# Introduction to Cosmology (in 3 lectures)

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# Cosmology

#### **Program:**

- Introduction, lightening fast background review (homeworks in yellow background)
- Inflation
- Dark energy

Lectures and additional material will appear at http://icc.ub.edu/~liciaverde/TAElectures.html

# Cosmology

```
Cosmos= Universe, Order, beauty
-logy= study
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Greek!

Study of the Universe as a whole Aim at getting an understanding of:

-its origin

-its structure and composition

(where do galaxies, stars, planets, people come from?) -its evolution

-its fate

Deep connections to fundamental physics

### Scales involved!



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### New units of measure

For distance, we use pc, Kpc & Mpc  $1 \text{ pc} = 3.086 \times 10^{16} \text{ m}$  $1 \text{ Mpc} = 3.086 \times 10^{22} \text{ m}$ 

For comparison, mean Earth-Sun  $1 \text{ AU} = 1.496 \times 10^{11} \text{m}$ distance (Astronomical Unit):  $1 \text{ pc} = 2.1 \times 10^5 \text{AU}$ 

- Cosmologists often express masses  $1 M_{\odot} = 1.99 \times 10^{30} kg$  - in units of the solar mass:

### Looking far away is looking back in time! 8 minutes ago





28000 years ago

#### Cosmic archeology

Andromeda, M31 2.2 million years ago

Looking far away in space= looking back in time



3 billion years ago

### stars



HST image of stars being born, but it has no direct use for Cosmology; exploding stars (supernovae) are very useful (you'll see later on)

By the way, we are star-dust

# galaxies

#### Collections of ~10<sup>11~</sup> 10<sup>12</sup> Stars







# galaxies







### The local group



Entering the regime of cosmology....

groups



### Groups and clusters





### Hubble deep field



# Not only pretty pictures

Nature is written in the mathematical language (Galileo)

The laws of physics are the same in the entire Universe

The universe is comprehensible (by us)

Deep connections to fundamental physics

FROW QUARKS TO THE COSMOS!

# Distances are difficult: velocities are "easy" Thank you Edwin Hubble





## REDSHIFT

$$z = rac{\lambda_{
m obsv} - \lambda_{
m emit}}{\lambda_{
m emit}}$$
 or

$$1 + z = rac{\lambda_{
m obsv}}{\lambda_{
m emit}}$$

#### In relativity:

$$1 + z = \gamma \left( 1 + \frac{v_{\parallel}}{c} 
ight)$$
  
 $z pprox rac{v_{\parallel}}{c}$  For small velocity

or 
$$1 + z = \frac{1 + v \cos(\theta)/c}{\sqrt{1 - v^2/c^2}}$$



Hubble's Law  $cz = v = H_0 d$ 

Ho=74.2 +- 3.8 km/s/Mpc



The universe is expanding! The universe had a beginning!

The extremely successful BIG BANG theory!



### The scale factor a



r(t)=r(t<sub>0</sub>) a(t) Comoving coordinates!

$$v_{12}=dr_{12}/dt=\dot{a} r_{12}(t_0)=\dot{a}/a r_{12}(t)$$
  
 $H=\frac{\dot{a}}{a}$  Important!







### How old is the Universe?

 $t_0 = r/v = r/(H_0 r) = 1/H_0$ 

Hubble time

The Hubble time is the age of the present day Universe assuming a constant expansion rate.



Hubble radius c/H<sub>0</sub> Also called Hubble horizon

Exercise: compute numerical values. Do they make sense? Why? Is that a coincidence?

### Some assumptions

The Universe is homogeneous and isotropic on large scales



This is supported by observations

HDF north and south



#### 2dF GRS

Large volumes of the sky in different directions, 100's of Mpc in size, look about the same.

### Geometry







Universe with *positive* curvature. Diverging line converge at great distances. Triangle angles add to more than 180°. Universe with *negative* curvature. Lines diverge at ever increasing angles. Triangle angles add to less than 180°.



