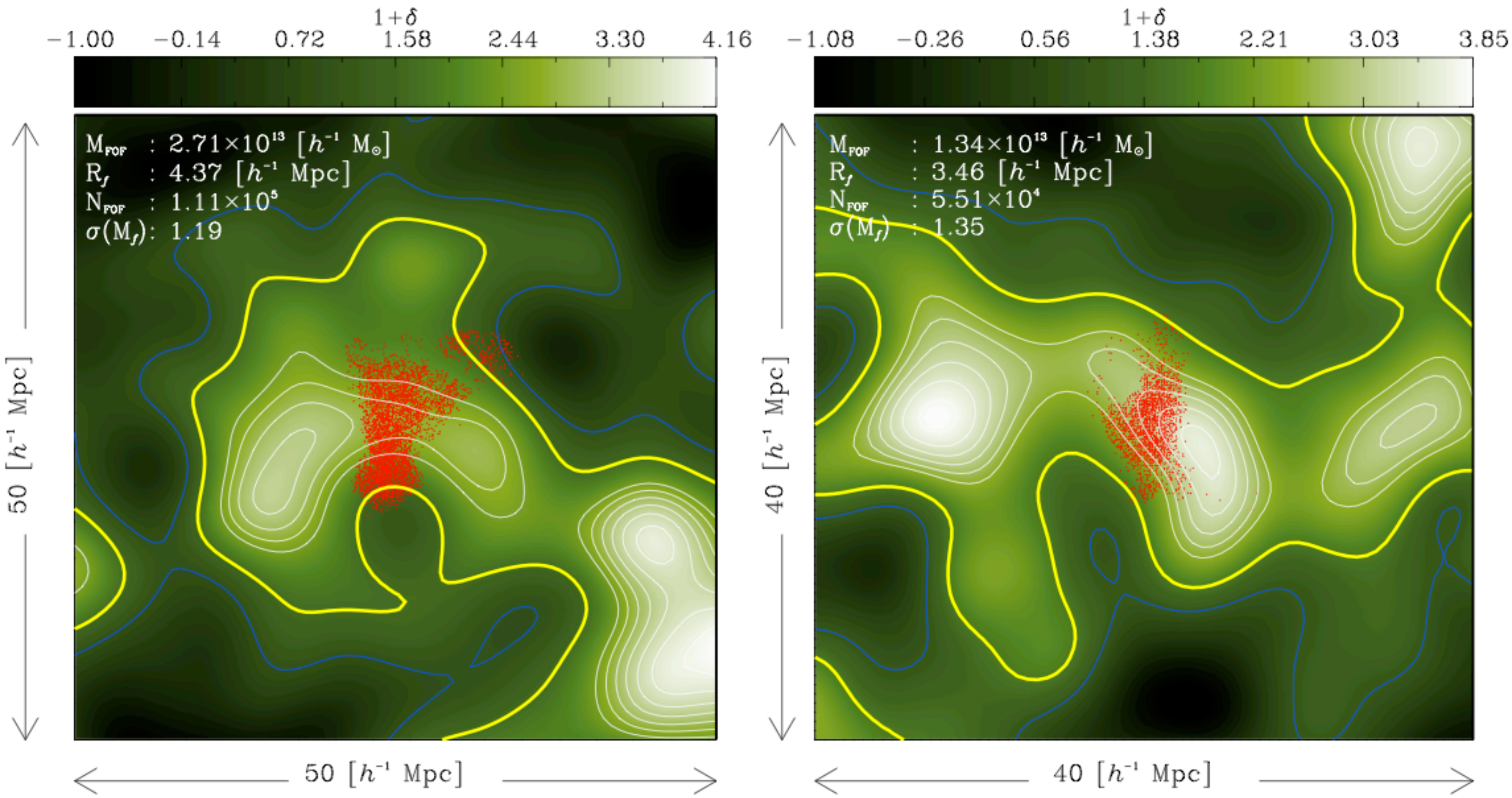
Halo biasing: peaks, scale dependence and primordial non-Gaussianity



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Overview

- Do dark-matter halos form out of high density peaks?
  (Ludlow & Porciani 2010)

- A new physical model for their scale-dependent bias
  (Elia, Kulkarni, Porciani, Pietroni & Matarrese 2010)

- Halo bias and primordial non-Gaussianity
  (Giannantonio & Porciani 2010, Phys. Rev. D, 81.063530)

The peaks formalism

- Key assumption: Halos of mass M form out of density maxima in the linear density field smoothed on the scale M where the peak height exceeds some threshold

- Combining this hypothesis with a dynamical model for the collapse of the density peaks determines the linear bias of the halos as a function of mass (Kaiser 1984, Cole & Kaiser 1989, Mo & White 1996, Desjacques 2008)

- And also their number density (Peacock & Heavens 1985, Bardeen et al. 1986, Bond & Myers 1996, Sheth & Tormen 1999)

THE ASTROPHYSICAL JOURNAL, **327**: 507–525, 1988 April 15

THE FORMATION OF DARK HALOS IN A UNIVERSE DOMINATED BY COLD DARK MATTER

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Received 1987 May 22; accepted 1987 October 13

N-body simulation: 64^3 particles, 14 comoving Mpc box, from z=6 to z=0

Conclusion: "The locations at which halos form are closely related to the peaks of a suitably smoothed representation of the initial *linear* density field"

Mon. Not. R. Astron. Soc. **265**, 689–705 (1993)

Galaxy formation and the peaks formalism

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Accepted 1993 June 9. Received 1993 May 5; in original form 1992 September 28

N-body simulation: 144^3 particles, 100 comoving Mpc box, from z=69 to z=0

Conclusion: "Peaks in the linear density field are not good indicators of the sites of galaxy formation as determined by the dissipationless collapse of halos"

Mon. Not. R. Astron. Soc. **332,** 339–351 (2002)

Testing tidal-torque theory – II. Alignment of inertia and shear and the characteristics of protohaloes

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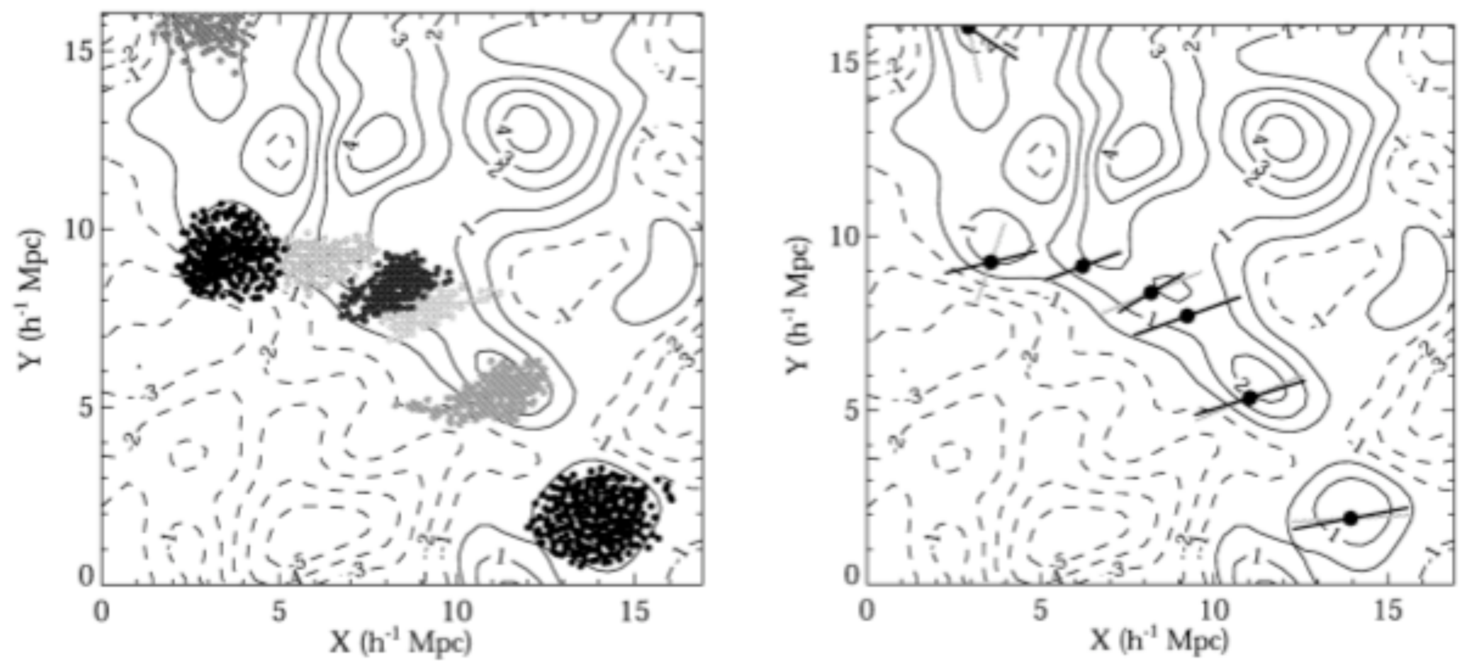
Accepted 2001 December 21. Received 2001 December 19; in original form 2001 May 9

N-body simulation: 256^3 particles, 170 comoving Mpc box, from z=50 to z=0

Conclusion: “The centres of protohalos tend to lie in ~1σ overdensity regions, but their association with linear overdensity maxima smoothed on galactic scales is vague: only ~40% of the halos contain peaks”

A few examples



Porciani, Dekel, Hoffman 2002

CDM halos vs density peaks

Ludlow & Porciani

Our new study

- N-body simulations: 1024^3 particles, 150 and 1200 h^{-1} Mpc boxes, WMAP5 cosmology (from Pillepich, Porciani & Hahn 2010)

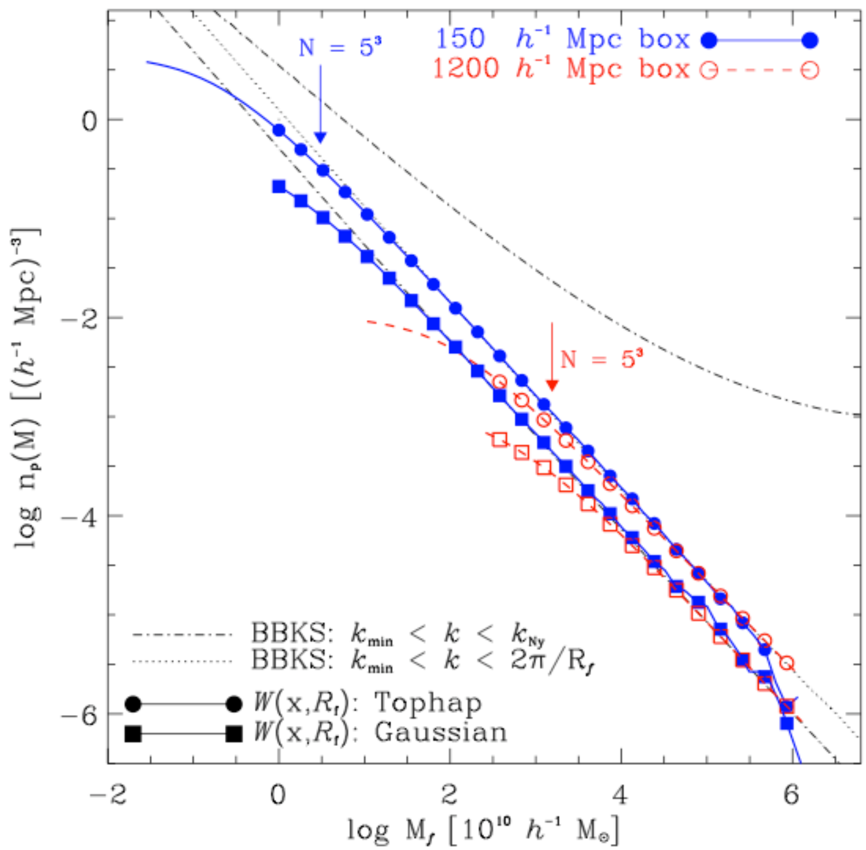
- Halos identified with different finders and traced back to the initial conditions

