Shape and topology optimization for piezo-mechanic-acoustic couplings

Günter Leugering, Andre Novotny, Gustavo Perla-Menzala and Jan Sokolowski (for the shape part), Bänsch, Kaltenbacher, Wein, Schury, Stingl (SIMP-part)

Shape sensitivity analysis of a quasi-electrostatic piezoelectric system in multilayered media

Mathematical Methods in the Applied Sciences, 2010, 33, 2118–2131 and AMO 2011

Optimization of Electro-mechanical Smart Structures

International Series of Numerical Mathematics, Vol. 160, 487–505

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On the effect of self-penalization of piezoelectric composites in topology optimization

Structural and Multidisciplinary Optimization, 2011, 43, 405–417

ENGINEERING OF ADVANCED MATERIALS

Benasque, 2.09.2011

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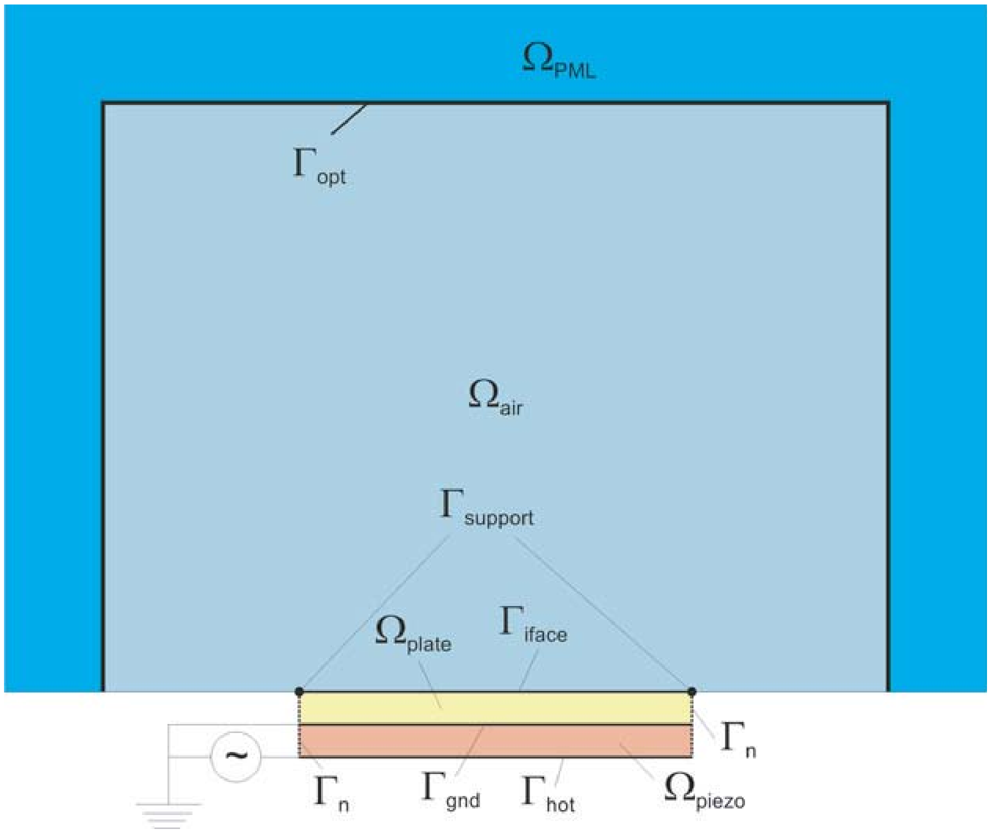
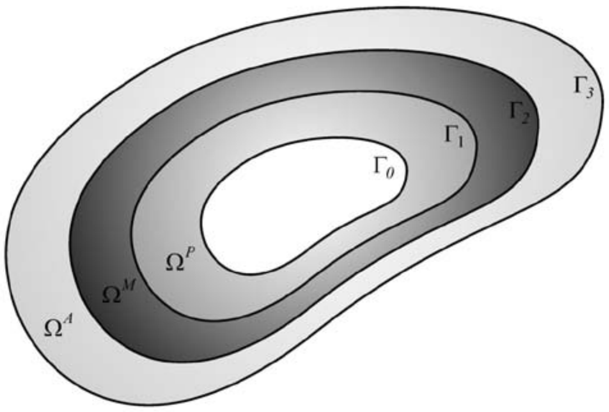
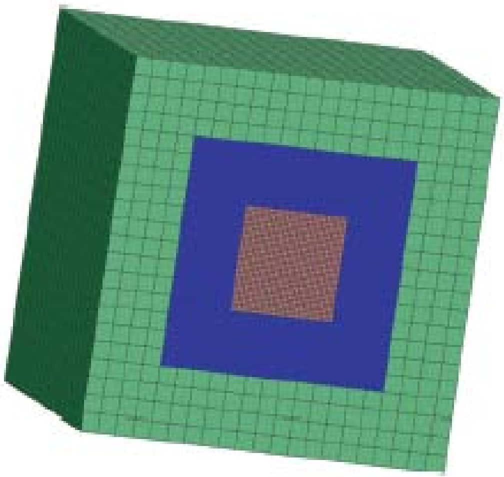
Geometries

