Dynamical Casimir effect with time dependent Robin boundary conditions in $3+1$ dimensions

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Introduction

The dynamical Casimir effect (DCE) is a quantum vacuum effect which consists, basically, of two closely related phenomena, namely, particle creation due to moving mirrors and radiation reaction forces acting on moving boundaries.

Analogously to what happens in ordinary quantum mechanics, where a system initially in its fundamental state can jump into an excited state due to the interaction with an external time-dependent potential, a quantized field can also leave its vacuum state and jump into an excited state due to the interaction with an external time-dependent potential.

In the DCE, moving boundaries can be considered as external time-dependent potentials (for example, a moving boundary described by a electric permittivity and a magnetic permeability changing in time). For this reason, the interaction between quantized fields and moving mirrors induces the field to go out of its vacuum state. In other words, moving boundaries are responsible by particle creation.

In the theoretical aspect of the DCE, Moore, DeWitt and Fulling-Davies [1] are considered the pioneers to discuss the DCE in the context of a real massless scalar field in two-dimensional spacetime. Some years later, Ford and Vilenkin [2] proposed a perturbative approach to DCE. One of the advantages of the Ford-Vilenkin's approach is the possibility of generation of radiation by moving mirrors in more realistic situations, such as in $3+1$ dimensions.

Exploring the method presented by Ford and Vilenkin, we found a series of papers devoted to the DCE from different boundary conditions (BC) and initial quantum states (See Refs. [3, 4]) to a generalization for the electromagnetic field [5]. In connection with the last ideas, the problem of Casimir forces and particle creation due a moving mirror with Robin BC was discussed by Mintz *et al* [4].

For a scalar field, the Robin BC is defined as:

$$\left(\phi - \gamma_0 \frac{\partial \phi}{\partial n}\right)\Big|_{\text{boundary}} = 0, \tag{1}$$

where γ_0 is the Robin parameter. The Robin BC has interesting properties (See, for instance, [4] an references therein). The parameter γ_0 interpolates continuously Dirichlet ($\gamma_0 \to 0$) and Neumann BC ($\gamma_0 \to \infty$). Robin BC can simulate the plasma model in real metals for low frequencies. For $\omega \ll \omega_P$, the parameter γ_0 plays the role of the plasma wavelength which is directly related to the penetration depth of the field.

In the present work, we consider a real massless scalar field in $3+1$ dimensions satisfying a time-independent Robin boundary condition at a moving mirror:

$$\left[\partial_z \phi\left(t,\vec{r}\right) + \delta\dot{q}(t)\partial_t \phi\left(t,\vec{r}\right) - \gamma_0^{-1}\phi\left(t,\vec{r}\right)\right]\Big|_{z=\delta q(t)} = 0. \tag{2}$$

The angular and spectral distributions for the created particles are computed, generalizing previous results obtained by Mintz *et al* [4]. We show that the suppression in the total number of created particles is still present in $3+1$ dimensions for particular values of the Robin parameter and of the mechanical frequency.

The Bogoliubov transformations

Considering the Ford and Vilenkin approach [2], the scalar field can be written as:

$$\phi\left(t,\vec{r}\right) = \phi_0\left(t,\vec{r}\right) + \delta\phi\left(t,\vec{r}\right). \tag{3}$$

The field perturbation obeys the Klein-Gordon equation $\partial^2 \delta\phi(t,\vec{r}) = 0$ with the following BC:

$$\left(\partial_z - \gamma_0^{-1}\right)\delta\phi\left(t,\vec{r}\right)\Big|_{z=0} = \Big[\delta q\left(t\right)\left(\gamma_0^{-1}\partial_z - \partial_z^2\right) - \delta\dot{q}\left(t\right)\partial_t\Big]\phi_0\left(t,\vec{r}\right)\Big|_{z=0}. \tag{4}$$

