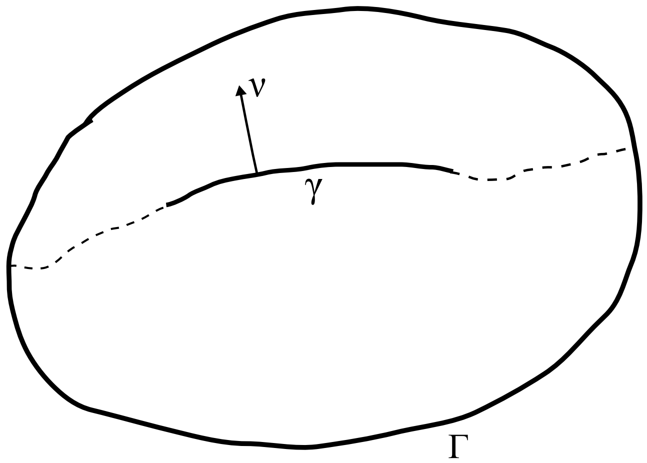

CRACK PROBLEMS WITH OVERLAPPING DOMAINS

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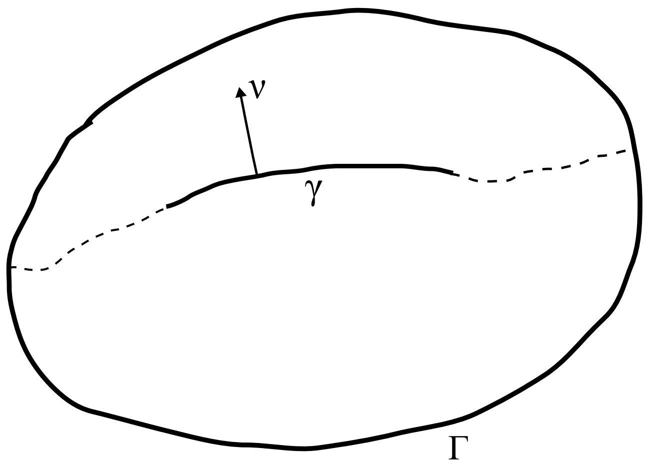
Formulation of crack problem



Find functions $\boldsymbol{u} = (\boldsymbol{u_1}, \boldsymbol{u_2})$, $\boldsymbol{\sigma} = \{\boldsymbol{\sigma_{ij}}\}, \boldsymbol{i}, \boldsymbol{j} = \mathbf{1}, \mathbf{2}$, such that

$$-\mathrm{div}\boldsymbol{\sigma} = \boldsymbol{f} \quad \text{in} \quad \boldsymbol{\Omega_\gamma}\ , \tag{1}$$

$$\boldsymbol{\sigma} = \boldsymbol{A\varepsilon(u)} \quad \text{in} \quad \boldsymbol{\Omega_\gamma}\ , \tag{2}$$

$$\boldsymbol{u} = \mathbf{0} \quad \text{on} \quad \boldsymbol{\Gamma}\ , \tag{3}$$

$$[\boldsymbol{u}]\boldsymbol{\nu} \geq \mathbf{0}, \quad [\boldsymbol{\sigma_\nu}] = \mathbf{0}, \quad [\boldsymbol{u}]\boldsymbol{\nu} \cdot \boldsymbol{\sigma_\nu} = \mathbf{0} \quad \text{on} \quad \boldsymbol{\gamma}\ , \tag{4}$$

$$\boldsymbol{\sigma_\nu} \leq \mathbf{0}, \ \boldsymbol{\sigma_\tau} = \mathbf{0} \quad \text{on} \quad \boldsymbol{\gamma^\pm}\ . \tag{5}$$

Here $\boldsymbol{u}$ displacement vector, $\boldsymbol{\sigma}$ stress tensor, $[\boldsymbol{v}] = \boldsymbol{v}^+ - \boldsymbol{v}^-$

$\boldsymbol{\varepsilon(u) = \{\varepsilon_{ij}(u)\}}$ strain tensor, $\boldsymbol{i, j = 1, 2}$

