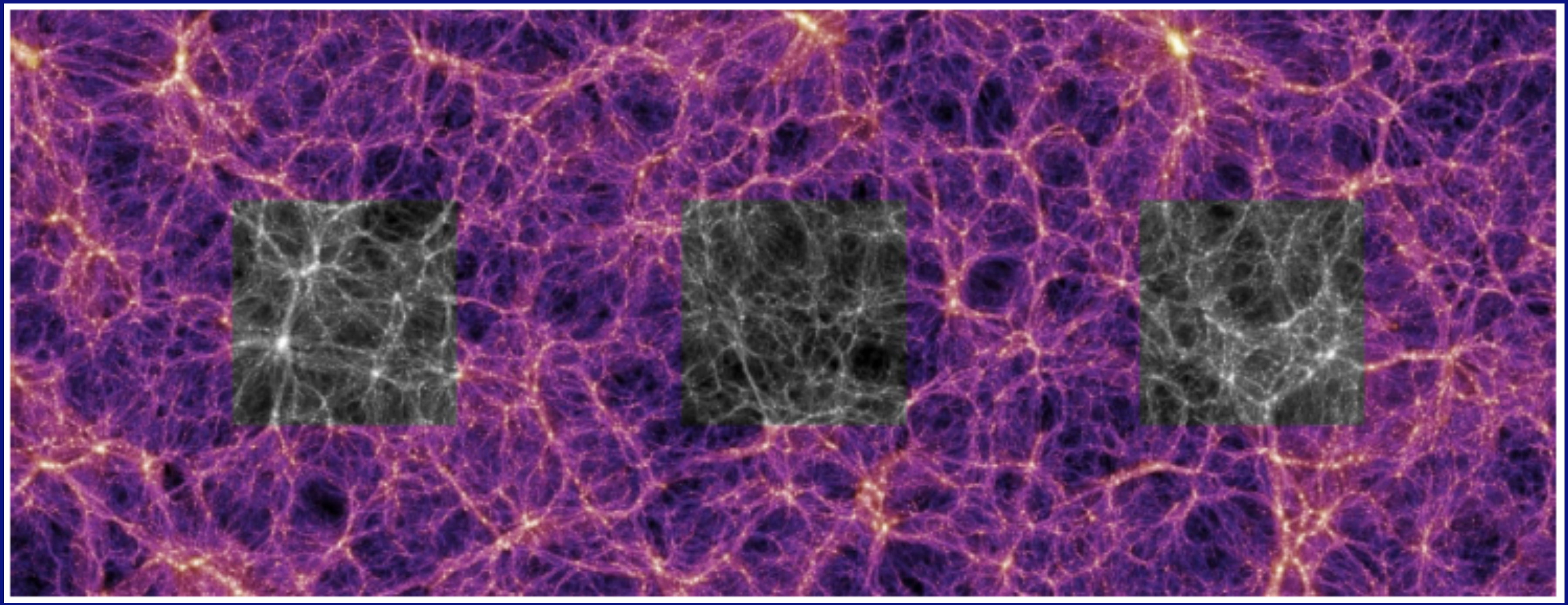
Is the Universe homogeneous?



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February 2011

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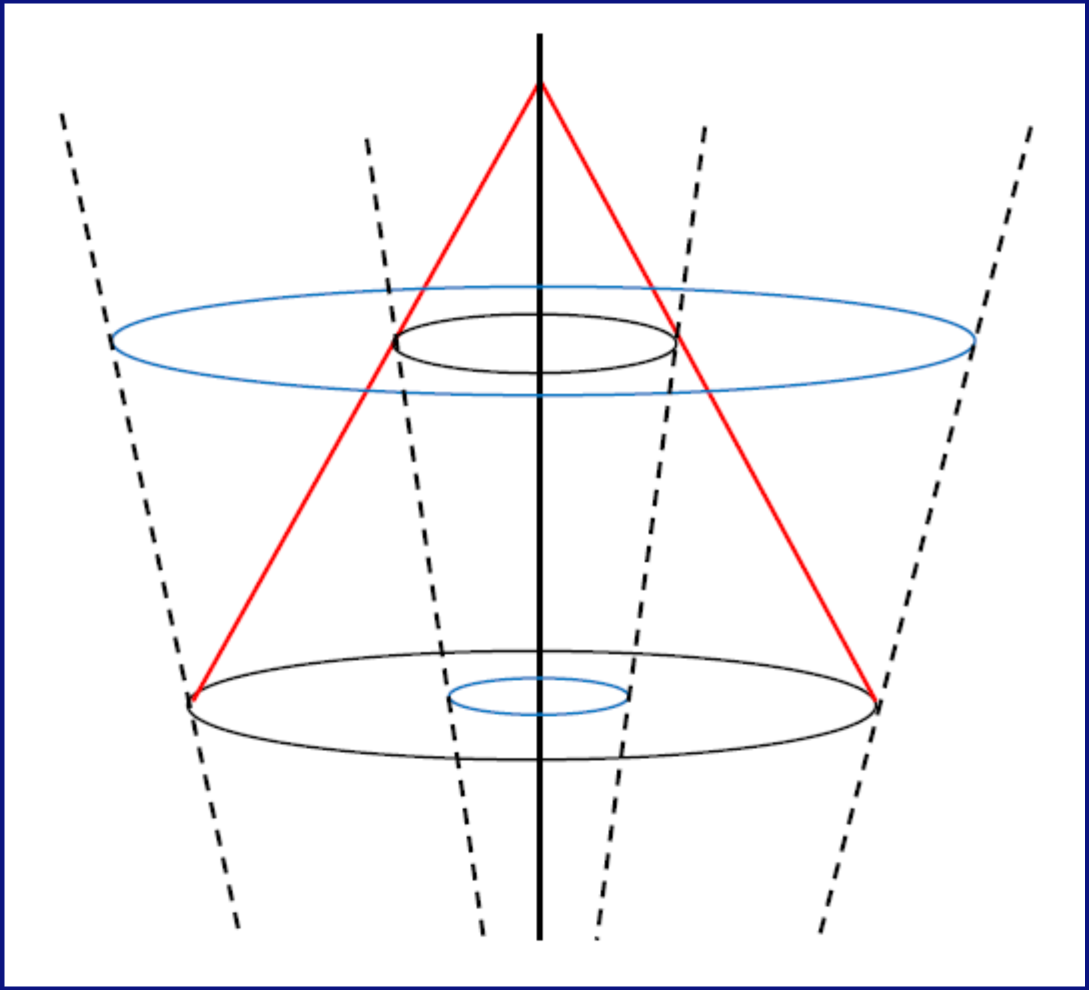
What is the basis for homogeneity?

