Subtly Broken Symmetries

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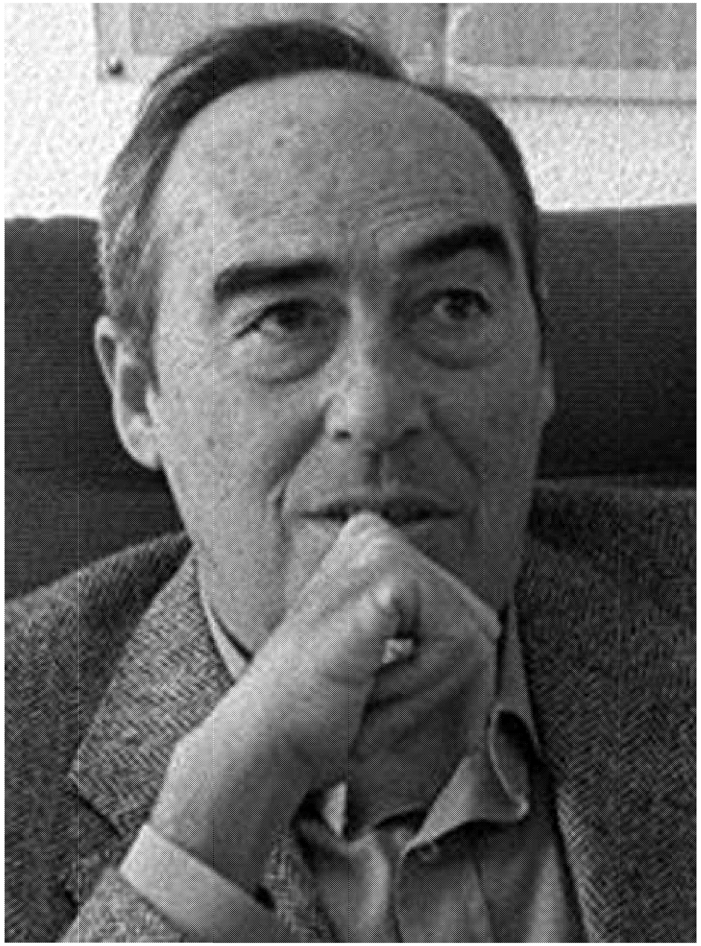
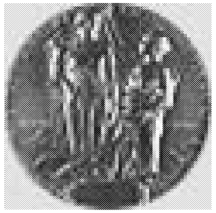
With thanks to Joan Soto





The Nobel Prize in Physics 2008

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

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- Symmetry group of the solution: $H = 1$

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- There are excitations whose energy is arbitrarily close to the ground state solution: magnons or spin waves

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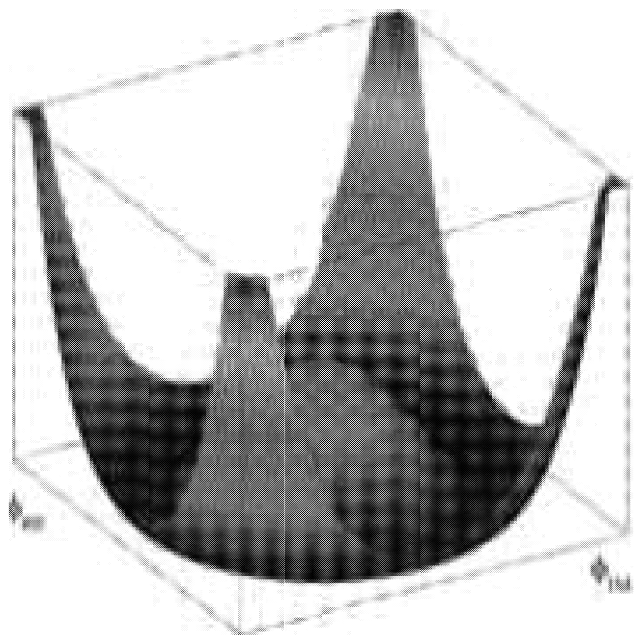
Nambu-Goldstone modes

Nambu (1960), Goldstone (1961)



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- There excitations whose energy is arbitrarily close to the ground state are now quantum states

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with respect to some arbitrary axis.

