Thermodynamic Bethe Ansatz and Y-system for AdS/CFT

Diego Bombardelli

IEU - Ewha Womans University, Seoul and University of Bologna

Benasque, Centro de Ciencias “Pedro Pascual” - 15 July 2010

based on:

- D. B., D. Fioravanti, R. Tateo, “Thermodynamic Bethe Ansatz for planar AdS/CFT: a proposal”, J. Phys. A 42:375401, 2009; arXiv:0902:3930 [hep-th]
- D. B., D. Fioravanti, R. Tateo, “TBA and Y-system for planar AdS_4/CFT_3”, Nucl. Phys. B 834:543-561, 2010; arXiv:0912.4715 [hep-th]

Outline

Introduction

- string theory: a quantum theory of gravity?
- discovered as description of the strong interactions
- abandoned in favor of QCD (SU(3) gauge group)
- **'t Hooft limit**: $N, g_{YM} \to \infty\,,\ \lambda \equiv g_{YM}^2 N$ fixed ['t Hooft '74]
  - **planar** SU(N) gauge theories $\Leftrightarrow$ **free** string theories
  - large N Feynman diagrams $\Leftrightarrow$ string perturbation theory with $g_s \sim 1/N$
- conjectures of precise equivalences:

AdS/CFT $\begin{cases} \text{conformal } \mathcal{N}=4 \text{ SUSY Yang-Mills} \Leftrightarrow \text{IIB superstring on } AdS_5 \times S^5 \\ \text{conformal } \mathcal{N}=6 \text{ SUSY Chern-Simons} \Leftrightarrow \text{IIA superstring on } AdS_4 \times \mathbb{CP}^3 \end{cases}$

[Maldacena '97; Gubser, Klebanov, Polyakov ; Witten '98;Aharony, Bergman, Jafferis, Maldacena '08]

$$g^2 = \frac{\lambda_{AdS_5}}{16\pi^2} = \frac{R^4_{AdS_5}}{16\pi^2\alpha'^2}\ ; \qquad \lambda_{AdS_4} = \frac{N}{k} = \frac{R^4_{AdS_4}}{2\pi^2\alpha'^2}$$

$$\boxed{\Delta(\lambda, N) = E(\lambda, g_s)}$$

where $\mathfrak{D}\,\mathcal{O} = \Delta\,\mathcal{O}$ such that $\langle \mathcal{O}(x)\,\mathcal{O}(y)\rangle \simeq \frac{1}{|x-y|^{2\Delta}}$

local gauge invariant operators $\mathcal{O} = Tr(\psi^A_\alpha D_\mu \phi_a D_\nu \bar{\psi}^{\bar{A}}_{\dot{\alpha}})$

- **strong/weak** coupling dualities $\Rightarrow$ hard to test for generic observables
- breakthrough: discovery of **integrability** on both sides of the correspondences
  - gauge operators $\leftrightarrow$ integrable spin chains [Minahan, Zarembo '02;'08]
  - string solutions $\leftrightarrow$ integrable non-linear sigma models [Bena, Polchinski, Roiban '03; Arutyunov, Frolov '08]

Motivations

- to **test** AdS/CFTs with the help of **integrability** methods
- to solve **exactly** the gauge and string theories involved
  - AdS/CFTs spectrum at **finite size**
  - to investigate string **quantum corrections**
  - to gain **non-perturbative** information on SYM ($\Rightarrow$ QCD?)
- how does string theory discretize into a spin chain?
- what kind of theories in AdS/CFTs?
- why integrability in AdS/CFTs?

Outline

Classical integrability

- def. **integrable systems** with N degrees of freedom and Hamiltonian H:
  - N conserved independent quantities in involution: $Q_j,\ j = 1, ..., N$ such that

$$\begin{cases} \{Q_i, Q_j\} = 0 \\ \{H, Q_j\} = 0 \end{cases} \quad \forall\, i, j = 1, ..., N-1$$

- in QFT: $N = \infty$
- a method to show integrability:
  - equations of motion $\Leftrightarrow$ Lax pair such that $[L, M] = 0$
    $\Rightarrow$ **flat connection** $L(\lambda)$
  - **monodromy matrix** $T(\lambda) = \overleftarrow{\exp} \int_\gamma L(\lambda)$
  - **transfer matrix** $t(\lambda) = trT(\lambda)$ satisfies

$$[tr\, T(\mu), tr\, T(\lambda)] = 0 \quad \forall \mu, \lambda$$

  $\Rightarrow$ a family of **conserved charges** can be obtained by $Q_j \propto \left(\frac{d}{d\lambda}\right)^j t(\lambda)\Big|_{\lambda=\lambda_0}$

- the eigenvalues $e^{ip_i(\lambda)}$ of $T(\lambda)$ define the **quasi-momenta** on a multi-sheeted Riemann surface with cuts C_k)(**algebraic curve**)

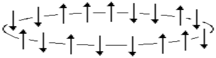
$$p(\lambda + i0) + p(\lambda - i0) = 2\pi n_k\,, \quad \lambda \in C_k$$

- adding poles and calculating the frequencies of the fluctuations
  $\Rightarrow$ one-loop energies (see Aniceto's and Kim's talks)

Quantum integrability: the Algebraic Bethe Ansatz

- an example: **Heisenberg chain**

$$H = \frac{1}{4}\sum_{n=1}^{L}\left(1 - \vec{\sigma}_n\vec{\sigma}_{n+1}\right)$$



- **Lax operator** $\psi_{n+1,a} = L_{n,a}\psi_{n,a}$
- **monodromy matrix** $T_{L,a} = \prod_{n=1}^{L} L_{n,a} = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}$
- **transfer matrix** $t(\lambda) = tr_a T(\lambda) = A(\lambda) + D(\lambda)$
- diagonalization of $t(\lambda)$: **BETHE ANSATZ** [Bethe '31]

$$\Phi(\{\lambda\}) = B(\lambda_1)B(\lambda_2)\cdots B(\lambda_M)|\uparrow\uparrow \ldots \uparrow\rangle$$

only if
$$\left(\frac{\lambda_k + i/2}{\lambda_k - i/2}\right)^L = \prod_{m=1;m\neq k}^{M}\frac{\lambda_k - \lambda_m + i}{\lambda_k - \lambda_m - i} \qquad k = 1, \ldots, M$$

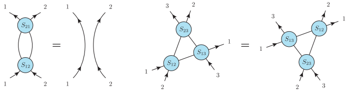
- in the limit $L \to \infty$, $x_m = L\,\lambda_m$, reduce to integral **classical Bethe equations** equivalent to $p(x + i0) + p(x - i0) = 2\pi n_k\,, \quad x \in C_k$ [Kazakov, Marshakov, Minahan, Zarembo '04]
- **observables** (conserved charges):
  - $\Pi\Phi = \sum_{m=1}^{M}\frac{1}{i}\ln\frac{\lambda_m + i/2}{\lambda_m - i/2}\Phi$
  - $H\Phi = \sum_{m=1}^{M}\frac{1}{2}\frac{1}{\lambda_m^2 + 1/4}\Phi$