It is convenient to express the field in the Fourier domain, such that $\Phi(\omega,\vec{k}_\parallel;z) = \Phi_0(\omega,\vec{k}_\parallel;z) + \delta\Phi(\omega,\vec{k}_\parallel;z)$ represents the Fourier transformation of Eq. (3). We can show that:

$$\Phi_0(\omega,\vec{k}_\parallel;z) = \sqrt{\frac{16\pi^3\left|\omega\right|}{k_z^2}\left(\frac{1}{1+\gamma_0^2 k_z^2}\right)}\left[\sin(k_z z) + \gamma_0 k_z \cos(k_z z)\right] \times \left[a(\vec{k})\Theta(\omega) - a\dagger(-\vec{k})\Theta(-\omega)\right]\Theta(k_z^2) \tag{5}$$

where

$$k_z = \left[(\omega + i\epsilon)^2 - k_\parallel^2\right]^{1/2}, \quad \epsilon \to 0^+. \tag{6}$$

The pertubation $\delta\Phi(\omega,\vec{k}_\parallel;z)$ satisfying Helmholtz equation, $\left(\omega^2 - k_\parallel^2 + \partial_z^2\right)\delta\Phi(\omega,\vec{k}_\parallel;z) = 0$, with the following BC:

$$\left(\partial_z - \gamma_0^{-1}\right)\delta\Phi(\omega,\vec{k}_\parallel;z)|_{z=0} = \gamma_0^{-1}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\delta Q(\omega-\omega') \times \left(\partial_z + \gamma_0^2\omega\omega'\right)\Phi_0(\omega',\vec{k}_\parallel;z)|_{z=0}, \tag{7}$$

where $\delta Q\left(\omega\right)$ is the Fourier transform of $\delta q\left(t\right)$. After a straightforward calculation, it is not difficult to obtain:

$$a_{out}(\vec{k}) = a_{in}(\vec{k}) - \frac{2ik_z}{\sqrt{|\omega|\left(1+\gamma_0^2 k_z^2\right)}}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\sqrt{\frac{|\omega'|}{(1+\gamma_0^2 k_z'^2)}}\Theta(k_z'^2) \times \delta Q(\omega-\omega')(1+\gamma_0^2\omega\omega')[a_{\text{in}}(\vec{k}')\Theta(\omega') - a_{\text{in}}\dagger(-\vec{k}')\Theta(-\omega')], \tag{8}$$

that is the Bogoliubov transformation between a_{out} and a_{in} and their hermitian conjugates.

Spectral distribution

The particle distribution of the created particles inside a volume $d^3\vec{k}$ in Fourier domain is defined as:

$$\frac{dN(\vec{k})}{d^3\vec{k}} = \left\langle 0_{\text{in}}\right| a^\dagger_{\text{out}}(\vec{k})a_{\text{out}}(\vec{k})\left|0_{\text{in}}\right\rangle. \tag{9}$$

By taking polar coordinates, we obtain the number of particles per unit frequency interval and solid angle:

$$\frac{dN(\omega,\theta)}{d\omega d\Omega} = \frac{2}{\pi}\frac{k_z^2}{|\omega|\left(1+\gamma_0^2 k_z^2\right)} \times \int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\omega'(1-\gamma_0^2\omega\omega')^2\left|\delta Q(\omega+\omega')\right|^2\Theta(\omega')}{(1+\gamma_0^2 k_z'^2)}, \tag{10}$$

Let us consider $\delta q(t)$ given by: $\delta q(t) = \epsilon_0\cos(\omega_0 t)e^{-|t|/\tau}$, with $\omega_0\tau >> 1$. Taking the Fourier transform of this expression, we get: $|\delta Q(\omega)|^2 \approx \frac{\pi}{2}\epsilon_0^2\tau[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$, and consequently:

$$\frac{dN(\omega,\theta)}{d\omega d\Omega} = \frac{\epsilon_0^2\tau}{2\pi}\frac{\omega(\omega_0-\omega)\cos^2\theta[1-\gamma_0^2\omega(\omega_0-\omega)]^2\Theta(\omega_0-\omega)}{[1+\gamma_0^2\omega^2\cos^2\theta][1+\gamma_0^2(\omega_0-\omega)^2\cos^2\theta]}. \tag{11}$$