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- Unbroken group: $H = SO(2)$

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The antiferromagnetic Heisenberg model, also with two broken generators, is described by an effective wave equation, which is real $(c \to v_s)$.

- The two broken generators correspond to two different quasi-particles

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Superconductivity

Autobiography in *Broken Symmetry: Selected Papers of Y. Nambu*, World Scientific

One day before publication of the BCS paper, Bob Schrieffer, still a student, came to Chicago to give a seminar on the BCS theory in progress. . . . I was very much disturbed by the fact that their wave function did not conserve electron number. It did not make sense. . . .At the same time I was impressed by their boldness and tried to understand the problem.

6

I will begin by a short story about my background. I studied physics at the University of Tokyo. I was attracted to particle physics because of the three famous names, Nishina, Tomonaga and Yukawa, who were the founders of particle physics in Japan. But these people were at different institutions than mine. On the other hand, condensed matter physics was pretty good at Tokyo. I got into particle physics only when I came back to Tokyo after the war. In hindsight, though, I must say that my early exposure to condensed matter physics has been quite beneficial to me.

History repeats itself

1960 Midwest Conference in Theoretical Physics, Purdue University

A 'SUPERCONDUCTOR' MODEL OF ELEMENTARY PARTICLES AND ITS CONSEQUENCES by Y. Nambu (University of Chicago)†

(In absence of the author the paper was presented by G. Jona-Lasinio.)

1

In recent years it has become fashionable to apply field-theoretical techniques to the many-body problems one encounters in solid state physics and nuclear physics. This is not surprising because in a quantized field theory there is always the possibility of pair creation (real or virtual), which is essentially a many-body problem. We are familiar with a number of close analogies between ideas and problems in elementary particle theory and the corresponding ones in solid state physics. For example, the Fermi sea of electrons in a metal is analogous to the Dirac sea of electrons in the vacuum, and we speak about electrons and holes in both cases. Some people must have thought of the meson field as something like the shielded Coulomb field. Of course, in elementary particles we have more symmetries and invariance properties than in the other, and blind analogies are often dangerous.

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- But if the vacuum is just the ground state of a quantum mechanical system, why not?
  Nambu, 1960

Linear sigma model

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$$L = i\bar{\psi}_L \not{\partial}\psi_L + i\bar{\psi}_R \not{\partial}\psi_R - g\bar{\psi}_L\Sigma\psi_R - g\bar{\psi}_R\sigma^\dagger\psi_L + V(\Sigma^\dagger\Sigma)$$

$$\Sigma = \sigma I + i\pi^a\tau^a$$

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- $\Sigma = \sigma U, \qquad U = e^{i\tilde{\pi}/f_\pi}$ and G acts as $U \to LUR^\dagger$

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- Fermions become massive: dynamical mass generation

The Nambu–Jona-Lasinio (NJL) model

Y. Nambu, G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)

The Lagrangian of the model is

$$L = -\bar{\psi}\gamma_\mu \partial_\mu \psi + g\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right]$$

It is invariant under ordinary and γ_5 gauge transformations

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$$
$$\psi \to e^{i\alpha\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5}$$

The spectrum of the NJL model

Mass equation

$$\frac{2\pi^2}{g\Lambda^2} = 1 - \frac{m^2}{\Lambda^2} \ln\left(1 + \frac{\Lambda^2}{m^2}\right)$$

where Λ is the invariant cut-off

Spectrum of bound states

nucleon number	mass μ	spin-parity	spectroscopic notation
0	0	0^-	$^1S_0$
0	$2m$	0^+	$^3P_0$
0	$\mu^2 > \frac{8}{3}m^2$	1^-	$^3P_1$
± 2	$\mu^2 > 2m^2$	0^+	$^1S_0$

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- Massless bosons $\rightarrow$ Pions
- Nucleons become eventually massless

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- In two dimensions and in the limit where the number of nucleons is $\to \infty$, the model is fully renormalizable and a bona fide QFT (Gross, Neveu, 1974)