The S-matrix

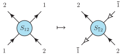
- general action of a two-particle **S-matrix** on **asymptotic states**:

$$|a(\theta_a)b(\theta_b)\rangle_{in} = \mathbf{S}_{ab}^{cd}|c(\theta_c)d(\theta_d)\rangle_{out}$$

- diagonal case: $|a(\theta_a)b(\theta_b)\rangle_{in} = \mathbf{S}_{ab}|a(\theta_a)b(\theta_b)\rangle_{out}$
- relativistic case: $S_{ab}(\theta_a,\theta_b) = S_{ab}(\theta_a - \theta_b)$
- **integrability** $\leftrightarrow$ **factorizability** $\Rightarrow S_{ab}S_{ac}S_{bc} = S_{bc}S_{ac}S_{ab}$ (**YBE**)



- physical requirements
  - unitarity: $S_{ab}^{cd}(S^\dagger)_{dc}^{ef} = \delta_a^e\delta_b^f$
  - CPT invariance
  - crossing symmetry: $S_{a\bar{b}}(\theta) = S_{ab}(i\pi - \theta)$



- diagonal asymptotic BAEs can be obtained by imposing

$$e^{ip_iL}\prod_{j\neq i}^{N} S_{ij}(p_i,p_j) = 1$$

Outline

Integrability in AdS/CFT

- **integrability in AdS**
  - IIB superstring formulated as a nonlinear sigma model on supercoset
$$\frac{PSU(2,2|4)}{SO(1,4)\times SO(5)} = AdS_5 \times S^5$$
  - IIA superstring formulated as a nonlinear sigma model on supercoset
$$\frac{Osp(2,2|6)}{SO(3,1)\times U(3)} = AdS_4 \times \mathbb{CP}^3$$

  are classically integrable: **Lax connection** and **classical Bethe equations** were found [Bena, Polchinski, Roiban '03; Kazakov, Marshakov, Minahan, Zarembo '04; Arutyunov, Frolov; Stefanski '08]
- **integrability in CFT**
  - $\mathcal{N}=4$ **SYM** one-loop **SU(2)** sector ↔ Heisenberg spin chain
$$tr\ ZZZWWZZZWWWZWZZZZ\ldots \quad \longleftrightarrow \quad |\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\ldots\rangle$$
  - $\mathcal{N}=6$ **SCS** one-loop SU(4) sector ↔ SU(4) alternating spin chain (reduces to two $XXX_{1/2}$ for the $SU(2)\times SU(2)$ subsector)
  - ABAEs lifted to the full $Osp(2,2|6)$ superalgebra at one-loop [Minahan, Zarembo '08]
  - **S-matrix** and **all-loop ABAEs** conjecture [Gromov, Vieira; Ahn, Nepomechie '08]

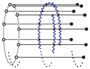
The AdS_5/CFT_4 Bethe Ansatz equations (1)

- all-loop: **asymptotic** BAEs for long range spin chain models (BDS) [Beisert, Dippel, Staudacher '04]

$$\left[\frac{x^+(u_j)}{x^-(u_j)}\right]^L = \prod_{\substack{k=1 \\ k\neq j}}^{M} \frac{u_j - u_k + i}{u_j - u_k - i}\,, \quad \text{where } x^{\pm}(u) = \frac{1}{2}\left[u \pm i/2 + \sqrt{u \pm i/2 - 4g^2}\right]$$

$$H = H_{Heis} + g^2 H_{next-to-nearest} + g^4 H_{next-to-next-to-nearest} + \dots$$
$$\Delta = L + g^2 E_{Heis} + g^4 E_2 + g^6 E_3 + \dots$$

- BDS BAEs are **asymptotic**: affected by **wrapping corrections** when interaction range (loop order) exceeds L: at order g^{2L-2}



- mismatches with string spectrum $\Rightarrow$ **dressing factor** for the correct interpolation weak/strong coupling:

$$\sigma(x_k^{\pm}, x_j^{\pm}; g) = \exp\left\{ i \sum_{r=2}^{\infty} \sum_{s=r+1}^{\infty} c_{r,s}(g)[q_r(x_k^{\pm})q_s(x_j^{\pm}) - q_r(x_j^{\pm})q_s(x_k^{\pm})]\right\}$$

[Arutyunov, Frolov, Staudacher '04; Beisert, Hernandez, Lopez '06]

- **conserved charges**: $q_r(u) = \frac{i}{r-1}\left[\left(\frac{1}{x^+(u)}\right)^{r-1} - \left(\frac{1}{x^-(u)}\right)^{r-1}\right]$

The AdS_5/CFT_4 Bethe Ansatz equations (2)

- using PABA [Staudacher '04] for various sectors
  $\Rightarrow$ **all-loop asymptotic** BAEs for the full $PSU(2,2|4)$ superalgebra of SYM
  [Beisert, Staudacher '05]

$$e^{ip_k L} = \prod_{\substack{l=1\\ l\neq k}}^{K^{\mathrm{I}}} \left[S_0(p_k, p_l) \left(\frac{x_k^+ - x_l^-}{x_k^- - x_l^+} \sqrt{\frac{x_l^+ x_k^-}{x_l^- x_k^+}} \right)^{\frac{1+\eta}{2}} \right]^2 \prod_{\alpha=1}^{2} \prod_{l=1}^{K^{\mathrm{II}}_{(\alpha)}} \left(\frac{x_k^- - y_l^{(\alpha)}}{x_k^+ - y_l^{(\alpha)}} \sqrt{\frac{x_k^+}{x_k^-}} \right)^{\eta}$$

$$(-1)^{\epsilon} = \prod_{l=1}^{K^{\mathrm{I}}} \frac{y_k^{(\alpha)} - x_l^+}{y_k^{(\alpha)} - x_l^-} \sqrt{\frac{x_l^-}{x_l^+}} \prod_{l=1}^{K^{\mathrm{III}}_{(\alpha)}} \frac{v_k^{(\alpha)} - w_l^{(\alpha)} + \frac{i}{g}}{v_k^{(\alpha)} - w_l^{(\alpha)} - \frac{i}{g}} \Leftrightarrow e^{-i\phi} e^{iq_k L} = \prod_{l=1}^{M} \frac{\lambda_l - \sin q_k - i\frac{U}{4}}{\lambda_l - \sin q_k + i\frac{U}{4}}$$

$$1 = \prod_{l=1}^{K^{\mathrm{II}}_{(\alpha)}} \frac{w_k^{(\alpha)} - v_l^{(\alpha)} - \frac{i}{g}}{w_k^{(\alpha)} - v_l^{(\alpha)} + \frac{i}{g}} \prod_{\substack{l=1\\ l\neq k}}^{K^{\mathrm{III}}_{(\alpha)}} \frac{w_k^{(\alpha)} - w_l^{(\alpha)} + \frac{2i}{g}}{w_k^{(\alpha)} - w_l^{(\alpha)} - \frac{2i}{g}} \Leftrightarrow \prod_{l=1}^{N} \frac{\lambda_k - \sin q_l - i\frac{U}{4}}{\lambda_k - \sin q_l + i\frac{U}{4}} = \prod_{\substack{l=1\\ l\neq l}}^{M} \frac{\lambda_k - \lambda_l - i\frac{U}{2}}{\lambda_k - \lambda_l + i\frac{U}{2}}$$