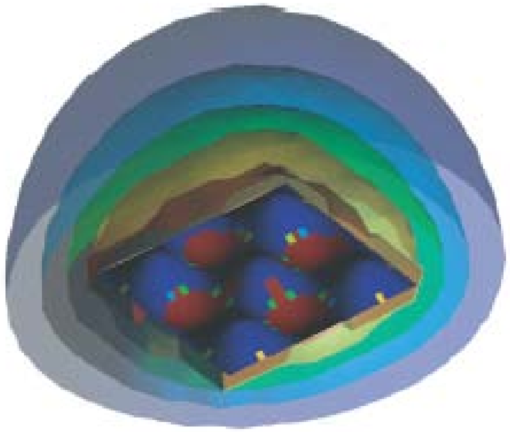




acoustic chambers

Numerical realization





Objectives are e.g.
- **change the topology of the piezo-patch such that the pressure in the acoustic chmaber is maximized**
- **use classical methods like SIMP for the Helmholtz problem**
- **use shape sensitivity calculus for the transient problem**

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The coupled system

$$
\left\{
\begin{array}{rcll}
\frac{1}{c^2}\varphi_{tt} - \Delta\varphi & = & f & \text{in} \quad \Omega^A \times (0,T) \\
w_{tt} - \mathrm{div} S & = & g & \text{in} \quad \Omega^M \times (0,T) \\
u_{tt} - \mathrm{div}\sigma & = & h & \multirow{2}{*}{$\left.\begin{array}{l} \\ \\ \end{array}\right\} \text{in} \quad \Omega^P \times (0,T)$} \\
-\mathrm{div}\psi & = & 0 &
\end{array}
\right.
$$

$$
\left\{
\begin{array}{rcl}
S(w) & = & A\varepsilon(w) \ , \\
\sigma(u,q) & = & C\varepsilon(u) - Pe(q) \ , \\
\psi(u,q) & = & P^\top \varepsilon(u) + De(q) \ ,
\end{array}
\right.
$$

$$
A_{ijkl}X_{ij}X_{kl} \geq a_0 X_{ij}^2, \quad C_{ijkl}Y_{ij}Y_{kl} \geq c_0 Y_{ij}^2, \quad D_{ij}z_i z_j \geq d_0 z_i^2,
$$

$$
\varepsilon(u) = \nabla^s u := \frac{1}{2}(\nabla u + \nabla u^\top), \quad \varepsilon(w) = \nabla^s w := \frac{1}{2}(\nabla w + \nabla w^\top) \quad \text{and} \quad e(q) = -\nabla q \ ,
$$

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Initial-, boundary- and transmission conditions

Initial conditions

$$\begin{cases} \varphi(x,0) = \varphi_0(x),\ \varphi_t(x,0) = \varphi_1(x), \\ w(x,0) = w_0(x),\ w_t(x,0) = w_1(x), \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), \end{cases}$$

and boundary conditions

$$\begin{cases} \psi \cdot n & = & 0 \\ u & = & 0 \end{cases} \text{ on } \Gamma_0 \times (0,T), \quad \frac{\partial \varphi}{\partial n} = -\frac{1}{c}\varphi_t \text{ on } \Gamma_3 \times (0,T),$$

Finally, we consider the following transmission conditions

$$\begin{cases} u & = & w \\ \sigma n & = & Sn \\ q & = & q^P \end{cases} \text{ on } \Gamma_1 \times (0,T) \quad \text{and} \quad \begin{cases} w_t \cdot n & = & -\dfrac{\partial \varphi}{\partial n} \\ Sn & = & -\varphi_t n \end{cases} \text{ on } \Gamma_2 \times (0,T),$$

Shape functions

We consider a shape functional of the form

$$\mathcal{J}_\Omega(\varphi_t, w) = \int_0^T J_\Omega(\varphi_t, w) \ ,$$

with $J_\Omega(\varphi_t, w)$ defined as

$$J_\Omega(\varphi_t, w) := \alpha \frac{1}{2} \int_{\Omega^A} (\varphi_t - p^\star)^2 - \beta \int_{\Omega^M} (\mathrm{div}(w)\eta + w \cdot \nabla\eta) \ ,$$

$$J_\Omega(\varphi_t, w) = \alpha \frac{1}{2} \int_{\Omega^A} (\varphi_t - p^\star)^2 - \beta \int_{\Gamma_2} w \cdot n \ ,$$

Weak solutions and spaces

$$
\begin{aligned}
a_A(\varphi,\varphi) &:= \langle \nabla\varphi, \nabla\varphi\rangle_{\Omega^A} \ , \\
a_M(w,w) &:= \langle A\nabla^s w, \nabla^s w\rangle_{\Omega^M} \ , \\
a_{MM}(u,u) &:= \langle C\nabla^s u, \nabla^s u\rangle_{\Omega^P} \ , \\
a_{EE}(q,q) &:= \langle D\nabla q, \nabla q\rangle_{\Omega^P} \ , \\
a_{ME}(u,q) &:= \langle P^\top \nabla^s u, \nabla q\rangle_{\Omega^P} \ , \\
a_{EM}(q,u) &:= \langle P\nabla q, \nabla^s u\rangle_{\Omega^P} \ ,
\end{aligned}
$$

and spaces

$$
\mathcal{W}_A = H^1(\Omega^A),\ \mathcal{W}_M = [H^1(\Omega^M)]^3,\ \mathcal{W}_P = [H^1(\Omega^P)]^3,\ \mathcal{W}_E = H^1(\Omega^P),
$$

as well as

$$
\begin{aligned}
\mathcal{W} &= \{(\varphi,w,u,q)(t) \in \mathcal{W}_A \times \mathcal{W}_M \times \mathcal{W}_P \times \mathcal{W}_E : \\
&\qquad u = 0 \text{ on } \Gamma_0, w = u \text{ on } \Gamma_1 \text{ and } q = q^P(t) \text{ on } \Gamma_1,\ \text{for each } t \in (0,T)\} \ , \\
\widetilde{\mathcal{W}} &= \{(\widetilde{\varphi},\widetilde{w},\widetilde{u},\widetilde{q}) \in \mathcal{W}_A \times \mathcal{W}_M \times \mathcal{W}_P \times \mathcal{W}_E : \\
&\qquad \widetilde{u} = 0 \text{ on } \Gamma_0, \widetilde{w} = \widetilde{u} \text{ on } \Gamma_1 \text{ and } \widetilde{q} = 0 \text{ on } \Gamma_1\} \ .
\end{aligned}
$$

