On random optimal design problems for elliptic problems

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Partial Differential Equations
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Random Optimal Design

Optimal design problems with perturbations

Classical design problems.

A general framework of a shape optimization problem:

- $D \subset \mathbb{R}^d$,
- a volume constraint,
- a source term f,
- for every subdomain $A \subset D$ a PDE

$$E_A u = f,$$

- the final cost

$$F(A) = \int_D j(x, u_A(x), Du_A(x))\, dx.$$

The shape optimization problem is

$$\min \left\{ F(A) \;:\; A \subset D,\ |A| \le m \right\}.$$

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Optimal design problems with perturbations

The new fact is that f is only known up to a random perturbation, i.e., if $(\Omega, \mathcal{F}, P)$ is a probability space,

$$f(x,\omega) = \bar{f}(x) + \xi(x,\omega).$$

$$\int_\Omega \xi(x,\omega)\,dP(\omega) = 0, \quad \int_\Omega |\xi(x,\omega)|^2\,dP(\omega) < \infty \qquad \text{for a.e. } x \in D.$$

the functional

$$F(a) = \int_\Omega \Big[\int_D j(x,\omega; u_a(x,\omega))\,dx \Big]\,dP(\omega) \tag{1}$$

with $j(x,\omega;u)$ is a Caratheodory function and such that for suitable $c > 0$ and $\Lambda \in L^1(D \times \Omega)$

$$|j(x,\omega;u)| \le \Lambda(x,\omega) + c|u|^2 \qquad \forall (x,\omega,u).$$

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Our problem.

Our optimal design problem consists in

$$\min F(a)$$

subject to,

$$\begin{cases} -\operatorname{div}\left(a(x)Du(x,\omega)\right) = f(x,\omega) \\ u = 0 \text{ on } \partial D, \end{cases} \tag{2}$$

with

$$\alpha \leq a(x) \leq \beta, \qquad \int_D a(x)\,dx \leq m$$

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  2. Pedregal, P. *Optimal Design in Two-Dimensional Conductivity for a General Cost Depending on the Field*, Arch. Rational Mech. Anal. **182** (2006) 367-385.
- Random problems
  1. Alvarez F. and Carrasco M., *Minimization of the expected compliance as an alternative approach to multiload truss optimization*, Struct. Multidiscip. Optim., **29** (2005), 470-476.

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Optimal design problems with perturbations

Definition

We say that a sequence of tensors $\{A_n\}_{n\in N}$ H-converges to the tensor $A_* \in L^\infty(D, M^{n\times n})$ if, for any f such that $f(\cdot,\omega) \in H^{-1}(D)$ P-a.e. $\omega \in \Omega$, the sequence $\{u_n\}$ of solutions of

$$\begin{cases} -\operatorname{div}\left(A_n(x)\nabla u_n(x,\omega)\right) &= f(x,\omega) \quad \text{in } D \\ u_n &= 0 \quad \text{on } \partial D. \end{cases}$$

satisfies

$$\begin{cases} u_n(\cdot,\omega) \rightharpoonup u(\cdot,\omega) \quad \textit{in } H^1_0(D), \qquad P\text{-}a.e.\ \omega \in \Omega \\ A_n\nabla u_n(\cdot,\omega) \rightharpoonup A_*\nabla u(\cdot,\omega) \quad \textit{in } L^2(D)^d, \qquad P\text{-}a.e.\ \omega \in \Omega \end{cases}$$

with $u(\cdot,\omega)$ solution of the homogenized equation P-a.e. $\omega \in \Omega$

$$\begin{cases} -\operatorname{div}\left(A_*(x)\nabla u(x,\omega)\right) &= f(x,\omega) \quad \text{in } D, \\ u &= 0 \quad \text{on } \partial D. \end{cases}$$

Optimal design problems with perturbations

$$(O_c) \qquad \min I(\chi) = \int_\Omega \Big[\int_D j(x, \omega, u_\chi(x,\omega))\, dx \Big]\, dP(\omega)$$

subject to

$$\begin{gathered} \chi \in L^\infty(\Omega; \{0,1\}), \text{ with } A = \alpha I_d \chi + \beta I_d (1-\chi), \\ -\operatorname{div}\big(A(x)\nabla u(x,\omega)\big) = f(x,\omega) \quad \text{in } D, \\ u = 0 \quad \text{on } \partial D, \end{gathered}$$

P-a.e. $\omega \in \Omega$, and to the volume constraint

$$\int_D \chi(x)\, dx \leq L,$$

with $L \in (0, |D|)$.

F. Murat, *Contre-exemples pour divers problèmes où le contrôle intervient dans les coefficients*, Ann. Mat. Pura Appl., **112** (1977), 49-68.

We denote G_θ and $\widetilde{G}_\theta$ the G-closure associated with the deterministic and random equations.

Proposition

$$G_\theta = \widetilde{G}_\theta$$

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Relaxed problem.

$$(O_r) \qquad \min I(\theta, A_*) = \int_\Omega \left[\int_D j(x,\omega,u(x,\omega))\,dx \right] dP(\omega)$$

subject to

$$\begin{array}{c} \theta \in L^\infty(D;[0,1]),\ \text{with } A_* \in G_\theta, \\ -\operatorname{div}\left(A_*(x)\nabla u(x,\omega)\right) = f(x,\omega) \quad \text{in } D, \\ u = 0 \quad \text{on } \partial D, \end{array} \tag{3}$$

P-a.e. $\omega \in \Omega$, and the volume constraint

$$\int_D \theta(x)\,dx \le L,$$

with $L \in (0,|D|)$.

