Aggregation versus Diffusion

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ICREA - Universitat Autònoma de Barcelona

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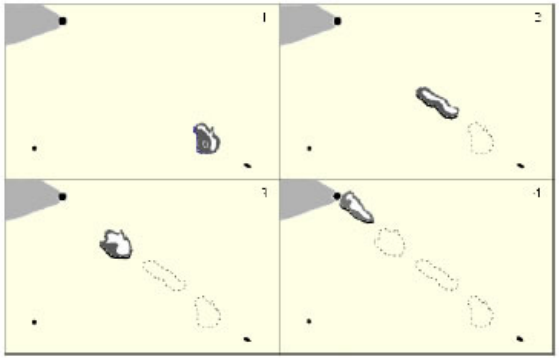
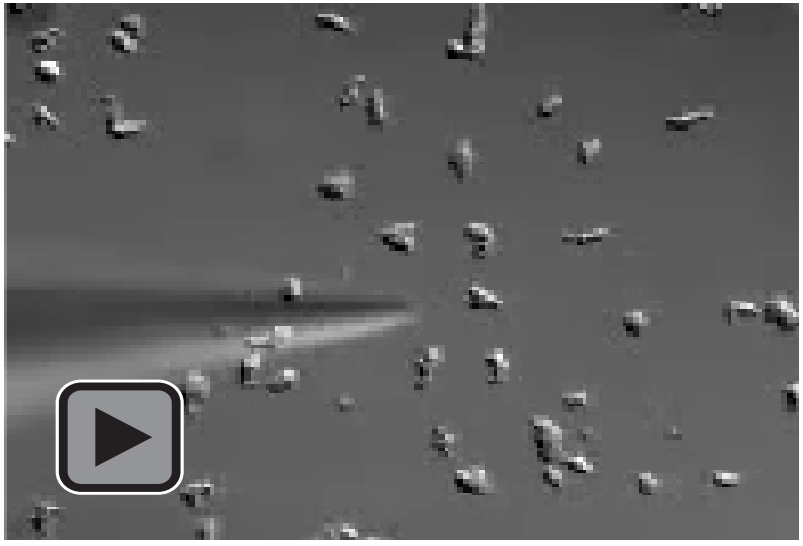
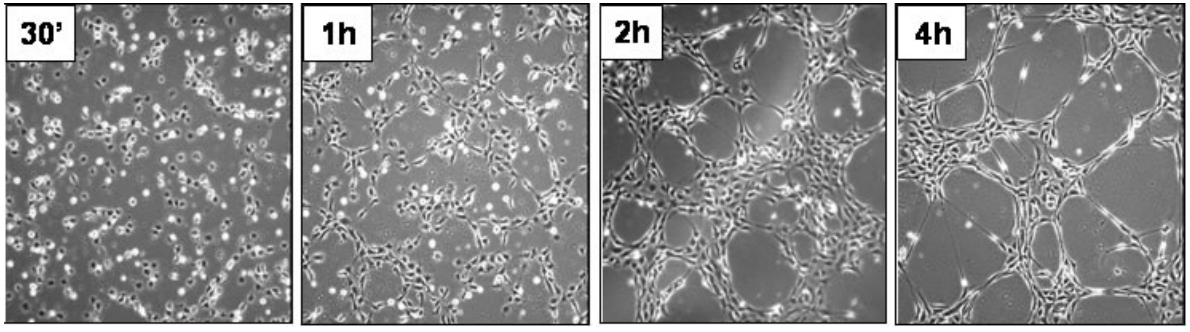
Outline