Variation of densities: SIMP

- We introduce a pseudodensity ρ, s.t. $\rho_\epsilon \leq \rho(x) \leq 1$, we then premultiply the bilinear forms a_{EE}, a_{EM}, a_{ME} by $\mu(\rho) = p^3$ (typically)

- We do not consider here the infinite-dimensional setting and rather discretize using FE

- On the FE-level we have for each FE such a pseudo-densitiy $\rho_e, e = 1, \ldots, N$

- Obtain new local and then global stiffness and mass matrices for the piezo, the mechanics and the coupling parts.

- Use the engineering approach of structural damping (Rayleigh)

$$S(\omega) = \boldsymbol{K} + j\omega \boldsymbol{C} - \omega^2 \boldsymbol{M} \quad \text{and} \quad \boldsymbol{C} = \alpha_K \boldsymbol{K} + \alpha_M \boldsymbol{M}.$$

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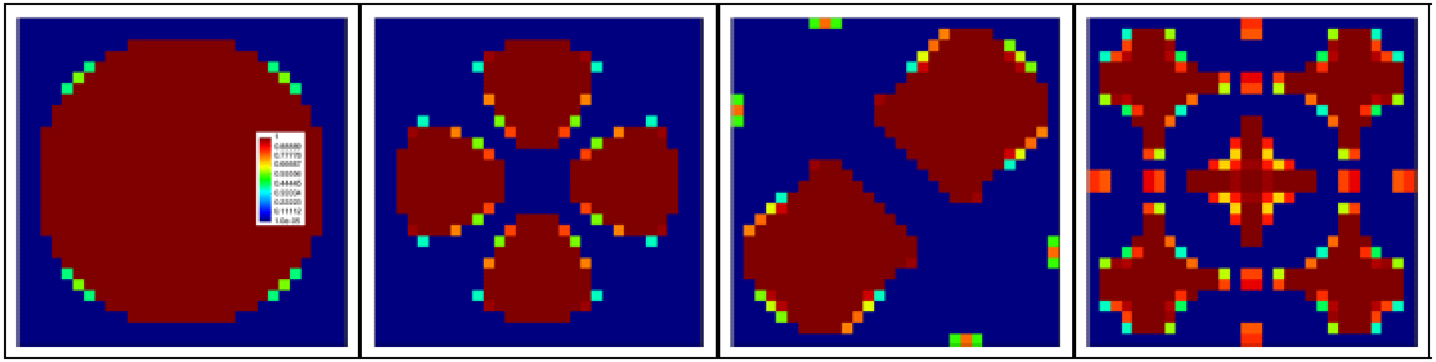
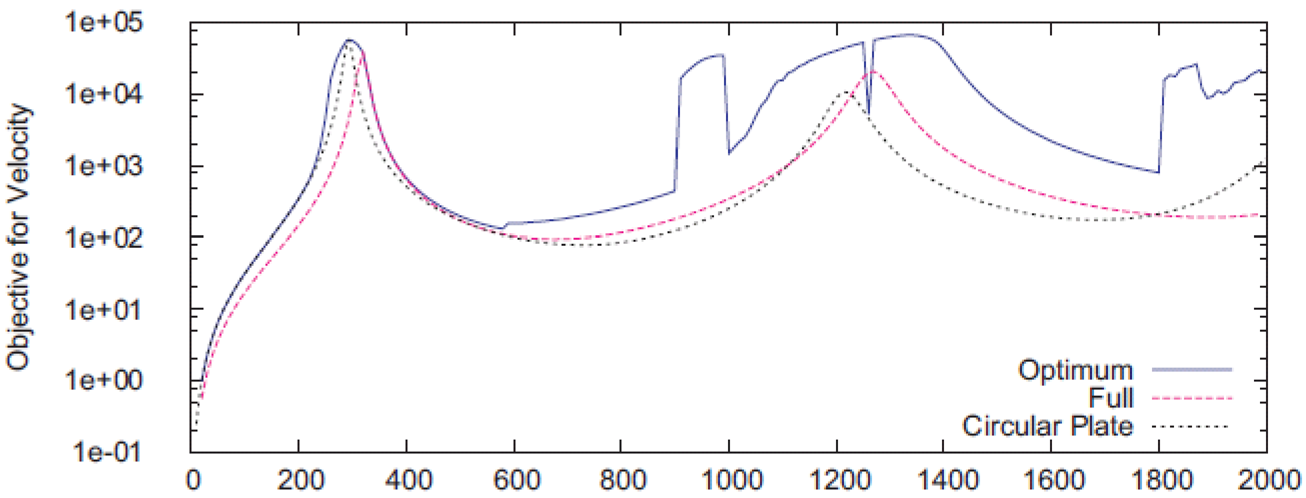
Optimal topologies for several frequencies



(A) 20 Hz (B) 255 Hz (C) 950 Hz (D) 1995 Hz



Variation of the domain

The perturbed domain is denoted as

$$\Omega_\tau = \{x_\tau \in \mathbb{R}^3 : x_\tau = x + \tau V,\ x \in \Omega,\ \tau \geq 0\}\ ,$$

Perturbations of the boundary Γ_0 of the electromechanical device and of the interface Γ_1 between the mechanical and electromechanical devices.

$$V = 0 \text{ on } \Gamma_2 \cup \Gamma_3 = \partial\Omega^A\ .$$

The shape functional defined in the perturbed domain reads

$$\mathcal{J}_{\Omega_\tau}(\varphi_{\tau,t}, w_\tau) = \int_0^T J_{\Omega_\tau}(\varphi_{\tau,t}, w_\tau)\ ,$$

Material and shape derivatives

In this section the normal component

$$v_n := V \cdot n$$

of the velocity vector field is nonnull only on the boundary Γ_0 and on the interface Γ_1. It means that only Γ_0 and Γ_1 are perturbed by an action of the shape velocity field V.