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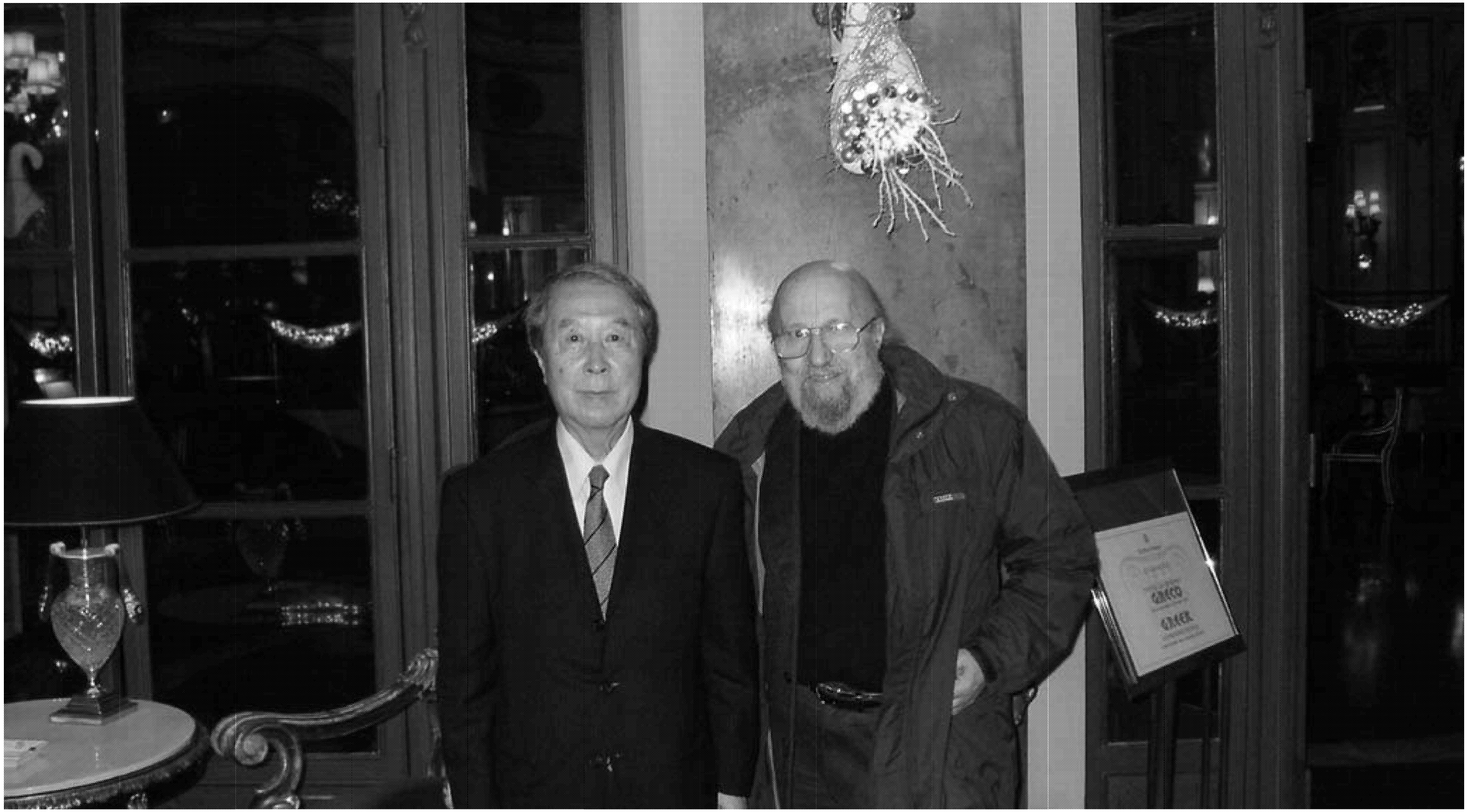
  - Physical results do not depend on the cut-off thanks to the renormalizability of Yang-Mills theories (see below...)



