Limiting Cosmological Magnetic fields

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astro-ph/0106244, astro-ph/0304556, astro-ph/0305059, astro-ph/0504553, astro-ph/0603476, astro-ph/0609216, arXiv:0906.4976, arXiv:1005.5322

Benasque, 12/08/2010

Outline

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- But there are other very interesting events which may have left observable traces, relics, in the universe.
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- Most notably confinement at $t \simeq 10^{-4}$ sec or the electroweak transition at $t \simeq 10^{-10}$ sec which may have led to the observed baryon asymmetry in the Universe.
- It has been proposed that confinement and, especially the electroweak phase transition but also inflation might generate primordial magnetic fields which represent seeds for the magnetic fields observed in galaxies and clusters.

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- Estimates by equi-partition (e.g. of magnetic field and thermal energy).

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- Intergalactic space, voids: The fact that certain blazars do emit TeV γ-radiation but not GeV, means that lower energy electrons which are produced by scattering with intergalactic background light and which then generate a cascade of GeV photons by inverse Compton scattering must be deflected out of the beam. This requires intergalactic fields of $B \gtrsim 3 \times 10^{-16}$Gauss with coherence scales of 1Mpc (Neronov & Vovk (2010).

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Because of the difficulty to permeate voids with magnetic fields at late times, we look only at primordial processes in this talk.

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- Since a constant magnetic field breaks parity, its Faraday rotation leads to correlations between E-polarization and temperature anisotropies (and E- and B-polarization) in the CMB (Scannapieco & Ferreira, '97; Kahniashvili et al. '09). Also this leads to limits of the order of $B < 10^{-8}$Gauss.

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- It is not surprising that all these limits are comparable, since

$$\Omega_B \simeq 10^{-5}\Omega_\gamma \left(\frac{B}{10^{-8}\mathrm{Gauss}}\right)^2$$

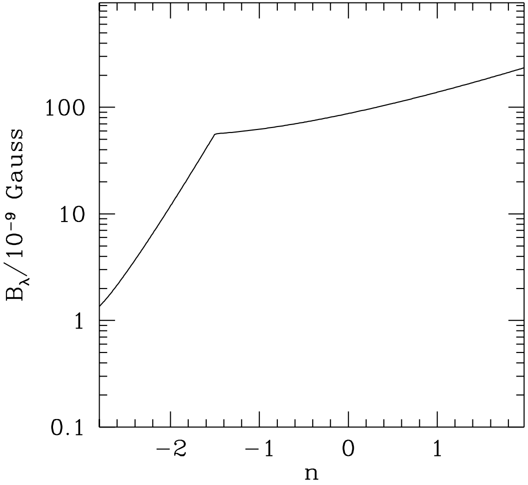
  Magnetic fields of the order 3×10^{-9}Gauss (on CMB scales) will leave 10% effects on the CMB anisotropies while 10^{-9}Gauss will leave 1% effects. It is thus clear that we can never detect magnetic fields of the order of 10^{-16} or even 10^{-20}Gauss (on galactic scales) in the CMB.

Other effects on the CMB

Magnetic fields effect the CMB via

- their energy-momentum tensor which leads to metric perturbations $\Rightarrow$ perturbed photon geodesics
- magnetosonic waves affect the acoustic peaks in the CMB spectrum
- Alvèn waves (vector perturbations)
- Faraday rotation can turn E-mode polarization into B-modes

All these lead to magnetic field limits on the order of 10^{-9}Gauss on CMB scales. Depending on the spectral index this leads to different limits on galactic scales $\lambda \sim 0.1$Mpc.



($\lambda = 0.1$Mpc, from: RD, Ferreira & Kahniashvili '98)

Generation of primordial magnetic fields

There are three ideas how magnetic fields may have formed in the early Universe:

- Second order perturbations: To generate magnetic fields in the cosmic fluid one needs vorticity and a charge and current density. The first can be obtained only in second order perturbation theroy (first order vector perturbations decay) and the second only in second order in the tight coupling limit. Estimates have shown that typical fields do not exceed 10^{-22}Gauss. This is far too small to be consistent with the Neronov-Vovk bound or with the minimal amplitude needed for dynamo amplification.