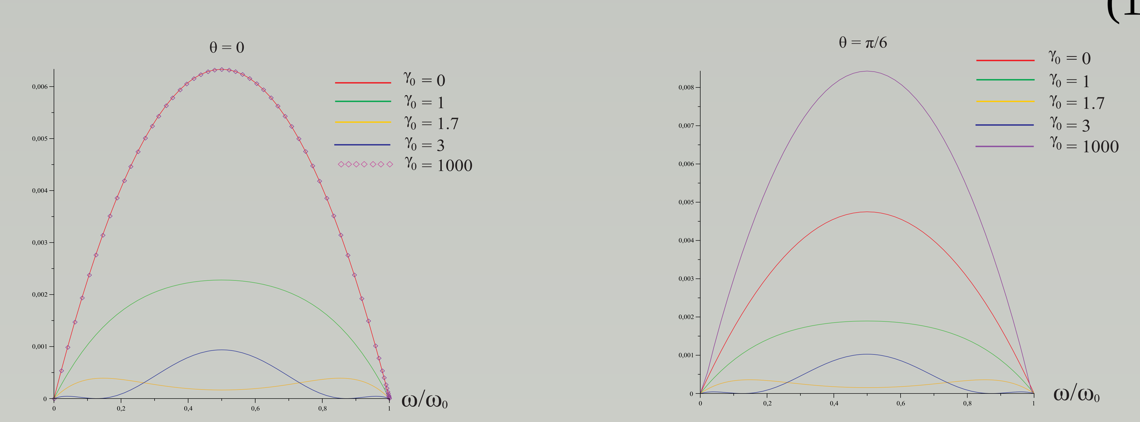

Figure 1: Spectral distribution per unit frequency and solid angle as a function of ω/ω_0 for and different values of θ and γ_0 ($\epsilon_0 = \tau = 1$).

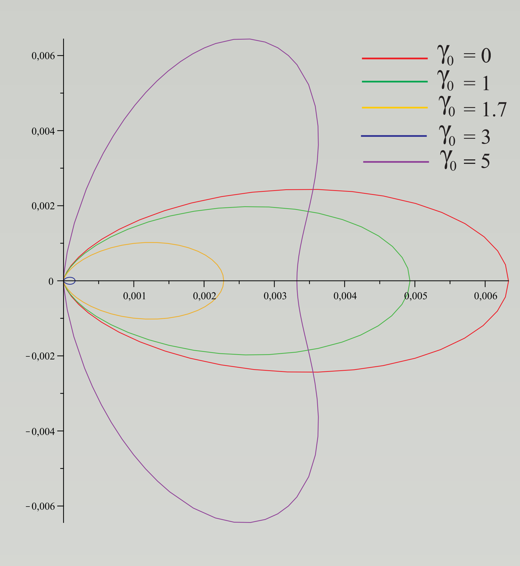

Figure 2: Polar diagram of the spectral distribution per unit frequency and solid angle ($\epsilon_0 = \tau = 1$).

In Eq. (11), after an integration over solid angle, we obtain

$$\frac{dN(\omega)}{d\omega} = \frac{\epsilon_0^2\tau}{2\pi}\frac{(\omega_0-\omega)\tan^{-1}(\gamma_0\omega) - \omega\tan^{-1}[\gamma_0(\omega_0-\omega)]}{\gamma_0^3\omega_0(\omega_0-2\omega)} \tag{12}$$

as being the particle distribuition per unit frequency. In the same way, after an integration over frequency, we can show that:

$$\frac{dN(\omega)}{d\Omega} = \frac{\epsilon_0^2\tau}{2\pi}\frac{\omega_0^3}{\xi^4\cos^2\theta(4+\xi^2\cos^2\theta)} \times \times(2+\xi^2\cos^2\theta)\ln(1+\xi^2\cos^2\theta) - 2\xi\cos\theta\tan^{-1}[\xi\cos\theta], \tag{13}$$

with $\xi = \gamma_0\omega_0$. In the next section, we present some numerical results for the total number of created particles.