- **non-diagonal** scattering $\Rightarrow$ auxiliary variables describing particles' inner degrees of freedom
- BAEs can be obtained by imposing momenta quantization conditions with the **transfer matrix** eigenvalues

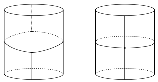
$$e^{ip_i L}\Lambda(p_i; p_1, p_2, \dots, p_N) = 1$$

- similar to (two copies of) inhomogeneous Hubbard BAEs

Outline

Lüscher corrections (1)

- from factorizable **S-matrix** (asymptotic info) → **finite-size** corrections for integrable theories
- Lüscher corrections originally for mass shift in **relativistic theories** [Lüscher '83,'86; Klassen, Melzer '91]



- generalisation to **non-relativistic** case to apply it in AdS/CFT [Ambjorn, Janik, Kristjansen '05; Janik, Lukowski '07]

$$E_\infty(p) = \sqrt{1 + 16g^2 \sin^2(p/2)} \neq m^2 + p^2$$

$$\textbf{F-term} \quad \delta E_a^F(p) = -\int_{-\infty}^{\infty} \frac{dq}{2\pi} \left(1 - \frac{E'_\infty(p)}{E'_\infty(q(q^*))}\right) e^{-iq^*(q)L} \sum_b (-1)^{F_b} (S_{ba}^{ba}(p, q^*(q))$$

$$\mu\textbf{-term} \quad \delta E_a^\mu(p) = \mathrm{Re}\left[-i\left(1 - \frac{E'_\infty(p)}{E'_\infty(\tilde{q}^*)}\right) e^{-i\tilde{q}^* L} \underset{q^*=\tilde{q}^*}{Res} \sum_b (-1)^{F_b} S_{ba}^{ba}(p, q^*)\right]$$

Lüscher corrections (2)

in AdS/CFT

- **strong coupling**: Lüscher corrections match
  - string [Arutyunov, Frolov, Zamaklar '06; Astolfi, Forini, Grignani, Semenoff '07]
  - and algebraic curve results [Gromov, Schäfer-Nameki, Vieira '08] also in AdS_4/CFT_3 [D. B., Fioravanti; Ohlsson-Sax, Lukowski; Ahn, Bozhilov '08; Ahn, Kim, Lee '10] (see Kim's talk)

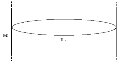
  for the giant magnon solution
- for **non-diagonal** scattering theories with more species a of particles there is a conjecture [Bajnok, Janik '08]:

$$E_a(L) \simeq \sum_{j=1}^{N} \nu_a(\theta_j) - \sum_{j,k=1}^{N} \frac{d\nu(\theta_k)}{d\theta_k} \left(\frac{\delta BY_k}{\delta\theta_j} \right)^{-1} \delta\phi_j - \int_{-\infty}^{\infty} d\theta \nu_a(\theta) e^{-L\nu_b(\theta)} \Lambda_a^b \left(\frac{i\pi}{2} + \theta|\{\theta_j\} \right)$$

where $\Lambda_a^b(\theta|\{\theta_j\}) = (-1)^F \left[S_{c_1 a}^{c_2 b}(\theta-\theta_1) S_{c_2 a}^{c_3 b}(\theta-\theta_1) \dots S_{c_N a}^{c_1 b}(\theta-\theta_1) \right]$

- **weak coupling**:
  - for Konishi operators match field theory diagrammatic computations [Bajnok, Janik, Lukowski '08; Fiamberti, Santambrogio, Sieg, Zanon '07,'08]
  - also in the β-deformed SYM [Ahn, Bajnok, D. B., Nepomechie] (see Ahn's talk)
  - and BFKL expectations in the case of general multi-particle states in $sl(2)$

Generalities on TBA [Zamolodchikov '90]



- (1+1)D **relativistic** QFT on a torus $\Rightarrow$ two **equivalent** choices (L, R) for the time

$$Z(R,L) = Tr_L\, e^{-LH_L} \underset{R\to\infty}{\approx} e^{-RE_0(L)}\ ;\ Z(R,L) = Tr_R\, e^{-RH_R} \underset{R\to\infty}{\approx} e^{-RLf(L)} \text{ with } T = 1/L$$

- from asymptotic (**diagonal**) BAEs

$$e^{ip(\theta_i)R} \prod_{j\neq i}^{N} S_{ij}(\theta_i - \theta_j) = 1 \Rightarrow Lp(\theta_i) + \sum_{j\neq i} \ln S_{ij}(\theta_i - \theta_j) = 2\pi n_i \quad \text{with } i = 1, \ldots, N$$

- thermodynamic limit $R, N \to \infty$ $\Rightarrow$ define (roots and holes) **densities**

$$\rho^r(\theta) + \rho^h(\theta) = \lim_{R\to\infty} \frac{n_{i+1} - n_i}{R(\theta_{i+1} - \theta_i)}$$

$$\Rightarrow \textbf{density equations} \quad \rho^r(\theta) + \rho^h(\theta) = \frac{d}{d\theta} p(\theta) + (\varphi * \rho^r)(\theta)$$

- minimization with constraint of the free energy $f[\rho] = E[\rho_r] - TS[\rho^r, \rho^h]$

$$\text{with} \quad S = \int d\theta \left[\rho^r(\theta) + \rho^h(\theta)\right] \ln\left[\rho^r(\theta) + \rho^h(\theta)\right] - \rho^r(\theta)\ln\rho^r(\theta) - \rho^h(\theta)\ln\rho^h(\theta)$$

Excited states TBA

$\Rightarrow$ **TBA equation** $\varepsilon(\theta) = \nu(\theta)/T - [\varphi * \ln(1+e^{-\varepsilon})](\theta)$ with $e^{\varepsilon(\theta)} \equiv \frac{\rho^h}{\rho^r}$

$\Rightarrow$ **minimum free energy** $f(T) = -T \int_{-\infty}^{\infty} d\theta \nu(\theta) \ln(1+e^{-\varepsilon(\theta)})$