- Linear density maxima identified by locating grid cells that are denser than all 26 neighboring points after smoothing (using Gaussian or top hat windows) with 60 different filtering radii equally spaced in log

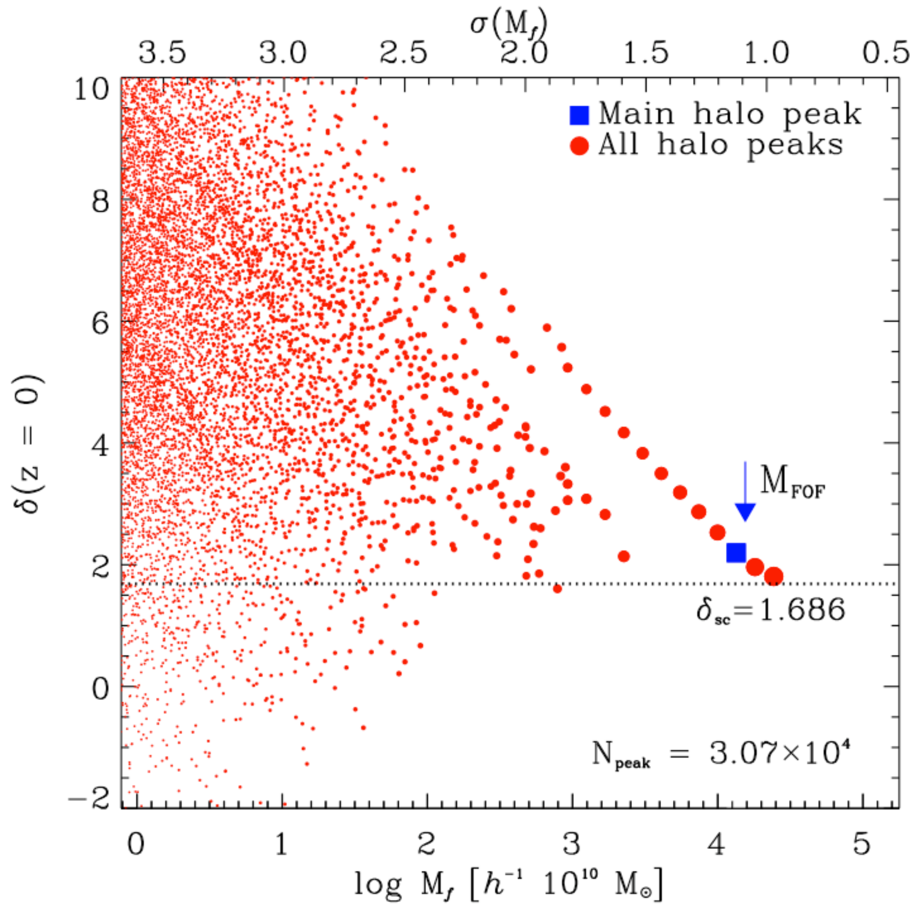
- Their evolution is followed by tagging the particle nearest to each peak grid point

- The subset of particles in a halo is scanned for matchings in the list of peak particles (the ejection of particles during mergers is also accounted for)

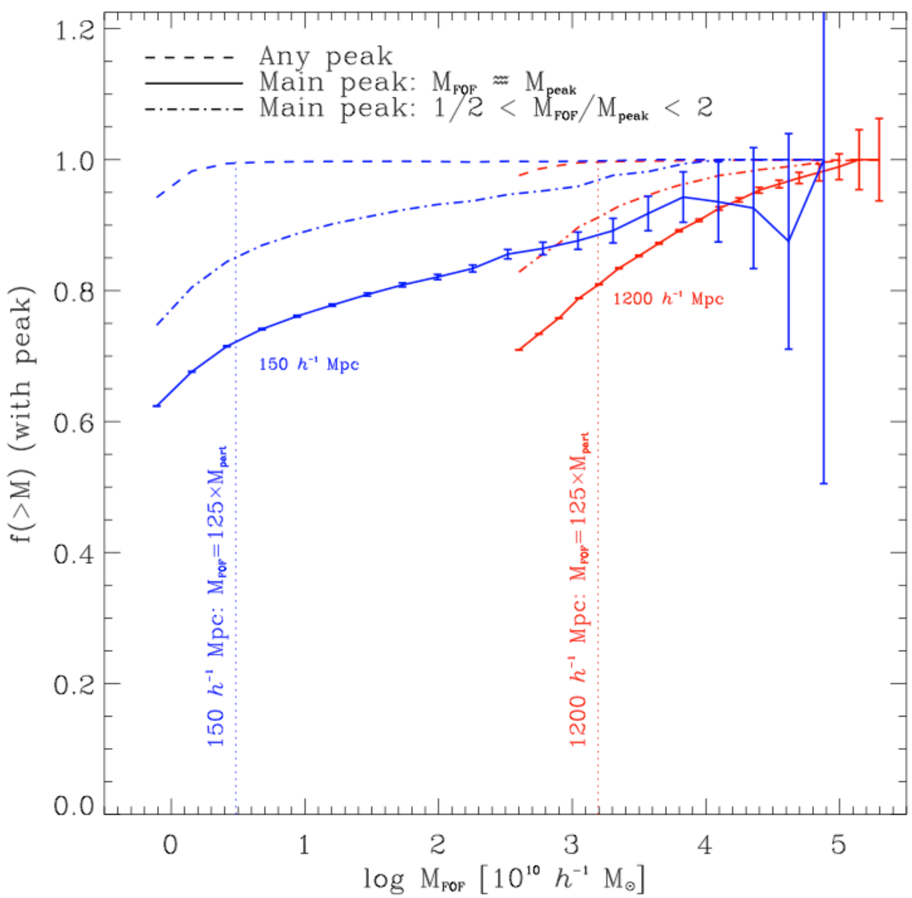
Density maxima in a box



Density peaks in a proto-halo

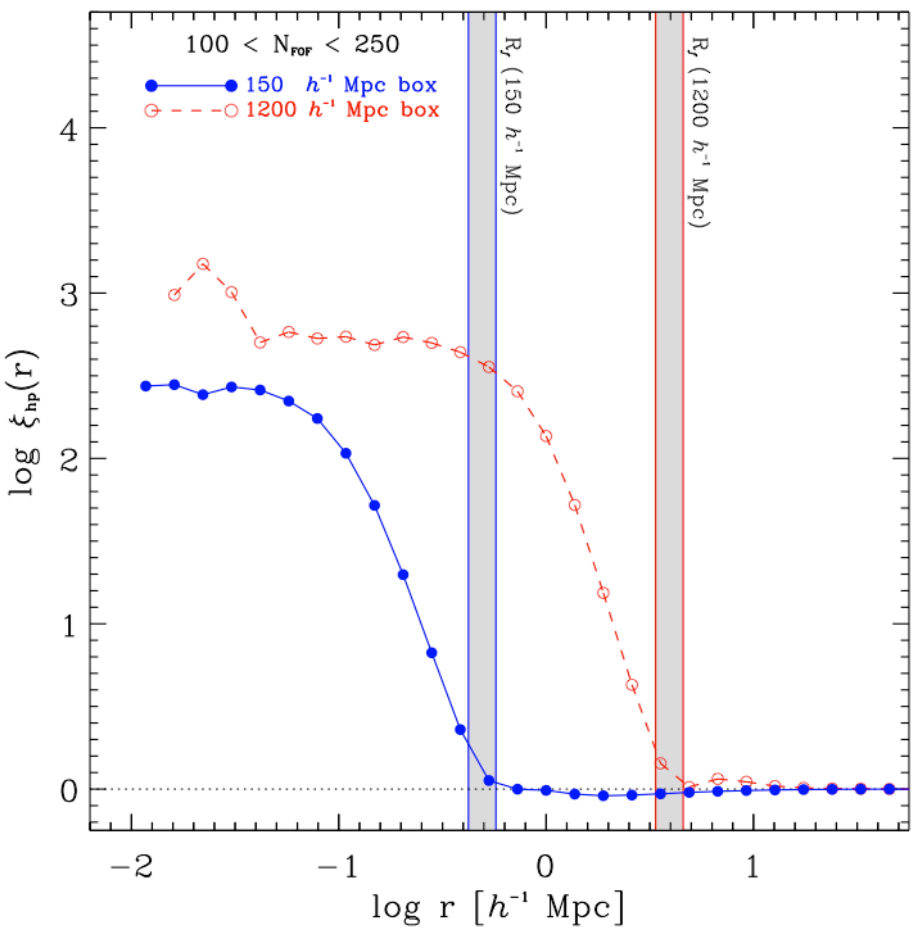


Peaks within protohalos: statistics

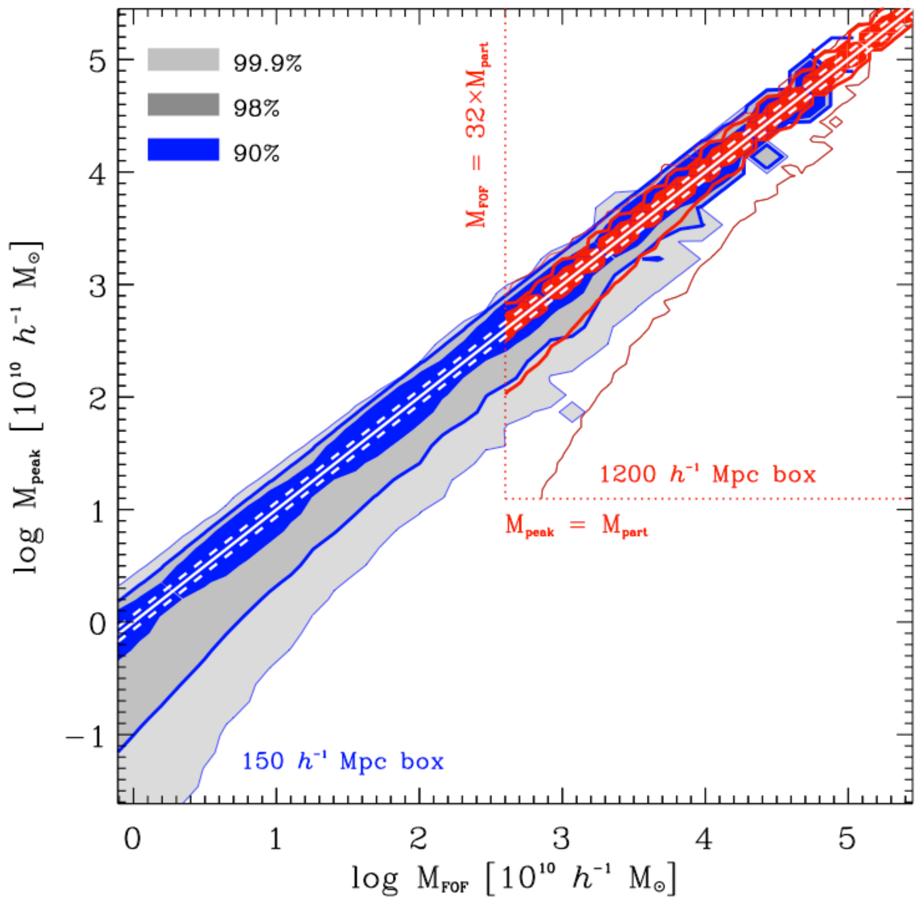


The fraction of halos with $M > 125\ m_{part}$ containing a main peak with $0.5 < M_{peak}/M < 2$ is 85% - 92%