Universe with no curvature. Lines diverge at constant angle. Triangle angles add to 180°.



Finite area, max separation

### In 3D and in general...

$$d\theta^2 - - - d\theta^2 + \sin^2\theta d\phi^2 = d\Omega^2$$

 $ds^2=dr^2+S_k(r)^2d\Omega^2$ 

$$S_{k}(r) = \begin{cases} \sqrt{k}^{-1} \sin(r\sqrt{k}), & k > 0\\ r, & k = 0\\ \sqrt{|k|}^{-1} \sinh(r\sqrt{|k|}), & k < 0. \end{cases}$$

*k* may be taken to belong to the set  $\{-1,0,+1\}$ 

Changing coordinate system  $x=S_k(r)$ :

$$ds^2 = \frac{dx^2}{1 - \kappa x^2/R^2} + x^2 d\Omega^2.$$

### Freedman-Robertson Walker metric

#### In 4 dimensions and introducing back the scale factor

$$ds^{2} = dt^{2} - a^{2}(t) \left[ \frac{dx^{2}}{1 - kx^{2}} + x^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}) \right]$$

$$c^{2} \qquad \qquad \text{If } k=0 \text{ Minkowski}$$

$$OR \quad ds^{2} = -c^{2}dt^{2} + a(t)^{2} \left[ dr^{2} + S_{\kappa}(r)^{2} d\Omega^{2} \right] \quad Comoving \ coords \ again!$$

*t is COSMIC TIME: time seen by an observer who sees the universe expanding uniformly* 

#### Knowing a(t),k, and R<sub>0</sub> is "all" you need!

### *Compute distances:*

At fixed time spatial geodesic (angles are fixed) ds=a(t)dr

Proper distance:

$$d_p = a(t) \int_0^r dr = a(t)r = a(t)r(x) = a(t)S_k(r)^{-1}$$

Hubble law again:

$$\dot{d}_p = \dot{a}r = \frac{\dot{a}}{a}d_p$$

$$H_o = \left(\frac{\dot{a}}{a}\right)_{t=t_0}$$

$$v_p(t_0) = H_0 d_p(t_0)$$

### Redshift, again

14

$$ds = 0 \longrightarrow c^{2}dt^{2} = a(t)^{2}dr^{2}$$
Photons travel along  
null geodesics
$$c\frac{dt}{a(t)} = dr$$
tonly
ronly
tonly
$$c\int_{t_{e}}^{t_{o}} \frac{dt}{a(t)} = \int_{0}^{r} dr = r$$

$$c\int_{t_{e}+\lambda_{e}/c}^{t_{o}+\lambda_{o}/c} \frac{dt}{a(t)} = \int_{0}^{r} dr = r$$

$$c\int_{t_{e}+\lambda_{e}/c}^{t_{o}+\lambda_{o}/c} \frac{dt}{a(t)} = \int_{0}^{r} dr = r$$

$$c\int_{t_{e}+\lambda_{e}/c}^{t_{o}+\lambda_{o}/c} \frac{dt}{a(t)} = c\int_{t_{e}}^{t_{o}} \frac{dt}{a(t)}$$

$$\frac{1}{a(t_e)} \int_{t_e}^{t_e + \lambda_e/c} dt = \frac{1}{a(t_o)} \int_{t_o}^{t_o + \lambda_o/c} dt \longrightarrow \frac{\lambda_e}{a(t_e)} = \frac{\lambda_o}{a(t_o)}$$
Should remind you of z
$$\frac{\lambda_o - \lambda_e}{\lambda_e} = (1+z) = \frac{a(t_o)}{a(t_e)}$$

If an object has z=3, what was the size of the Universe?

#### How do we measure the geometry then?

 $\begin{array}{ll} \alpha + \beta + \gamma = \pi + kA/R_0 \\ \mbox{K=+1--> finite size: circumference} & 2\pi R_0 & \mbox{In the past even smaller} \\ \mbox{If} & 2\pi R_0 << ct_0 \sim c/H_0 & \mbox{????} \end{array}$ 

Angular size of objects If I happen to know dl....

$$d heta = rac{dl}{a(t)S_k(r)}$$
 Ah!