- in the convolution terms, consider the singularities θ_j of $\ln(1+e^{-\varepsilon(\theta)})$, corresponding to the particle rapidities [Dorey, Tateo '96]

$$\varepsilon(\theta_j + i\pi/2) = i(2n_j+1)\pi \; ; \quad j = 1, \ldots, N$$

$$\Rightarrow \varepsilon(\theta) = \nu(\theta)/T + \sum_{j=1}^{N} \ln S(\theta - \theta_j - i\pi/2) - [\varphi * \ln(1+e^{-\varepsilon})](\theta)$$

$\Rightarrow$ **energy** $E(L) = \sum_{j=1}^{N} \nu(\theta_j) - \int_{-\infty}^{\infty} d\theta \nu(\theta) \ln(1+e^{-\varepsilon(\theta)})$

- usually the convolution terms are negligible at leading order in L

$\Rightarrow$ **Lüscher corrections** $E(L) \simeq \sum_{j=1}^{N} \nu(\theta_j) - \int_{-\infty}^{\infty} d\theta \nu(\theta) e^{-L\nu(\theta)} \prod_{j=1}^{N} S\left(\frac{i\pi}{2} + \theta - \theta_j\right)$

Universal TBA

- with more species $a = 1, ..., N$ of particles , TBA equations read

$$\varepsilon_a(\theta) = \nu_a(\theta)/T - \sum_{b=1}^{N} [\varphi_{ab} * \ln(1 + e^{-\varepsilon_b})](\theta)$$

- via the general identity

$$\left(\delta_{ab} - \frac{\hat{\varphi}_{ab}(k)}{2\pi}\right)^{-1} = \delta_{ab} - \frac{I_{ab}}{2\cosh(\pi k/h)}$$

TBA equations for certain diagonal, relativistic, scattering theories can be written in a **universal form** [Zamolodchikov '91]

$$\varepsilon_a(\theta) = \nu_a(\theta)/T - \sum_{b=1}^{N} I_{ab}[s * (\nu_b - \ln(1 + e^{\varepsilon_b})](\theta)$$

where $I_{ab}(e.g. = \delta_{a,b+1} + \delta_{a,b-1})$ is called incidence matrix

and the universal kernel $s(\theta) = \frac{h}{2\cosh(h\theta/2)}$

Y-system

- by exploiting the universal kernel's property
$$s(\theta + i\pi/h - i0) + s(\theta - i\pi/h + i0) = \delta(\theta)$$
- or manipulating directly the canonical TBA equations [Ravanini, Tateo, Valleriani '92]
$$\varepsilon_a(\theta + i\pi/h) + \varepsilon_a(\theta - i\pi/h) - I_{ab}\varepsilon_b(\theta) - I_{ab}\ln(1 + e^{-\varepsilon_b})(\theta) = 0$$
- defining $Y_a(\theta) = e^{\varepsilon_a(\theta)}$
$$Y_a(\theta + i\pi/h)Y_a(\theta - i\pi/h) = \prod_b (1 + Y_b(\theta))^{I_{ab}}$$
- exact BAes are given by $Y_a(\theta_k + i\pi/2) = -1$
- the **Y-system** encodes the TBA equations for the **excited states**
- in **non-diagonal** case we can be helped by **T-system**
$$T_a(\theta + i\pi/h)T_a(\theta - i\pi/h) = T_{a+1}(\theta)T_{a-1}(\theta) + T_0(\theta + i\pi(a+1)/h)T_0(\theta - i(a+1)\pi/h)$$
via
$$Y_a(\theta) = \frac{T_{a+1}(\theta)T_{a-1}(\theta)}{T_0(\theta + i\pi(a+1)/h)T_0(\theta - i(a+1)\pi/h)}$$
- asymptotic solution
$$Y_0(\theta) \simeq e^{-L\cosh(\theta)}\phi(\theta)T_1(\theta)$$
$\Rightarrow$ Lüscher corrections
$$E(L) \simeq \sum_{j=1}^{N} \nu(\theta_j) - \int_{-\infty}^{\infty} d\theta \nu(\theta) Y_0(\theta)$$

Outline

The mirror theory

- **non-relativistic** case: the choices are **not equivalent** $\Rightarrow$ define two different theories (physical and mirror) related by analytic continuation

$$E(p)=\sqrt{1+4g^2\sin^2(p/2)} \quad\rightarrow\quad i\tilde{p}$$
$$p \quad\rightarrow\quad i\tilde{E}(\tilde{p}) = 2\,i\,\text{arcsinh}\left(\sqrt{1+\tilde{p}^2}/2g\right)$$

- from double Wick rotation on AdS/CFT S-matrix $\Rightarrow$ **mirror S-matrix**

$$\tilde{S}(z_1,z_2)=S(z_1+\omega_2/2, z_2+\omega_2/2)$$

$\Rightarrow$ **mirror asymptotic BAEs** [Arutyunov, Frolov '07]:

- defined in the **mirror** kinematic region: $|x^{\pm}_{ph}(u)|>1\rightarrow \mathcal{I}m[x^{\pm}_{mir}(u)]<0$

$$x_{mir}(u)=\frac{1}{2}(u-i\sqrt{4-u^2})\,;\quad \tilde{p}=gx_{mir}(u-i/g)-gx_{mir}(u+i/g)+i$$

- and on the fermionic reference states ("$sl(2)$" grading $\eta=-1$):

only in this grading there are **mirror massive bound states** (strings)!

$$e^{i\tilde{p}_1 R}=\frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\cdots \qquad e^{i\tilde{p}_2 R}=\frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\cdots$$

when $R\rightarrow\infty\ \Rightarrow\ x_1^+ - x_2^- = 0$ solved by $\tilde{p}^A=p/2-iq, \tilde{p}^B=p/2+iq$, with $\mathcal{R}e(q)>0$

The string hypothesis

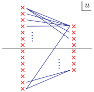
- **thermodynamic limit** $R, K^I, K^{II}_\alpha, K^{III}_\alpha \to \infty$ on mirror AdS_5/CFT_4 BAEs
  $\Rightarrow$ Bethe roots organize in $SL(2)$ **Bethe strings** (bound states):

$$u_k^{Q,j} = u_k^Q + i(Q+1-2j)/g\,,\ j=1,\dots,Q$$

  up to exponential corrections in R: **string hypothesis**

- remaining (magnonic) strings configuration similar to those of (two copies of) the Hubbard model
- y-particles with real momenta q_k
- vw-strings $v_k^{M,j} = v_k^M \pm i(M+2-2j)/g\,,\ j=1,\dots,M$
  $w_k^{M,j} = w_k^M + i(M+1-2j)/g\,,\ j=1,\dots,M$
- w-strings $w_k^{N,j} = w_k^N + i(N+1-2j)/g\,,\ j=1,\dots,N$