We cannot *observe* homogeneity, only isotropy

SDSS: $z \sim 0.3$

CMB: $z \sim 1100$



What do (perfect) observations tell us?

(I) Without assuming the Copernican Principle:

(I.1) What do observations tell us directly

- for any gravity theory?
- for GR?

(I.2) What can we say from isotropy of

- matter observations?
- the CMB?

CP: we are not at a special position in the universe

(II) With the Copernican Principle:

(II.1) What do isotropic matter observations tell us?

(II.2) What do isotropic CMB observations tell us?

(III) Testing the Copernican Principle

(IV) Towards the real Universe

(I) Without the Copernican Principle

(I.1) What do observations tell us *directly*?

Try to determine spacetime geometry from lightcone observations. (Ellis 1975; Ellis, Nel, RM et al 1985)

Problem: CDM and DE cannot be directly observed.

(Gravitational lensing? – only determines matter if geometry is assumed a priori.)

Assume we the know 'missing' baryon distribution, and

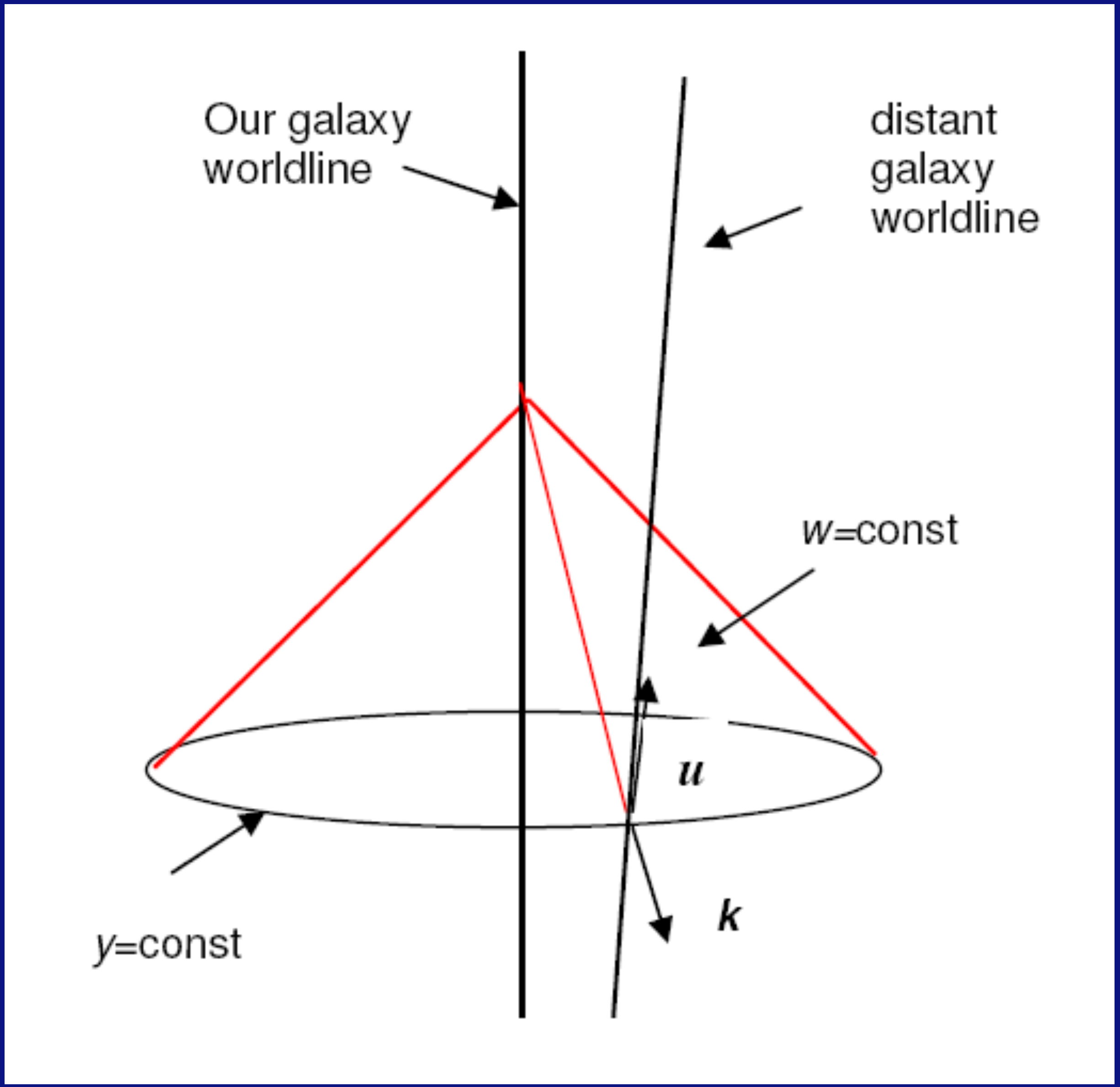
CDM $\rho_{\rm c}$ known from $\rho_{\rm b}$, $u^a_{\rm b} = u^a_{\rm c} := u^a$