We have the following relation between material and shape derivatives, since in general case the material derivative of a function ξ can be written as

$$\dot{\xi} = \xi' + \langle \nabla \xi, V \rangle \,. \tag{1}$$

Shape derivative

The shape derivatives satisfy

$$\left\{\begin{array}{rclll} \varphi'_{tt} - c^2\Delta\varphi' & = & 0 & \text{in} & \Omega^A \times (0,T) \\ w'_{tt} - \mathrm{div} S' & = & 0 & \text{in} & \Omega^M \times (0,T) \\ u'_{tt} - \mathrm{div}\sigma' & = & 0 & \multirow{2}{*}{\Big\}\ \text{in}} & \multirow{2}{*}{$\Omega^P \times (0,T)$} \\ -\mathrm{div}\psi' & = & 0 & & \end{array}\right. \tag{1}$$

along with the homogeneous initial conditions,

$$\left\{\begin{array}{l} \varphi'(x,0) = 0,\ \varphi'_t(x,0) = 0, \\ w'(x,0) = 0,\ w'_t(x,0) = 0, \\ u'(x,0) = 0,\ u'_t(x,0) = 0\,. \end{array}\right. \tag{2}$$

and nonhomogeneous boundary and interface conditions obtained from

BCs on Γ_0

- The homogeneous Dirichlet boundary condition for the displacement field $u = 0$ leads to

$$u' = -\frac{\partial u}{\partial n} V \cdot n = -v_n \frac{\partial u}{\partial n} \text{ on } \Gamma_0 \times (0,T)\,, \tag{1}$$

- The homogeneous Dirichlet boundary condition for the normal component of the vector field ψ written in the form $\psi_\tau(x_\tau) \cdot n_\tau(x_\tau) = 0$,

$$\psi' \cdot n + v_n n \cdot D\psi \cdot n - \psi_\Gamma \cdot \nabla_\Gamma v_n = 0 \text{ on } \Gamma_0 \times (0,T)\,, \tag{2}$$

where we denote by $\psi_\Gamma := \psi - (\psi \cdot n)n$ the tangential component of the field ψ on the moving boundary $\Gamma_0 \times (0,T)$

- The third condition on φ remains for φ' since the boundary $\Gamma_3 \times (0,T)$ is independent of the shape parameter τ.

Transmission conditions on Γ_1

- The transmission condition for displacement fields $u = w$ leads to

$$u' + v_n \frac{\partial u}{\partial n} = w' + v_n \frac{\partial w}{\partial n} \text{ on } \Gamma_1 \times (0,T)\,,$$

- In the similar way the boundary value for the shape derivative q' of the potential q is obtained

$$q' + v_n \frac{\partial q^P}{\partial n} = 0 \text{ on } \Gamma_1 \times (0,T)\,,$$

- The equality of normal stresses $\sigma n = Sn$ on the interface $\Gamma_1 \times (0,T)$ leads to

$$\sigma' n - v_n(h + 2\kappa S n) + \mathrm{div}_\Gamma(v_n \sigma_\Gamma) = S' n - v_n(g + 2\kappa \sigma n) + \mathrm{div}_\Gamma(v_n S_\Gamma) \text{ on } \Gamma_1 \times (0,T)\,,$$

where κ is the mean curvatrure of Γ_1, $\sigma_\Gamma = \sigma n - (\sigma n \cdot n)n$ is the tangential stress on Γ_1, div_Γ is the tangential divergence on Γ_1, and $S_\Gamma = Sn - (Sn \cdot n)n$ is the tangential stress on Γ_1.

Shape calculus

We are going to denote by $\varphi_{\tau,t} := \dfrac{\partial \varphi_\tau}{\partial t}$ the time derivative of the function φ_τ which is defined in Ω_τ.

Let us perform the shape sensitivity analysis of the functional $\mathcal{J}_{\Omega_\tau}(\varphi_{\tau,t}, w_\tau)$. Thus, we need to calculate its derivative with respect to the parameter τ at $\tau = 0$, that is

$$\int_0^T \dot{J}_\Omega(\varphi_t, w) = \dot{\mathcal{J}}_\Omega(\varphi_t, w) := \frac{d}{d\tau} \mathcal{J}_{\Omega_\tau}(\varphi_{\tau,t}, w_\tau)\Big|_{\tau=0} \quad .$$

the shape derivative of the functional $J_\Omega(\varphi_t, w)$ is given by

$$\dot{J}_\Omega(\varphi_t, w) = \langle D_\Omega(J_\Omega(\varphi_t, w)), V\rangle + \langle D_{\varphi_t}(J_\Omega(\varphi_t, w)), \dot{\varphi}_t\rangle + \langle D_w(J_\Omega(\varphi_t, w)), \dot{w}\rangle \ ,$$

