetism





Example: hexaborides (BCS)

A. McDonald et al, '00



N = paramagnetic (semi)metal;

EI = excitonic insulator;

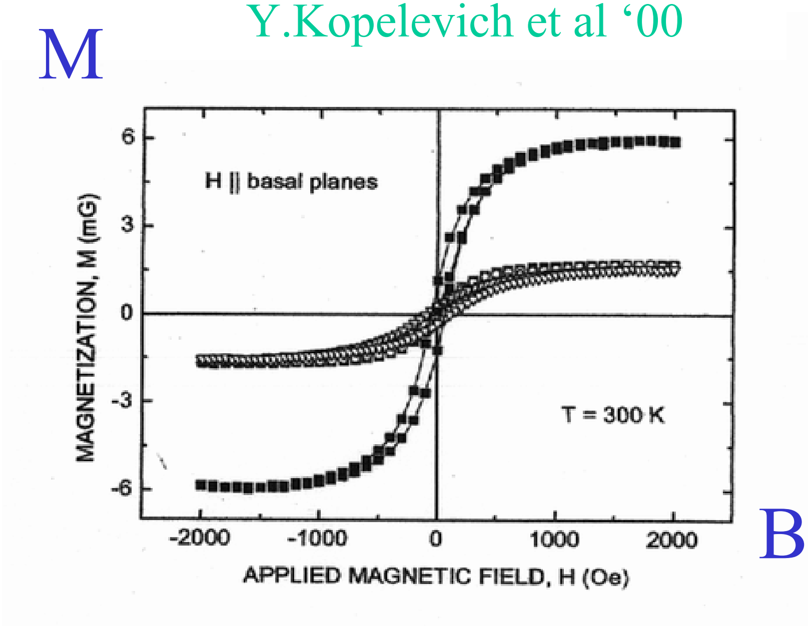
COL/NC = (non-)collinear ferromagnet

Weak ferromagnetism in HOPG

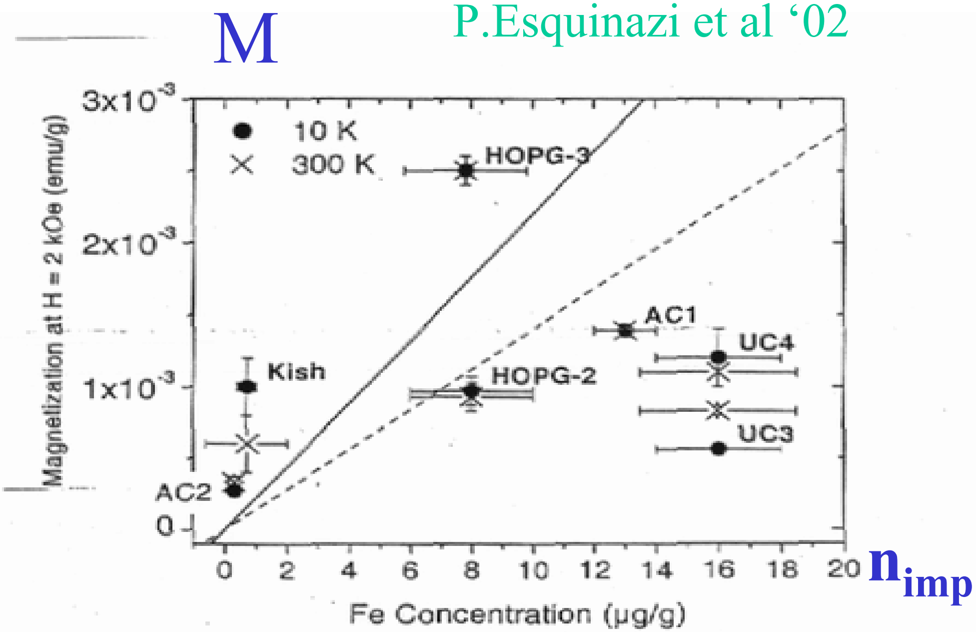
•Small, yet robust, magnetic moment:

M ~ 0.03-0.05 μ_B/carrier, Tc~500K

Y.Kopelevich et al '00



P.Esquinazi et al '02



• Possible mechanisms:

-Single-particle (magnetic impurities; structural defects, edges, H-bonds)

-Many-body (Coulomb interactions) ?

Dirac fermion-phonon coupling: Cooper pairing

- Elastic energy:

$$F = \frac{\rho}{2}[(\partial_i \vec{u})^2 + (\partial_i h)^2 - \kappa^2(\partial_i^2 h)^2 - c^2(\partial_i u_j + \partial_j u_i + \partial_i h \partial_j h)^2]$$

$$D_a(\omega, q) = \frac{\Omega_a(q)}{\omega^2 - \Omega_a^2(q) + i0}$$

- Effective e-e interaction:

$$V_{ph}(\omega, q) = \sum_{a=0,1,2} (D_a(\omega, q)|M_q^a|^2 + \sum_{q'} \int \frac{d\omega'}{2\pi} D_z(\omega'_+, q'_+)|M_{q'_-}^a|^2 D_z(\omega'_-, q'_-)|M_{q'_+}^a|^2)$$

$$V_{ph,\parallel}(q) = -\sum_{a=0.1} \frac{|M_q^a|^2}{\Omega_a} = -(V_0 + V_1)$$

$$V_{ph,z}(q) = -\int \frac{d\omega}{2\pi} \sum_k e|M_k|^2 D(\omega, k+q) D(\omega, k) = -V_2 \ln \frac{\Lambda}{q}$$

- E-ph coupling:

$$\frac{\lambda(p)}{\lambda} = (\frac{v(p)}{v})^2 \qquad \lambda_0 = \frac{\sqrt{27} D^2 a^2}{4\pi m \Omega_0 v^2} \approx 0.04 \longrightarrow \mathbf{0.4}$$