$\boldsymbol{\varepsilon_{ij}(u) = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}), \ \Omega_\gamma = \Omega \setminus \bar{\gamma}}$

$\boldsymbol{A = \{a_{ijkl}\}}$ known elasticity tensor, $\boldsymbol{i, j, k, l = 1, 2}$

$\boldsymbol{\sigma_\nu = \sigma_{ij}\nu_j\nu_i}$ normal stress, $\boldsymbol{\nu = (\nu_1, \nu_2)}$,

$\boldsymbol{\sigma_\tau}$ tangential stresses, $\boldsymbol{f = (f_1, f_2)}$ external force

DIRECTIONS OF INVESTIGATIONS

1. Solvability of boundary value problems, solution smoothness (elastic, viscoelastic, thermoelastic, electro-thermoelastic bodies)

2. Dependence on parameters, shape sensitivity analysis, differentiability of energy functionals

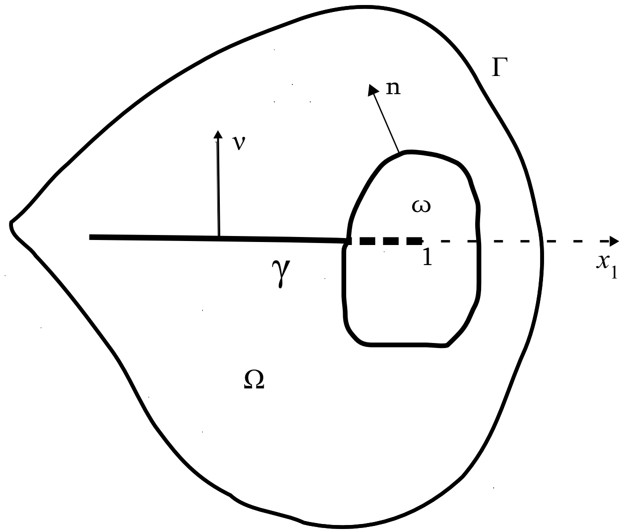
3. Optimal control problems

4. Smooth domain method. Fictitious domain method

5. Overlapping domain problems

6. Rigid inclusions in elastic bodies

"Patch" problem



$$-\mathrm{div}\boldsymbol{\sigma}^{\delta} = \boldsymbol{f} \quad \text{in} \quad \boldsymbol{\Omega}_{\gamma} \setminus \boldsymbol{\partial\omega}, \tag{6}$$

$$\boldsymbol{\sigma}^{\delta} = \boldsymbol{A\varepsilon}(\boldsymbol{u}^{\delta}) \quad \text{in} \quad \boldsymbol{\Omega}_{\gamma}, \tag{7}$$

$$-\mathrm{div}\boldsymbol{p}^{\delta} = \boldsymbol{0} \quad \text{in} \quad \boldsymbol{\omega}, \tag{8}$$

$$\boldsymbol{p}^{\delta} = \frac{\boldsymbol{1}}{\boldsymbol{\delta}}\boldsymbol{B\varepsilon}(\boldsymbol{v}^{\delta}) \quad \text{in} \quad \boldsymbol{\omega}, \tag{9}$$

$$\boldsymbol{u}^{\delta} = \boldsymbol{0} \quad \text{on} \quad \boldsymbol{\Gamma}, \tag{10}$$

$$[\boldsymbol{u}^{\delta}]\boldsymbol{\nu} \geq \boldsymbol{0}, \ [\boldsymbol{\sigma}^{\delta}_{\boldsymbol{\nu}}] = \boldsymbol{0}, \ \boldsymbol{\sigma}^{\delta}_{\boldsymbol{\nu}} \leq \boldsymbol{0}, \ \boldsymbol{\sigma}^{\delta}_{\boldsymbol{\tau}} = \boldsymbol{0}, \ \boldsymbol{\sigma}^{\delta}_{\boldsymbol{\nu}} \cdot [\boldsymbol{u}^{\delta}]\boldsymbol{\nu} = \boldsymbol{0} \quad \text{on} \quad \boldsymbol{\Gamma_0}, \tag{11}$$

$$\boldsymbol{u}^{\delta} = \boldsymbol{v}^{\delta}, \ [\boldsymbol{\sigma}^{\delta}\boldsymbol{n}] = \boldsymbol{p}^{\delta}\boldsymbol{n} \quad \text{on} \quad \boldsymbol{\partial\omega}. \tag{12}$$