A reason for cell's motility: Chemotaxis

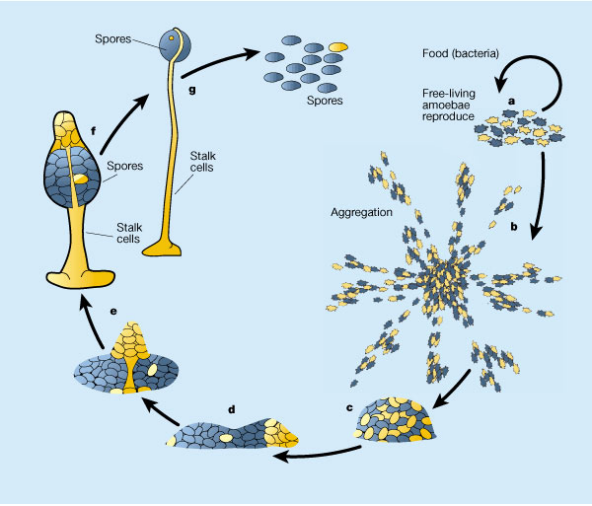


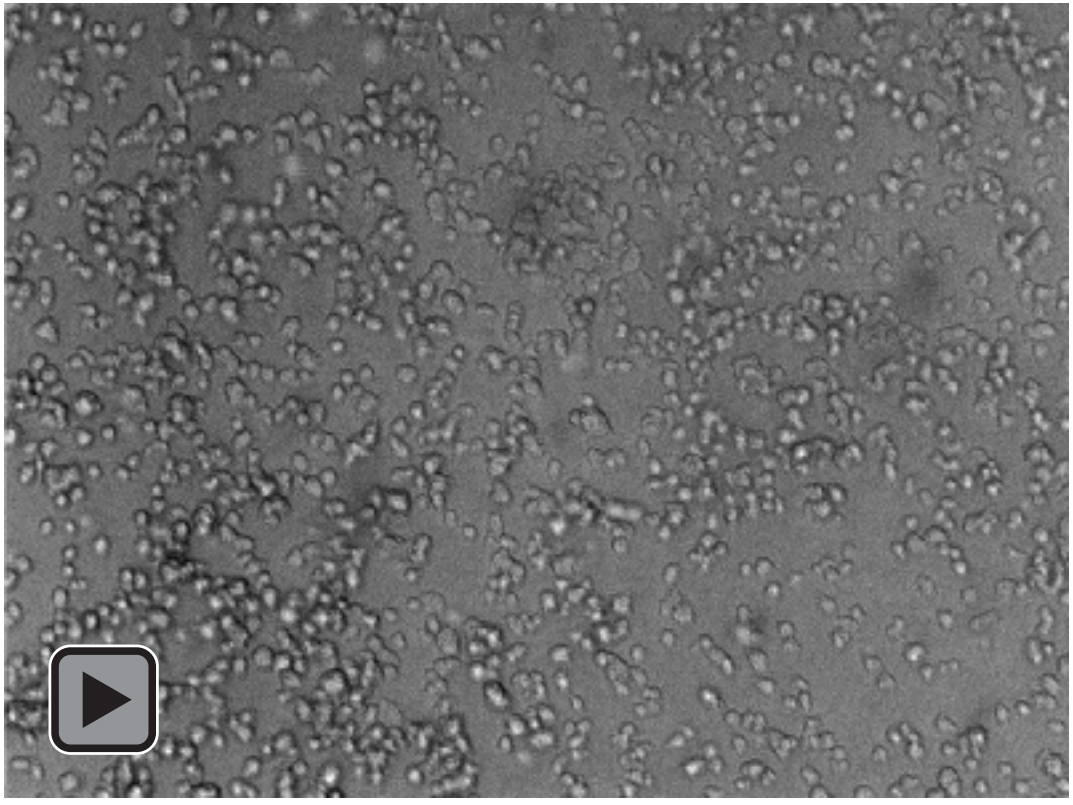




Cell movement and aggregation by chemical interaction.

Dicty's Life cycle





PKS System

$$
\begin{cases}
\dfrac{\partial \rho}{\partial t}(x,t) = \Delta \rho(x,t) - \nabla \cdot (\rho(x,t) \nabla c(x,t)) & x \in \mathbb{R}^2 \,,\ t > 0 \,, \\
c(x,t) = -\dfrac{1}{2\pi} \displaystyle\int_{\mathbb{R}^2} \log |x-y| \, \rho(y,t) \, dy & x \in \mathbb{R}^2 \,,\ t > 0 \,, \\
\rho(x, t=0) = \rho_0 \geq 0 & x \in \mathbb{R}^2 \,.
\end{cases}
$$

Aggregation Versus Diffusion: Open Problems

- Sharp Conditions on Interaction Potential for linear diffusion to win: Several partial answers by (Karch-Suzuki, Nonlinearity 2010) and (Cañizo-C.-Schonbek, preprint).
- Asymptotics for small linear diffusion: Several partial answers by (Primi-Stevens-Velázquez, CPDE 2009).
- Sharp results in any dimension with nonlinear diffusions:
  Fair competition $m = 2 - \frac{2}{d}$: (Blanchet-C.-Laurençot, CVPDE 2008).
  Diffusion-Dominated case $m > 2 - \frac{2}{d}$: (Sugiyama, 2008), (Calvez-C., JMPA 2006), (Bedrossian-Rodríguez-Bertozzi, Nonlinearity 2011), (Bedrossian, to appear in CMS), (Kim-Yao, preprint).
  Aggregation-Dominated Case $m < 2 - \frac{2}{d}$: very few results.
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- Full-Parabolic system: (Calvez-Corrias, CMS 2008), (DiFrancesco-Rosado, Nonlinearity 2009, Carrillo-Lisini, in preparation).
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