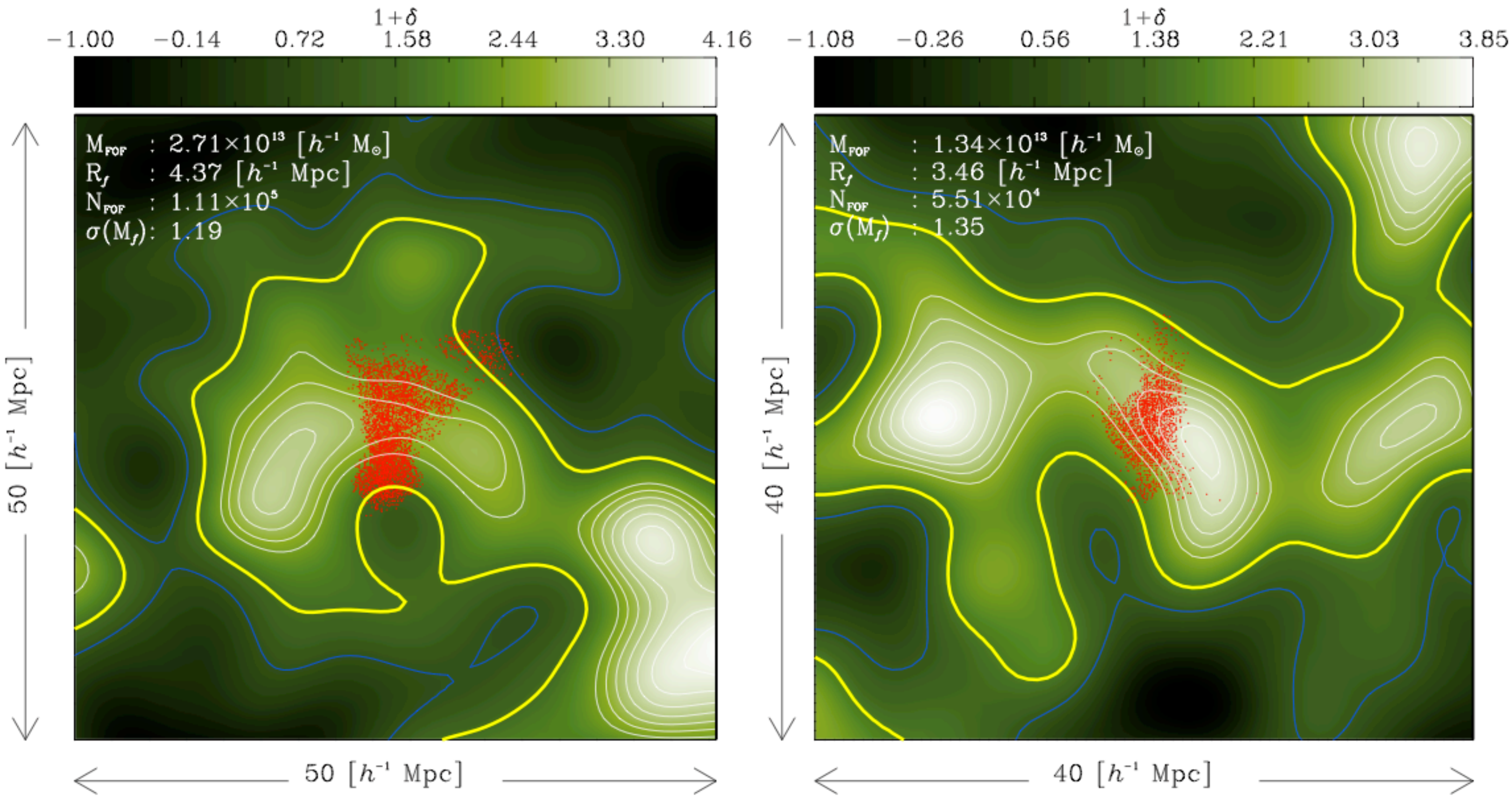
Are proto-halos and peaks correlated?



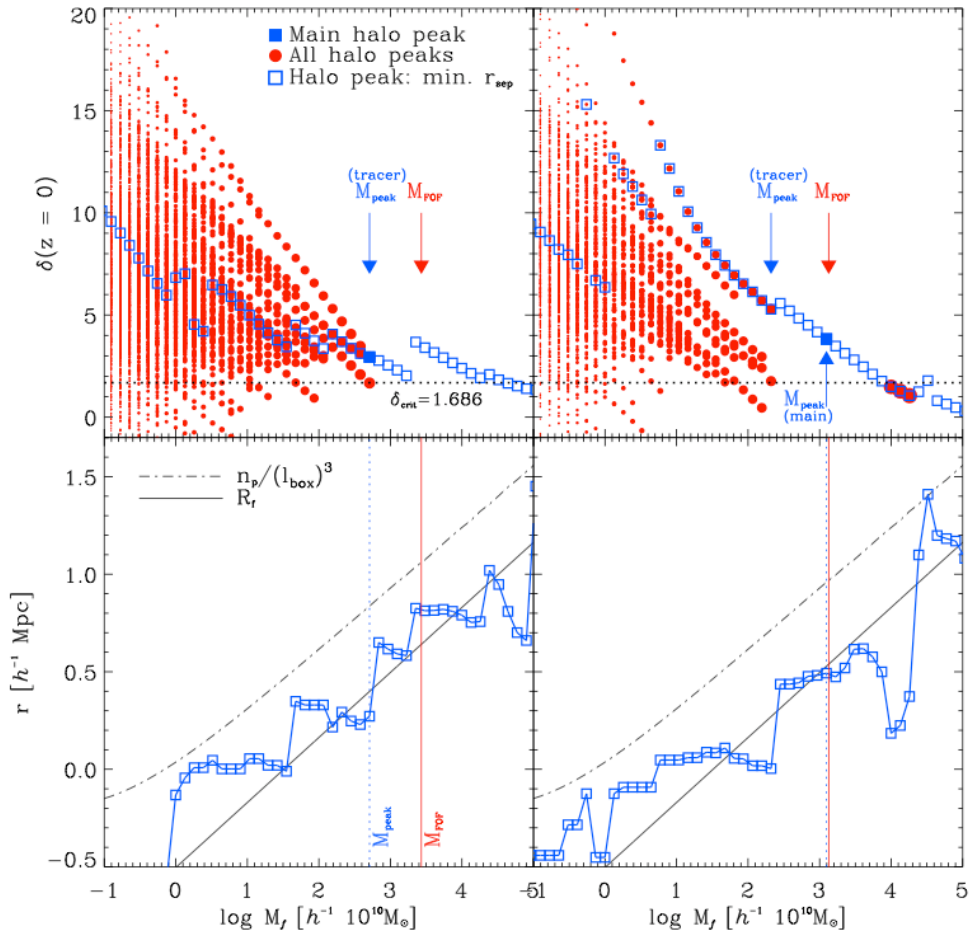
Halos and peaks: statistics



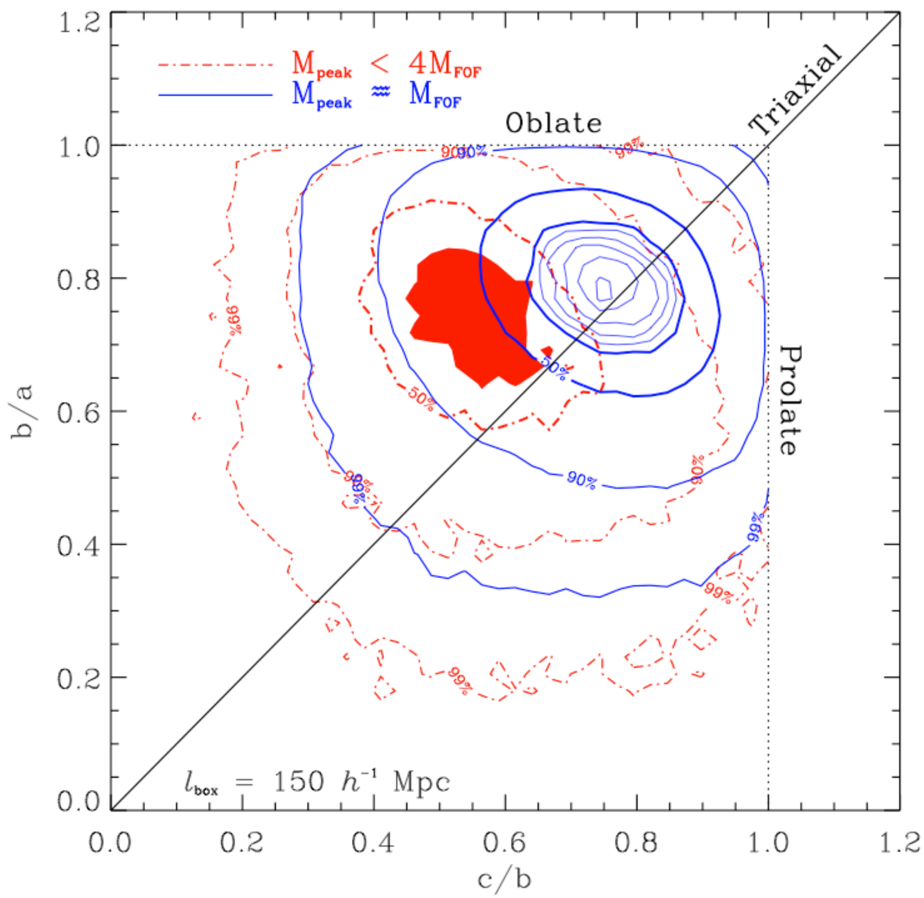
Some examples



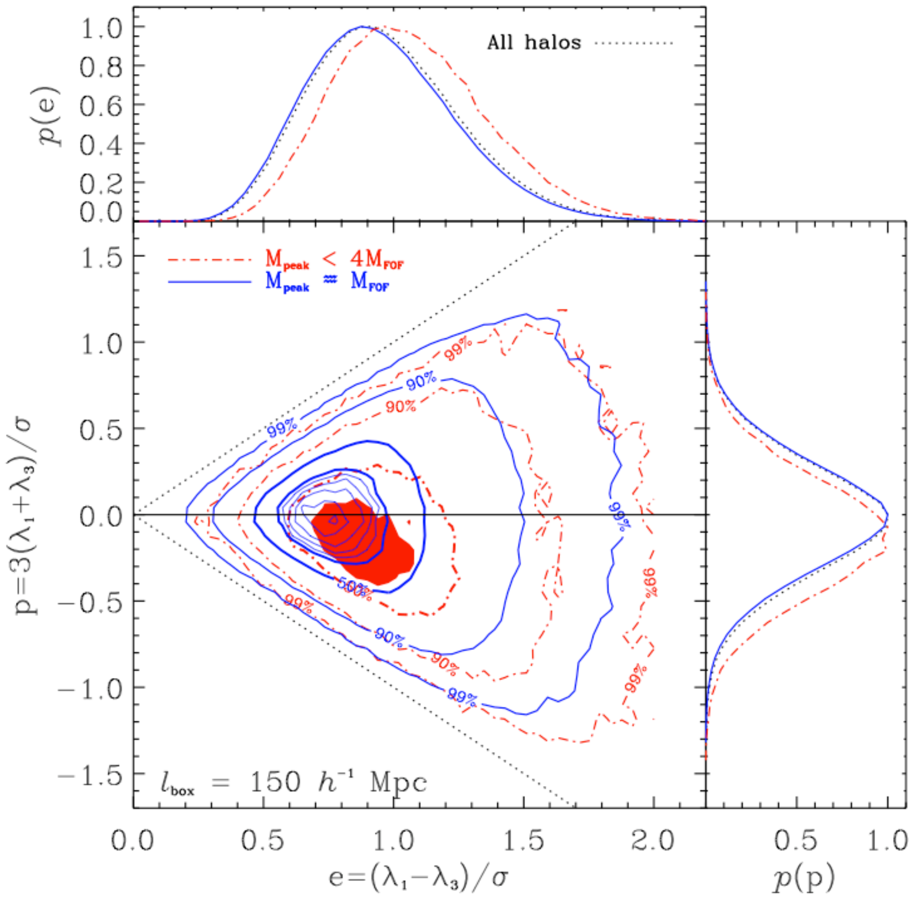
Are they really “peakless”?