Limit problem

$$-\mathrm{div}\boldsymbol{\sigma} = \boldsymbol{f} \ \text{ in } \ \boldsymbol{\Omega}_\gamma \setminus \boldsymbol{\partial\omega}, \tag{13}$$

$$\boldsymbol{\sigma} = \boldsymbol{A\varepsilon}(\boldsymbol{u}) \ \text{ in } \ \boldsymbol{\Omega}_\gamma, \tag{14}$$

$$\boldsymbol{u} = \boldsymbol{0} \ \text{ on } \ \boldsymbol{\Gamma}, \tag{15}$$

$$\boldsymbol{u} = \boldsymbol{\rho}_0 \ \text{ on } \ \boldsymbol{\partial\omega}, \tag{16}$$

$$[\boldsymbol{u}]\boldsymbol{\nu} \geq \boldsymbol{0},\ [\boldsymbol{\sigma}_\nu] = \boldsymbol{0},\ \boldsymbol{\sigma}_\nu \leq \boldsymbol{0},\ \boldsymbol{\sigma}_\tau = \boldsymbol{0},\ \boldsymbol{\sigma}_\nu \cdot [\boldsymbol{u}]\boldsymbol{\nu} = \boldsymbol{0} \ \text{ on } \ \boldsymbol{\Gamma}_0, \tag{17}$$

$$\int\limits_{\partial\omega} [\boldsymbol{\sigma n}]\boldsymbol{\rho} = \boldsymbol{0} \ \ \forall \boldsymbol{\rho} \in \boldsymbol{R}(\boldsymbol{\omega}), \tag{18}$$

where

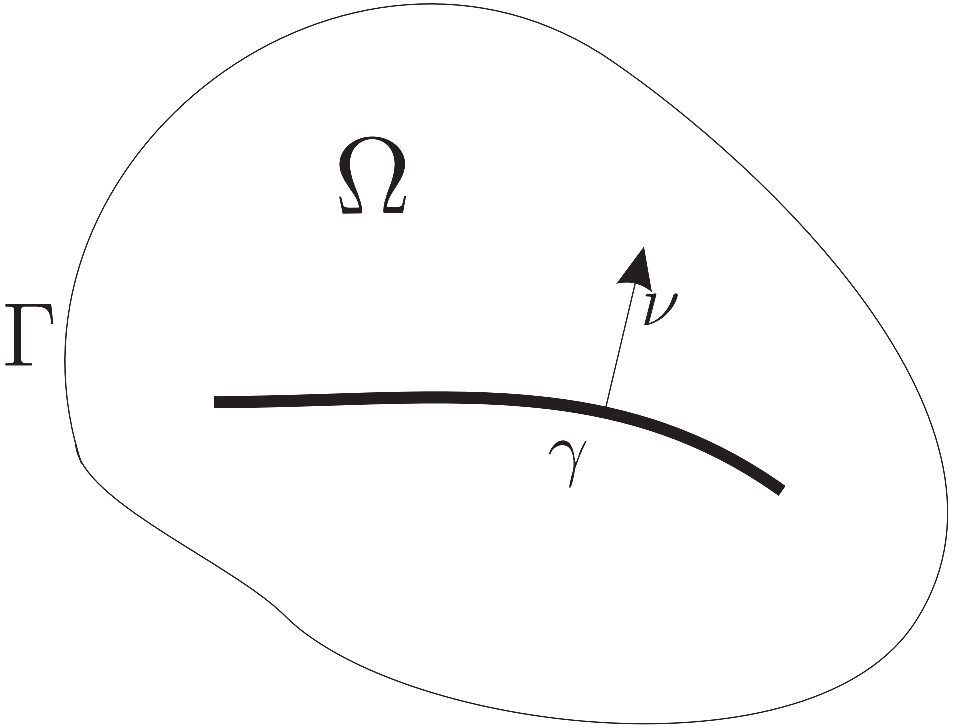
$$\boldsymbol{R}(\boldsymbol{\omega}) = \{\boldsymbol{\rho} = (\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \mid \boldsymbol{\rho}(\boldsymbol{x}) = \boldsymbol{Bx} + \boldsymbol{C},\ \boldsymbol{x} \in \boldsymbol{\omega}\},$$