DE Λ known independently of cosmological observations

Observational coordinates in a general spacetime

$$x^{\mu} = (w, y, x^{I})$$

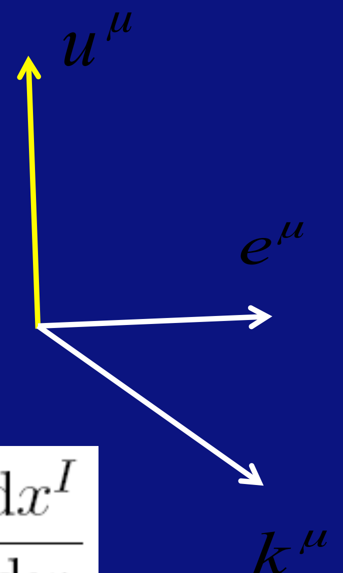
$$x^{I} = (\theta, \phi)$$



Metric in observational coordinates

$$\mathrm{d}s^2 = -A^2\mathrm{d}w^2 + 2B\mathrm{d}y\mathrm{d}w + 2C_I\mathrm{d}x^I\mathrm{d}w + D^2\left(\mathrm{d}\Omega^2 + L_{IJ}\mathrm{d}x^I\mathrm{d}x^J\right)$$

past lightcone	$C^-(w_0):\ w = w_0$ and $y = z$
lightray 4 - vector	$k_\mu = \partial_\mu w,\ k^\mu = B^{-1}\delta^\mu_y$
angular distance	$D = D_A$
lensing distortion	L_{IJ}



Matter 4-velocity

$$u^0 = 1 + z\,, \quad u^1 = 0\,, \quad u^I = (1+z)V^I\,, \quad V^I := \frac{\mathrm{d}x^I}{\mathrm{d}w}$$

transverse velocities

Lensing convergence and shear in a general spacetime:

$$\widehat{\Theta} = \frac{1}{BD}\frac{\partial D}{\partial y}\,, \quad \widehat{\sigma}_{\mu\nu} = \delta_\mu{}^I \delta_\nu{}^J \frac{D^2}{2B}\frac{\partial}{\partial y} L_{IJ}\,.$$

$$\frac{\mathrm{d}}{\mathrm{d}v}\widehat{\Theta} = -\tfrac{1}{2}\widehat{\Theta}^2 - \widehat{\sigma}_{ab}\widehat{\sigma}^{ab} - R_{ab}k^a k^b$$ (Ricci)

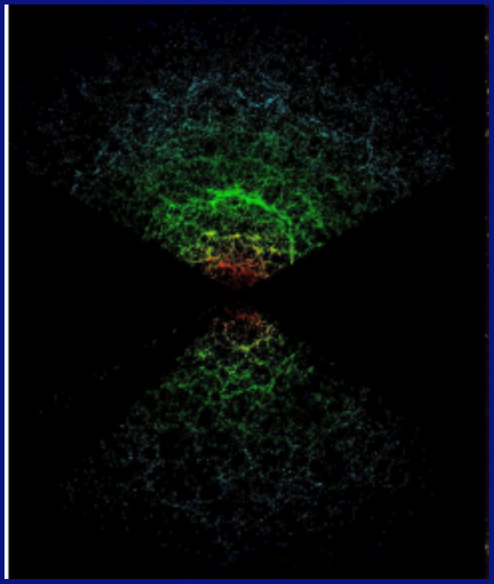
$$\frac{\mathrm{d}}{\mathrm{d}v}\widehat{\sigma}_{ab} = -\widehat{\Theta}\widehat{\sigma}_{ab} - C_{acbd}k^c k^d$$ (Weyl)

Number counts in a general spacetime:

$$\mathrm{d}N = f_d n D^2 (1+z) B \,\mathrm{d}\Omega_0 \,\mathrm{d}z$$ *$B=dv/dz$, $n=\rho/m$*

(I.1a) What do observations tell us directly - without field equations?