D.Basko and I.Aleiner, '07

- Gap equation:

$$\Delta_{SC}(p) = \sum_{a=0,1,2,q} \frac{|M_{p-q}^a|^2}{\Omega_a(p-q) + E(q)} \frac{\Delta_{SC}(q)}{2E(q)} \tanh \frac{E(q)}{2T}$$

$$\Delta_{SC} \approx \sum_{a=0,1,2} E_a e^{-1/\lambda_a} \qquad E_0 \sim min[\Omega_0, \mu], \quad E_1 \sim \frac{c\mu}{v}, \quad E_2 \sim \mu(\frac{\Lambda\kappa}{v})^{1/2}$$

Dirac fermion-phonon coupling: Cooper pairing

•Elastic energy:

$$F = \frac{\rho}{2}[(\partial_t \vec{u})^2 + (\partial_t h)^2 - \kappa^2(\partial_i^2 h)^2 - c^2(\partial_i u_j + \partial_j u_i + \partial_i h \partial_j h)^2]$$

$$D_a(\omega, q) = \frac{\Omega_a(q)}{\omega^2 - \Omega_a^2(q) + i0}$$

•Effective e-e interaction:

$$V_{ph}(\omega, q) = \sum_{a=0,1,2} (D_a(\omega, q)|M_q^a|^2 + \sum_{q'} \int \frac{d\omega'}{2\pi} D_z(\omega'_+, q'_+)|M^a_{q'_-}|^2 D_z(\omega'_-, q'_-)|M^a_{q'_+}|^2)$$

$$V_{ph,\parallel}(q) = -\sum_{a=0.1} \frac{|M_q^a|^2}{\Omega_a} = -(V_0 + V_1)$$

$$V_{ph,z}(q) = -\int \frac{d\omega}{2\pi} \sum_k e|M_k|^2 D(\omega, k+q) D(\omega, k) = -V_2 \ln\frac{\Lambda}{q}$$

•E-ph coupling:

$$\frac{\lambda(p)}{\lambda} = (\frac{v(p)}{v})^2 \qquad \lambda_0 = \frac{\sqrt{27} D^2 a^2}{4\pi m \Omega_0 v^2} \approx 0.04 \longrightarrow \textbf{0.4}$$

D.Basko and I.Aleiner, '07

•Gap equation:

$$\Delta_{SC}(p) = \sum_{a=0,1,2,q} \frac{|M^a_{p-q}|^2}{\Omega_a(p-q) + E(q)} \frac{\Delta_{SC}(q)}{2E(q)} \tanh\frac{E(q)}{2T}$$

• Maximum gap: $\Delta \sim 30$ meV (no Coulomb repulsion!)

•KT critical temperature:

$$T_{KT} = \frac{\pi}{2}\rho_s \approx \frac{1}{8}\Delta$$

Dirac fermion-phonon coupling: Cooper pairing

• Elastic energy:

$$F = \frac{\rho}{2}[(\partial_i \vec{u})^2 + (\partial_i h)^2 - \kappa^2(\partial_i^2 h)^2 - c^2(\partial_i u_j + \partial_j u_i + \partial_i h \partial_j h)^2]$$

$$D_a(\omega, q) = \frac{\Omega_a(q)}{\omega^2 - \Omega_a^2(q) + i0}$$

• Effective e-e interaction:

$$V_{ph}(\omega, q) = \sum_{a=0,1,2} (D_a(\omega, q)|M_q^a|^2 + \sum_{q'} \int \frac{d\omega'}{2\pi} D_z(\omega'_+, q'_+)|M^a_{q'_-}|^2 D_z(\omega'_-, q'_-)|M^a_{q'_+}|^2)$$

$$V_{ph,\parallel}(q) = -\sum_{a=0,1} \frac{|M_q^a|^2}{\Omega_a} = -(V_0 + V_1)$$

$$V_{ph,z}(q) = -\int \frac{d\omega}{2\pi} \sum_k e|M_k|^2 D(\omega, k+q) D(\omega, k) = -V_2 \ln \frac{\Lambda}{q}$$

• E-ph coupling:

$$\frac{\lambda(p)}{\lambda} = (\frac{v(p)}{v})^2 \qquad \lambda_0 = \frac{\sqrt{27} D^2 a^2}{4\pi m \Omega_0 v^2} \approx 0.04 \longrightarrow \mathbf{0.4}$$

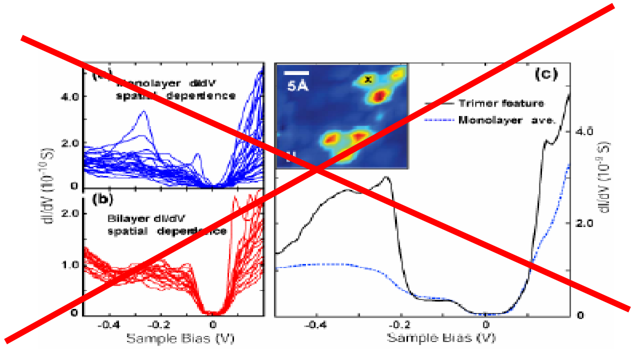
D.Basko and I.Aleiner, '07

• Gap equation:

$$\Delta_{SC}(p) = \sum_{a=0,1,2,q} \frac{|M^a_{p-q}|^2}{\Omega_a(p-q) + E(q)} \frac{\Delta_{SC}(q)}{2E(q)} \tanh \frac{E(q)}{2T}$$

V.Brar et al, '07

Strong Coulomb repulsion →



Excitonic and Cooper instabilities in real-life graphene

•Electron-hole puddles: $n_e \sim 10^{11} cm^{-2}$

would destroy the excitonic gap $\Delta \sim 10$ meV

•Ripples: $B_{eff} \sim 5T$

would destroy the Cooper gap $\Delta \sim 30$ meV

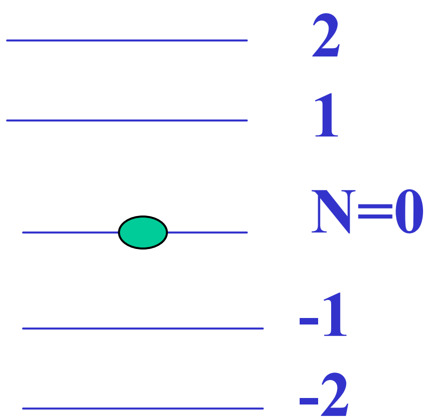
Coulomb interacting Dirac fermions in magnetic field