$$\boldsymbol{B} = \begin{pmatrix} \boldsymbol{0} & \boldsymbol{b} \\ -\boldsymbol{b} & \boldsymbol{0} \end{pmatrix},\ \boldsymbol{C} = (\boldsymbol{c}^1, \boldsymbol{c}^2);\ \boldsymbol{b}, \boldsymbol{c}^1, \boldsymbol{c}^2 = \boldsymbol{const.}$$

Formulas for the derivative

$$\boldsymbol{G}^{\delta} = \frac{1}{2}\int\limits_{\Omega_\gamma} \{\mathrm{div}\boldsymbol{V}\cdot\boldsymbol{\sigma}_{ij}(\boldsymbol{u}^{\delta})\boldsymbol{\varepsilon}_{ij}(\boldsymbol{u}^{\delta}) - 2\boldsymbol{\sigma}_{ij}(\boldsymbol{u}^{\delta})\boldsymbol{E}_{ij}(\boldsymbol{V};\boldsymbol{u}^{\delta})\} - \int\limits_{\Omega_\gamma} \mathrm{div}(\boldsymbol{V}\boldsymbol{f}_i)\boldsymbol{u}_i^{\delta}. \tag{19}$$

$$\boldsymbol{G} = \frac{1}{2}\int\limits_{\Omega_\gamma} \{\mathrm{div}\boldsymbol{V}\cdot\boldsymbol{\sigma}_{ij}(\boldsymbol{u})\boldsymbol{\varepsilon}_{ij}(\boldsymbol{u}) - 2\boldsymbol{\sigma}_{ij}(\boldsymbol{u})\boldsymbol{E}_{ij}(\boldsymbol{V};\boldsymbol{u})\} - \int\limits_{\Omega_\gamma} \mathrm{div}(\boldsymbol{V}\boldsymbol{f}_i)\boldsymbol{u}_i. \tag{20}$$



Thin rigid inclusion with delamination

Find functions $\boldsymbol{u} = (\boldsymbol{u_1}, \boldsymbol{u_2}), \boldsymbol{\rho_0} \in \boldsymbol{R}(\boldsymbol{\gamma}),\ \boldsymbol{\sigma} = \{\boldsymbol{\sigma_{ij}}\}, \boldsymbol{i}, \boldsymbol{j} = \mathbf{1}, \mathbf{2},$ such that

$$-\mathrm{div}\boldsymbol{\sigma} = \boldsymbol{f} \quad \text{in} \quad \boldsymbol{\Omega_\gamma}, \tag{21}$$

$$\boldsymbol{\sigma} - \boldsymbol{A\varepsilon}(\boldsymbol{u}) = \mathbf{0} \quad \text{in} \quad \boldsymbol{\Omega_\gamma}, \tag{22}$$

$$\boldsymbol{u} = \mathbf{0} \quad \text{on} \quad \boldsymbol{\Gamma}, \tag{23}$$

$$[\boldsymbol{u}]\boldsymbol{\nu} \geq \mathbf{0},\ \boldsymbol{u}^- = \boldsymbol{\rho_0},\ \boldsymbol{\sigma}_{\boldsymbol{\nu}}^+ \leq \mathbf{0},\ \boldsymbol{\sigma}_{\boldsymbol{\tau}}^+ = \mathbf{0} \quad \text{on} \quad \boldsymbol{\gamma}, \tag{24}$$

$$\boldsymbol{\sigma}_{\boldsymbol{\nu}}^+ \cdot [\boldsymbol{u}]\boldsymbol{\nu} = \mathbf{0} \quad \text{on} \quad \boldsymbol{\gamma}, \tag{25}$$