Shape derivative

Thus, since the acoustic chamber remains fixed, we have

$$\begin{aligned}
\dot{\mathcal{J}}_\Omega(\varphi_t, w) &= \beta \int_0^T \langle \nabla w^\top \eta + \nabla \eta \otimes w, \nabla V \rangle_{\Omega^M} - \beta \int_0^T \langle \mathrm{div}(w)\eta + w \cdot \nabla \eta, \mathrm{div} V \rangle_{\Omega^M} \\
&- \int_0^T \alpha \langle (\varphi_{tt} - p_t^\star), \dot{\varphi} \rangle_{\Omega^A} - \int_0^T \beta(\langle \eta, \mathrm{div}(\dot{w}) \rangle_{\Omega^M} + \langle \nabla \eta, \dot{w} \rangle_{\Omega^M} \\
&+ \alpha \langle (\varphi_t(T) - p^\star(T)), \dot{\varphi}(T) \rangle_{\Omega^A} \; .
\end{aligned}$$

$$\dot{\mathcal{J}}_\Omega(\varphi_t, w) = \int_0^\top \left(\int_{\Omega^M} \Sigma^M \cdot \nabla V + \int_{\Omega^P} \Sigma^P \cdot \nabla V \right) \; ,$$

where the Eshelby tensors Σ^M and Σ^P are respectively given by

$$\begin{aligned}
\Sigma^M &= -(w_t \cdot w_t^a - S \cdot \nabla^s w^a + \beta(\mathrm{div}(w)\eta + w \cdot \nabla \eta))I \\
&\quad -(\nabla w^\top S^a + (\nabla w^a)^\top S - \beta(\nabla w^\top \eta + \nabla \eta \otimes w) \; , \\
\Sigma^P &= -(u_t \cdot v_t - \sigma \cdot \nabla^s v + \psi \cdot \nabla p)I \\
&\quad -(\nabla u^\top \sigma^a + \nabla v^\top \sigma - \nabla q \otimes \psi^a - \nabla p \otimes \psi) \; ,
\end{aligned}$$

Shape and topology optimizaiton optimization for optical devices

Günter Leugering, Frantisek Seifrt et.al

Painting by Numbers: Nanoparticle-Based Colorants in the Post-Empirical Age

Advanced Materials, 2011, 23, 2554–2570

http://dx.doi.org/10.1002/adma.201100541

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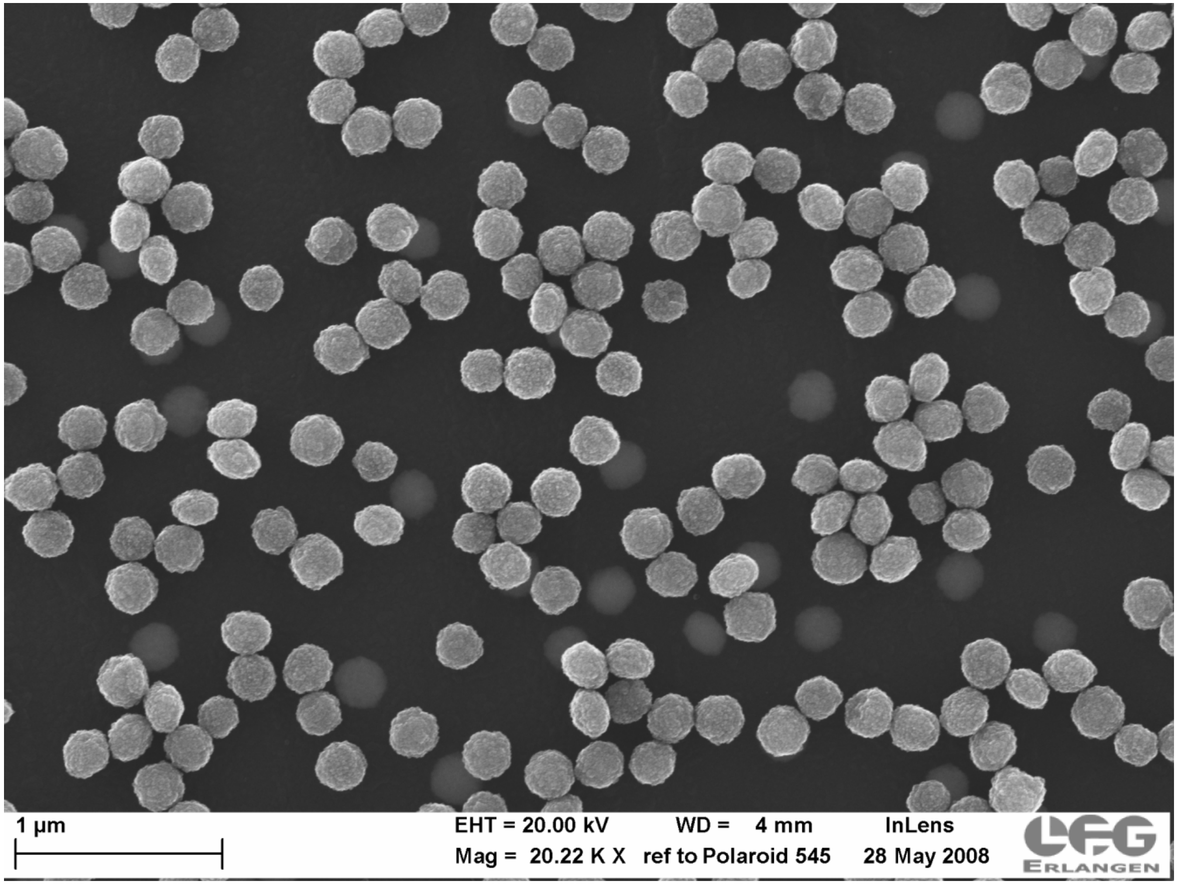
Particles in colors