- Relativistic analog of FQHE: **magnetic catalysis**

$$\Delta(p) = i\int \frac{d\omega\, d\mathbf{k}}{(2\pi)^3}\frac{\Delta(k+p)}{(\epsilon+\omega+i\delta)^2-\Delta^2(k+p)}$$
$$\frac{g e^{-((\mathbf{k}+\mathbf{p})^2+\mathbf{p}^2)/B}}{|\mathbf{k}|+\sqrt{B}\, g N \mathbf{k}^2 e^{-\mathbf{k}^2/2B}(B-\omega^2/2)^{-1}}$$

DVK, cond-mat/0106261
V.Gorbar et al, cond-mat/0202422

-Coulomb interaction:
screening is even weaker than at B=0
-No threshold for g



- Field-induced gap at the N=0 LL:

$\Delta \sim f(\nu)\, B^{1/2}$ $\quad$ **f(0)=f(1)=0**

- A magnetic field-induced fermion mass can provide a means of **spatially confining** the Dirac fermions (cf. electrostatic potential – Klein's tunneling).

Moderately strong fields: (Half)Integer Quantum Hall Effect

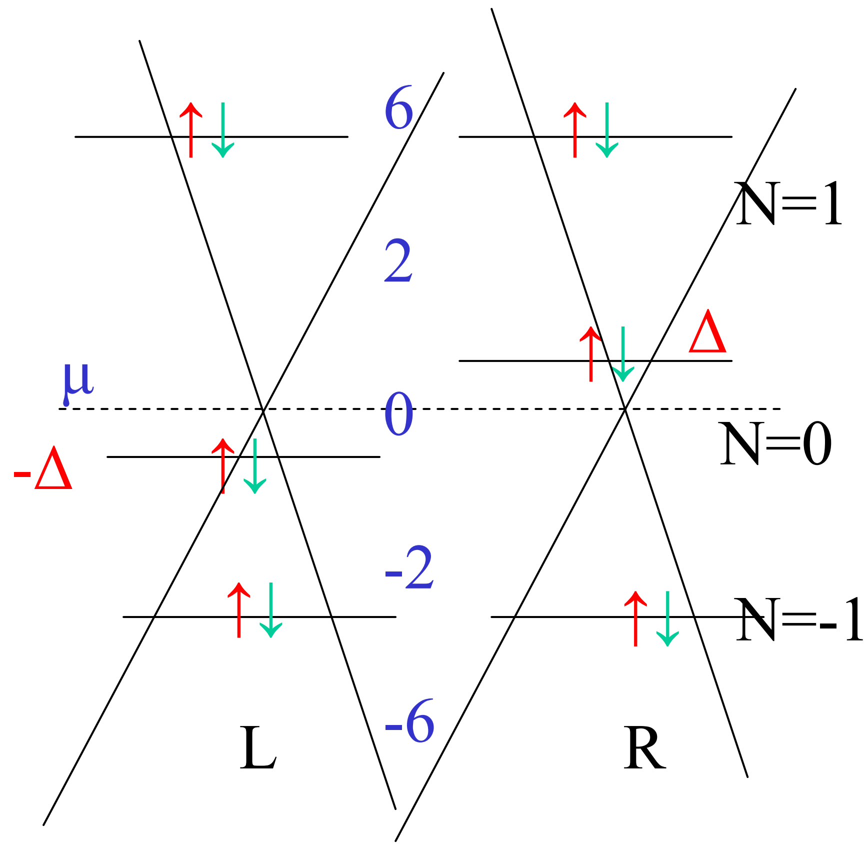
- Dirac fermions' Landau levels:

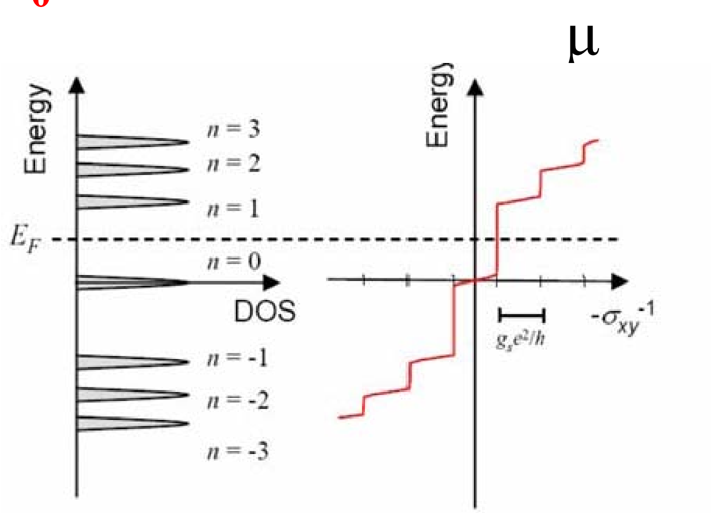
$$E_N = \pm (2 v_F^2 N B + \Delta^2)^{1/2}$$