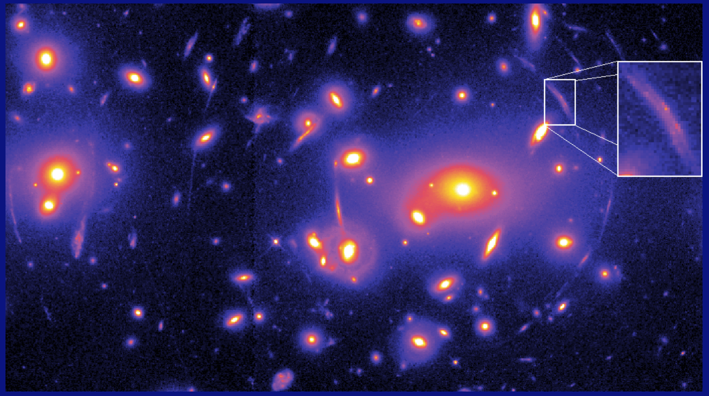
In principle, for ideal observations:



* standard candles/ sirens/ rulers give D

* number counts give $B\rho_{\mathrm{m}}$
from galaxy surveys on lightcone
(+ assumptions on CDM)

* lensing shear gives L_{IJ}
if we know intrinsic shapes



* transverse motions give V^{I}
?

The maximum achievable in principle

$$\text{Idealized data} \Rightarrow \{u^\mu, B\rho_\mathrm{m}, g_{IJ}\} \text{ on } C^-(w_0)$$

We cannot determine our past lightcone without field equations. Thus:

- Observations cannot directly test GR on cosmological scales (or any modified gravity)
- We need to assume the spacetime geometry first

We also get an interesting test for transverse velocities:

Anisotropy in the observed Hubble parameter implies that the transverse velocities are nonzero

$$\frac{\partial}{\partial x^I} H_0^{\mathrm{obs}} = -\frac{1}{3}\frac{\partial}{\partial x^I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial x^J}\left(\sin\theta\, V_0^J\right)\right]$$

(RM 1980; RM, Matravers 1994)

(I.1b) What do observations tell us directly - with GR?

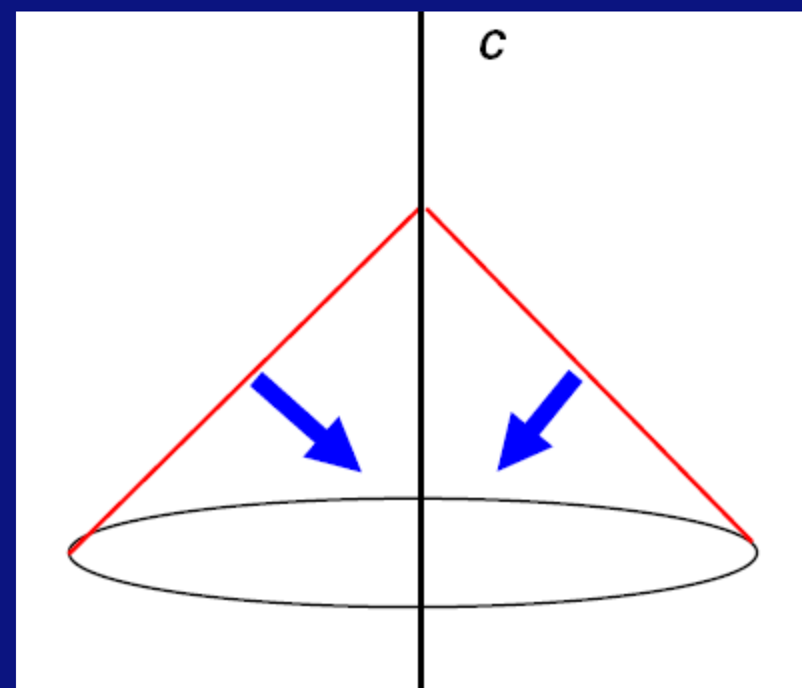
1. $\{u^\mu\,, B\rho_{\rm m}\,, g_{IJ}\}$ on $C^-(w_0)$

   is *exactly* the data needed on the past lightcone for EFE to uniquely determine the

   matter distribution (i.e. $\rho_{\rm m}, u^\mu$) and geometry (i.e. $g_{\mu\nu}$)