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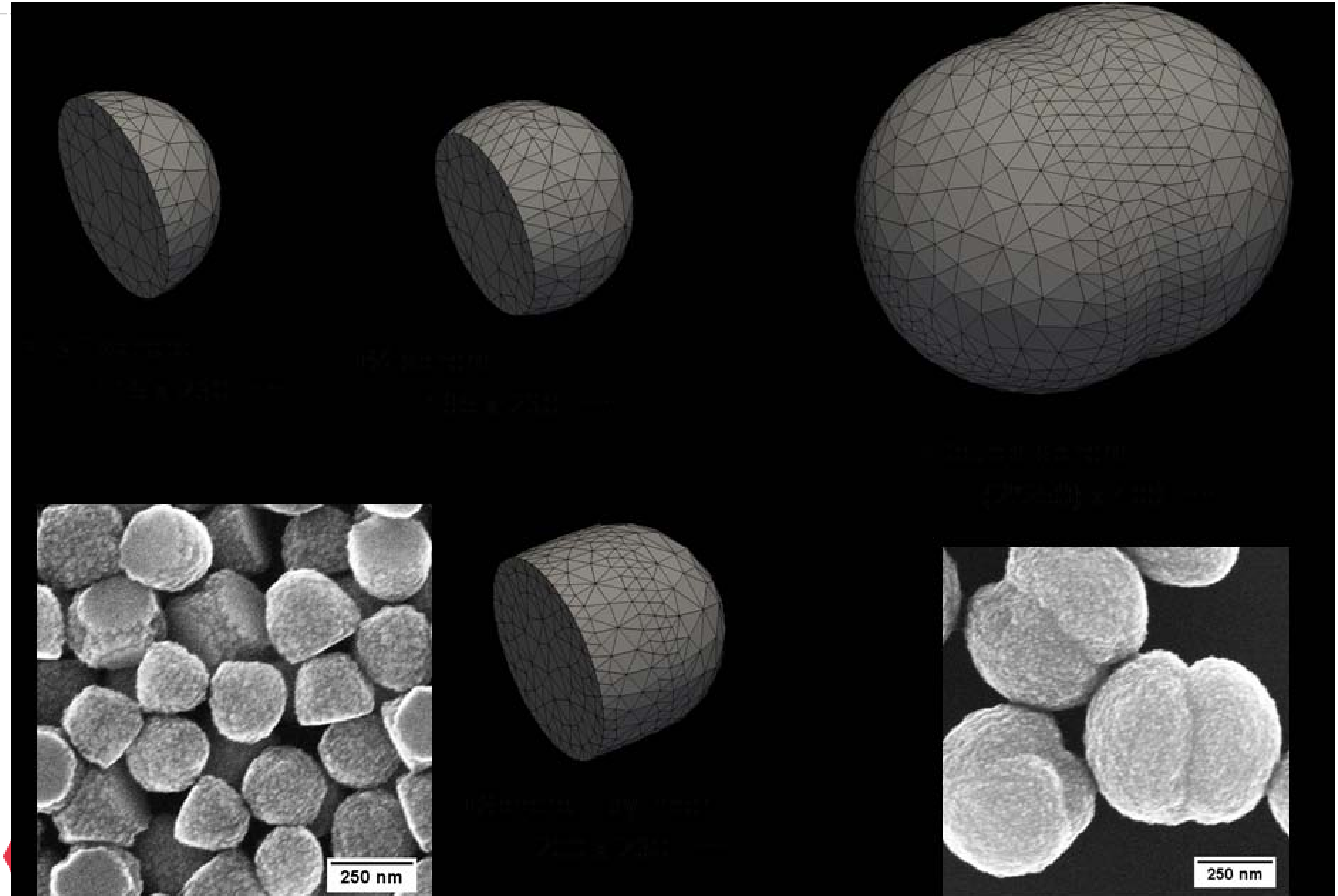
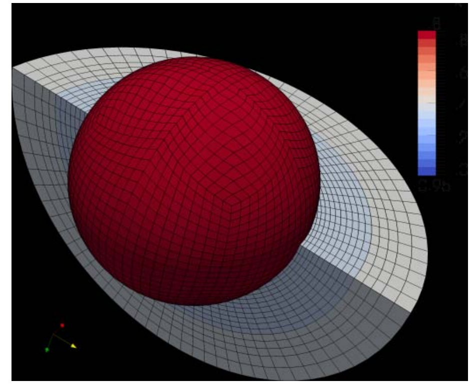
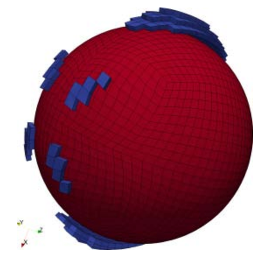
Different shapes of particles

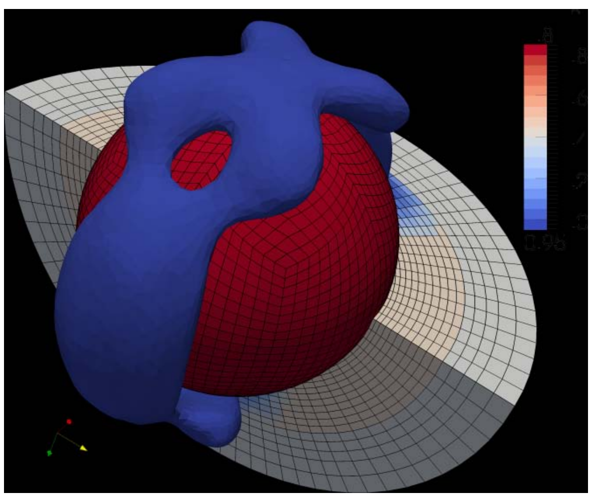


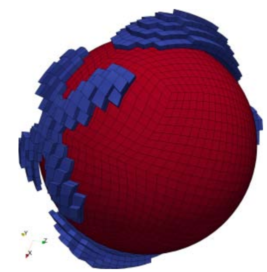
Topology optimization

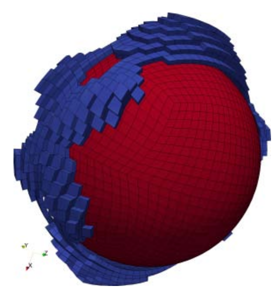
Minimization of extiction eciency in the VIS region
ZnO sphere 80 nm
matrix medium: ethanol
nd optimal distribution of low refr. index medium - air and ethanol in a 30nm thick surrounding layer