- "Anomalous" IQHE:

$$\sigma_{xy}(T) = 4(e^2/h)(N+1/2)$$

$B < B_0 \sim 10T$: $\Delta = 0$





A.Geim et al '05
P. Kim, et al '05

Stronger fields: magnetic field-induced mass

•New plateaus: B > ~10T

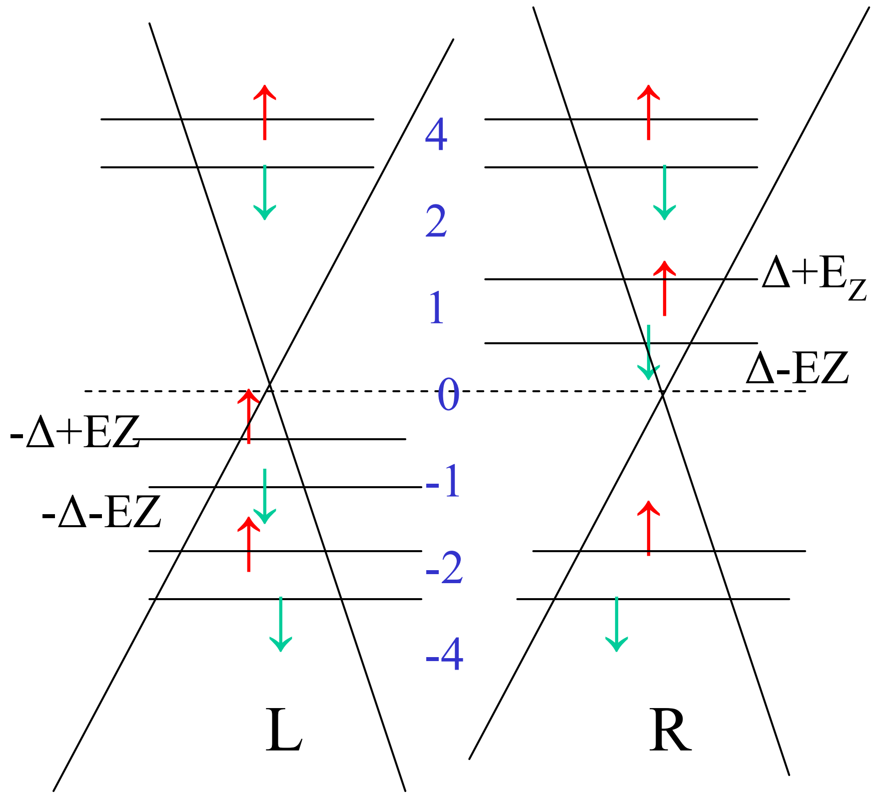
$\sigma_{xy}(T) = \pm(e^2/h)(0, 1, 4)$

Y.Zhang et al,'06

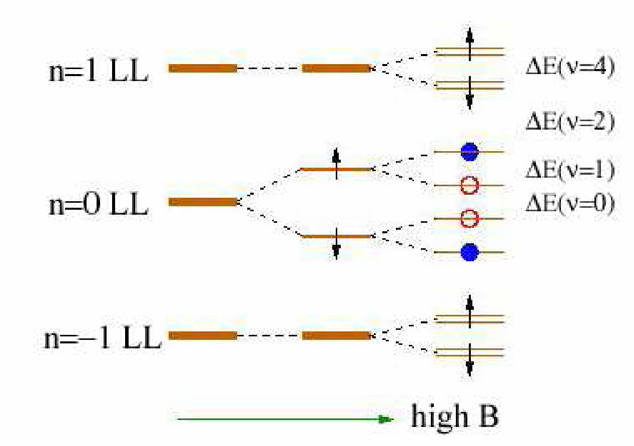
•Spin and valley splitting at LLL (N=0)

•Valley degeneracy remains intact for N≠0

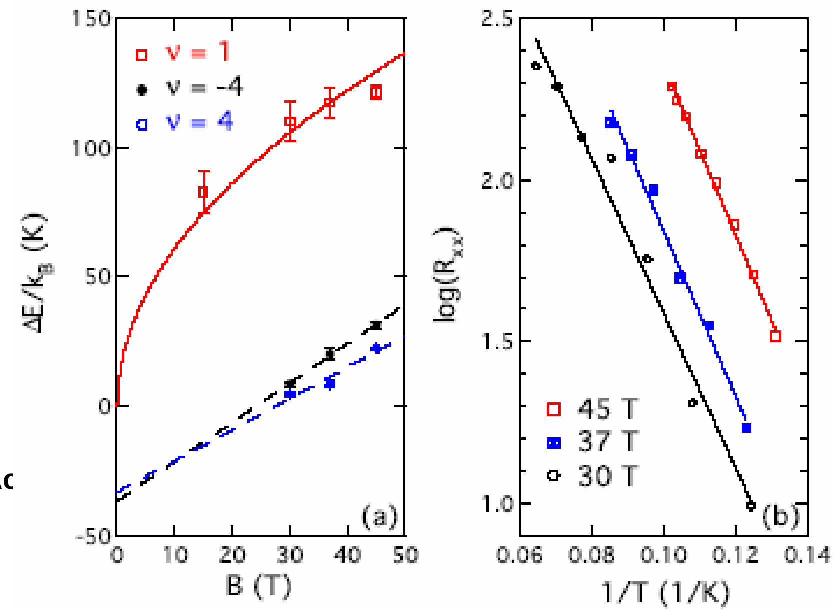
•**NO** plateaus observed at ±3, ±5,..
(until recently)