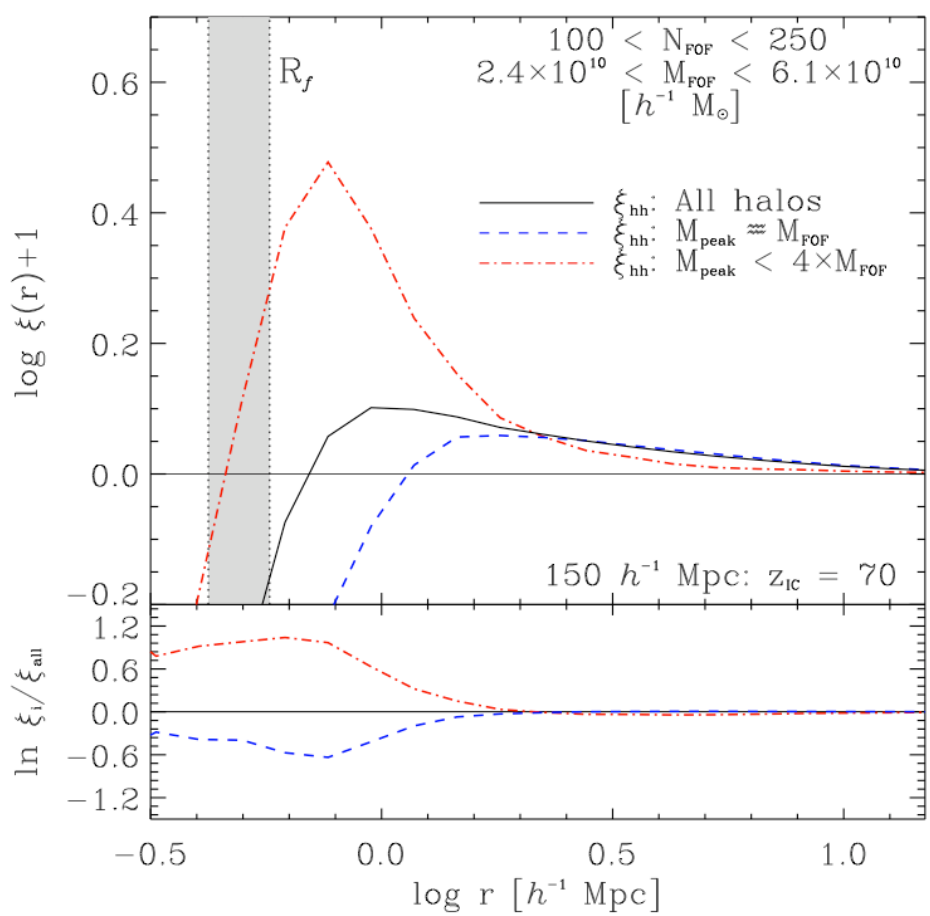
Lagrangian shapes



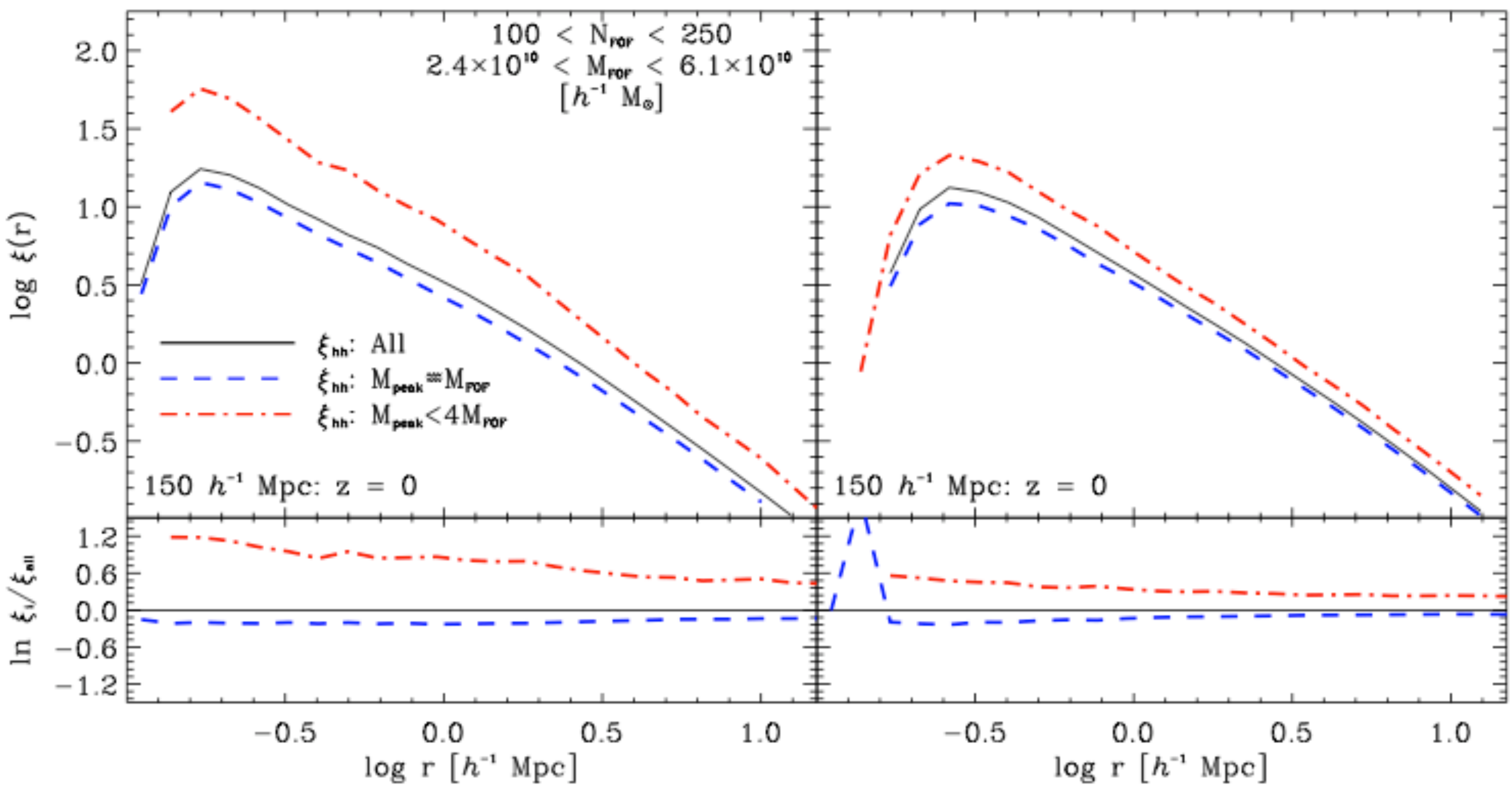
Tides and velocity shear



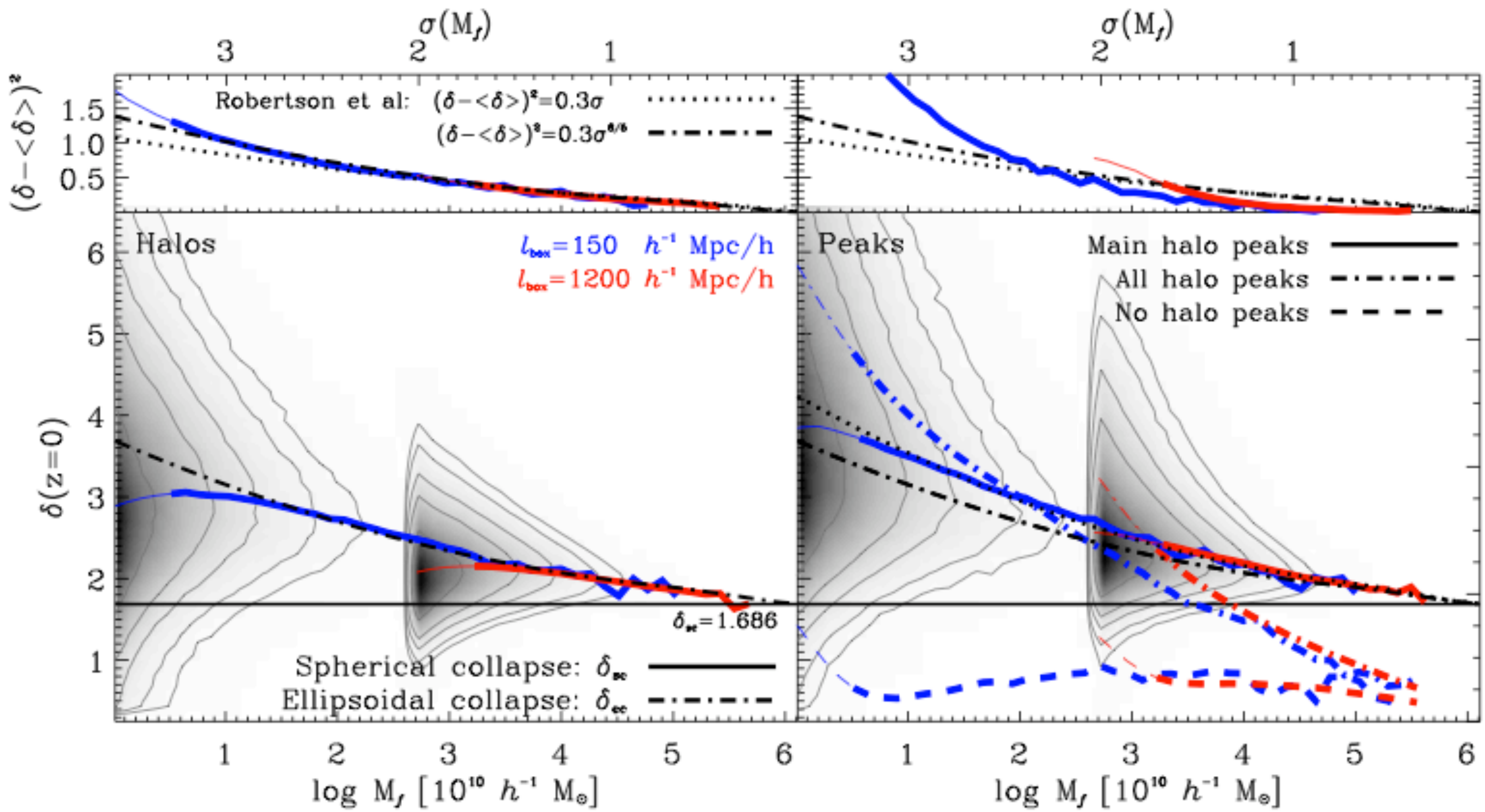
Lagrangian autocorrelation function



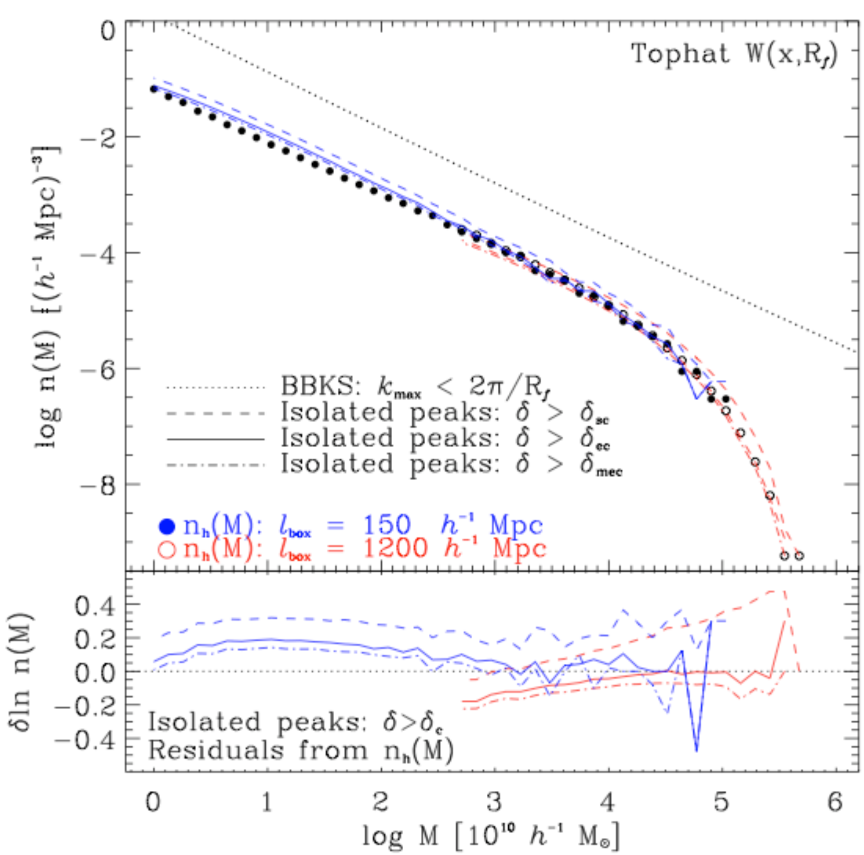
Eulerian correlations



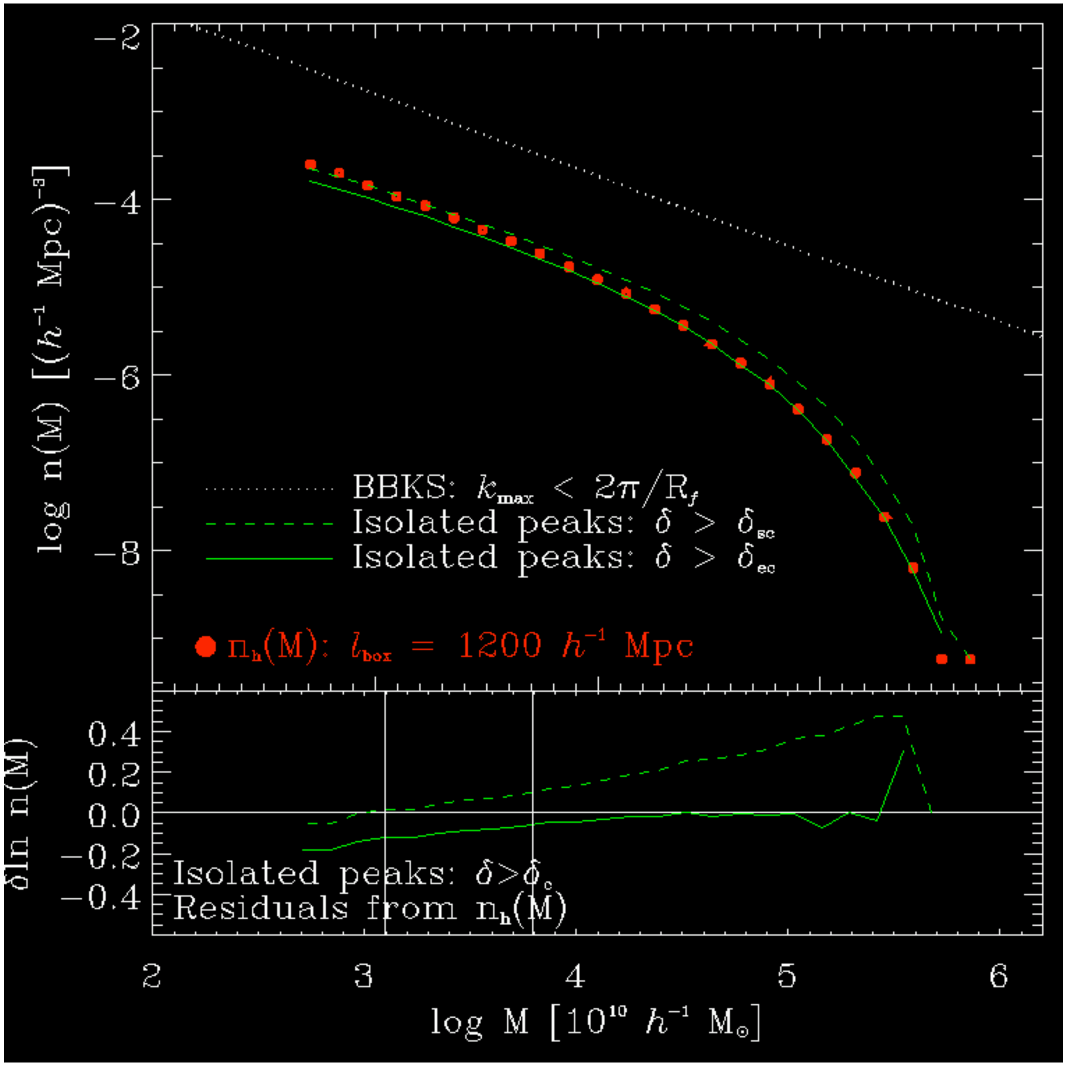
Collapse threshold



Halo mass function from peaks?



Mass function from isolated peaks?