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The spectrum

We assume that the process leading to a magnetic field is statistically homogeneous and isotropic. A magnetic field spectrum generated by such a process is of the form

$$\langle B_i(\mathbf{k}) B_j^*(\eta, \mathbf{q})\rangle = \frac{(2\pi)^3}{2}\delta(\mathbf{k}-\mathbf{q})\left\{(\delta_{ij} - \hat{k}_i\hat{k}_j)P_S(k) - i\epsilon_{ijn}\hat{k}_n P_A(k)\right\}$$

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$P_A \propto |B_+|^2 - |B_-|^2$ determines the helicity of the magnetic field. Its integral is the helicity density while the integral of $P_S \propto |B_+|^2 + |B_-|^2$ determines the energy density in magnetic fields.
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On small scales the magnetic field is damped by the viscosity of the cosmic plasma, $P_S = P_A = 0$ for $k > k_d(t)$. Here $k_d(t)$ is a time-dependent damping scale.

Magnetic field evolution

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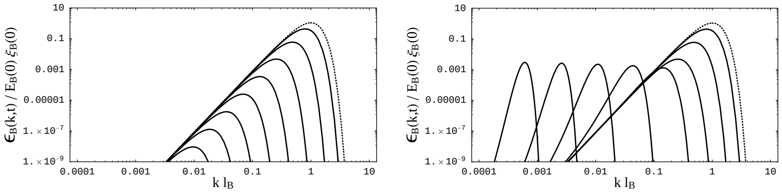
However, if the field is helical, helcity conservation leads to an inverse cascade which moves the correlation scale to larger and larger scales (numerical simulations by Jedamzik et al. '00-'05, Campanelli, '07)



Campanelli, 2007

If the magnetic field is generated during a 1st order phase transition, its correlation length is finite. It is typically of the size of the largest bubbles when they coalesce and the phase transition terminates. This is a fraction of the Hubble scale at the transition. On scales larger than the Hubble scale, correlation vanish by causality.

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Hence the correlation function is a function of compact support; and therefore its Fourier transform is analytic. Usually this signifies white noise (flat) on large scales but since we have to additional condition $\nabla \cdot \mathbf{B} = 0$, for magnetic fields we must require $P_S \propto k^2$ on large scales. Correspondingly n_A must be odd and the physical requirement $|P_A| \leq P_S$ then implies $P_A \propto k^3$.

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Limits on magnetic fields from phase transitions

As we now show this already implies very stringent limits on magnetic fields from phase transitions. Be $\epsilon = \Omega_B^*/\Omega_r^*$ the ratio of the magnetic field to the radiation energy density at the moment of formation and k_* the cutoff scale. Since radiation and magnetic fields scale the same way, at later times and scales larger than the cutoff, the magnetic field to radiation density is given by

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$$\left(\frac{B(k_1)}{10^{-6}\mathrm{Gauss}}\right)^2 \simeq \Omega_r^{-1}\frac{d\Omega_B}{d\log(k_1)} < \epsilon \times 10^{-50}$$

Since also $\epsilon < 1$ this implies $B(k_1) < 10^{-31}$Gauss. Using slightly more model dependent but also more realistic numbers (e.g. $k_* \simeq 100\mathcal{H}_*$ one arrives at $B(k_1) < 10^{-36}$Gauss.

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The limits from the QCD phase transition are somewhat less stringent but still discouraging, $B(k_1) < 10^{-30}$Gauss.

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$B(k_1) < 10^{-21}$Gauss for the QCD transition. This result could be marginally sufficient dynamo amplification, but is still several orders of magnitude below the Neronov-Vovk-bound.

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Let us discuss a simple case where we couple the inflation to the electromagnetic field.

$$S = \int d^4x\sqrt{-g}\left[R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{f(\phi)}{4}F^2\right]$$

Magnetic fields from inflation

These results from phase transition motivate to enquire what can be done at inflation. There, scales which were initially inside the horizon grow very big and the annoying 'causality constraint' does not apply.
Let us discuss a simple case where we couple the inflation to the electromagnetic field.

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This is a wave equation with a time-dependent mass term. We know how to calculate the generation of its modes out of the vacuum. This case has been discussed for the first time in (Ratra '92).

Magnetic fields from inflation: example

For example if $f \propto a^{\gamma}$ is a simple power law, we can compute the resulting magnetic fields spectrum to

$$P_S \propto k^n \quad \text{with} \quad n = \begin{cases} 1 - 2\gamma & \text{if } \gamma \geq -1/2 \\ 3 + 2\gamma & \text{if } \gamma \leq -1/2 \end{cases}$$

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During inflation we cannot assume that the Universe is highly conducting and the electric field is damped. We therefore also have to compute the electric field spectrum. One finds (Martin & Yokoyama '08, Subramanian '10)

$$P_E \propto k^m \quad \text{with} \quad m = \begin{cases} 3 - 2\gamma & \text{if } \gamma \geq 1/2 \\ 1 + 2\gamma & \text{if } \gamma \leq 1/2 \end{cases}$$

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Since there is no evident infrared cutoff, we require that the spectral index be larger than -3 otherwise $\frac{d\rho_B}{d\log(k)} \propto k^3 P_S \propto k^{3+n}$ or $\frac{d\rho_E}{d\log(k)} \propto k^3 P_E \propto k^{3+m}$ diverges.