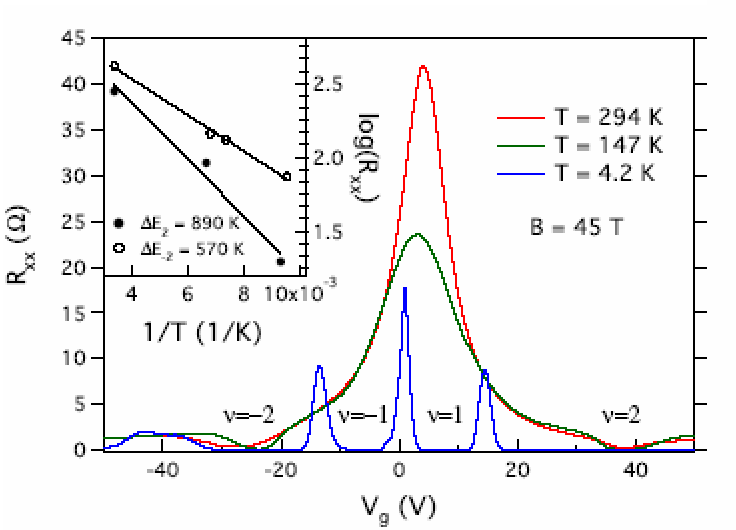


Field dependence of spectral gaps



Ac





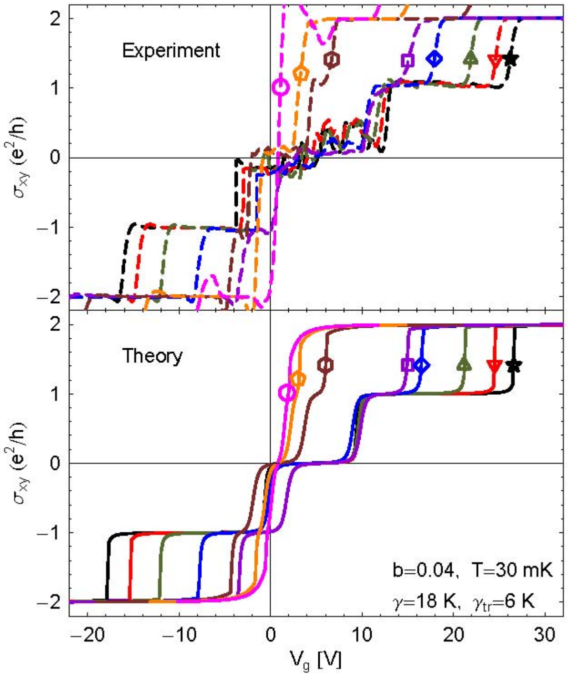
P.Kim et al, '07

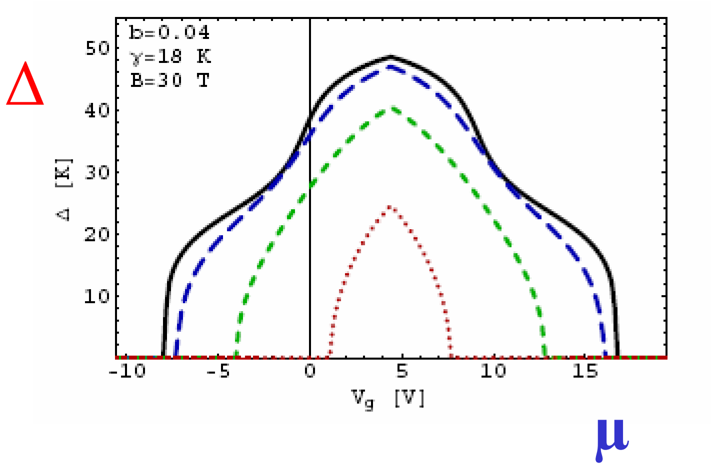
$\nu = \pm 4$ $\Delta \sim B$ Zeeman?

$\nu = \pm 1$ $\Delta \sim B^{1/2}$ Coulomb?

Magnetic catalysis scenario: data fitting

V. Gusynin et al, ‘06





Δ~50K
B=30T

Alternative mechanisms:

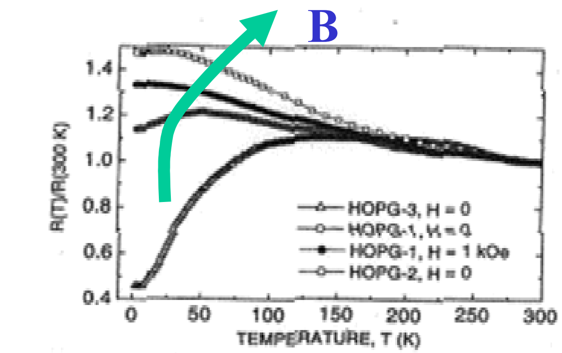
•Many-body: QH Ferromagnetism

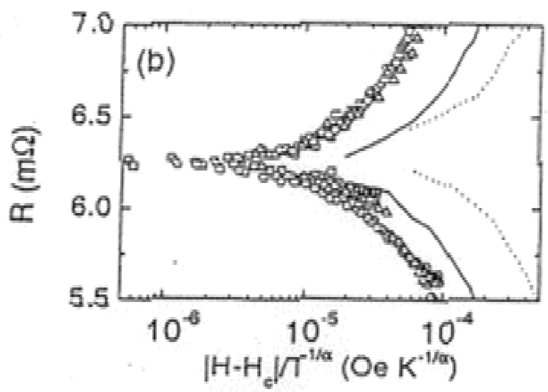
K.Nomura, A.McDonald, ’06; J.Alicea, M.P.E. Fisher, ‘06; M.Goerbig et at, ‘06, K.Yang et al, ‘06.

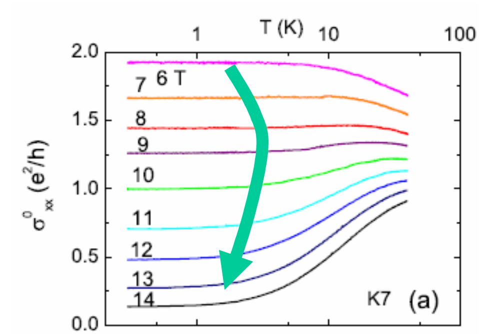
•Single-particle: Peierls distortion

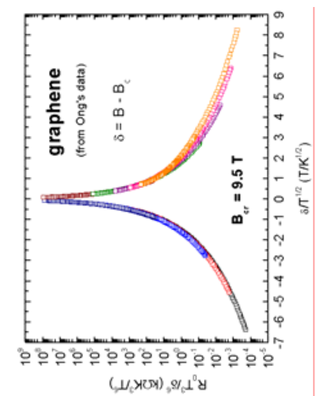
J. Fuchs and P. Lederer, ‘06

Field-induced MIT in HOPG and graphene?