Standard ruler, angular diameter distance

brightness  
**Standard candle** 
$$F = \frac{L}{4\pi r^2} \longrightarrow F = \frac{L}{4\pi S_k^2 (1+z)^2}$$
  
Ah! ah!

- In GR space tells mass how to move, mass tells space how to curve.
- Suspicion: a(t) related to content of Universe?
- Really need GR but we do Newton...

Friedmann equations (1)  

$$a_{cc} = \frac{d^2 r_s(t)}{dt^2} = -\frac{GM_s}{r_s^2(t)}$$

$$\frac{dr_s}{dt} \frac{d^2 r_s(t)}{dt^2} = -\frac{GM_s}{r_s^2(t)} \frac{dr_s}{dt} = \frac{1}{2} \frac{d}{dt} \left(\frac{dr_s}{dt}\right)^2 \text{ integrate } \frac{1}{2} \left(\frac{dr_s}{dt}\right)^2 = \frac{GM_s}{r_s(t)} + U$$

$$Ms = \frac{4\pi}{3} \rho(t) r_s^3(t) \equiv \frac{4\pi}{3} \rho(t) a^3(t) r_s^3 \text{ Substitute and divide each side by } \frac{r_s^2 a^2}{2}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho(t) + \frac{2U}{r_s^2 a^2}$$
Symmetric under time reversal

Start with an expanding sphere: How does it evolve of U>0 ? How does it evolve of U<0 ?

Need GR to go further

### Friedmann equations 1



### Friedmann equations 2

One eq.; 2 unknowns a(t),  $\rho(t)$ , need a relation between the two

 $dQ = dE + PdV \quad \text{Expanding universe, adiabatic} \quad \dot{E} + P\dot{V} = 0$ Consider a spherical chunk of Universe  $\dot{V} = \frac{4\pi}{3} \frac{d}{dt} (ar_s)^3 = \frac{4\pi}{3} r_s^3 3a^2 \dot{a} = V \left(3\frac{\dot{a}}{a}\right)$  $\dot{E} = \frac{d}{dt} \epsilon(t) V(t) = \dot{V} \epsilon + V \dot{\epsilon} = V \left(\dot{\epsilon} + 3\frac{\dot{a}}{a}\epsilon\right)$  $0 = \dot{E} + P\dot{V} = V \left(\dot{\epsilon} + 3\frac{\dot{a}}{a}\epsilon\right) + PV3\frac{\dot{a}}{a}$ 

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}\left(\epsilon + P\right) = 0$$

Fluid equation

Friedman and fluid equations are ENERGY CONSERVATION

### Friedmann Equations 3

Let's combine the two to get a useful eqn

Multiply Friedmann by a²  
derive wrt t  
Divide by 2àa
$$\dot{a}^2 = \frac{8\pi G}{3c^2}\epsilon a^2 - \frac{kc^2}{R_0^2}$$
  
 $2\dot{a}\ddot{a} = \frac{8\pi G}{3c^2}(2\epsilon a\dot{a} + \dot{\epsilon}a^2)$   
 $\frac{\ddot{a}}{a} = \frac{8\pi G}{3c^2}(\dot{\epsilon}\frac{a}{\dot{a}} + 2\epsilon)$ Use fluid eq. $\dot{\epsilon}\frac{a}{\dot{a}} = -3(\epsilon + P)$ 

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P)$$
 Acceleration equation!

If P=0 and  $\varepsilon$ >0 ...

So if you start with an expanding Universe....

And what would it take to make it accelerate?

If the Universe started off expanding stuff should slow down the expansion!

#### Friedmann equations NOT independent!

$$H^{2} = \left(\frac{\dot{a}}{a}\right)^{2} = \frac{8\pi G}{3}\rho(t) - \frac{kc^{2}}{R_{0}^{2}a^{2}}$$
$$\rho(t)c^{2} = \epsilon(t)$$
$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^{2}}(\epsilon + 3P)$$

If you want to solve for

$$a(t), \epsilon(t), P(t)$$

need more info...