Total number

The total number of particles created by the movement of the boundary can be found by integrating either Eqs. (12) or (13).

$$N = \int_0^\infty \frac{dN(\omega)}{d\omega}d\omega. \tag{14}$$

Unfortunately, we could not yet find an analytical expression for the total number. However, it could be calculated numerically, as displayed below.

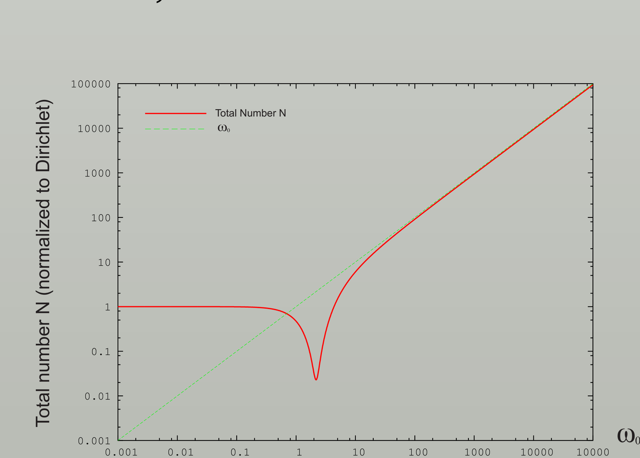

Figure 3: Total number of particles as a function of ω_0 ($\gamma_0 = 1$, $\epsilon_0 = \tau = 1$). The result is normalized by the number of particles in the Dirichlet ($\gamma_0 = 0$) case.

We see that, with low oscillation frequencies ω_0, the creation of particles is similar to the $\gamma_0 = 0$ case. However, for $\omega_0 \sim 2\gamma_0^{-1}$, there is a *suppression* of particle creation. This feature of the Robin boundary condition has already been noticed in [4] for the 1+1 case. It can be interpreted as a frequency-dependent decoupling between the moving boundary and the quantum field. For higher frequencies,

$$\frac{N_{\gamma_0\omega_0\gg 1}}{N_{\gamma_0=0}} = \frac{3\pi}{10}\gamma_0\omega_0 \tag{15}$$

the particle number grows faster than in the $\gamma_0 = 0$ case by one power of $\gamma_0\omega_0$, as can be seen also from the $\gamma_0\omega_0 \gg 1$ limit in (12).

Final comments

In this work, we generalize to $3+1$ dimensions the results obtained by Mintz *et al* [4] for a moving mirror with time-independent Robin BC. The angular distribution of the created particles was investigated. We have seen that the supression is still present in the total number of the particle creation rate for particular values of the Robin parameter and of the mechanical frequency.

Acknowledgments

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References

[1] G.T. Moore, *J Math. Phys.* **11**, 2679, (1970); B. DeWitt, *Phys. Rep.* **19**, 295 (1975); S.A. Fulling and P.C.W. Davies, *Proc. R. Soc. London* A **348**, 393, (1976).
[2] L.H. Ford and A. Vilenkin, *Phys. Rev.* D **25**, 2569 (1982).
[3] D.T. Alves, C. Farina and P.A. Maia Neto, *J. Phys. A: Math. Gen.* **36**, 11333 (2003).
[4] B. Mintz, C. Farina, P.A. Maia Neto and R.B. Rodrigues, *J. Phys.* A **39**, 6559 (2006); B. Mintz, C. Farina, P.A. Maia Neto and R.B. Rodrigues, *J. Phys.* A **39**, 11325 (2006).
[5] P.A. Maia Neto and L.A.S. Machado, *Phys. Rev.* A **54**, 3420 (1996).