Theorem

(O_r) is a relaxation of (O_c) in the sense that

1. *the infima of both problems coincide*
2. *there are optimal solutions for the relaxed problem (O_r).*

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We analize the special case when the cost functionals is the *compliance*

$$j(x, \omega, u) = f(x, \omega)u$$

and P is a $d-1$ sum of Dirac masses.

Theorem

In the case of the compliance energy the original optimization problem (2) admits a solution.

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Numerical Analysis

We propose the numerical analysis of the following problem $D \subset \mathbb{R}^2$ (d=2):

$$(O) \qquad \min I(a) = \int_\Omega \left[\int_D f(x,\omega) \cdot u(x,\omega) dx \right] dP$$

subject to,

$$\begin{gathered} a \in L^\infty(D; [\alpha, \beta]), \\ -\operatorname{div}\big(a(x)\nabla u(x,\omega)\big) = f \quad \text{in } D, \\ u = u_0 \quad \text{on } \partial D, \\ \int_D a(x)\, dx = L \end{gathered}$$

Optimal design problems with perturbations

We use a gradient descent algorithm.
We consider $0 < \alpha \leq \beta$, $m \in (\alpha|D|, \beta|D|)$ and $\varepsilon < 1$, $\varepsilon_1 \ll 1$ data of the problem, the structure of the algorithm is as follows.

- Initialization of the density $a^0 \in L^\infty(D; [\alpha, \beta])$;
- for $k \geq 0$, iteration until convergence (i.e., $|I_\gamma(a^{k+1}) - I_\gamma(a^k)| \leq \varepsilon_1 |I_\gamma(a^0)|$) as follows:
  - compute the state u_{a^k} and then the co-state p_{a^k}, both corresponding to $a = a^k$;
  - compute the descent direction $\tilde{a}$,and the multiplier γ;
  - update the density a^k in D:

$$a^{k+1} = a^k + \varepsilon(a^k - \alpha)(\beta - a^k)\tilde{a}^k,$$

    with $\varepsilon \in \mathbb{R}^+$ small enough to ensure the decrease of the cost function, $a^{k+1} \in L^\infty(D, [\alpha, \beta])$.

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Optimal design problems with perturbations

We consider:

- The domain $D = (0,1)^2$,
- Two phases $\alpha = 1$ and $\beta = 2$,
- the volume constraint $m = \frac{\alpha+\beta}{2} = 1{,}5$
- the source term, $f(x,y) = (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2$.

By simplicity we choose the random variable ξ with a discrete distribution of probability. We consider two different cases for ξ:

- **Case 1:** $\xi(x) = \pm\chi_{D_0}$ where $D_0 = [\frac{1}{4}, \frac{3}{4}]^2 \subset D$
- **Case 2:** $\xi(x) = \pm\chi_{D_1}$ where $D_1 = D \setminus D_0$

and in both cases $P(\{\xi = \chi\}) = P(\{\xi = -\chi\}) = \frac{1}{2}$.
We show pictures for compliance minimization.

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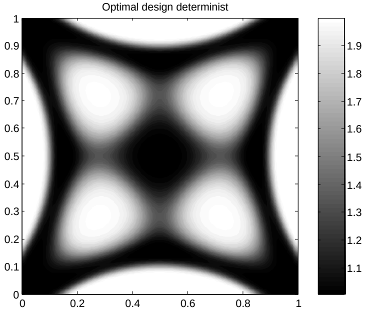
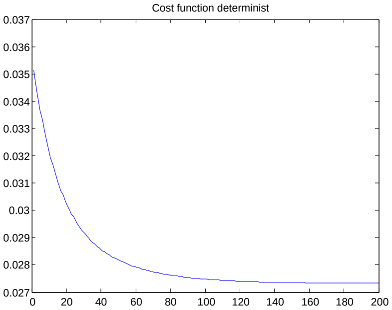
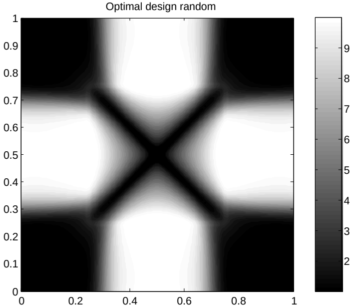

Optimal design determinist
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1


Cost function determinist
0.037
0.036
0.035
0.034
0.033
0.032
0.031
0.03
0.029
0.028
0.027
0
20
40
60
80
100
120
140
160
180
200


Optimal design random
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
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0.6
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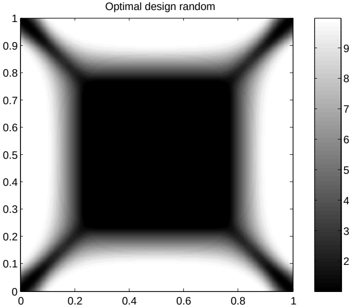
2


Optimal design random
1
0.9
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THANK YOU

FOR YOUR

ATTENTION!!!