```
P = P(\epsilon)
```

$$P=w\epsilon$$

#### Interesting cases:

Non-relativistic matter	$w\simeq 0$	
Radiation	w = 1/3	
Accelerating fluid	w < -1/3	
Cosmological constant	w = -1	

This is weird..... but looks like we are stuck with it

### Einstein and the cosmological constant $\nabla^2 \Phi = 4 \pi G \rho$ Poisson equation $a_{cc} = -\vec{\nabla} \Phi \equiv 0$ If static If static $ho = rac{1}{4\pi G} abla^2 \Phi = 0$ Empty... humm.... or $abla^2 \Phi + \Lambda = 4\pi G \rho$ Einstein.... $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{R_0^2 a^2} + \frac{\Lambda}{3}$ $\dot{\epsilon} + 3\frac{\dot{a}}{c}(\epsilon + P) = 0$ Does not get diluted... $\epsilon_{\Lambda} = \frac{c^2}{8\pi G}\Lambda$

$${\ddot a\over a}=-{4\pi G\over 3c^2}(\epsilon+3P)~+{\Lambda\over 3}$$

 $w = -1, P_{\Lambda} = -\epsilon$ 

Vacuum energy...

I told you it was weird....

### The universe composition



Use Friedmann equation

$$\dot{a}^{2} = \frac{8\pi G}{3c^{2}}\epsilon_{0}a^{-(1+3w)}$$

You can solve it!

$$\begin{split} & \text{If it was only one component} \\ \dot{a}^2 &= \frac{8\pi G}{3c^2} \epsilon_0 a^{-(1+3w)} \quad \text{Ansatz:} a \sim t^q \quad \text{If} \quad w \neq -1 \quad q = \frac{2}{3(1+w)} \\ & a(t) &= \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} \\ & a(t) &= \sqrt{\frac{c^2}{6\pi G \epsilon_0}} \frac{1}{(1+w)} \\ & t_0 &= \sqrt{\frac{c^2}{6\pi G \epsilon_0}} \frac{1}{(1+w)} \\ & H_0 &= \frac{\dot{a}}{a}\Big|_{t_0} = t_0^{-1} \frac{2}{3(1+w)} \\ & H_0 &= c \int_{t_e}^{t_o} \frac{dt}{a(t)} = \dots = \frac{c}{H_0} \frac{2}{1+3w} \left[1 - (1+z)^{-(1+3w)/2}\right] \end{split}$$

The farther object you can see.... Horizon! (we'll get back to this later)

If it was only 
$$\Lambda$$
  
 $\dot{a}^2 = \frac{8\pi G}{3c^2} \epsilon_0 a^{-(1+3w)}$   
 $H = \left(\frac{8\pi G\epsilon_\Lambda}{3c^2}\right)^{1/2} = H_0$   
 $\dot{a} = const \longrightarrow a(t) = \exp[H_0(t - t_0)]$ 

$$PHYSICAL: \ d_p(t) = a(t) \int \frac{dt}{a(t)} = \exp(H_\Lambda t) \int_{t_i}^{t_f} \exp(-H_\Lambda t) dt$$
$$= H_\Lambda^{-1}(\exp(H_\Lambda (t - t_i)) - 1)$$

Grows exponentially and c/H<<dp!

#### Back to Friedmann

$$H(t)^{2} = \frac{8\pi G}{3c^{2}}\epsilon - \frac{kc^{2}}{R_{0}^{2}a^{2}} \longrightarrow (1 - \Omega) = -\frac{kc^{2}}{R_{0}^{2}a^{2}H_{0}^{2}}$$
$$\rho_{c,0} = \frac{3H_{0}^{2}}{8\pi G} \qquad \Omega = \frac{\rho_{tot}}{\rho_{c}}$$

$$\frac{H(t)^2}{H_0^2} = \frac{8\pi G}{3c^2} \frac{\epsilon}{H_0^2} - \frac{(\Omega_0 - 1)}{a^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda_0} + \frac{\Omega_{k,0}}{a^2}; \qquad \Omega_k = 1 - \Omega_{k,0}$$

If you have a mix of components, chances are that at different times in the life of the Universe different components dominate





### Composition of the Universe



Radiation...