  - S-matrix factors in the BAEs' r.h.s. obtained by fusion procedure



$$S^{QQ'}_{sl(2)}(x_k,x_l) = \prod_{j=1}^{Q}\prod_{h=1}^{Q'} \frac{x^+(u_k^{Q,j}) - x^-(u_l^{Q,j})}{x^-(u_k^{Q,j}) - x^+(u_l^{Q,j})}\, \frac{1-\frac{1}{x^-(u_k^{Q,j})x^+(u_l^{Q,j})}}{1-\frac{1}{x^+(u_k^{Q,j})x^-(u_l^{Q,j})}}\, \sigma(u_k^{Q,j},u_l^{Q,j})^{-2}$$

$\Rightarrow$ BAEs for the real centers of the Bethe strings [Arutyunov, Frolov '09]

$$1 = e^{i\tilde{p}_k^Q R} \prod_{Q'=1}^{\infty} \prod_{\substack{l=1\\ l\neq k}}^{N_{Q'}} S_{sl(2)}^{QQ'}(x_k, x_l) \prod_{\alpha=1}^{2} \prod_{l=1}^{N_y^{(\alpha)}} \frac{x_k^- - y_l^{(\alpha)}}{x_k^+ - y_l^{(\alpha)}} \sqrt{\frac{x_k^+}{x_k^-}} \prod_{M=1}^{\infty} \prod_{l=1}^{N_{M|vw}^{(\alpha)}} S_{xv}^{QM}(x_k, v_{l,M}^{(\alpha)}) \,,$$

$$-1 = \prod_{Q=1}^{\infty} \prod_{l=1}^{N_Q} \frac{y_k^{(\alpha)} - x_l^+}{y_k^{(\alpha)} - x_l^-} \sqrt{\frac{x_l^-}{x_l^+}} \prod_{M=1}^{\infty} \prod_{l=1}^{N_{M|vw}^{(\alpha)}} \frac{v_k^{(\alpha)} - v_{l,M}^{(\alpha)-}}{v_k^{(\alpha)} - v_{l,M}^{(\alpha)+}} \prod_{N=1}^{\infty} \prod_{l=1}^{N_{N|w}^{(\alpha)}} \frac{v_k^{(\alpha)} - w_{l,N}^{(\alpha)-}}{v_k^{(\alpha)} - w_{l,N}^{(\alpha)+}} \,,$$

$$\prod_{Q=1}^{\infty} \prod_{l=1}^{N_Q} S_{xv}^{QK}(x_l, v_{k,K}^{(\alpha)}) = \prod_{M=1}^{\infty} \prod_{l=1}^{N_{M|vw}^{(\alpha)}} S_{vv}^{KM}(v_{k,K}^{(\alpha)}, v_{l,M}^{(\alpha)}) \prod_{N=1}^{\infty} \prod_{l=1}^{N_y^{(\alpha)}} S_{vy}^{K}(v_{k,K}^{(\alpha)}, w_l^{(\alpha)}) \,,$$

$$(-1)^K = \prod_{l=1}^{N_y^{(\alpha)}} \frac{w_{k,K}^{(\alpha)-} - v_l^{(\alpha)}}{w_{k,K}^{(\alpha)+} - v_l^{(\alpha)}} \prod_{N=1}^{\infty} \prod_{l=1}^{N_{N|w}^{(\alpha)}} S_{ww}^{KN}(w_{k,K}^{(\alpha)}, w_{l,N}^{(\alpha)}) \qquad \text{with } \alpha = 1, 2 \text{ and } Q, K, M = 1, 2, \ldots$$

TBA equations for planar AdS_5/CFT_4 [D.B, Fioravanti, Tateo; Gromov, Kazakov, Vieira; Arutyunov, Frolov '09]

- taking the logarithm and the thermodynamic limit $R, N_Q, N_y^\alpha, N_{K|vw}^\alpha, N_{K|w}^\alpha$ of the mirror BAEs
  $\Rightarrow$ we derive an infinite set of **density equations** for $\rho_Q, \rho_y^\alpha, \rho_{v,K}^\alpha, \rho_{w,K}^\alpha$:

$$\rho_A^r(u) + \rho_A^h(u) = \frac{d\tilde{p}(u)}{du} + \sum_B \phi_{AB}(u,v) * \rho_B(v)$$

- by standard procedure (minimization of the free energy)
  $\Rightarrow$ we derive an infinite set of **TBA equations** for the pseudoenergies $\varepsilon_Q, \varepsilon_y^\alpha, \varepsilon_{v,K}^\alpha, \varepsilon_{w,K}^\alpha$

$$\varepsilon_A(u) = LE_Q\delta_{A,Q} - \sum_B \ln(1 + e^{-\varepsilon_B(v)}) * \phi_{BA}(u,v)$$

- part of them is very similar to the TBA equations for the Hubbard model
- we obtain the **ground state energy** as

$$E_0(L) = Lf(1/L) = -\sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} \frac{d\tilde{p}}{2\pi} \ln(1 + e^{-\varepsilon_Q(\tilde{p})}) + i\mu(N_F^{(1)} - N_F^{(2)})$$

- in order to keep the densities finite we introduce **chemical potentials**: $\varepsilon_A \to \mu_A/T + \varepsilon_A$
- SUSY case: $E_0(L) = 0$ (BPS ground state) is determined by Witten's index $Tr(-1)^F e^{-\beta H}$ with $\mu \equiv \mu_{y,y*}^{(1),(2)} = \pi$

[D.B, Fioravanti, Tateo '09]

$$\begin{aligned}
\varepsilon_Q(\tilde{p}) &= 2L\,\mathrm{arcsinh}\left(\frac{\sqrt{Q^2+\tilde{p}^2}}{2g}\right) - \sum_{Q'=1}^{\infty}(\phi_{sl(2)}^{Q'Q} * L_{Q'})(\tilde{p}) \\
&- \sum_{\alpha=1}^{2}(\phi_{yx}^{Q} * L_y^{\alpha})(\tilde{p}) - \sum_{\alpha=1}^{2}\sum_{M=1}^{\infty}(\phi_{vx}^{MQ} * L_{v,M}^{\alpha})(\tilde{p}) \, , \\
\varepsilon_y^{\alpha}(q) &= -\sum_{Q=1}^{\infty}(\phi_{xy}^{Q} * L_Q)(q) - \sum_{M=1}^{\infty}(\phi_{wy}^{M} * L_{w,M}^{\alpha})(q) - \sum_{N=1}^{\infty}(\phi_{vy}^{N} * L_{v,N}^{\alpha})(q) \, , \\
\varepsilon_{v,K}^{\alpha}(\lambda) &= -\sum_{Q=1}^{\infty}(\phi_{xv}^{QK} * L_Q)(\lambda) - (\phi_{yv}^{K} * L_y^{\alpha})(\lambda) - \sum_{M=1}^{\infty}(\phi_{vv}^{MK} * L_{v,M}^{\alpha})(\lambda) \, , \\
\varepsilon_{w,K}^{\alpha}(\lambda) &= -(\phi_{yw}^{K} * L_y^{\alpha})(\lambda) - \sum_{M=1}^{\infty}(\phi_{ww}^{MK} * L_{w,M}^{\alpha})(\lambda)
\end{aligned}$$