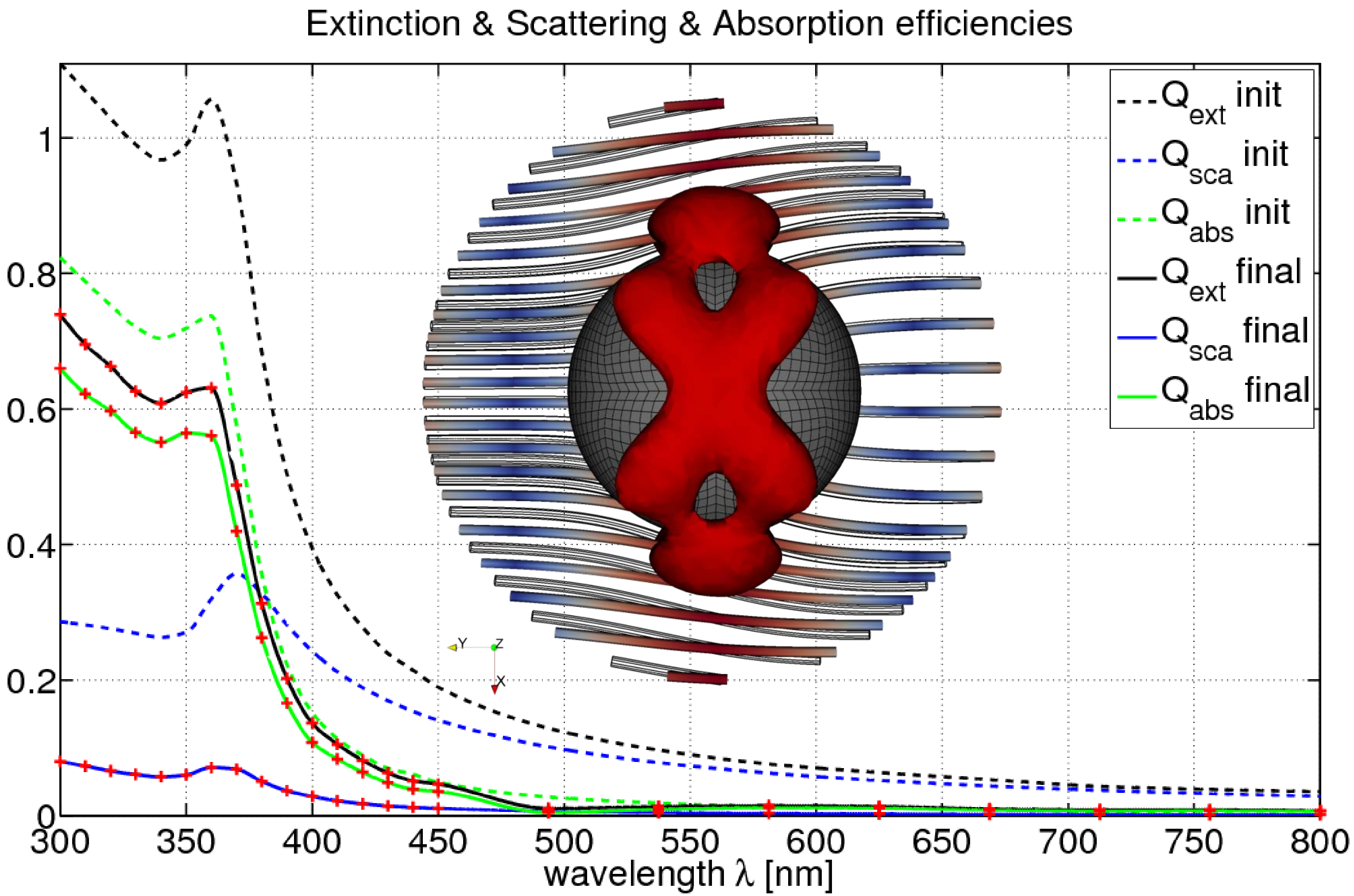




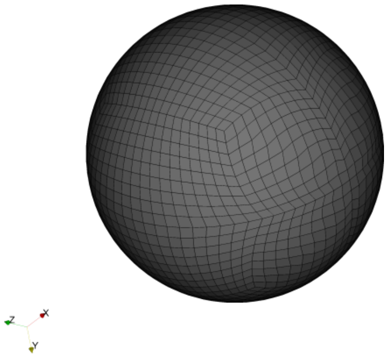


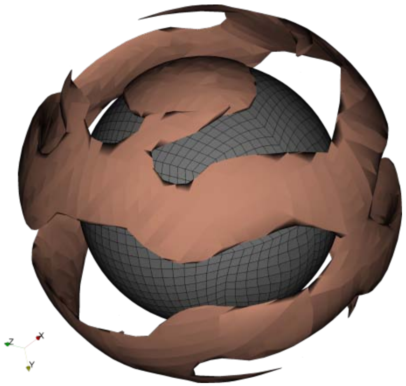


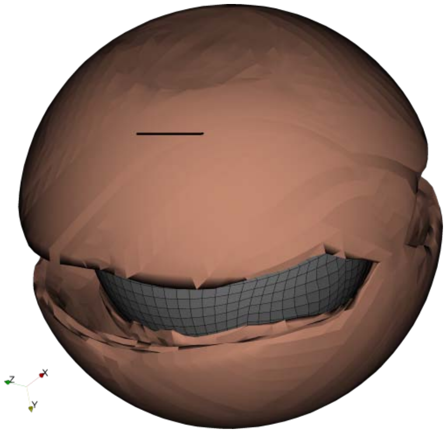
Minimizing in the VIS

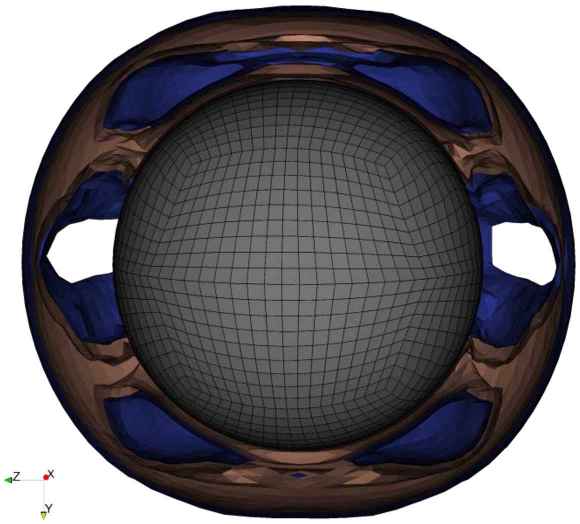


Maximizing in the IR region