#### Evolution....



Exercise: take a point in this plot and show that the arrow direction is OK

This is a BIG puzzle

#### Distances, for completeness

Luminosity distance: In an expanding universe, distant galaxies are much dimmer than you would normally expect because the photons of light become stretched and spread out over a wide area.

Angular diameter distance: In an expanding universe, we see distant galaxies when they were much younger and much closer to us

**Comoving Distance** is the opposite of the Angular Diameter Distance - it tells us where galaxies are now rather than where they were when they emitted the light that we now see

The Light Travel Time Distance represents the time taken for the light from distant galaxies to reach us



For small distances all four distance scales converge and become the same

#### Distances in useful format

$$1+z=rac{a(t_0)}{a(t)}$$

$$d_c = \int_{t_e}^{t_o} \frac{a_0}{a} dt$$

 $\frac{dz}{da} = -\frac{a_0}{a^2}$ 

$$\frac{H(z)}{H_0} = E(z) = \left(\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda\right)^{\frac{1}{2}}$$

$$\begin{array}{rcl} \frac{dz}{E(z)} & = & -\frac{da \ a_0}{a(t)^2} \frac{1}{E(z)} \\ & = & -\frac{da \ a_0}{a(t)^2} \frac{a(t)}{\dot{a}(t)} H_0 \\ & = & -\frac{a_0}{a(t)} H_0 \ dt. \end{array}$$

Comoving, line-of-sight distance

$$d_c = D_c = D_H \int_0^z \frac{dz'}{E(z)} \qquad D_H = \frac{c}{H_0}$$

Comoving transverse distance

$$D_M = \begin{cases} D_H \frac{1}{\sqrt{\Omega_k}} \sinh\left[\sqrt{\Omega_k} D_C / D_H\right] & \text{for } \Omega_k > 0\\ D_C & \text{for } \Omega_k = 0\\ D_H \frac{1}{\sqrt{|\Omega_k|}} \sin\left[\sqrt{|\Omega_k|} D_C / D_H\right] & \text{for } \Omega_k < 0 \end{cases}$$

$$D_M = D_H \frac{2 \left[2 - \Omega_M \left(1 - z\right) - \left(2 - \Omega_M\right) \sqrt{1 + \Omega_M z}\right]}{\Omega_M^2 \left(1 + z\right)} \text{ for } \Omega_\Lambda = 0$$

#### Distances, useful format

Angular diameter distance

$$D_A = \frac{D_M}{(1+z)}$$

#### Warning: not additive!

$$D_{A12} = \frac{1}{1+z_2} \left[ D_{M2} \sqrt{1 + \Omega_k \frac{D_{M1}^2}{D_H^2}} - D_{M1} \sqrt{1 + \Omega_k \frac{D_{M2}^2}{D_H^2}} \right]$$



Distances, useful format

Luminosity distance

 $D_L = (1+z)D_M = (1+z)^2 D_A$ 



### Composition of the Universe



Radiation...

# There is more than meets the eye

In the solar system sun + planets Mass-to-light ratio

#### Let's consider galaxies





And clusters: Xrays

$$K.E. \sim \frac{3}{2}kT = \frac{3}{2}m_H\sigma_v^2$$

$$T \sim 6 \times 10^7 K; \lambda \sim \frac{c}{\nu} = \frac{ch}{KT}$$

### **Gravitational lensing**



### **Gravitational lensing**



#### Reconstructed dark matter distribution of cluster Abell 2218



Abell 2218

## New evidence REAL DATA!



**Bullet cluster** 



### **DIY dark matter**



Elastic WIMP

Nucleus

(v ~ 250 km/s)

(v = 0 km/s)

 $\theta_{_{\text{Recoil}}}$ 

E(recoil) ~ 20 keV

# Computer simulation of dark matter distribution







NASA, ESA and R. Massey (California Institute of Technology)

# It gets worst!

- The universe is undergoing an accelerated expansion
- It may not have been the first time

### Key concepts today

The expansion of the Universe Hubble's Law Redshift Geometry FRW metric Universe composition and background evolution

Take a good look at: <u>http://arxiv.org/pdf/astro-ph/9905116v4</u> Play with icosmo