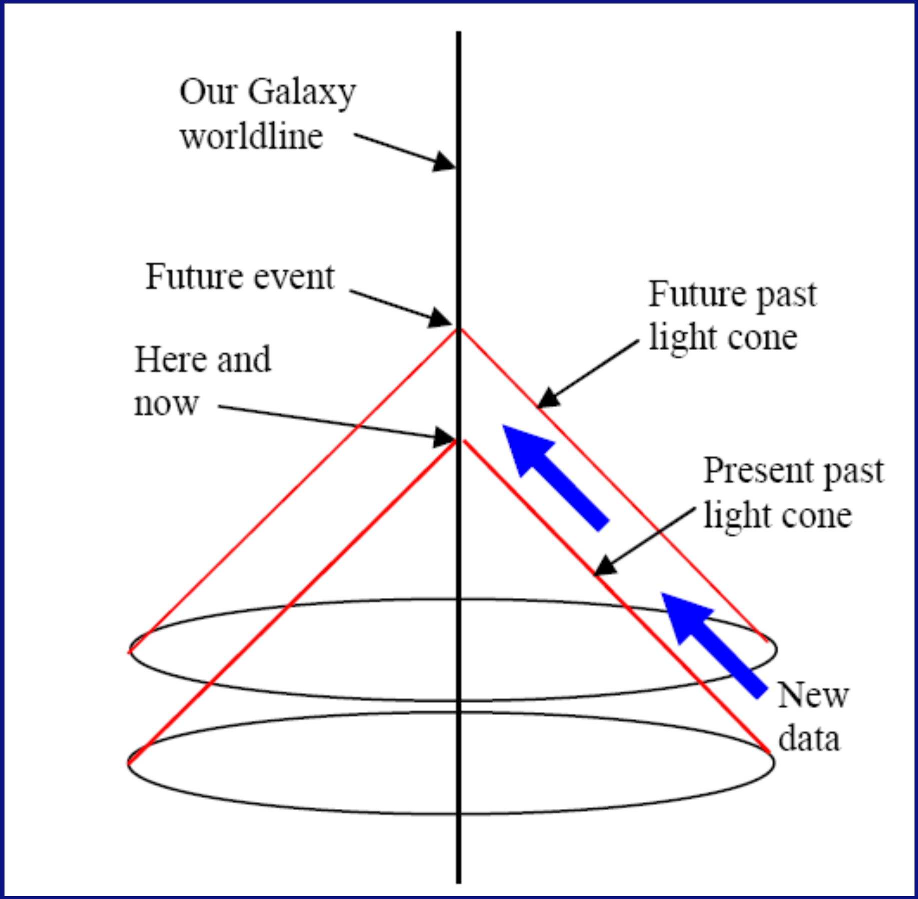
   on the past lightcone.

2. Then EFE propagate off the lightcone to determine the *interior* (past)



(Ellis, Nel, RM et al 1985)

3. EFE *cannot* propagate to the future since new data can destroy the predictions



(I.2a) What can we say from isotropy of matter observations?

Isotropic matter observations:

$$V^I = 0 = L_{IJ}, \ \frac{\partial D}{\partial x^I} = 0 = \frac{\partial n}{\partial x^I}$$

This is exactly enough to produce isotropic geometry:

Matter isotropy on lightcone gives isotropy of geometry

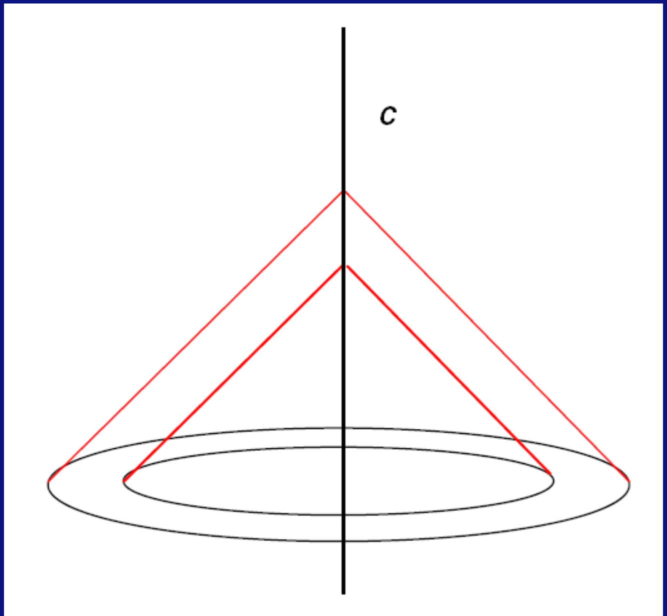
- If one observer comoving with matter sees isotropic angular distances, number counts, bulk velocities and lensing, in a dust Universe with Λ, then spacetime is isotropic, i.e. LTB

(Ellis, Nel, RM et al 1984; RM, Matravers 1994)

(I.2b) What can we say from isotropy of the CMB?

Seems obvious that this enforces isotropy of the spacetime.
It is plausible: we expect isotropic decoupling surfcae, which then evolves to isotropic future.
But this does not follow from Einstein-Liouville equations (at least not in any obvious way)

We *cannot* deduce isotropy of the geometry, without further assumptions on the matter



(II) With the Copernican Principle

Without the CP, we cannot establish homogeneity:
homogeneity cannot be directly observed in the matter or CMB

(II.1) What do isotropic matter observations tell us?

Isotropy about all observers implies homogeneity:

Matter isotropy on light-cones → FLRW

In a dust region of a universe with Λ, if all fundamental observers measure isotropic area distances, number counts, bulk velocities, and lensing, then the spacetime is FLRW in that region.

An observational basis for the Cosmological Principle

A more powerful result

Isotropy of area distances alone, and for small z, about all observers - implies homogeneity:

Isotropic distances to 3rd order in z imply FLRW

- In a dust region of a Universe with Λ, if all fundamental observers measure isotropic distances to $O(z^3)$, then spacetime is FLRW in that region

(Hasse, Perlick 1999; Clarkson 2000; Clarkson, RM 2010)

Series expansion (Kristian, Sachs 1966):

$$z = \left[K^a K^b \nabla_a u_b\right]_O D + \frac{1}{2}\left[K^a K^b K^c \nabla_a \nabla_b u_c\right]_O D^2$$

$$+ \frac{1}{6}\left[K^a K^b K^c K^d \nabla_a \nabla_b \nabla_c u_d + \frac{1}{2} K^a K^b K^c K^d R_{cd} \nabla_a u_b\right]_O D^3 + \ldots.$$

where $\nabla_a u_b = \frac{1}{3}\Theta h_{ab} + \sigma_{ab} - \omega_{ab} - u_a \dot{u}_b$