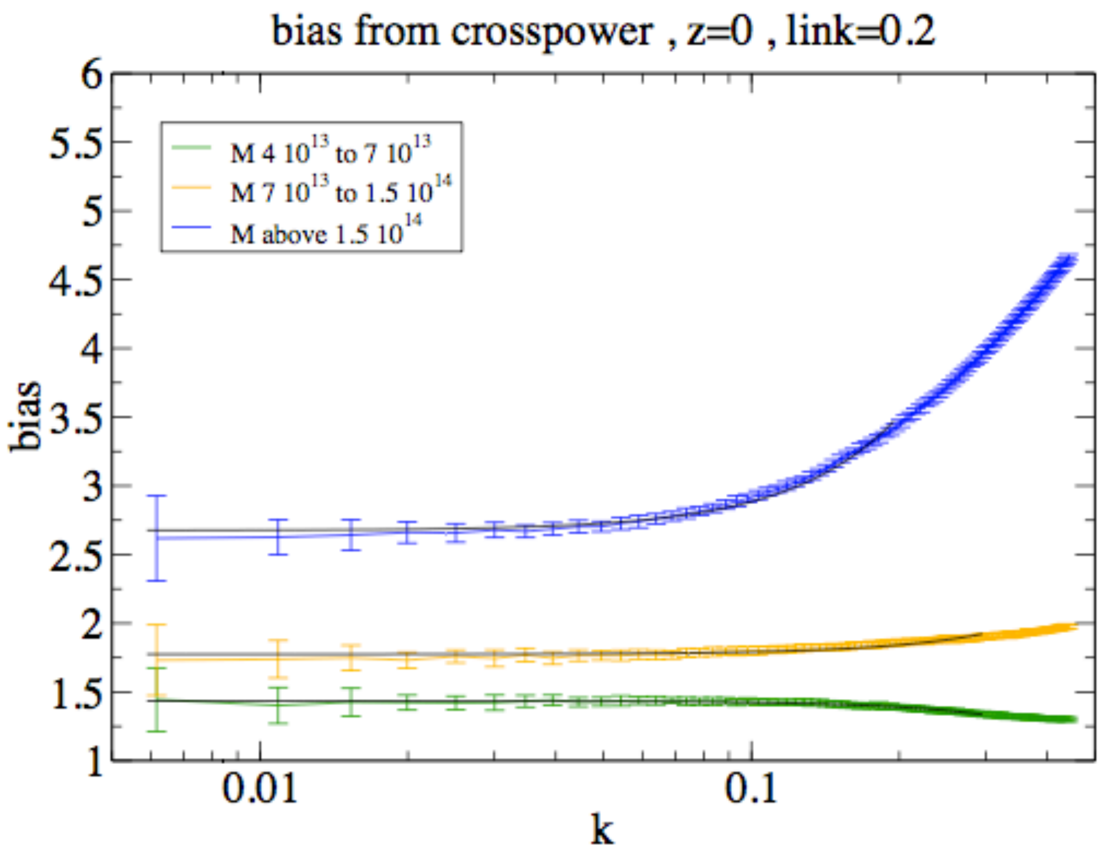


Modeling scale-dependent halo biasing

Elia, Kulkarni, Porciani, Pietroni & Matarrese

Scale-dependent biasing

Manera, Scoccimarro & Sheth 2009

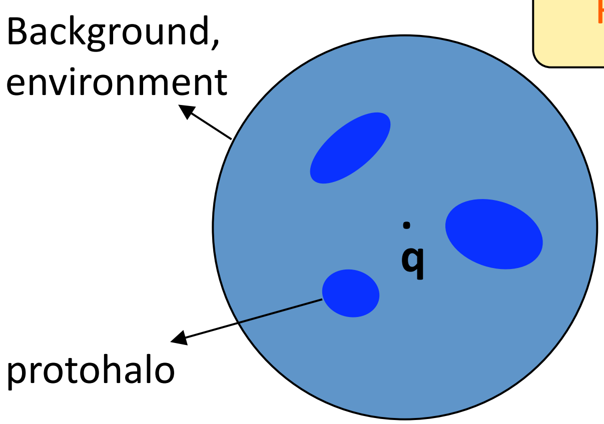


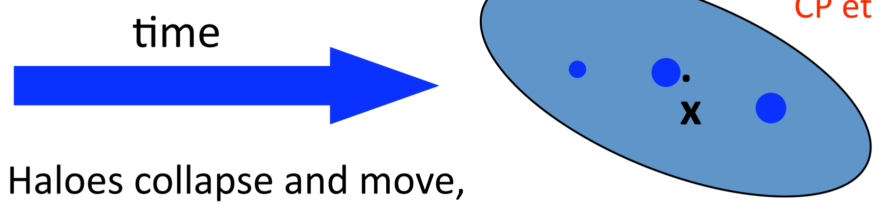
A model is needed if we want to study galaxy clustering at larger wavenumbers or build more accurate halo models