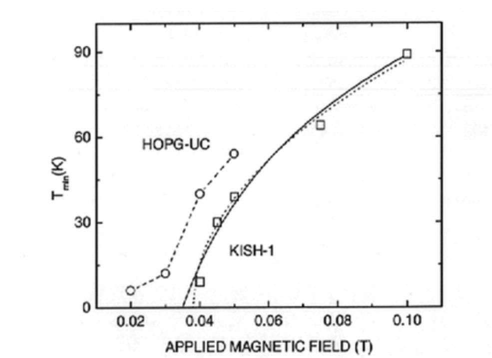






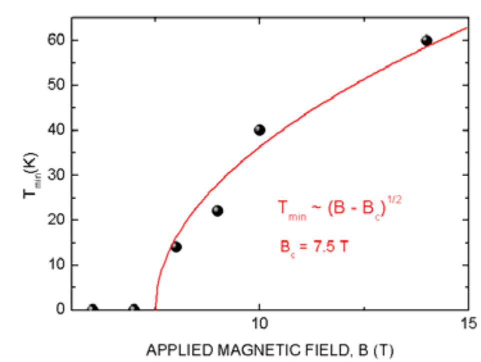
Y.Kopelevich et al, ‘00

Y.Kopelevich, ‘08



$$\Delta \sim (B - B_0)^{1/2}$$

B_0 (μ)



FQHE in graphene

•Standard (Jain's) fractions: $\sigma_{xy} = (\pm)\nu^{\pm} = (\pm)\frac{m}{2m \pm 1}$

spin and valley polarized

•Composite Dirac fermions: new fractions $\sigma_{xy} = (\pm)\frac{2m}{2m \pm 1}$

spin and/or valley singlets **DVK, '06**

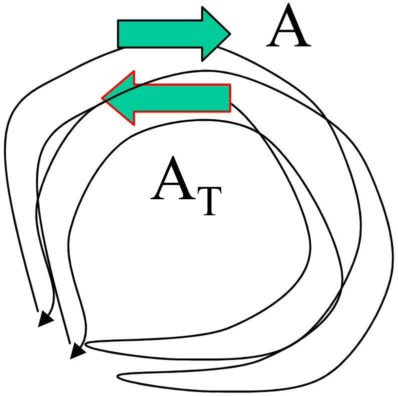
$$\sigma_{xy} = (\pm)\frac{2}{2m \pm 1}$$

•Also found numerically:

V.Apalkov and T.Chakraborty, '06; C.Toke et al, '06

Effects of disorder on Dirac fermions

Negative interference → WAL→ Positive MR



Intrinsic Berry phase π

$A_T = - A$

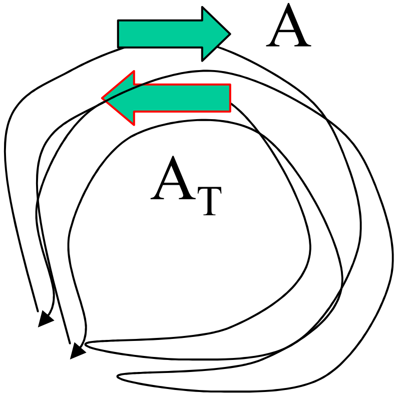
No inter-valley scattering:
WAL

T. Ando and H. Suzuura, '02

$\Delta\sigma_{WL}(H) < 0$

Effects of disorder on Dirac fermions

Negative interference → WAL→ Positive MR



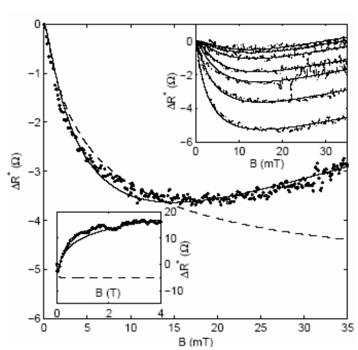
Intrinsic Berry phase π

$A_T = - A$

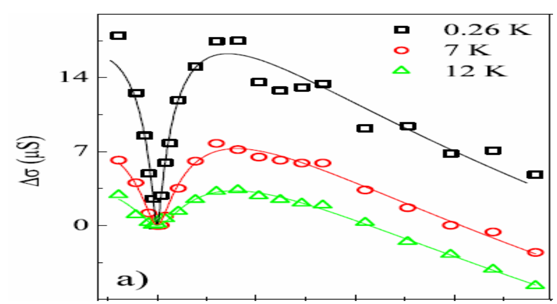
Intra- and inter-valley scattering: crossover between WL and WAL

DVK, PRL 97, 036802,'06 (0602398)

E. McCann et al, PRL 97, 146805,'06 (0604015)



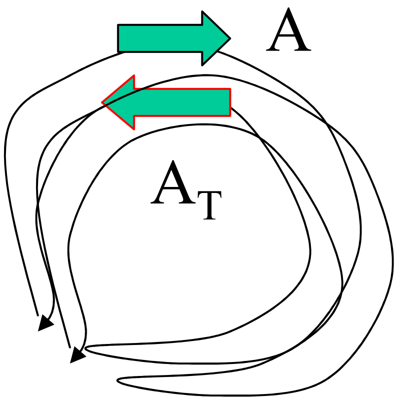
X.Wu et al, '07



V.Tikhonenko et al '07

Effects of disorder on Dirac fermions

Negative interference → WAL→ Positive MR



Intrinsic Berry phase π

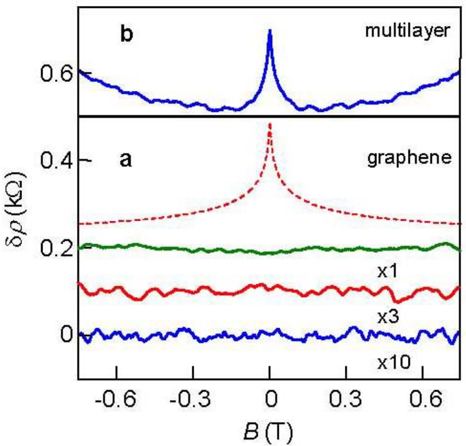
$A_T = - A$

Special disorder models:
(commensurate substrate potential, chiral disorder,…)