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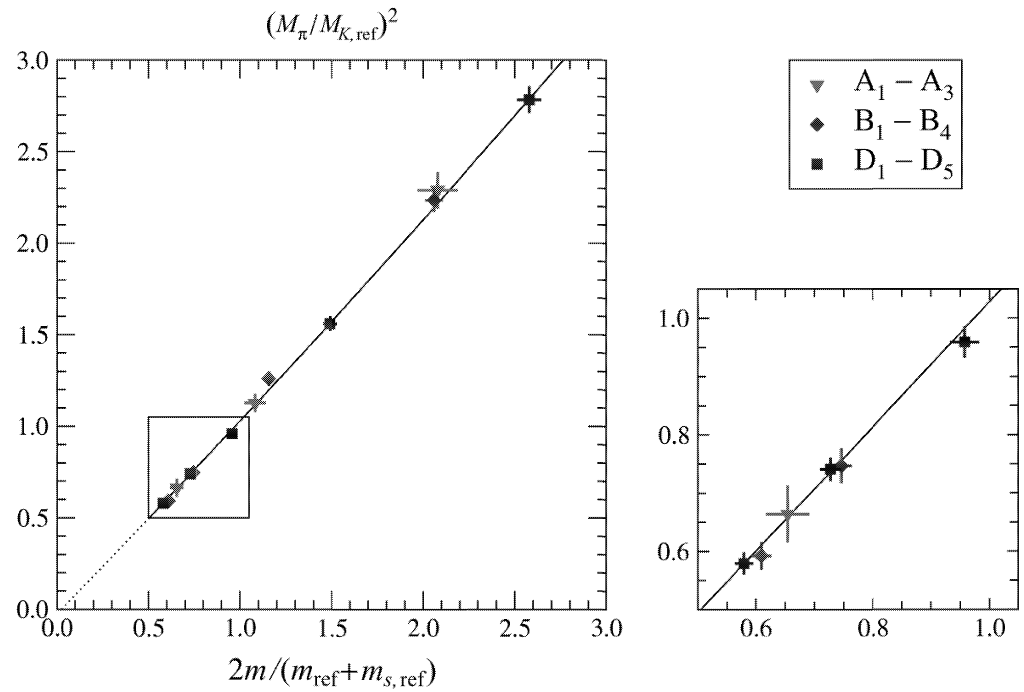

Giusti, 2006

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Nambu's comment

Y. Nambu, preliminary Notes for the Nobel Lecture

In hindsight I regret that I should have explored in more detail the general mechanism of mass generation for the gauge field. But I thought the plasma and the Meissner effect had already established it. I also should have paid more attention to the Ginzburg-Landau theory which was a forerunner of the present Higgs description.

Someone we knew well was in Nambu’s group ...

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Pere Pascual de Sans

Sevilla, 1934 - Barcelona, 2006

Someone we knew well ...

Self-consistent models of strong interactions with chiral symmetry
Y. Nambu, P. Pascual, Nuovo Cimento 30 (1963) 354

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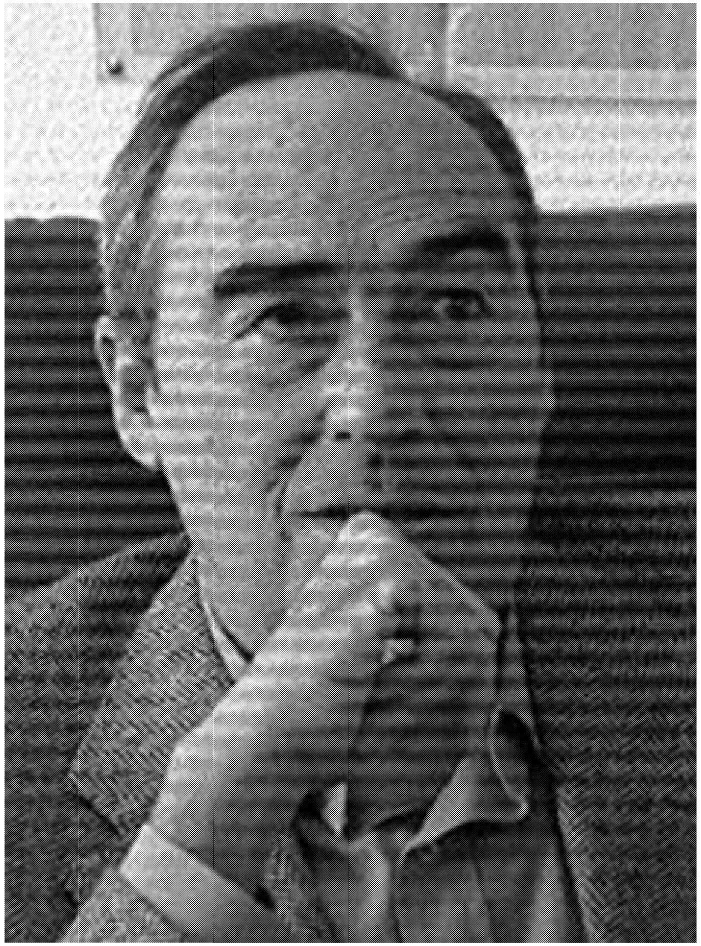
... both of them eventually proved to be crucial papers in the history of particle physics