with $L_A(\tilde{p}) = \ln(1+e^{-\varepsilon_A(\tilde{p})})$ and the integration limits on $\phi_{yz}^{M} * L_y$ are $-\pi, \pi$ if $y_k = ie^{-iq_k}$ or $\int_{-2}^{2} dz \left[L_{y|-}(z)\,\phi_{(y|-),M}(z,u) - L_{y|+}(z)\,\phi_{(y|+),M}(z,u)\right]$ if

$$y(u) = \begin{cases} x(u) & \text{for} \quad \Im m(y) < 0 \, , \\ 1/x(u) & \text{for} \quad \Im m(y) > 0 \, . \end{cases}$$

with $u = 2\sin(q)$, $x(u) = u/2 - i\sqrt{1-u^2/4}$, $\varepsilon_{y|+}(u) = \varepsilon_{y|-}((u+2)e^{i2\pi} - 2)$

Hubbard Y-system

- the TBA equations of the Hubbard model can be written in their universal form via the Zamolodchikovs' identity
  $\Rightarrow$ the universal kernel $\varphi_g(\theta) = \frac{g}{2\cosh(\pi\theta g)}$ satisfies the property

$$\varphi_g\left(\theta + \frac{i}{2g}\right) + \varphi_g\left(\theta - \frac{i}{2g}\right) = \delta(\theta)$$

- then leads to the functional relations

$$Y^+_{a,b} Y^-_{a,b} = \frac{(1+Y_{a+1,b})(1+Y_{a-1,b})}{\left(1+Y^{-1}_{a,b+1}\right)\left(1+Y^{-1}_{a,b-1}\right)}$$

with $a, b = 1, 2, \ldots$ and

$$Y_{11}(q) = e^{-\varepsilon_y(\pi - q)};\ Y_{22}(q) = e^{\varepsilon_y(q)};\ Y_{1,b+1}(\lambda) = e^{-\varepsilon_{w,b}(\lambda)};\ Y_{a+1,1}(\lambda) = e^{\varepsilon_{v,a}(\lambda)}$$

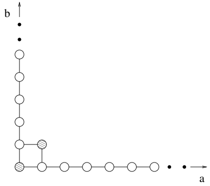


Figure: Hubbard Y-system

AdS_5/CFT_4 Y-system [D.B, Fioravanti, Tateo; Gromov, Kazakov, Vieira; Arutyunov, Frolov '09]

- thanks to the property satisfied by the kernels of the TBA equations: [Ravanini, Tateo, Valleriani '92]

$$\phi_{ww}^{KM}\left(\lambda', \lambda + \frac{i}{2g}\right) + \phi_{ww}^{KM}\left(\lambda', \lambda - \frac{i}{2g}\right) - \sum_{K'=1}^{\infty} I_{KK'}\,\phi_{ww}^{K'M}(\lambda', \lambda) = -I_{KM}\delta(\lambda' - \lambda)$$

- manipulating the TBA equations we obtain relations like

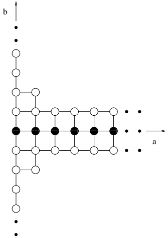
$$\varepsilon_{w,K}^{\alpha}\left(\lambda + \frac{i}{2g}\right) + \varepsilon_{w,K}^{\alpha}\left(\lambda - \frac{i}{2g}\right) - \sum_{K'=1}^{\infty} I_{KK'}\,\varepsilon_{w,K}^{\alpha}(\lambda) = -I_{KM}$$

and defining

$$Y_{Q,0}(u) = e^{\varepsilon_Q(u)}\,;\; Y_{1,\pm1}(q) = e^{\varepsilon_y^{1,2}(q)}\,;\; Y_{2,\pm2}(q) = e^{-\varepsilon_y^{1,2}(\pi - q)}\,;$$

$$Y_{K+1,\pm1}(\lambda) = e^{\varepsilon_{v,K}^{1,2}(\lambda)}\,;\; Y_{1,\pm K-1}(\lambda) = e^{-\varepsilon_{w,K}^{1,2}(\lambda)}$$

$$\Rightarrow \qquad Y_{a,b}^{+}Y_{a,b}^{-} = \frac{(1 + Y_{a+1,b})(1 + Y_{a-1,b})}{\left(1 + Y_{a,b+1}^{-1}\right)\left(1 + Y_{a,b-1}^{-1}\right)}$$



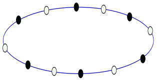
asymptotic solution $Y_{Q,0}(u) \simeq e^{-\tilde{E}_Q(u)L} \prod_{j=1}^{N} S_{sl(2)}^{1Q}(u_j, u)\, T_{Q,1}^2$

- the equation for $Y_{2,\pm2}$ cannot be included in this system, but $Y_{2,\pm2}^{\alpha}(q) = \frac{1}{Y_{1,\pm1}^{\alpha}(\pi - q)}$
- the AdS_5/CFT_4 Y-system corresponds to **two Hubbard Y-systems** coupled by the "massive" equations (inhomogenities)

Outline

The AdS_4/CFT_3 Bethe Ansatz equations

- Minahan and Zarembo ('08) formulated BAEs describing 1-loop anomalous dimensions of $\mathcal{N}=6$ SCS operators



$\Rightarrow$ algebraic curve and **all-loop asymptotic** BAEs for the full $Osp(2,2|6)$ superalgebra of SCS [Gromov,Vieira '08]

$$e^{ip_k^\alpha J} = \prod_{\substack{l=1\\ l\neq k}}^{K^{\mathrm{I}}_\alpha} \frac{u_k^\alpha - u_l^\alpha + \frac{2i}{h}}{u_k^\alpha - u_l^\alpha - \frac{2i}{h}} \left(\frac{x_k^{\alpha-} - x_l^{\alpha+}}{x_k^{\alpha+} - x_l^{\alpha-}}\right)^{\frac{1-\eta}{2}} \left(\sqrt{\frac{x_l^{\alpha+}}{x_l^{\alpha-}}\frac{x_k^{\alpha-}}{x_k^{\alpha+}}}\right)^{\frac{1+\eta}{2}} \sigma(p_k^\alpha, p_l^\alpha)$$

$$\prod_{l=1}^{K^{\mathrm{I}}_\beta} \left(\frac{x_k^{\alpha-} - x_l^{\beta+}}{x_k^{\alpha+} - x_l^{\beta-}}\right)^{\frac{1-\eta}{2}} \left(\sqrt{\frac{x_l^{\beta+}}{x_l^{\beta-}}\frac{x_k^{\beta-}}{x_k^{\beta+}}}\right)^{\frac{1+\eta}{2}} \sigma(p_k^\alpha, p_l^\beta) \prod_{j=1}^{K^{\mathrm{II}}} \frac{x_k^{\alpha-} - y_j}{x_k^{+\alpha} - y_j} \sqrt{\frac{x_k^{\alpha+}}{x_k^{\alpha-}}}$$

