## **Cosmological Simulations**

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- Simulation predictions for the dark sector of ΛCDM
- Can we falsify ΛCDM with simulations?
- Exaflop computing
- Beyond the dark sector: Hydrodynamic simulations
- Challenges for galaxy formation simulations



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Universität Heidelberg Unsolved Problems in Cosmology and Astrophysics Benasque, February 2011

#### The first slice in the CfA redshift survey



de Lapparent, Geller, Huchra (1986)

## Current galaxy redshift surveys map the Universe with several hundred thousand galaxies



# The initial conditions for cosmic structure formation are directly observable

THE MICROWAVE SKY

WMAP Science Team (2003, 2006, 2008, 2010)

### The basic dynamics of structure formation in the **dark matter** BASIC EQUATIONS AND THEIR DISCRETIZATION

Gravitation (Newtonian approximation

to GR in an expanding space-time)



Dark matter is collisionless

Monte-Carlo integration as **N-body System** 

3N **coupled**, non-linear differential equations of second order

Friedmann-Lemaitre model

$$H(a) = H_0 \sqrt{a^{-3}\Omega_0 + a^{-2}(1 - \Omega_0 - \Omega_\Lambda) + \Omega_\Lambda}$$

Collisionless Boltzmann equation with self-gravity

$$\frac{\mathrm{d}f}{\mathrm{d}t} \equiv \frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0$$
$$\nabla^2 \Phi(\mathbf{r}, t) = 4\pi G \int f(\mathbf{r}, \mathbf{v}, t) \mathrm{d}\mathbf{v}$$

Hamiltonian dynamics in expanding space-time  $H = \sum_{i} \frac{\boldsymbol{p}_{i}^{2}}{2 m_{i} a(t)^{2}} + \frac{1}{2} \sum_{ij} \frac{m_{i} m_{j} \varphi(\boldsymbol{x}_{i} - \boldsymbol{x}_{j})}{a(t)}$   $\nabla^{2} \varphi(\boldsymbol{x}) = 4\pi G \left[ -\frac{1}{L^{3}} + \sum_{\boldsymbol{n}} \tilde{\delta}(\boldsymbol{x} - \boldsymbol{n}L) \right]$ 

#### **Problems:**

- N is very large
- All equations are coupled with each other

1 Gpc/h

**'Millennium' simulation** Springel et al. (2005)

 $\Lambda$ CDM

10.077.696.000 particles m=8.6 x 10<sup>8</sup> M<sub>o</sub>/h

# Why are **cosmological simulations** of structure formation **useful for studying the dark universe**?



Simulations are the theoretical tool of choice for calculations in the non-linear regime.

They connect the (simple) cosmological initial conditions with the (complex) present-day universe.

#### **Predictions from N-body simulations:**

- Abundance of objects (as a function of mass and time)
- Their spatial distribution
- Internal structure of halos (e.g. density profiles, spin)
- Mean formation epochs
- Merger rates
- Detailed dark matter distribution on large and fairly small scales
- Galaxy formation models
- Gravitational lensing
- Baryonic acoustic oscillations in the matter distribution
- Integrated Sachs-Wolfe effect
- Dark matter annihilation rate
- Morphology of large-scale structure ("cosmic web")
- ....

## Simulations provide accurate measurements for halo abundance as a function of time

**CONVERGENCE RESULTS FOR HALO ABUNDANCE** 



Boylan-Kolchin, Springel, White, et al. (2009)

Simulated and observed largescale structure in the galaxy distribution

**MOCK PIE DIAGRAMS COMPARED TO** SDSS, 2DFGRS, AND CFA-2



## The two-point correlation function of galaxies in the Millennium run is a very good power law

GALAXY TWO-POINT FUNCTION COMPARED WITH 2dFGRS



#### The galaxy distribution is **biased** with respect to the mass distribution GALAXY AND MASS CLUSTERING AT DIFFERENT EPOCHS



The large-scale clustering pattern of halos and galaxies is already imprinted on the initial conditions