O(z): $\left(K^a K^b \nabla_a u_b\right)_o = \left[\frac{1}{3}\Theta + \dot{u}_a e^a + \sigma_{ab} e^a e^b\right]_o = H_o^{\mathrm{obs}}$

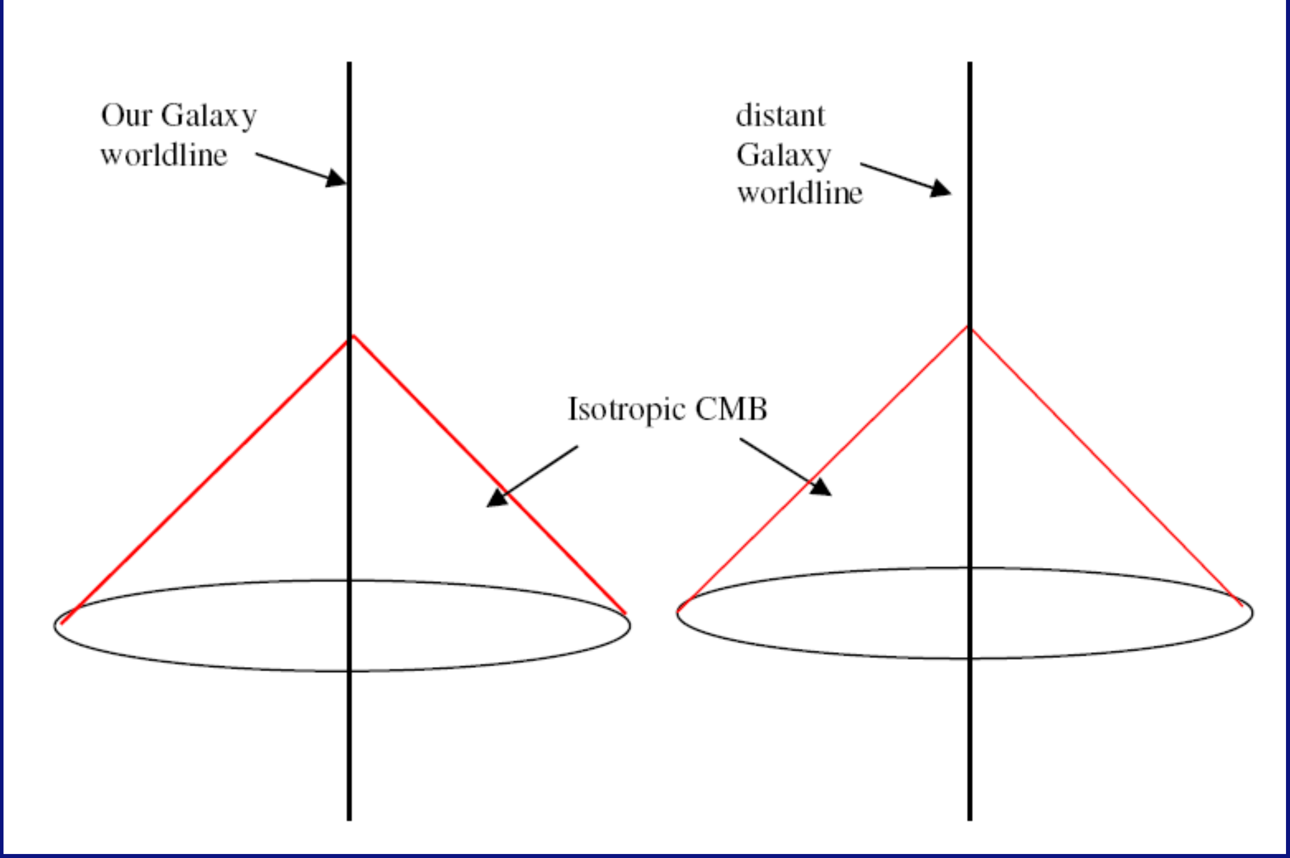
Isotropy: $\dot{u}_a = 0, \ \ \sigma_{ab} = 0$ etc.

(II.2) What do isotropic CMB observations tell us?

CMB isotropy for all fundamental observers gives the strongest basis that we have for homogeneity

History: 1968 theorem by Ehlers, Geren, Sachs (EGS)

Update: generalize to include baryons, CDM and DE



It seems obvious that we should get FLRW – but we have to *show* it using the *general, fully nonlinear* Einstein-Liouville equations.

Nonlinear perturbations are not an option – we cannot assume the FLRW background that we are trying to prove.

$\boxed{\textit{CMB isotropy + Copernican Principle} \to \textit{FLRW}}$

In a region, if

- collisionless radiation is exactly isotropic,
- the radiation four-velocity is geodesic and expanding,
- there are pressure-free baryons and CDM, and dark energy in the form of Λ, quintessence or a perfect fluid,

then the metric is FLRW in that region.