$$(-1)^\epsilon = \prod_{l=1}^{K^{\mathrm{I}}} \frac{y_k - x_l^+}{y_k - x_l^-}\sqrt{\frac{x_l^-}{x_l^+}} \prod_{l=1}^{K^{\mathrm{III}}} \frac{v_k - w_l + \frac{i}{h}}{v_k - w_l - \frac{i}{h}} \,; \qquad 1 = \prod_{l=1}^{K^{\mathrm{II}}} \frac{w_k - v_l - \frac{i}{h}}{w_k - v_l + \frac{i}{h}} \prod_{\substack{l=1\\ l\neq k}}^{K^{\mathrm{III}}} \frac{w_k - w_l + \frac{2i}{h}}{w_k - w_l - \frac{2i}{h}}$$

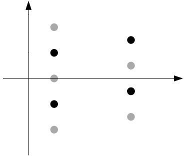
- 2 different kind of momentum carrying particles $\alpha, \beta = A, B$
- in $SU(2)\times SU(2)$ grading A and B eqs related only by dressing factor;

  in $SL(2|1)$ grading by $\frac{x_k^{\alpha-} - x_l^{\beta+}}{x_k^{\alpha+} - x_l^{\beta-}}\sigma(p_k^\alpha, p_l^\beta)$

TBA for planar AdS_4/CFT_3 [D. B., Fioravanti, Tateo; Gromov, Levkovich Maslyuk '09]

- two different kinds of particles (A and B) in AdS_4/CFT_3 BAEs
- we have proposed **mirror BAEs** in the $\eta = -1$ grading ("$SL(2|1)$")

$$e^{i\tilde{p}_A R} = \frac{x_A^- - x_B^+}{x_A^+ - x_B^-}\cdots \qquad e^{i\tilde{p}_B R} = \frac{x_A^- - x_B^+}{x_A^+ - x_B^-}\cdots$$



$\Rightarrow$ Bethe roots organize in $SL(2|1)$ **Bethe strings** [Pozsgay, Saleur '09] with mixed alternated type A and type B roots

$\Rightarrow$ 4 different kinds of strings (wide-I, wide-II, strange-I, strange-II)

- we obtained 2 TBA equations for massive pseudo-energies of type I and II

$$\varepsilon_{\mathcal{Q}|I}(u) = L E_{\mathcal{Q}}(u) - \sum_{\mathcal{Q}'=1}^{\infty} L_{\mathcal{Q}'|I} * \phi_{(\mathcal{Q}'|I),(\mathcal{Q}|I)}(u) - \sum_{\mathcal{Q}'=1}^{\infty} L_{\mathcal{Q}'|II} * \phi_{(\mathcal{Q}'|II),(\mathcal{Q}|I)}(u) + \sum_{M=1}^{\infty} L_{v|M} * \phi_{(v|M),\mathcal{Q}}(u) + \dots$$

$$\varepsilon_{\mathcal{Q}|II}(u) = L E_{\mathcal{Q}}(u) - \sum_{\mathcal{Q}'=1}^{\infty} L_{\mathcal{Q}'|II} * \phi_{(\mathcal{Q}'|II),(\mathcal{Q}|II)}(u) - \sum_{\mathcal{Q}'=1}^{\infty} L_{\mathcal{Q}'|I} * \phi_{(\mathcal{Q}'|I),(\mathcal{Q}|II)}(u) + \sum_{M=1}^{\infty} L_{v|M} * \phi_{(v|M),\mathcal{Q}}(u) + \dots$$

$$\varepsilon_{y|\pm}(u) = \dots \qquad \varepsilon_{v|K}(u) = \dots \qquad \varepsilon_{w|K}(u) = \dots$$

- remaining TBA eqs are very similar to AdS_5/CFT_4 case (with $\alpha = 1$) $\Rightarrow$ only **one copy of the Hubbard** TBA eqs, with 2 types of inhomogeneities
- **free energy** $f(T) = -T\sum_{\mathcal{Q}=1}^{\infty}\int_{\mathbb{R}} \frac{du}{2\pi}\frac{d\tilde{p}^{\mathcal{Q}}}{du} \ln\left(1 + e^{-\varepsilon_{\mathcal{Q}|I}(u)}\right) + \ln\left(1 + e^{-\varepsilon_{\mathcal{Q}|II}(u)}\right)$

Y-system for planar AdS_4/CFT_3 [D. B., Fioravanti, Tateo; Gromov, Levkovich Maslyuk '09]

- peculiar property of kernels describing interactions between massive pseudoenergies:

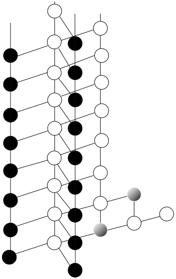
$$\phi_{(\mathcal{Q}'|I),(\mathcal{Q}|I)}\left(z,u+\frac{i}{h}\right)+\phi_{(\mathcal{Q}'|I),(\mathcal{Q}|II)}\left(z,u-\frac{i}{h}\right)=\left(\phi_{(\mathcal{Q}'|I),(\mathcal{Q}+1|I)}+\phi_{(\mathcal{Q}'|I),(\mathcal{Q}-1|II)}\right)(z,u)-\delta(z-u)\delta_{\mathcal{Q}',\mathcal{Q}+1}$$

$$\phi_{(\mathcal{Q}'|I),(\mathcal{Q}|I)}\left(z,u-\frac{i}{h}\right)+\phi_{(\mathcal{Q}'|I),(\mathcal{Q}|II)}\left(z,u+\frac{i}{h}\right)=\left(\phi_{(\mathcal{Q}'|I),(\mathcal{Q}-1|I)}+\phi_{(\mathcal{Q}'|I),(\mathcal{Q}+1|II)}\right)(z,u)-\delta(z-u)\delta_{\mathcal{Q}',\mathcal{Q}-1}$$

- lead to strange Y-system equations

$$Y_{\mathcal{Q}|I}(u+\frac{i}{h})Y_{\mathcal{Q}|II}(u-\frac{i}{h}) = (1+Y_{\mathcal{Q}+1|I}(u))(1+Y_{\mathcal{Q}-1|II}(u))\left(1+\frac{1}{Y_{v|\mathcal{Q}-1}(u)}\right)^{-1}$$

$$Y_{\mathcal{Q}|II}(u+\frac{i}{h})Y_{\mathcal{Q}|I}(u-\frac{i}{h}) = (1+Y_{\mathcal{Q}+1|II}(u))(1+Y_{\mathcal{Q}-1|I}(u))\left(1+\frac{1}{Y_{v|\mathcal{Q}-1}(u)}\right)^{-1}$$