TIME EVOLUTION OF THE MATTER AND GALAXY DISTRIBUTION



The baryonic wiggles remain visible in the galaxy distribution down to low redshift and may serve as a "standard ruler" to constrain dark energy

DARK MATTER AND GALAXY POWER SPECTRA FROM THE MILLENNIUM SIMULATION IN THE REGION OF THE WIGGLES



Springel et al. (2005)





Structure of the central cusp

10<sup>9</sup> Spherically averaged density profiles of dark matter halos 10<sup>8</sup> have a nearly universal shape 10<sup>7</sup> **DENSITY PROFILE AS A FUNCTION OF RADIUS** 10<sup>6</sup>  $\rho(r) / <\rho>$ 10<sup>5</sup> 10<sup>4</sup> Fundamental importance for:



- Internal structure of galaxy clusters
- Gravitational lensing
- DM annihilation
- Galaxy mergers



## The logarithmic slope of the density profile does not show asymptotic behavior towards the core

SLOPE OF THE DENSITY PROFILE AS A FUNCTION OF RADIUS



A consensus on the central structure of the cups seems to be emerging RECENT RESULTS FROM THE 'GHALO' SIMULATION OF THE ZURICH GROUP

Stadel et al. (2009)

"The logarithmic slope of the radial density profile is close to a power law, gradually turning over to a slope of -0.8 at our innermost resolved point."

"The Einasto profile also provides an excellent fit to the density profiles of the two simulations."



#### Our simulations allow us to study the convergence of **subhalo density profiles**

SPHERICALLY AVERAGED DENSITY PROFILES IN THE AQ-A HALO AT DIFFERENT RESOLUTION



Dark matter substructure

Zooming in on dark matter halos reveals a huge abundance of dark matter substructure

DARK MATTER DISTRIBUTION IN A MILKY WAY SIZED HALO AT DIFFERENT RESOLUTION





## Zooming in on dark matter halos reveals a huge abundance of dark matter substructure

DARK MATTER IN A MILKY WAY SIZED HALO AT ULTRA-HIGH RESOLUTION



## The subhalo abundance per unit halo mass is surprisingly uniform **VELOCITY FUNCTION IN OUR DIFFERENT HALOS**



# The cumulative mass fraction in **resolved substructures** reaches about 12-13%, we expect up to ~18% down to the thermal limit **FRACTION OF MASS IN SUBSTRUCTURES AS A FUNCTION OF MASS LIMIT**



The radial distribution of substructures is strongly antibiased relative to all dark matter, and independent of subhalo mass

RADIAL SUBSTRUCTURE DISTRIBUTION IN Aq-A-1

Most subhalos are at large radii, subhalos are more effectively destroyed near the centre

Subhalos are far from the Sun





## The local mass fraction in substructures is a strong function of radius MASS FRACTION IN SUBSTRUCTURES AS A FUNCTION OF RADIUS IN HALO AQ-A



## Dark matter annihilation predictions

## Is the dark matter annihilation flux boosted significantly by dark matter substructures?

SIMULATED ALL-SKY MAP OF THE DM ANNIHILATION FLUX AROUND THE SUN IN THE MILKY WAY

 $L \propto \rho^2 \, \mathrm{d}V$ 

Aq-A-2

14.



## The annihilation luminosity from main halo and subhalos has a very different radial distribution

THE RELATIVE DISTRIBUTION OF MASS, MAIN HALO, AND SUBHALO LUMINOSITY



Surface brightness profile of a typical subhalo with  $V_{max}$ =10 km/s at different distances from the galactic center

SURFACE BRIGHTNESS PROFILE OF DIFFERENT SUBHALO COMPONENTS

The sub-sub component appears as a (extended) "disk" on the sky

The central surface brightness of the smooth component actually increases with smaller distance (because the concentration increases)



#### Dark matter annihilation can be best discovered with an optimal filter against a bright background

THE SIGNAL-TO-NOISE FOR DETECTION WITH AN OPTIMAL FILTER

The optimal filter  
is proportional to  
the signal 
$$S/N = \sqrt{\tau A_{\text{eff}}} \left[ \int \frac{n_{\gamma}^2(\theta, \phi)}{n_{\gamma}(\theta, \phi) + b_{\gamma}(\theta, \phi)} d\Omega \right]^{1/2}$$