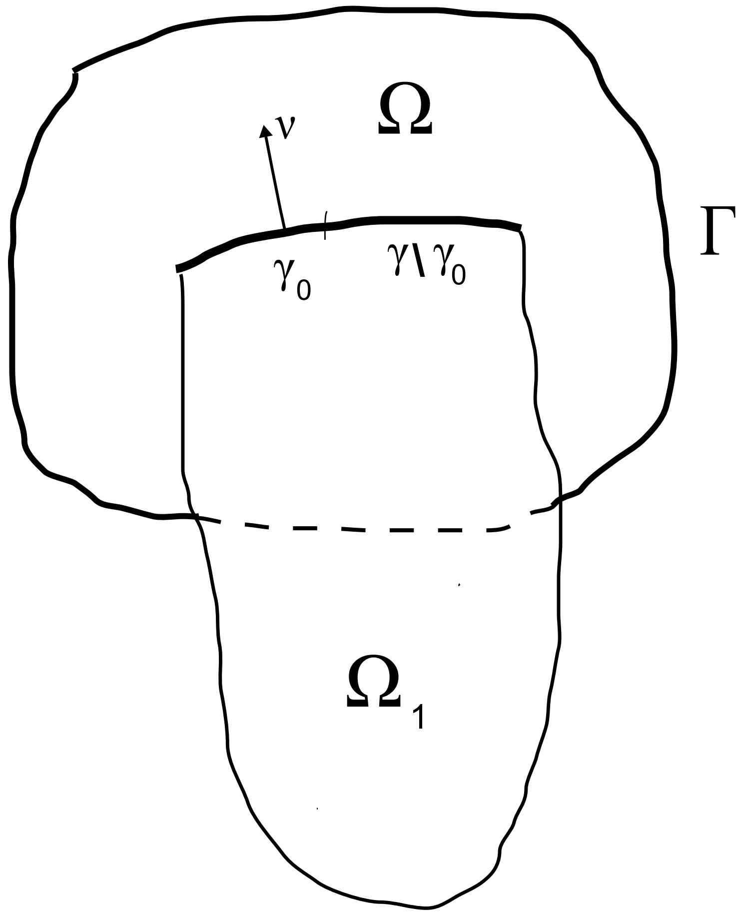
$$\int\limits_{\gamma} [\boldsymbol{\sigma\nu}]\boldsymbol{\rho} = \mathbf{0} \quad \forall \boldsymbol{\rho} \in \boldsymbol{R}(\boldsymbol{\gamma}), \tag{26}$$

where

$$\boldsymbol{R}(\boldsymbol{\gamma}) = \{\boldsymbol{\rho} = (\boldsymbol{\rho_1}, \boldsymbol{\rho_2}) \mid \boldsymbol{\rho}(\boldsymbol{x}) = \boldsymbol{Bx} + \boldsymbol{C},\ \boldsymbol{x} \in \boldsymbol{\gamma}\}$$

$$\boldsymbol{B} = \begin{pmatrix} \mathbf{0} & \boldsymbol{b} \\ -\boldsymbol{b} & \mathbf{0} \end{pmatrix},\ \boldsymbol{C} = (\boldsymbol{c}^1, \boldsymbol{c}^2);\ \boldsymbol{b}, \boldsymbol{c}^1, \boldsymbol{c}^2 = \boldsymbol{const.}$$

Bilayer structure



$$-\mathrm{div}\boldsymbol{\sigma} = \boldsymbol{f} \ \text{ in } \ \boldsymbol{\Omega_\gamma}, \tag{27}$$

$$\boldsymbol{\sigma} = \boldsymbol{A\varepsilon(u)} \ \text{ in } \ \boldsymbol{\Omega_0}, \tag{28}$$

$$-\mathrm{div}\boldsymbol{p} = \boldsymbol{g} \ \text{ in } \ \boldsymbol{\Omega_1}, \tag{29}$$

$$\boldsymbol{p} = \boldsymbol{B\varepsilon(v)} \ \text{ in } \ \boldsymbol{\Omega_1}, \tag{30}$$