$$\Delta\sigma_{WL}(H) = 0$$

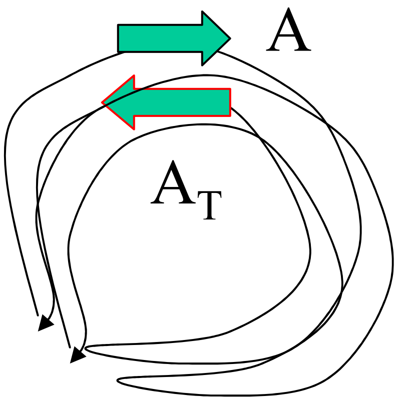
DVK,'06,
P.Ostrovsky et al, '07

Morozov et al '06



Effects of disorder on Dirac fermions

Negative interference → WAL → Positive MR



Intrinsic Berry phase π

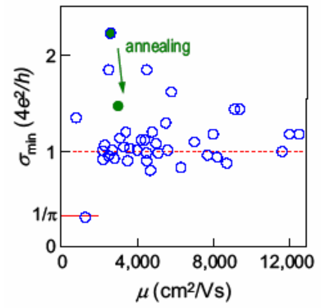
$A_T = -A$

Theory: momentum-independent,
yet predominantly intra-valley, scattering

No such scattering mechanism in **undoped** graphene

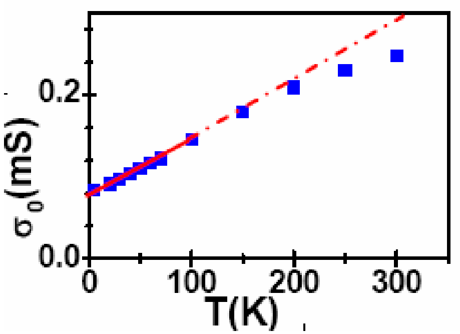
Experimentally relevant disorder

- (Non)universal minimal conductivity:



A.Geim et al, '06

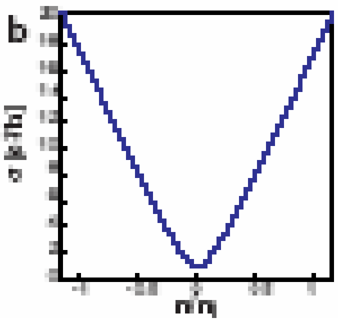
- Linear T-dependence:



G.Li et al, '08

- Linear density dependence:

→ long-range-correlated disorder



Long-range-correlated disorder

- Scalar vs vector disorder: $< V_q V_{-q} >= \Gamma_s / q^{2\eta}$ intra-valley

$$< A_q A_{-q} >= \Gamma_v / q^{2\eta}$$

"T-reversal": even (V) vs odd (A)

NOT <VV>=const

<AA>=const

η =0

$$H = \sigma_2 H^T \sigma_2$$

Long-range-correlated disorder

- Scalar vs vector disorder: $< V_q V_{-q} >= \Gamma_s / q^{2\eta}$

  $< A_q A_{-q} >= \Gamma_v / q^{2\eta}$
- Experiment: linear conductivity of graphene **(σ~n)** → η=1
- RP:

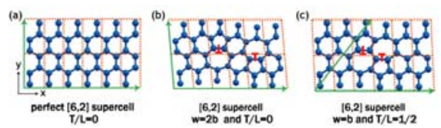
-Coulomb impurities: η=1

A.McDonald and K. Nomura,'06; S. Das Sarma et al, '06

-Cf. short-range potential disorder: η=0

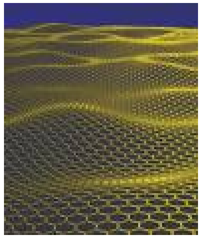
- RMF:

-Disclinations (topological defects): η=1



F. Guinea et al,'93

-Cf. Dislocations (pentagon/heptagon pairs): η=0



- Ripples (asymptotic regime): η=0.2

M.Katsnelson et al, '07; N.Abedpour et al, '07

Non-linear conductivity at n→0 ?

Scalar vs vector disorder with η=1: perturbation theory

- Self-consistent Born approximation, **doped** case:

$$\hat{\Sigma}^R_\alpha(\epsilon, \mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^2} \frac{w_\alpha(\mathbf{q})}{\hat{G}^R(\epsilon, \mathbf{p}+\mathbf{q})^{-1} + \hat{\Sigma}^R_\alpha(\epsilon, \mathbf{p}+\mathbf{q})}$$

$$\hat{G}_R(\omega, \mathbf{p}) = [(\epsilon + i0)\hat{\gamma}_0 - p_\mu \hat{\gamma}_\mu]^{-1}$$

$$w(q) = g/(lq)^{2\eta}$$

- Fermion lifetimes:

$$\gamma_s = ImTr\hat{\gamma}_0 \hat{\Sigma}^R_s(\epsilon, \epsilon/v) \sim \frac{v^2 \Gamma_s}{\epsilon} min[\frac{1}{g}, \frac{1}{g^2}] \quad \gamma_v = ImTr\hat{\gamma}_0 \hat{\Sigma}^R_v(\epsilon, \epsilon/v) \sim v\Gamma_v^{1/2}\sqrt{\ln L}$$

- Failure of perturbation theory (genuine IR divergence due to a gauge non-invariant nature of G)

Scalar vs vector disorder with η=1: perturbation theory

- Self-consistent Born approximation, **doped** case:

$$\hat{\Sigma}^R_\alpha(\epsilon,\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^2}\frac{w_\alpha(\mathbf{q})}{\hat{G}^R(\epsilon,\mathbf{p}+\mathbf{q})^{-1}+\hat{\Sigma}^R_\alpha(\epsilon,\mathbf{p}+\mathbf{q})} \qquad \hat{G}_R(\omega,\mathbf{p}) = [(\epsilon+i0)\hat{\gamma}_0 - p_\mu\hat{\gamma}_\mu]^{-1}$$

