Observational cosmology – Exercise

Taller de Altas Energías 2010

You are designing a space mission to map the cosmic microwave background radiation over the full sky with a sensitivity of 5 microK per 10' resolution element. The instrument uses an array of focal plane detectors with equivalent system temperature of 12K and bandwidth 20 GHz. The cryogenic system allows a detectors lifetime of 2 years.

How many detectors do you need to implement in the focal plane? How many would you need if the sensitivity requirement was 5 times more stringent? (Neglect foreground emissions and systematic effects).

SOLUTION:

The white noise of the array is given by (assuming radiometer constant equal to unity):

$$\delta T_{rms} \approx \frac{T_{sys} + T_{sky}}{\sqrt{\Delta\nu \cdot (N\tau)}}$$

Where *N* is the number of detectors to be found. The time necessary to reach the desired sensitivity in each 10' pixel is:

$$\tau_{pix} \approx \frac{1}{N}\left(\frac{T_{sys} + T_{sky}}{\sqrt{\Delta\nu \cdot}}\frac{1}{\delta T_{rms}}\right)^2 = \frac{1}{N\Delta\nu}\left(\frac{T_{sys} + T_{sky}}{\delta T_{rms}}\right)^2 =$$

$$= \frac{1}{N}\frac{1}{20\cdot 10^9\,\mathrm{Hz}}\left(\frac{(12+2.7)\mathrm{K}}{5\cdot 10^{-6}\mathrm{K}}\right)^2 = \frac{1}{N}432.2\,\mathrm{sec}$$

The number 10' square pixels in the full sky is:

$$n_{pix} = \frac{\Omega_{sky}}{\Omega_{beam}} = \frac{4\pi}{(10/60)^2}\frac{180^2}{\pi^2} \cong 1.5\times 10^6,$$

and the total observing time must be:

$$\tau_{tot} \approx \tau_{pix} n_{pix} = \frac{1}{N}432.2\cdot 1.5\times 10^6\,\mathrm{sec}$$

The lifetime of the mission is $\tau_{tot} = 2\ \mathrm{yrs} = 6.3\times 10^7\ \mathrm{sec}$, so we have

$$N = \frac{432.2\cdot 1.5\times 10^6\,\mathrm{sec}}{6.3\times 10^7\,\mathrm{sec}} \approx 10$$

A requirement δT_{rms} = 1 microK (instead of 5microK) per 10' pixel, implies that τ_{pix} is 25 times larger, and therefore N = 10 x 25 = 250.