$$\boldsymbol{u} = \boldsymbol{0} \ \text{ on } \ \boldsymbol{\Gamma}, \tag{31}$$

$$\boldsymbol{v} = \boldsymbol{0} \ \text{ on } \ \boldsymbol{\partial\Omega_1 \setminus \gamma}, \tag{32}$$

$$\boldsymbol{u} = \boldsymbol{v}, \ \ [\boldsymbol{\sigma\nu}] = \boldsymbol{p\nu} \ \text{ on } \ \boldsymbol{\gamma \setminus \gamma_0}, \tag{33}$$

$$[\boldsymbol{u}]\boldsymbol{\nu} \geq \boldsymbol{0}, \ \boldsymbol{u}^- = \boldsymbol{v}, \ \boldsymbol{\sigma}_{\boldsymbol{\nu}}^+ \leq \boldsymbol{0}, \ \boldsymbol{\sigma}_{\boldsymbol{\tau}}^+ = \boldsymbol{0} \ \text{ on } \ \boldsymbol{\gamma_0}, \tag{34}$$

$$\boldsymbol{\sigma}_{\boldsymbol{\nu}}^+ = \boldsymbol{\sigma}_{\boldsymbol{\nu}}^- + \boldsymbol{p}_{\nu}, \ \boldsymbol{\sigma}_{\boldsymbol{\tau}}^- + \boldsymbol{p}_{\boldsymbol{\tau}} = \boldsymbol{0}, \ \boldsymbol{\sigma}_{\boldsymbol{\nu}}^+ \cdot [\boldsymbol{u}]\boldsymbol{\nu} = \boldsymbol{0} \ \text{ on } \ \boldsymbol{\gamma_0}. \tag{35}$$

Limit problem

Find functions $\boldsymbol{u}(\boldsymbol{x}) = (\boldsymbol{u}_1(\boldsymbol{x}), \boldsymbol{u}_2(\boldsymbol{x})), \boldsymbol{\rho}_0 \in \boldsymbol{R}(\boldsymbol{\Omega}_1), \boldsymbol{x} \in \boldsymbol{\Omega}_0,$ such that

$$-\mathrm{div}\boldsymbol{\sigma} = \boldsymbol{f} \ \text{ in } \ \boldsymbol{\Omega}_\gamma, \tag{36}$$

$$\boldsymbol{\sigma} = \boldsymbol{A}\boldsymbol{\varepsilon}(\boldsymbol{u}) \ \text{ in } \ \boldsymbol{\Omega}_0, \tag{37}$$

$$\boldsymbol{u} = \boldsymbol{0} \ \text{ on } \ \boldsymbol{\Gamma}, \tag{38}$$

$$\boldsymbol{u} = \boldsymbol{\rho}_0 \ \text{ on } \ \boldsymbol{\gamma} \setminus \boldsymbol{\gamma}_0, \tag{39}$$

$$(\boldsymbol{u}^+ - \boldsymbol{\rho}_0)\boldsymbol{\nu} \geq \boldsymbol{0}, \ \boldsymbol{u}^- = \boldsymbol{\rho}_0, \ \ \boldsymbol{\sigma}_{\boldsymbol{\nu}}^+ \leq \boldsymbol{0}, \ \boldsymbol{\sigma}_{\boldsymbol{\tau}}^+ = \boldsymbol{0} \quad \text{on} \quad \boldsymbol{\gamma}_0, \tag{40}$$

$$\boldsymbol{\sigma}_{\boldsymbol{\nu}}^+ \cdot (\boldsymbol{u}^+ - \boldsymbol{\rho}_0)\boldsymbol{\nu} = \boldsymbol{0} \ \text{ on } \ \boldsymbol{\gamma}_0. \tag{41}$$

$$\int_\gamma [\boldsymbol{\sigma}\boldsymbol{\nu}]\boldsymbol{\rho} = \boldsymbol{0} \quad \forall \boldsymbol{\rho} \in \boldsymbol{R}(\boldsymbol{\Omega}_1). \tag{42}$$