This limits $\qquad -2 \lesssim \gamma \lesssim 2.$

Problems

On the other hand, if the spectrum is too blue, the fact that magnetic fields should not dominate the energy density of the Universe leads to very stringent constraints on small scales. Since the Hubble scale at the end of inflation is so small, the spectrum needs not be very blue for this to happen.

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Hence during inflation the electron charge was much larger than 1. In this regime we cannot trust perturbation theory and our calculation does actually not apply...
(Demozzi et al. '09).
Note that choosing $f_i = 1$ does not help since then $f_0 \gg 1$ and the presently measured electron charge is $e_N = e/f_0$, hence we still need $e/f_i \gg 1$.

These problems motivated us to see what can be gained from a helical coupling of the inflaton to the magnetic field. The idea is that this does not mix with the electron charge. Furthermore, because of the inverse cascade invoked by helicity conservation in the evolution of the field after inflation, a somewhat bluer spectral index might be admissible.

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where $\tilde{F}^{\mu\nu} = \frac{1}{2}\eta^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual of the electromagnetic field tensor.

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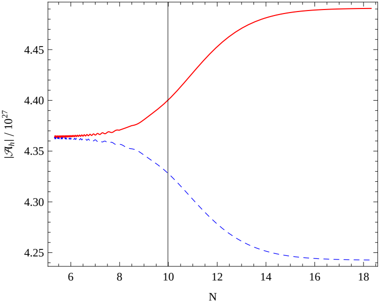
Again, a wave equation with time-dependent mass term. There are two main differences to the non-helical case: Now one of the helicity modes is amplified while the other is reduced depending on the sign of f', and the mass-term is proportional to k.

Generation of helical magnetic fields during inflation

Both difference are very important: the first leads to helicity and the second is the cause of the short duration of the amplification phase



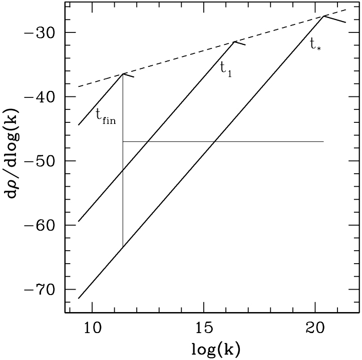
RD, Hollenstein & Jain, 2010

Because the duration during which the mass term is relevant is always just the Hubble exit time $k \sim \mathcal{H}$, before the $\ddot{A}$ term is much larger and later the $k^2\mathcal{A}$ term dominates, the vacuum fluctuation of one mode are always amplified while those of the other mode are suppressed. And this by the same, k-independent factor.

Since the vacuum fluctuations of the vector potential behave like k^{-1}, this yields $P_S \propto P_A \propto k$,

$$\frac{d\rho_B}{d\log(k)} \propto k^4.$$

Despite the inverse cascade, this spectrum is too steep to satisfy both, $\frac{d\rho_B}{d\log(k_*)} < \rho_r$ and $B(k_1, t_0) > 10^{-20}$Gauss.



(ρ_B in Gauss^2 and k in 1/Mpc)

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Conclusions

- Magnetic fields are observed on all cosmological scales (galaxies, clusters, filaments and probably even voids) with significant amplitudes. Intergalactic fields with coherence length of about 1Mpc and amplitudes of 10^{-20}Gauss (for dynamo amplification) or even 3×10^{-16}Gauss (Neronov-Vovk-bound) are required.

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- Fields from phase transition are too blue, they do not have enough power on large scales.
- Inverse cascade of helical magnetic fields can mitigate this problem but seems not quite sufficient to solve it.

Conclusions

- Magnetic fields from inflation can have many different spectra. They can actually be scale invariant leading to sufficient fields on large scales, but in this case, the effective electron charge during inflation must have been much larger than today and perturbation theory cannot be trusted.

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- Or is there a loop-hole in my argumentation and the the observed fields are nevertheless a window to the early Universe...