(EGS 1968; Ellis, Treciokas 1971; Stoeger, RM, Ellis 1995; Ferrando, Morales, Portilla 1999; Clarkson, Barrett 1999; Clarkson, Coley 2001; Rasanen 2009; Clarkson, RM 2010)

Note: It follows that (1) matter and DE have the same 4-velocity as radiation (2) matter, DE anisotropic stress =0

Liouville equation in any spacetime

Liouville:

$$\frac{\mathrm{d}f}{\mathrm{d}\tau} = p^a \frac{\partial f}{\partial x^a} + \frac{\mathrm{d}p^a}{\mathrm{d}\tau}\frac{\partial f}{\partial p^a} = 0$$

$$p^a = E(u^a + e^a) \qquad (= hk^a)$$

Covariant harmonics:

$$f(x,p) = \sum_{\ell=0}^{\infty} F_{A_\ell}(x,E)e^{A_\ell} = F(x,E) + F_a(x,E)e^a + F_{ab}(x,E)e^a e^b + \ldots$$

$$\left(A_\ell \equiv a_1 \ldots a_\ell\right)$$

Integrated multipoles:

$$\rho_\gamma \propto \int dE E^3 F(x,E)$$

$$q_\gamma^a \propto \int dE E^3 F^a(x,E) = 0$$

$$\pi_\gamma^{ab} \propto \int dE E^3 F^{ab}(x,E) = 0$$

General intensity multipoles:

$$I_{A_\ell}(x) = \Delta_\ell \int_0^\infty E^3\, F_{A_\ell}(x,E)\,\mathrm{d}E$$

Lowest ones

$$I = \rho,\ I_a = q_a,\ I_{ab} = \pi_{ab}.$$

Then Liouville becomes:

$$\begin{aligned} 0 = \dot{I}_{\langle A_\ell\rangle} + \frac{4}{3}\Theta I_{A_\ell} + \frac{\ell}{(2\ell+1)}\overline{\nabla}_{\langle a_\ell} I_{A_{\ell-1}\rangle} + \overline{\nabla}^b I_{bA_\ell} + \frac{\ell(\ell+3)}{(2\ell+1)}\dot{u}_{\langle a_\ell} I_{A_{\ell-1}\rangle} \\ - (\ell-2)\dot{u}^b I_{bA_\ell} - \ell\omega^b \eta_{bc(a_\ell} I_{A_{\ell-1})}{}^c - (\ell-1)\sigma^{bc} I_{bcA_\ell} \\ + \frac{5\ell}{(2\ell+3)}\sigma^b{}_{\langle a_\ell} I_{A_{\ell-1}\rangle b} - \frac{(\ell-1)\ell(\ell+2)}{(2\ell-1)(2\ell+1)}\sigma_{\langle a_\ell a_{\ell-1}} I_{A_{\ell-2}\rangle}\,, \end{aligned} \tag{11.1}$$

(Ellis, Treciokas, Matravers 1984; RM, Gebbie, Ellis 1999)

Quadrupole evolution in a general spacetime:

$$\dot{\pi}_\gamma^{\langle ab\rangle} + \frac{4}{3}\Theta\pi_\gamma^{ab} + \frac{8}{15}\rho_\gamma\sigma^{ab} + \frac{2}{5}\overline{\nabla}^{\langle a}q_\gamma^{b\rangle} + 2\dot{u}^{\langle a}q_\gamma^{b\rangle} - 2\omega^c\eta_{cd}{}^{(a}\pi_\gamma^{b)d} + \frac{10}{7}\sigma_c{}^{\langle a}\pi_\gamma^{b\rangle c} + \overline{\nabla}_c I^{abc} - \sigma_{cd}I^{abcd} = 0.$$

where $\overline{\nabla}_a = (\nabla_a)_\perp$

Then we get zero shear: $\sigma_{ab} = 0$

Momentum conservation: $\overline{\nabla}_a\rho_\gamma = 0$

Then

$$\mathrm{curl}\,\overline{\nabla}_a\rho_\gamma = -2\dot{\rho}_\gamma\omega_a \;\Rightarrow\; \Theta\rho_\gamma\omega_a = 0 \rightarrow \omega_a = 0$$

etc

More powerful result:

CMB partial isotropy + Copernican Principle → FLRW

In a region, if

- collisionless radiation has vanishing dipole, quadrupole and octupole, $\boxed{F_a = F_{ab} = F_{abc} = 0,}$
- the radiation four-velocity is geodesic and expanding,
- there are pressure-free baryons and CDM, and dark energy in the form of Λ, quintessence or a perfect fluid,

then the metric is FLRW in that region.

Ellis, Treciokas, Matravers 1985 (ETM theorem – generalized in Clarkson, RM 2010)