Revised 9 February 2004
FTUAM 03-19
hep-ph/0312187

Regge analysis of pion-pion (and pion-kaon) scattering for energy $s^{1/2} > 1.4$ GeV

J. R. Peláez

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and

F. J. Ynduráin

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Universidad Autónoma de Madrid,
Canto Blanco,
E-28049, Madrid, Spain.

Abstract

We perform a detailed Regge analysis of NN, πN, KN, $\pi\pi$ and πK scattering. From it, we find expressions that represent the $\pi\pi$ scattering amplitudes with an accuracy of a few percent, for exchange of isospin zero, and $\sim 15\%$ for exchange of isospin 1, and this for energies $s^{1/2} > 1.4\,\mathrm{GeV}$ and for momentum transfers $|t|^{1/2} \lesssim 0.4\,\mathrm{GeV}$. These Regge formulas are perfectly compatible with the low energy ($s^{1/2} \sim 1.4\,\mathrm{GeV}$) scattering amplitudes deduced from $\pi\pi$ phase shift analyses as well as with higher energy ($s^{1/2} \gtrsim 1.4\,\mathrm{GeV}$) experimental $\pi\pi$ cross sections. They are also compatible with NN, KN and πN experimental cross sections using factorization, a property that we check with great precision. This contrasts with results from current phase shift analyses of the $\pi\pi$ scattering amplitude which bear little resemblance to reality in the region $1.4 < s^{1/2} < 2$ GeV, as they are not well defined and increasingly violate a number of physical requirements when the energy grows. πK scattering is also considered, and we present a Regge analysis for these processes valid for energies $s^{1/2} > 1.7$ GeV.

As a byproduct of our analysis we present also a fit of NN, πN and KN cross sections valid from c.m. kinetic energy $E_{\mathrm{kin}} \simeq 1$ GeV to multi TeV energies.

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2 Feb 2008 8:22 a.m.
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Experimental status of the $\pi\pi$ isoscalar S wave at low energy: $f_0(600)$ pole and scattering length

R. García-Martín and J. R. Peláez
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Universidad Complutense de Madrid,
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Departamento de Física Teórica, C-XI
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Abstract

The experimental results obtained in the last few years on kaon decays (K$\rightarrow 2\pi$ and, above all, Ke4 decays) allow a reliable, model independent determination of low energy $\pi\pi$ scattering in the S0 wave. Using them and, eventually, other sets of data, it is possible to give a precise parametrization of the S0 wave as well as to find the scattering length and effective range parameter. One can also perform an extrapolation to the pole of the "σ resonance" [$f_0(600)$]. We obtain the results

$$a_0^{(0)} = 0.233 \pm 0.013\ M_\pi^{-1}, \quad b_0^{(0)} = 0.285 \pm 0.012\ M_\pi^{-3}$$

and, for the σ pole,

$$M_\sigma = 484 \pm 17\ \mathrm{MeV}, \quad \Gamma_\sigma/2 = 255 \pm 10\ \mathrm{MeV}.$$

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Conclusions

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Nature is indeed subtle

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Thanks Paco!