- very different from that conjectured by [Gromov, Kazakov, Vieira '09] and from that for the direct theory: nodes on massive lines are strongly correlated

$$Y_{Q|A}(u+\frac{i}{h})Y_{Q|A}(u-\frac{i}{h}) = (1+Y_{Q+1|A}(u))(1+Y_{Q-1|A}(u))\left(1+\frac{1}{Y_{v|Q-1}(u)}\right)^{-1}$$

  - in symmetric sector ($Y_{\mathcal{Q}|I}=Y_{\mathcal{Q}|II}$) they match
  - magnonic Y-system eqs $\Leftrightarrow$ **one Hubbard Y-system**

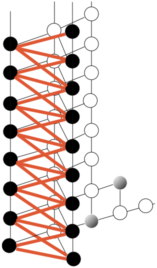
Y-system for planar AdS_4/CFT_3 [D. B., Fioravanti, Tateo; Gromov, Levkovich Maslyuk '09]

- peculiar property of kernels describing interactions between massive pseudoenergies:

$$\phi_{(\mathcal{Q}'|I),(\mathcal{Q}|I)}\left(z, u+\frac{i}{h}\right) + \phi_{(\mathcal{Q}'|I),(\mathcal{Q}|II)}\left(z, u-\frac{i}{h}\right) = \left(\phi_{(\mathcal{Q}'|I),(\mathcal{Q}+1|I)} + \phi_{(\mathcal{Q}'|I),(\mathcal{Q}-1|II)}\right)(z,u) - \delta(z-u)\delta_{\mathcal{Q}',\mathcal{Q}+1}$$

$$\phi_{(\mathcal{Q}'|I),(\mathcal{Q}|I)}\left(z, u-\frac{i}{h}\right) + \phi_{(\mathcal{Q}'|I),(\mathcal{Q}|II)}\left(z, u+\frac{i}{h}\right) = \left(\phi_{(\mathcal{Q}'|I),(\mathcal{Q}-1|I)} + \phi_{(\mathcal{Q}'|I),(\mathcal{Q}+1|II)}\right)(z,u) - \delta(z-u)\delta_{\mathcal{Q}',\mathcal{Q}-1}$$

- lead to strange Y-system equations



$$Y_{\mathcal{Q}|I}(u+\frac{i}{h})Y_{\mathcal{Q}|II}(u-\frac{i}{h}) = (1+Y_{\mathcal{Q}+1|I}(u))(1+Y_{\mathcal{Q}-1|II}(u))\left(1+\frac{1}{Y_{v|\mathcal{Q}-1}(u)}\right)^{-1}$$

$$Y_{\mathcal{Q}|II}(u+\frac{i}{h})Y_{\mathcal{Q}|I}(u-\frac{i}{h}) = (1+Y_{\mathcal{Q}+1|II}(u))(1+Y_{\mathcal{Q}-1|I}(u))\left(1+\frac{1}{Y_{v|\mathcal{Q}-1}(u)}\right)^{-1}$$

- very different from that conjectured by [Gromov, Kazakov, Vieira '09] and from that for the direct theory: nodes on massive lines are strongly correlated

$$Y_{Q|A}(u+\frac{i}{h})Y_{Q|A}(u-\frac{i}{h}) = (1+Y_{Q+1|A}(u))(1+Y_{Q-1|A}(u))\left(1+\frac{1}{Y_{v|Q-1}(u)}\right)^{-1}$$

  - in symmetric sector ($Y_{\mathcal{Q}|I} = Y_{\mathcal{Q}|II}$) they match
  - magnonic Y-system eqs $\Leftrightarrow$ **one Hubbard Y-system**

Outline

Results...

- we have formulated **ground state TBA** equations and **Y-systems**, by generalising the method originally developed for relativistic theories, in order to give the exact finite-size spectrum of $\mathbf{AdS_5/CFT_4}$ and $\mathbf{AdS_4/CFT_3}$ correspondences
- $\mathbf{AdS_5/CFT_4}$
  - quantization of L [Frolov, Suzuki '09]
  - 2 different **excited states TBA** for the sl(2) sector [Gromov, Kazakov, Vieira; Arutyunov, Frolov, Suzuki '09] inspired by standard techniques [Dorey, Tateo '96]
  - **strong coupling** numerical solutions for 2-particle (Konishi) $sl(2)$ states [Gromov, Kazakov, Vieira '09; Frolov '10] give 1-loop mismatch with string results [Roiban, Tseytlin '09]
  - at **weak coupling**: confirmation [Arutyunov, Frolov, Suzuki; Balog, Hegedus '10] of 5-loop Lüscher prediction [Bajnok, Janik, Hegedus, Lukowski '09]; conjectured **generalized Lüscher formula** derived analytically (see Balog's talk)
  - from **Y-system**'s asymptotic solutions: Konishi states 4-loop finite-size an. dim. matching Lüscher [Bajnok, Janik, Lukowski '08] and diagrammatic [Fiamberti, Santambrogio, Sieg, Zanon '07,'08] results
  - at **strong coupling** matching quasi-classical BAEs from **algebraic curve** [Gromov '09, Gromov, Kazakov, Vieira '10]
  - **analytical properties of the Y-system** $\Rightarrow$ TBA [Cavagliá, Fioravanti, Tateo '10]
- $\mathbf{AdS_4/CFT_3}$
  - from **Y-system**'s asymptotic solutions: 2-loop finite-size an. dim. of $SU(2) \times SU(2)$ symmetric operator [Gromov, Kazakov, Vieira '09] confirmed by diagrammatic calculations [Minahan, Ohlsson-Sax, Sieg '09]
  - at **strong coupling** matching quasi-classical BAEs from **algebraic curve** [Gromov, Levkovich Maslyuk '09]

...and Outlook

- **TBA for all the excited states** $\Rightarrow$ exact AdS/CFT spectra
  - to solve $sl(2)$ **excited states TBA** eqs at **any** g (numerically and analytically)
  - to formulate **excited states TBA eqs for all the other sectors** (string solutions) of AdS_5/CFT_4 and for AdS_4/CFT_3
  - to investigate **non-symmetric sectors** with asymptotic solution of the AdS_4/CFT_3 Y-system
- to formulate a finite set of NLIE or DdV equations
- generalization to the case with **boundaries** (open spin chain/string)
- generalization to the β-deformed case
  $\Rightarrow$ definitive test of the correspondences $\Delta = E$
  $\Rightarrow$ exact solution of SYM and SCS (and β-SYM,...)
  $\Rightarrow$ exact solution of $AdS_5 \times S^5$ and $AdS_4 \times \mathbb{CP}^3$ superstring theories
- which kind of theories under $AdS/CFTs$?
  ...a single and two coupled inhomogeneous Hubbard models?
- why integrability in $AdS/CFTs$?