The distribution of dark matter haloes

Peak-background split

Cole & Kaiser 1988
Mo & White 1996
CP, PhD thesis, 1999
Catelan, CP et al. 1998a,b
CP et al. 1998





Haloes collapse and move, background evolves dynamically

Lagrangian world
(initial conditions)

Eulerian world
(evolved distribution)

Exact solution:

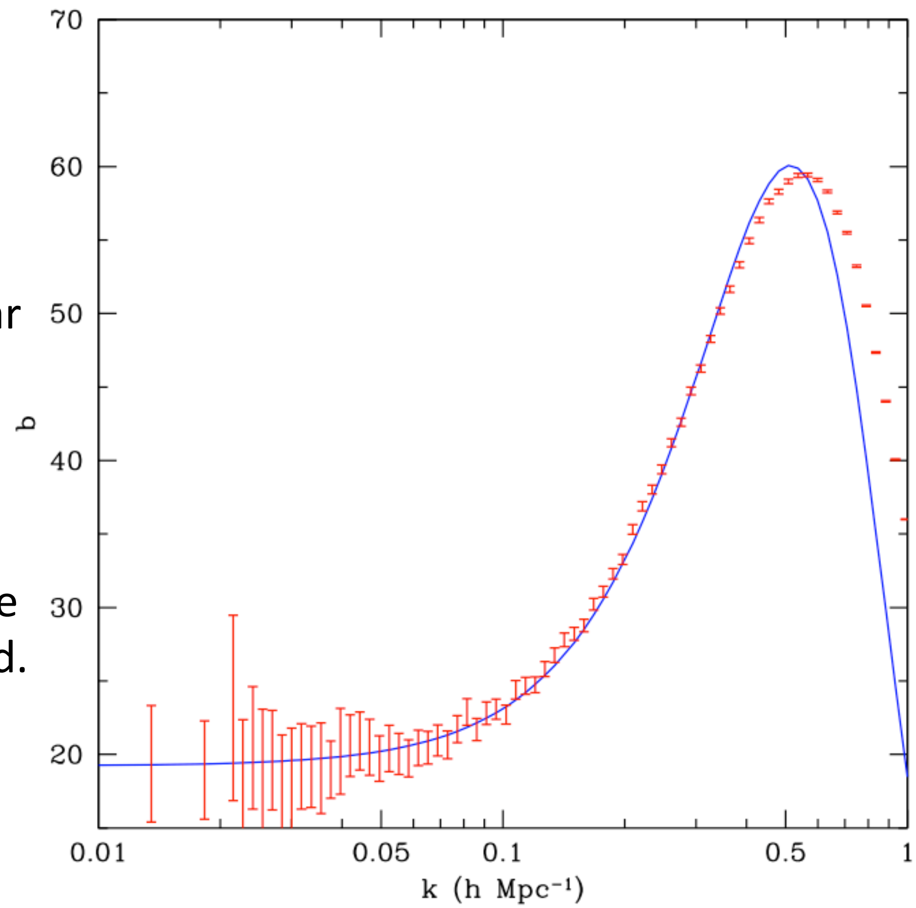
$$1 + \delta_h(\mathbf{x},z) = [1 + \delta_h(\mathbf{q})]\ [1 + \delta(\mathbf{x},z)]$$

The bias of dark matter haloes is intrinsically stochastic, non-linear and non-local!

Lagrangian halo bias

Dark-matter halos with $M > 1.24\ 10^{13}\ h^{-1}\ M_\odot$
Identified at z=0 and traced back to the linear density field.

Lagrangian bias determined from the cross-spectrum with the linear mass density field.



$$P_{mh}(k) \quad = \quad (b_1 + b_2 k^2) P_m(k) \mathrm{e}^{-k^2 R^2/2}$$

Matsubara 1999
Desjacques 2008

Mathematical formulation

Matter fluid

$$\frac{\partial \delta_m}{\partial \tau} + \nabla \cdot [(1+\delta_m)\mathbf{v}] = 0\,,$$
$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\,\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$
$$\nabla^2\phi = \frac{3}{2}\,\mathcal{H}^2\,\Omega_m\,\delta_m\,,$$

Proto-halo fluid

$$\frac{\partial \delta_{\rm h}}{\delta \tau} + \nabla \cdot [(1+\delta_{\rm h})\mathbf{v}_{\rm h}] = 0$$

$$\frac{\partial \mathbf{v}_{\rm h}}{\partial \tau} + \mathcal{H}\,\mathbf{v}_{\rm h} + (\mathbf{v}_{\rm h}\cdot\nabla)\mathbf{v}_{\rm h} = -\nabla\phi$$

Mathematical formulation II

In the absence of velocity bias:

$$\eta \equiv \ln(D_+/D_{+in}), \qquad \begin{pmatrix} \varphi_1(\mathbf{k},\eta) \\ \varphi_2(\mathbf{k},\eta) \\ \varphi_3(\mathbf{k},\eta) \end{pmatrix} \equiv e^{-\eta} \begin{pmatrix} \delta_m(\mathbf{k},\eta) \\ -\theta(\mathbf{k},\eta)/(\mathcal{H} f_+) \\ \delta_h(\mathbf{k},\eta) \end{pmatrix}$$

$$\partial_\eta \varphi_a(\mathbf{k},\eta) = -\Omega_{ab}(\eta)\varphi_b(\mathbf{k},\eta) + e^\eta \gamma_{abc}(\mathbf{k}, -\mathbf{p}, -\mathbf{q})\varphi_b(\mathbf{p},\eta)\,\varphi_c(\mathbf{q},\eta),$$

Vertex with non-vanishing components:

$$\gamma_{121}(\mathbf{k}, \mathbf{p}, \mathbf{q}) = \frac{1}{2}\delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q})\,\alpha(\mathbf{p},\mathbf{q}),$$

$$\gamma_{222}(\mathbf{k}, \mathbf{p}, \mathbf{q}) = \delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q})\,\beta(\mathbf{p},\mathbf{q}),$$

$$\gamma_{112}(\mathbf{k}, \mathbf{q}, \mathbf{p}) = \gamma_{121}(\mathbf{k}, \mathbf{p}, \mathbf{q}),$$

$$\gamma_{323}(\mathbf{k}, \mathbf{p}, \mathbf{q}) = \gamma_{332}(\mathbf{k}, \mathbf{q}, \mathbf{p}) = \gamma_{121}(\mathbf{k}, \mathbf{p}, \mathbf{q})$$

$$\Omega(\eta) = \begin{pmatrix} 1 & -1 & 0 \\ -\frac{3}{2}\frac{\Omega_m}{f_+^2} & \frac{3}{2}\frac{\Omega_m}{f_+^2} & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

Matrix containing cosmology information

Linear solution

$$\varphi_a(\mathbf{k};\eta) = g_{ab}(\eta)\varphi_b(\mathbf{k};0)\,, \qquad g_{ab}(\eta) = \left[\left(\begin{array}{ccc} 3/5 & 2/5 & 0 \\ 3/5 & 2/5 & 0 \\ 3/5 & 2/5 & 0 \end{array}\right)\right.$$

Growing mode

Std. decaying mode

$$+e^{-5/2\eta}\left(\begin{array}{ccc} 2/5 & -2/5 & 0 \\ -3/5 & 3/5 & 0 \\ 2/5 & -2/5 & 0 \end{array}\right)$$

New decaying mode

$$\left.+e^{-\eta}\left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{array}\right)\right]\theta(\eta)\,,$$

$$\varphi_a(\mathbf{k};0) = \left(\begin{array}{c} \varphi(\mathbf{k}) \\ \varphi(\mathbf{k}) \\ \varphi_h(\mathbf{k}) \end{array}\right) \qquad \varphi_a(\mathbf{k};\eta) = \left(\begin{array}{c} \varphi(\mathbf{k}) \\ \varphi(\mathbf{k}) \\ \varphi(\mathbf{k}) + e^{-\eta}(\varphi_h(\mathbf{k}) - \varphi(\mathbf{k})) \end{array}\right)$$

Initial conditions → Time → Evolved fields: "debiasing", the initial bias is progressively erased (see also Fry 1996)

Non-linear solution

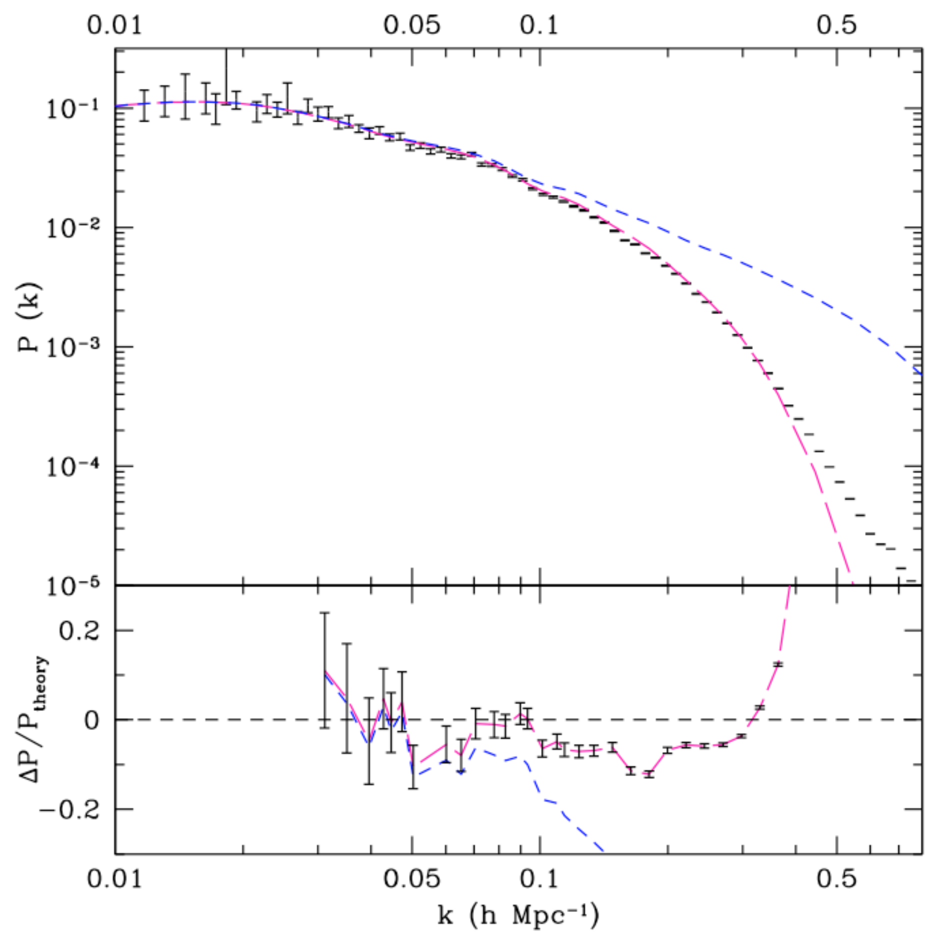
- We computed analytical 1-loop corrections to the propagator and the power spectrum

