Open Session on Inverse Problems

Alberto Mercado & Jaime H. Ortega

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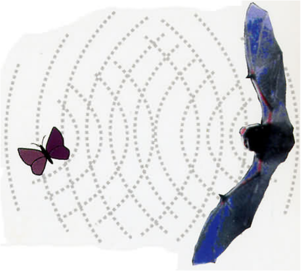
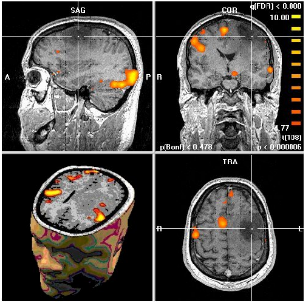
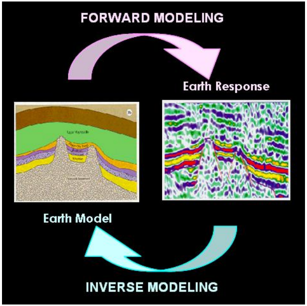
Inverse Problems I

In the nature



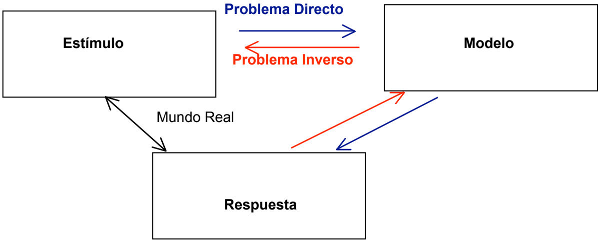
Some applications





Problemas Inversos III

In the inverse problems we know te answer but we do not know the question.



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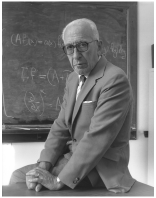

- Let $\Omega \subset \mathbb{R}^N$ be a regular domain. Let $\gamma(x)$ the electrical conductivity, which is unknown.
- $\gamma(x) \in L^\infty(\Omega)$ is strictly positive. the potential u in Ω with voltage f on $\partial\Omega$ satisfy:

$$\begin{aligned} \operatorname{div}\,(\gamma(x)\nabla u) &= 0 \qquad \text{in } \ \Omega \\ u &= f \qquad \text{on } \partial\Omega \end{aligned}$$

Calderón's Problem (I)

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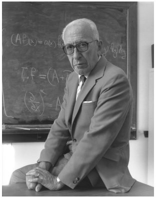

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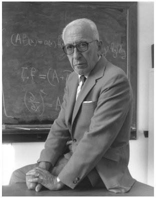

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Calderón's Problem (II)

- We consider the Steklov-Poincaré map :

$$f \longrightarrow \Lambda_\gamma(f) = \gamma \frac{\partial u}{\partial n} \quad \text{on} \quad \partial\Omega.$$

- The Calderón's Problem is to recover γ from the map

$$\Lambda : \gamma \longrightarrow \Lambda_\gamma$$

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Calderón's Problem (III)

We have the following questions:

- Inyectivity of Λ (Identifiability)
- Continuity of the maps Λ and Λ^{-1} (Stability)
- To obtain a formula to recover γ from Λ_γ (Reconstruction)
- To develop a numerical algorithm to obtain an aproximation of γ (Numerical Reconstruction)

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