- Fermion lifetimes:

$$\gamma_s = ImTr\hat{\gamma}_0\hat{\Sigma}^R_s(\epsilon,\epsilon/v) \sim \frac{v^2\Gamma_s}{\epsilon}min[\frac{1}{g},\frac{1}{g^2}] \qquad \gamma_v = ImTr\hat{\gamma}_0\hat{\Sigma}^R_v(\epsilon,\epsilon/v) \sim v\Gamma_v^{1/2}\sqrt{\ln L}$$

- Transport times: $(\varepsilon_F >> \Gamma_{s,v}^{1/2})$

$$\gamma^{tr}_\alpha = \int \frac{d\mathbf{q}}{(2\pi)^2}\delta(\epsilon_F - v|\mathbf{p}+\mathbf{q}|)w_\alpha(\mathbf{q})\sin^2\theta \qquad \gamma^{tr}_s \sim \frac{v^2\Gamma_s}{\epsilon_F}min[1,\frac{1}{g^2}], \quad \gamma^{tr}_v \sim \frac{v^2\Gamma_v}{\epsilon_F}$$

- Can't discriminate between RP and RMF: $\sigma \sim \varepsilon_F/\gamma \sim \varepsilon^2{}_F \sim n$

DVK, 0607174
A.Geim and M.Katsnelson, 0706.2490

Scalar vs vector disorder: characteristic cyclotron rates

- Envelope function of the SdH/dHvA oscillations:

$$\nu(\epsilon|B) = \nu(\epsilon|0) \sum_{n=-\infty}^{\infty} e^{2\pi i n A(\epsilon) - n^2 \delta S_1(\epsilon)}$$

$$\delta S_1(\epsilon) = -\sum_{\alpha=s,v} \ln W_\alpha = \pi[\frac{\Gamma_s}{B} + \frac{\Gamma_v \epsilon^2}{v^2 B^2}]$$

$$\nu(\epsilon|B) \propto \sum_{n=0}^{\infty} \exp[-\pi \frac{(\epsilon^2 - \omega_n^2)^2}{v^2 B (\gamma_s^{cycl})^2 + (\gamma_v^{cycl})^4}]$$

- Characteristic cyclotron times ($\varepsilon >> \Gamma^{1/2}$):

$$\gamma_s^{cycl} \sim v \Gamma_s^{1/2}, \quad \gamma_v^{cycl} \sim (\epsilon^2 v^2 \Gamma_v)^{1/4}$$

- Scalar vs vector disorder: different energy (=density) dependences

Scalar vs vector disorder: decay of Friedel oscillations

- Wave functions' correlation function:

$$L^4 < |\psi^2(\mathbf{r})\psi^2(0)| > -1 = \frac{< Im\hat{G}^R(\epsilon,\mathbf{r}) Im\hat{G}^R(\epsilon,-\mathbf{r}) >}{(\pi\nu(\epsilon))^2}$$
$$\sim (\frac{\gamma_\alpha^{FO}}{\epsilon^2 r})^{1/2} \cos(2\epsilon r) e^{-r\gamma_\alpha^{FO}}$$

- Characteristic rates of the Friedel oscillations' spatial decay:

($\varepsilon >> \Gamma^{1/2}$)

$$\gamma_s^{FO} \sim v\Gamma_s^{1/2}, \quad \gamma_v^{FO} \sim v^{4/3}\frac{\Gamma_v^{2/3}}{\epsilon^{1/3}}$$

$$\delta\rho(r) \propto (\frac{\gamma_\alpha^{FO}}{r^5})^{1/2} \cos(2\epsilon r) e^{-r\gamma_\alpha^{FO}}$$

- STM probe can distinguish between RP and RMF, too.

Long-vs short-range correlated RMF: density of states

- Chiral order parameter:

$$m^2_{\varphi,\theta} = \frac{1}{L^2} < \frac{\delta^2(S-S_0)}{\delta|\varphi,\theta|^2} >_0 = \pm \int D[\varphi,\theta] \cos 2[\varphi,\theta] e^{-S_0} \qquad \ln\frac{m(\epsilon)}{\epsilon} = \frac{1}{2}\int \frac{d^2q}{(2\pi)^2}\frac{q^2 w(q)}{[m^2(\epsilon)+q^2]^2}$$

- Short-range correlated RMF ($\eta=0$): $m(\varepsilon) \sim \varepsilon^{1/z}$

$$\nu_0(\epsilon) \propto \epsilon^{2/z-1} \qquad z = 1 + g \quad \text{for } g<2$$

$$z = (8g)^{1/2} - 1 \quad \text{for } g>2$$

A.Ludwig et al'94

- Long-range correlated RMF ($\eta>0$): $m(\varepsilon) \sim l^{-1}|\ln \varepsilon l|^{-2/\eta}$

$$\nu_\eta(\varepsilon) = \frac{1}{\pi}\mathrm{Im} < \overline{\psi}\psi > = \frac{\partial m^2}{\partial\varepsilon} \sim \frac{1}{\varepsilon l^2 |\ln \varepsilon l|^{2/\eta+1}}$$

Can η be measured directly (STM)?

Quenched Schwinger model ($\eta=1$): **A. Smilga, '92**

Conclusions

•Owing to the linear dispersion and **unscreened** Coulomb interactions, 2D Dirac fermions in graphene are prone to **excitonic pairing** for sufficiently **strong** Coulomb couplings;

-**evidence**: gap eq., MC simulations, experiment??
-**relevance**: intrinsic spectral gap

•**Magnetic field** facilitates an emergence of the fermion mass even at **weak** Coulomb couplings;
-**evidence**: gap eq., experiment?
-**relevance**: tunable gap
•New techniques? AdS/CFT('gravity dual of graphene', in progress).

•Dirac fermions in graphene exhibit novel **disorder effects,** the physically relevant disorder being **of long-range-correlated** nature;
-**evidence**: experiment
-**relevance**: probes for ascertaining the **nature of disorder**