- As in Crocce & Scoccimarro (2006), the leading order corrections in the large k limit can be resummed at all orders in perturbation theory, giving a well-behaved propagator

- Comparison with N-body simulations shows excellent agreement for $k<0.35\ h\ \mathrm{Mpc}^{-1}$

Results for the propagators

Cross spectrum between the evolved δ_h (at z=0) and the initial δ_m



N-body
Linear
Resummed

$P_{31}(k,\eta,0)=G_{31}(k,\eta)\ P_{11}^{(0)}(k) + G_{32}(k,\eta)\ P_{21}^{(0)}(k) + G_{33}(k,\eta)\ P_{31}^{(0)}(k)$

TRG equations

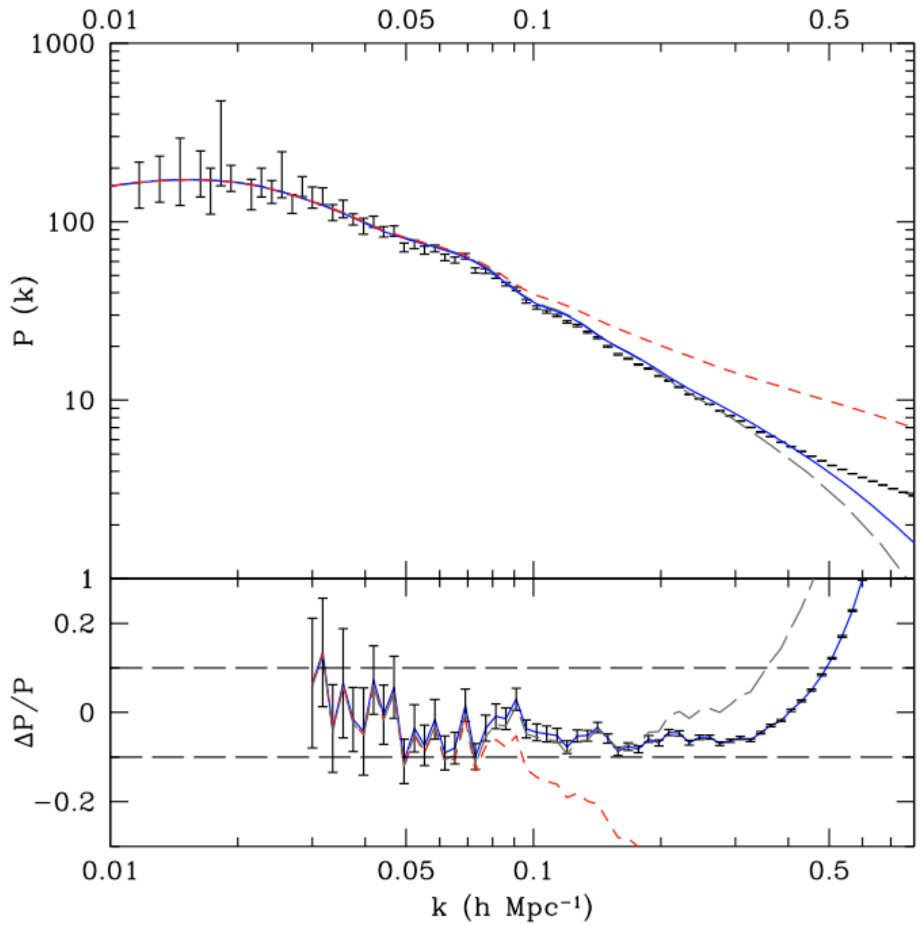
Unlike the propagator, the power spectrum cannot be resummed analytically at large k. To study its evolution we have used the time renormalization group (TRG) approach by Pietroni (2008).

$$
\begin{aligned}
&\partial_\eta\, P_{ab}(k;\eta) = \\
&\quad -\Omega_{ac}(\eta)P_{cb}(k;\eta) - \Omega_{bc}(\eta)P_{ac}(k;\eta) \\
&\quad + e^\eta \int d^3q\, \left[\gamma_{acd}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k})\, B_{bcd}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k};\,\eta)\right. \\
&\quad \left. + B_{acd}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k};\,\eta)\,\gamma_{bcd}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k})\right] ,
\end{aligned}
$$

$$
\begin{aligned}
&\partial_\eta\, B_{abc}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k};\,\eta) = \\
&\qquad -\Omega_{ad}(\eta)B_{dbc}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k};\,\eta) \\
&\qquad -\Omega_{bd}(\eta)B_{adc}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k};\,\eta) \\
&\qquad -\Omega_{cd}(\eta)B_{abd}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k};\,\eta) \\
&\qquad +2e^\eta \left[\gamma_{ade}(\mathbf{k},\,-\mathbf{q},\,\mathbf{q}-\mathbf{k})P_{db}(q;\,\eta)P_{ec}(|\mathbf{k}-\mathbf{q}|;\,\eta)\right. \\
&\qquad +\gamma_{bde}(-\mathbf{q},\,\mathbf{q}-\mathbf{k},\,\mathbf{k})P_{dc}(|\mathbf{k}-\mathbf{q}|;\,\eta)P_{ea}(k;\,\eta) \\
&\qquad \left. +\,\gamma_{cde}(\mathbf{q}-\mathbf{k},\,\mathbf{k},\,-\mathbf{q})P_{da}(k;\,\eta)P_{eb}(q;\,\eta)\right] , \qquad (30)
\end{aligned}
$$

Results for the halo-matter cross-power

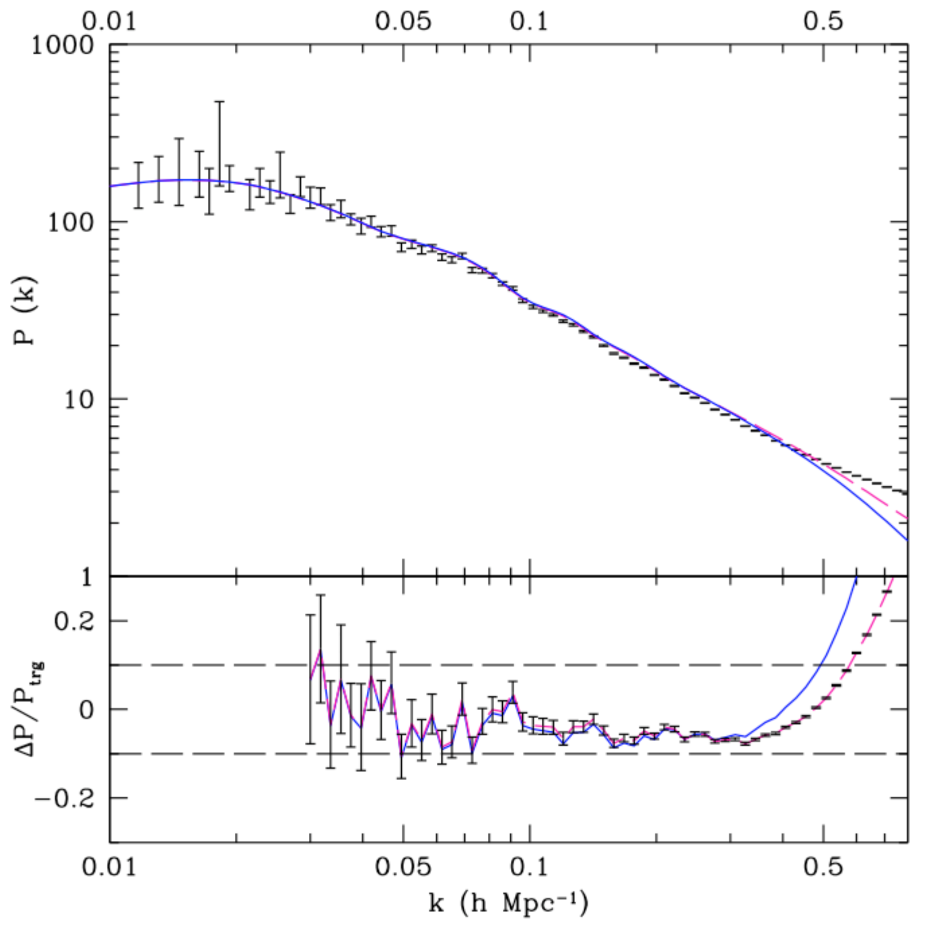
Cross spectrum between the evolved δ_h and the evolved δ_m both at z=0



N-body
1-loop
TRG (initial B=0)

Results for the halo-matter cross-power

Cross spectrum between the evolved δ_h and the evolved δ_m both at z=0



N-body

TRG (initial B=0)

TRG (initial B≠0)

Halo bias and primordial non-Gaussianity

Giannantonio & Porciani

2010, Phys. Rev. D, 81.063530

A particularly simple model

Most of the local models can be reduced to the simple form (Salopek & Bond 1990; Falk et al. 1993; Gangui et al. 1994):

$$\Phi(\vec{x}) = \phi(\vec{x}) + f_{NL}\left[\phi^2(\vec{x}) - \left\langle \phi^2 \right\rangle\right]$$

Bardeen's potential

auxiliary Gaussian field

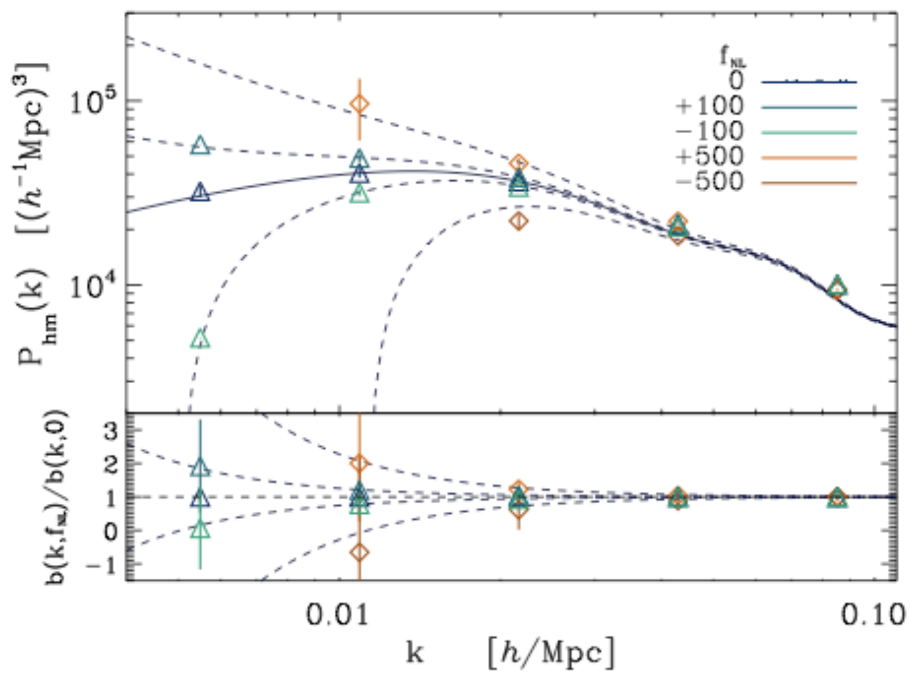
non-linearity parameter (a real number)