#### The background dominates, then:

Main halo's smooth component:  $(S/N)_{\text{MainSm}} = f_{\text{MainSm}} \left[ \frac{\tau A_{\text{eff}}}{b_{\gamma}} \right]^{1/2} \frac{F}{\theta_h}$ 

Subhalo's smooth component:  

$$(S/N)_{\text{SubSm}} = f_{\text{SubSm}} \left( \theta_h / \theta_{\text{psf}} \right) \left[ \frac{\tau A_{\text{eff}}}{b_{\gamma}(\vec{\alpha})} \right]^{1/2} \frac{F}{(\theta_h^2 + \theta_{\text{psf}}^2)^{1/2}}$$

$$S/N \sim F / \theta$$
$$S/N \sim L / r_h d$$

Sub-substructure of a subhalo:  $(S/N)_{\text{SubSub}} = f_{\text{SubSub}} \left( \theta_h / \theta_{\text{psf}} \right) \left[ \frac{\tau A_{\text{eff}}}{b_{\gamma}(\vec{\alpha})} \right]^{1/2} \frac{F}{(\theta_h^2 + \theta_{\text{psf}}^2)^{1/2}}$ 

background noise

## **Detectability of different annihilation emission components in the Milky Way**

S/N for detecting subhalos in units of that for the main halo

30 highest S/N objects, assuming the use of optimal filters

$$S/N \propto C V_{\rm max}^4 / (r_{\rm half}^2 d)$$



Highest S/N subhalos have 1% of S/N of main halo Highest S/N subhalos have 10 times S/N of known satellites Substructure of subhalos has no influence on detectability The velocity distribution of dark matter at the Sun's position shows residual structure DISTRIBUTION OF VELOCITY COMPONENTS AND VELOCITY

MODULUS



Vogelsberger et al. (2009)

Wiggles in the distribution of the modulus of the velocity point to residual structure in energy space



The wiggles are the same in wellseparated boxes at the same radial distance, and are reproduced in simulations of different resolution

DISTRIBUTION OF THE VELOCITY MODULUS IN DIFFERENT WELL-SEPARATED BOXES




#### Millennium-XXL

Largest N-body simulation ever

303 billion particles

L = 3 Gpc/h

~700 million halos at z=0

~25 billion (sub)halos in mergers trees

 $m_p = 6.1 \text{ x } 10^9 \text{ M}_{\odot}/\text{h}$ 

12288 cores, 30 TB RAM on Supercomputer JuRoPa in Juelich

2.7 million CPU-hours



Are the presently known high-mass clusters still consistent with ΛCDM? CLUSTER MASS FOR GIVEN ABUNDANCE AS A FUNCTION OF TIME







## The existence of a firm upper limit for the properties of clusters is a strong and falsifiable prediction of $\Lambda$ CDM

ABUNDANCE OF CLUSTERS USING DIFFERENT OBSERVATIONAL PROBES



no objects above these limits should be detectable by any technique

Trouble ahead in the Exaflop regime



ENIAC, 1946



Zuse Z3, 1941

## **Performance:** ~ 1000 Flops

### Jaguar: Department of Energy Leadership computer Designed for science from the ground up

Peak performance	1.645 petaflops
System memory	362 terabytes HUGE!
Disk space	10.7 petabytes
Disk bandwidth	240 gigabytes/second
Interconnect BW	532 terabytes/second
Number cores	181504

60 years later - 10<sup>12</sup> times faster



2 Managed by UT-Battelle

Currently the fastest supercomputers carry out about ~1 Petaflop, which are one thousand billion floating point operations per second



How long would the Millennium-XXL take on a Exaflop Supercomputer at peak performance?

15 min

One of the main problems: *Power Consumption* 

Petaflop Computer: 6 MW

Exaflop Computer: ~ GW ?

Need to get this down to 20-40 MW



The number of cores on the top supercomputers grows exponentially **EXTREME GROWTH OF PARALLELISM** 



## **Challenges in exascale computing**

CAN WE HAVE THE CAKE AND EAT IT?