This is the best basis we have for (exact) homogeneity

(III) Testing the Copernican Principle

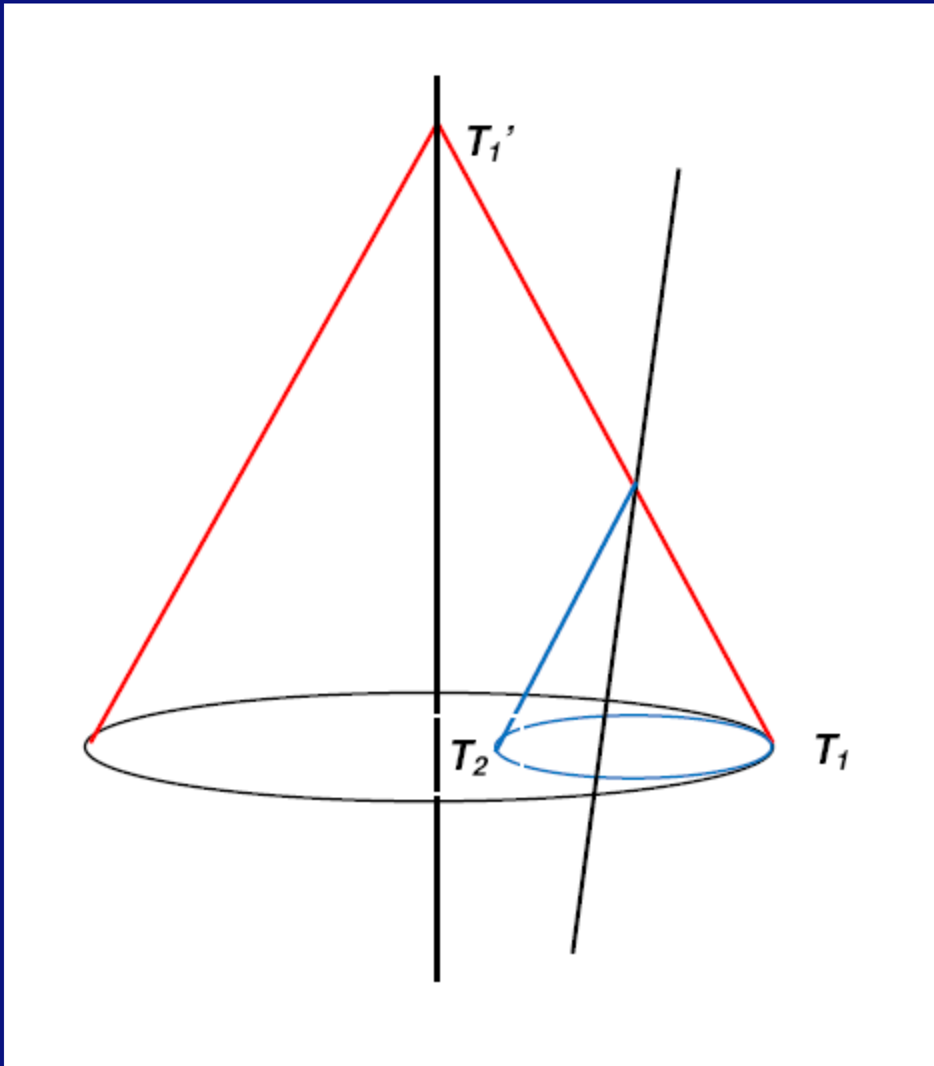
The CP is the foundation of theoretical homogeneity.

Can we test it?

* Sunyaev-Zeldovich tests

Scattered photons
at distant clusters
will distort the CMB
blackbody spectrum
if there is anisotropy
at the cluster

(Goodman 1995; Caldwell, Stebbins 2008)



* Constancy of curvature

A consistency test for homogeneity (Clarkson, Bassett, Lu 2009)

Luminosity distance

$$D_L(z) = \frac{(1+z)}{H_0\sqrt{-\Omega_{K0}}} \sin\left(\sqrt{-\Omega_{K0}} \int_0^z \frac{\mathrm{d}z'}{H(z')/H_0}\right)$$

implies

$$\Omega_{K0} = \frac{[H(z)D'_L(z)]^2 - 1}{H_0 D_L(z)^2}$$

Differentiate the constant curvature. Then the quantity

$$\mathcal{C}_1(z) := 1 + H^2(z)\left[D_L(z)D''_L(z) - D'_L(z)^2\right] + H(z)H'(z)D_L(z)D'_L(z)$$

vanishes identically in Robertson-Walker spacetimes (for *any* matter content and *any* field equations)

(IV) Towards the realistic situation

(IV.1) The CMB is almost isotropic

Partial result – we get almost-homogeneity if we make additional assumptions

CMB almost-isotropy+Copernican Principle → almost-FLRW
In a region of an expanding universe with cosmological constant, if all observers comoving with the matter measure an almost isotropic distribution of collisionless radiation, and if some of the time and spatial derivatives of the covariant multipoles are also small, then the region is almost FLRW.

(Stoeger, RM, Ellis 1995)

Open question: can we remove the assumptions on derivatives using other observations?

(Nilsson et al 1999; Clarkson et al 2003; Rasanen 2009; Clarkson, RM 2010)

(IV.2) We also need:

- A statistical approach to isotropy
- A statistical formulation of the CP
- A better understanding of light propagation in a lumpy Universe
- A better understanding of how we average over inhomogeneities

Summary

What is the basis for homogeneity in the idealized case?

(I) Without assuming the Copernican Principle:

(I.1) What do observations tell us directly

- for any gravity theory? *VERY LITTLE*
- for GR? *PAST LIGHTCONE + INTERIOR*

(I.2) What can we say from isotropy of

- matter observations? *LTB GEOMETRY*
- the CMB? *VERY LITTLE*

(II) With the Copernican Principle: *WE CAN TEST IT*

(II.1) What do isotropic matter observations tell us? *FLRW*

(II.2) What do isotropic CMB observations tell us? *FLRW*

Open problems of the realistic case: almost-isotropy, etc.