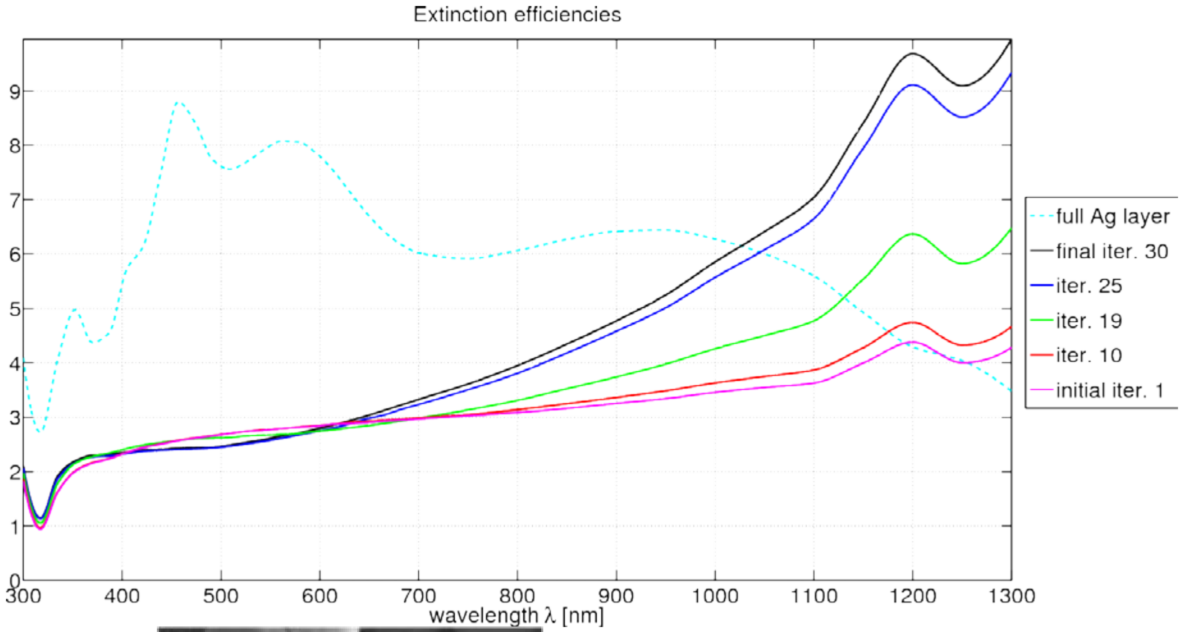


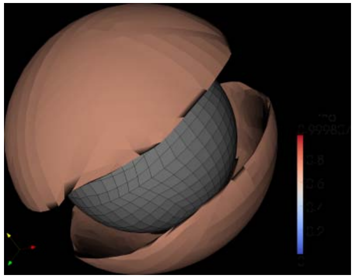


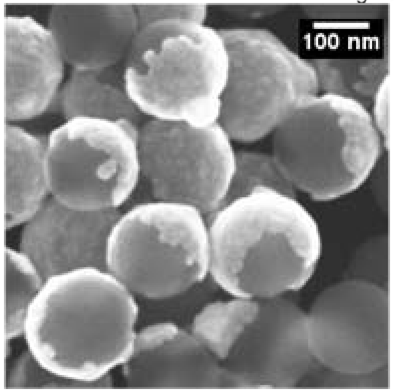


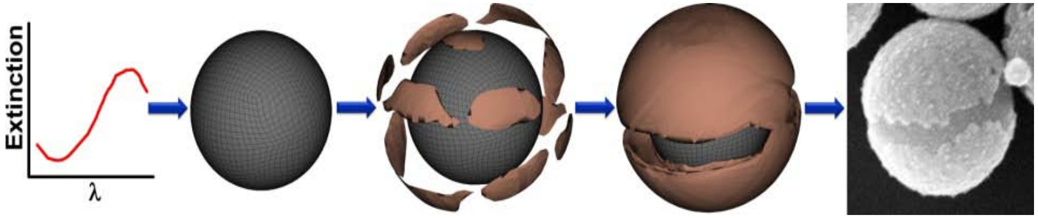


Maximizing extiction in the IR region









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