Memory per core decreasing.

Applications need to deal with multiple hierarchies of memory. (especially on GPU-accelerated or hybrid systems)

On systems with >10<sup>6</sup> cores, need fault-tolerant algorithms and codes. (resiliance has to be built into simulation codes)

Cost of data access relative to floating point ops drastically increasing.

Typical astrophysics codes run only at  $\sim 10\%$  of the peak performance – and its getting worse with time.

## None of our existing codes will survive and run on exascale platforms.

Astrophysics may be left behind in using these systems.

Hydrodynamical simulations

### Dynamics of structure formation in baryonic matter BASIC EQUATIONS

Astrophysical plasmas are extremely thin, with (usually) negligible viscosity

$$\begin{aligned} \frac{\partial \rho_c}{\partial t} &+ \frac{1}{a} \boldsymbol{\nabla}_c(\rho_c \boldsymbol{v}) = 0 \\ \frac{\partial (\rho_c \boldsymbol{v})}{\partial t} &+ \frac{1}{a} \boldsymbol{\nabla}_c[(\rho_c \boldsymbol{v} \boldsymbol{v}^T + P_c) \boldsymbol{v}] = -H(a) \rho_c \boldsymbol{v} - \frac{\rho_c}{a^2} \boldsymbol{\nabla}_c \Phi_c \\ \frac{\partial (\rho_c e)}{\partial t} &+ \frac{1}{a} \boldsymbol{\nabla}_c[(\rho_c e + P_c) \boldsymbol{v}] = -2H(a) \rho_c e - \frac{\rho_c \boldsymbol{v}}{a^2} \boldsymbol{\nabla}_c \Phi_c \\ \boldsymbol{\nabla}_c^2 \Phi_c &= 4\pi G \left[\rho_c(\boldsymbol{x}) - \overline{\rho_c}\right] \end{aligned}$$

Euler equations of inviscid ideal gas dynamics

#### Important hydrodynamical processes

Shock waves Turbulence Radiative transfer Magnetic fields Star formation Supernova explosions Black holes, etc...

#### Supersonic motion creates shock waves SHOCK WAVES OF A BULLET TRAVELLING IN AIR







Weak lensing mass reconstructions have confirmed an offset between mass peaks and X-ray emission MASS CONTOURS FROM LENSING COMPARD TO X-RAY EMISSION

**Clowe et al. (2006)** 



500 ksec Chandra exposure

Magellan Optical Image

weak lensing mass contours overlaid

NASA Press Release Aug 21, 2006:

## 1E 0657-56: NASA Finds Direct Proof of Dark Matter



## Fitting the density jump in the X-ray surface brightness profile allows a measurement of the shock's Mach number X-RAY SURFACE BRIGHTNESS PROFILE



### How rare is the bullet cluster?

DISTRIBUTION OF VELOCITIES OF THE MOST MASSIVE SUBSTRUCTURE IN THE MILLENNIUM RUN



# A simple toy merger model of two NFW halos on a zero-energy collision orbit

PARAMETERS OF A BASIC TOY MODEL



Mass model from Clowe et al. (2006):

 $M_{200} = 1.5 \times 10^{14} M_{\odot}$  $M_{200} = 1.5 \times 10^{15} M_{\odot}$  $R_{200} = 1.1 \text{ Mpc}$  $R_{200} = 2.3 \text{ Mpc}$ c = 7.2c = 2.0 $V_{200} = 780 \text{ km/sec}$  $V_{200} = 1680 \text{ km/sec}$ 

#### VIDEO OF THE TIME EVOLUTION OF A SIMPLE BULLET CLUSTER MODEL



Drawing the observed X-ray map and the simulation images with the same color-scale simplifies the comparison SIMULATED X-RAY MAP COMPARED TO OBSERVATION

Candra 500 ks image

bullet cluster simulation



Springel & Farrar (2007)

## The model also matches the observed temperature and mass profiles comparison of simulated temperature and mass profile with observations





