On the controllability of the Korteweg-de Vries equation

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Colaborators
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Colin and Ghidaglia in 2001

$$\begin{cases} y_t + y_x + y_{xxx} + yy_x = 0, \quad x \in [0, L], \ t \in (0, T), \\ y(t, 0) = h_1(t), \ y_x(t, L) = h_2(t), \ y_{xx}(t, L) = h_3(t), \\ y(0, x) = y_0(x), \ , x \in [0, L] \end{cases}$$

where the state is $y(t, \cdot) : [0, L] \to \mathcal{R}$ and the control inputs are $h_1(t), h_2(t), h_3(t)$

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Don't get confuse

$$\begin{cases} y_t + y_x + y_{xxx} + yy_x = 0, \quad x \in [0, L], \ t \in (0, T), \\ y(t, 0) = g_1(t), \ y(t, L) = g_2(t), \ y_x(t, L) = g_3(t), \\ y(0, x) = y_0(x), \ \ , x \in [0, L] \end{cases}$$

Well-posedness Results

Theorem (Zhang B-Z, Usman M., R.I.)
Let $s > -\frac{3}{4}$ *and* $T > 0$ *and* $r > 0$ *be given. If*

$$\phi \in H^s(0, L), \quad h_1 \in H^{\frac{s+1}{3}}(0, T), \quad h_2 \in H^{\frac{s}{3}}(0, T), \quad h_3 \in H^{\frac{s-1}{3}}(0, T)$$

satisfying

$$\|\phi\|_{H^s(0,L)} + \|h_1\|_{H^{\frac{s+1}{3}}(0,T)} + \|h_2\|_{H^{\frac{s}{3}}(0,T)} + \|h_3\|_{H^{\frac{s-1}{3}}(0,T)} \leq r,$$

There exists a $T^* > 0$ *such that the IBVP admits a unique solution*

$$u \in C([0, T^*]; H^s(0, L)) \cap L^2(0, T^*; H^{s+1}(0, L)).$$

Moreover, the solution u *depends Lipschitz continuously on* ϕ *and* $h_j, j = 1, 2, 3$ *in the corresponding spaces.*

Well-posedness Results

Theorem (Asymptotic behavior, Zhang B-Z, Kramer E. R.I)

If $s > 0$, there exist $\gamma_1 > 0,\ C_1 > 0$ and $g \in B_T^s$ such that

$$\|\vec{h}\| \leq g(t)e^{-\gamma_1 t} \quad \textit{for}\ \ t \geq 0,$$

then there exists γ with $0 < \gamma \leq \gamma_1$ and $C_2 > 0$ such that the corresponding solution u of the IBVP satisfies

$$\|u\|s \leq C_2\|(\phi, \vec{h})\|e^{-\gamma t} \quad \textit{for}\ \ t \geq 0.$$

and the IBVP is Global well-posed in $H^s(0, L)$.

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Controls used	Type of Controllability	info
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$g_2 \neq 0, g_3 \neq 0, \; g_1 = 0$	E.L.C	–
$g_3 \neq 0, g_1 \neq 0, \; g_2 = 0$	-	–
$g_1 \neq 0, g_2 \neq 0, g_3 \neq 0$	E.L.C	–

Control Results

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