Fractional non-Gaussian corrections are $\approx 10^{-5} f_{NL}$

Scale-dependent biasing

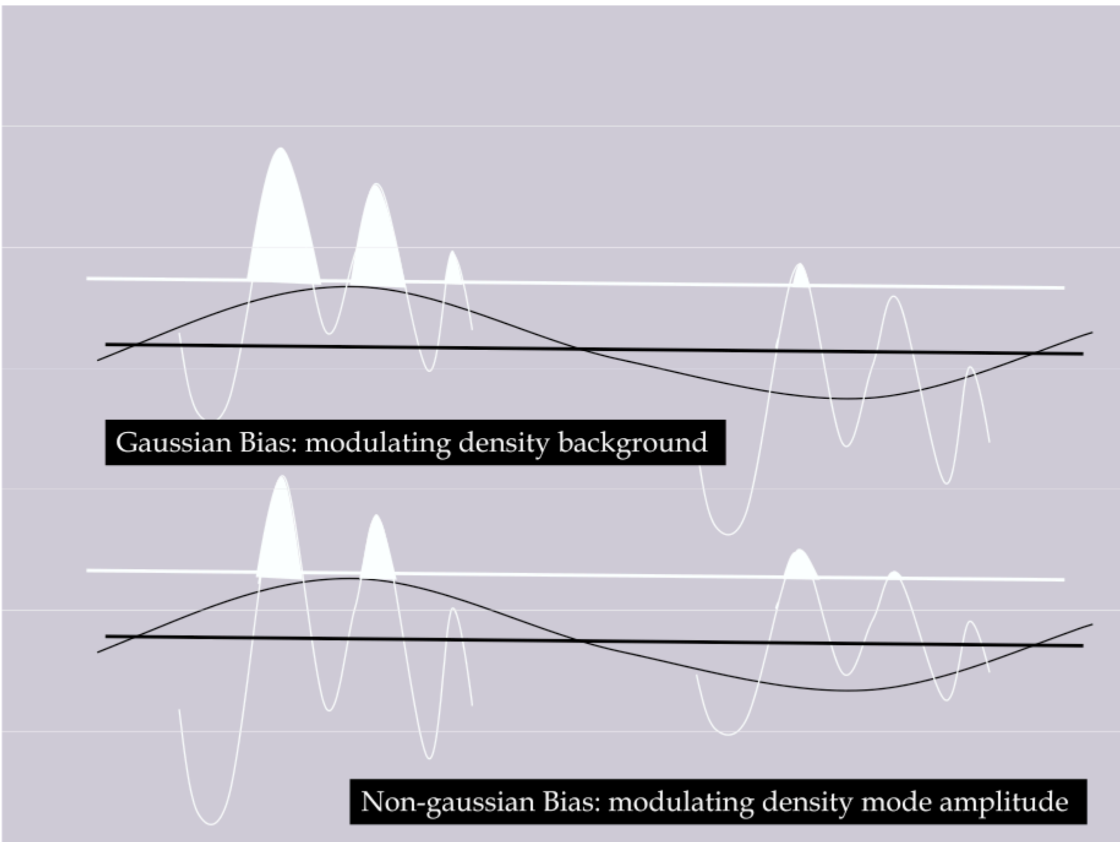
- The large-scale clustering of collapsed objects (galaxies, galaxy clusters) as measured by the power spectrum depends linearly on f_{NL}!!!
- An approximated model based on linear theory captures the most relevant physics (Dalal et al. 2008, Matarrese & Verde 2008, Slosar et al. 2008, Afshordi & Tolley 2008, McDonald 2008, Desjacques & Seljak 2009)
- However, recent simulations have evidenced some departures from the simple model predictions (Pillepich, Porciani & Hahn 2010)

Dalal et al. 2008



How does it work?

Afshordi & Tolley 2008



- Gaussian case:
  long and short wavelength modes of the density field are independent

- Non-Gaussian case:
  the long modes modulate the amplitude of the short ones via the gravitational potential

$$\delta_s = \delta_G [1 + 2 f_{NL} \phi_l / g(z)]$$

A non-local biasing scheme

- Using the peak-background split, we have shown that, in general:

$$\delta_h(x) = F[\delta_m(x),\ \phi(x),\ [\nabla\phi(x)]^2]$$

- Since the potential and the density field are linked by the Poisson equation, this generates a non-local biasing scheme in terms of the mass density

- When $f_{NL} \neq 0$, this is not compatible with the standard local, deterministic biasing scheme by Fry & Gaztañaga (1993)

The bias expansion

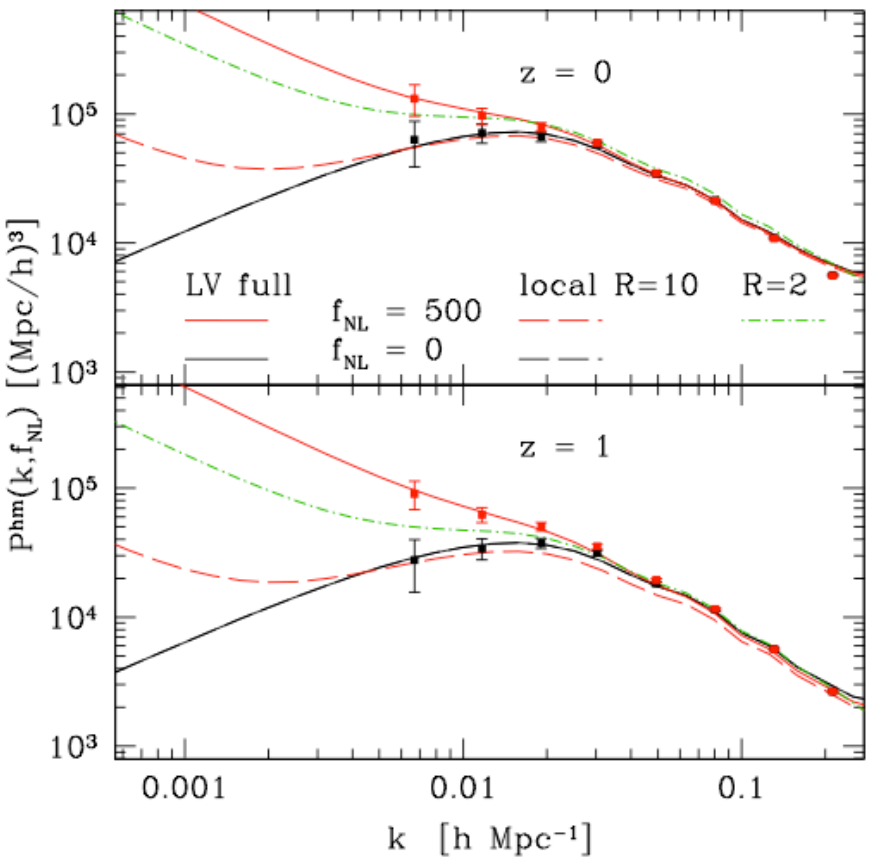
For galaxy and cluster sized halos, we can expand the halo overdensity as:

$$\begin{aligned}\delta_h(\mathbf{x}) &= b_0 + b_{10}\,\delta + b_{01}\,\varphi + \\ &+ \frac{1}{2!}\left(b_{20}\,\delta^2 + 2\,b_{11}\,\delta\varphi + b_{02}\,\varphi^2\right) + \\ &+ \frac{1}{3!}\left(b_{30}\,\delta^3 + 3\,b_{21}\,\delta^2\varphi + 3\,b_{12}\,\delta\varphi^2 + b_{03}\,\varphi^3\right),\end{aligned}$$

This term generates the leading order scale-dependence in the bias

and we provide explicit expression for the bias coefficients as a function of f_{NL} (and g_{NL}). All terms including the Gaussian potential vanish when f_{NL}=0 and the bias reduces to the model by Fry & Gaztañaga (1993).

Halo-matter cross spectrum

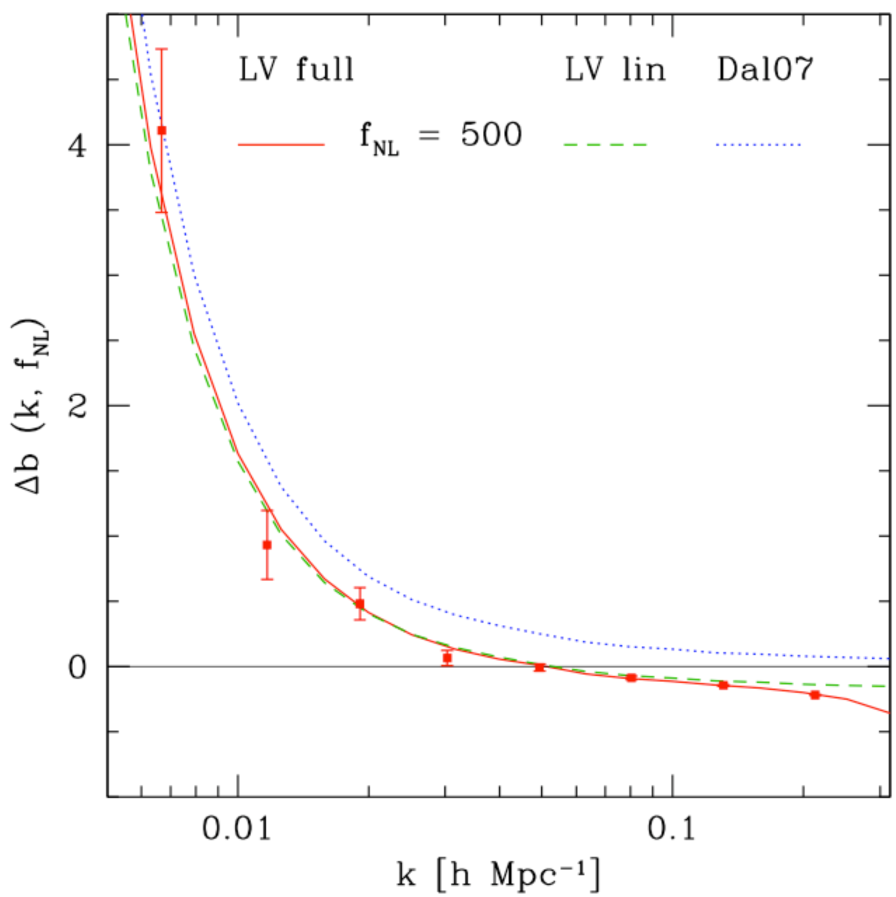


Solid lines: perturbative calculations (up to second next to leading order) by Giannantonio & Porciani 2010

Points with errorbars: N-body simulations by Pillepich, Porciani & Hahn 2010

Halo bias

$$\Delta b_{\text{linear}}(k) = b_{10}(f_{\text{NL}}) - b_{10}(f_{\text{NL}} = 0) + 2 f_{\text{NL}} \delta_c \left[b_{10}(f_{\text{NL}}) - 1\right] / \alpha(k)$$



$$\alpha(k) = \frac{2c^2 k^2 T(k) D(z)}{3\Omega_m H_0^2} \frac{g(0)}{g(\infty)}$$

Conclusions

- Most dark-matter halos originate from local density maxima but some of them might not
- This distinguishes two populations of halos with different Lagrangian and Eulerian clustering properties
- The TRG approach can be successfully used to model the non-linear evolution of the scale-dependent halo bias
- Primordial non-Gaussianity of the local type generates non-local biasing which modifies the statistical properties of the halo population (e.g. power spectrum, bispectrum)