Uncertainties and errors in hydrodynamical numerical techniques

### A cloud moving through ambient gas shows markedly different longterm behavior in SPH and Eulerian mesh codes

#### DISRUPTION OF A CLOUD BY KELVIN-HELMHOLTZ INSTABILITIES



Agertz et al. (2007)





Voronoi and Delaunay tessellations provide unique partitions of space based on a given sample of mesh-generating points BASIC PROPERTIES OF VORONOI AND DELAUNAY MESHES

#### Voronoi mesh



#### **Delaunay triangulation**



#### both shown together



Each Voronoi cell contains the **space closest** to its generating point

The Delaunay triangulation contains only triangles with an **empty circumcircle**. The Delaunay tiangulation maximizes the minimum angle occurring among all triangles.

The centres of the circumcircles of the Delaunay triangles are the vertices of the Voronoi mesh. In fact, the two tessellations are the topological **dual graph** to each other.

The fluxes are calculated with an exact Riemann solver in the frame of the moving cell boundary **SKETCH OF THE FLUX CALCULATION** 





On large scales, the code produces very similar results as standard SPH techniques

GAS AND TEMPERATURE FIELDS IN A COSMOLOGICAL HYDRODYNAMIC SIMULATION





#### AREPO produces much better galaxy morphologies than SPH for identical initial conditions

GAS AND TEMPERATURE FIELDS IN A COSMOLOGICAL HYDRODYNAMIC SIMULATION







The moving-mesh approach can also be used to realize arbitrarily shaped, moving boundaries STIRRING A COFFEE MUG







The challenge to simulate galaxy formation



Hydrodynamical simulations aim to predict:

- Morphology of galaxies
- Fate of the diffuse gas, WHIM, metal enrichment
- X-ray atmospheres in halos
- Turbulence in halos and accretion shocks
- Large-scale regulation of star formation in galaxies through feedback processes from stars and black holes
- Transport processes (e.g. conduction)
- Radiative transfer
- Dynamical transformations (e.g. ram-pressure stripping)
- Magnetic fields
A long standing issue in galaxy formation theory: The shapes of the CDM halo mass function and the galaxy luminosity function are very different THE OBSERVED LF COMPARED TO THE SHAPE OF THE CDM HALO MASS FUNCTION



van den Bosch et al. (2004)

Abundance matching gives the expected halo mass – stellar mass relation in ΛCDM

STELLAR MASSES FROM SDSS/DR7 MATCHED TO ACDM SIMULATION EXPECTATIONS

## **Assumption:**

Stellar mass is monotonically increasing with halo mass



Guo, White & Boylan-Kolchin (2010)

## Current cosmological hydrodynamic simulations have trouble to explain such a low galaxy formation efficiency GALAXY FORMATION EFFICIENCY AS A FUNCTION OF HALO MASS



Guo, White & Boylan-Kolchin (2010)

Sawala & White (2010)

## **Summary points**

- Direct numerical simulations have become indispensable for studying the non-linear growth of structures in ΛCDM cosmologies.
- Current numerical techniques allow high-resolution simulations with an unprecedented dynamic range.

One presently reaches N>10<sup>11</sup>, with a dynamic range of  $10^5 - 10^7$  in 3D.

- The future observation of a sufficiently massive cluster may easily rule out the ΛCDM model. The predicted satellite population may still be in tension with the observations.
- Understanding galaxy formation physics remains a serious challenge in ΛCDM, both at the faint and the bright end.
- Radiative magneto-hydrodynamics codes that follow structure formation still in their infancy.
- Exaflop computers arrive at the end of the decade, but it is highly questionable whether astrophysics can use them at scale.

## **Discussion points**

Future codes cannot be written by lonely graduate students any more...

They require large, interdisciplinary teams with sustained funding.

**Biggest challenges in arriving at better codes for galaxy formation:** 

- Cope with huge dynamic range in time and space.
- Avoid work-load imbalance losses when on 10<sup>3</sup> to 10<sup>7</sup> cores.
- Code validation.
- Simulation data management.
- Dragging bright physics students into computational cosmology.