Cosmic Strings Stabilized by Fermion Fluctuations

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Presentation is based on:

HW, M. Quandt, N. Graham, and O. Schröder, Nucl. Phys. **B831** (2010) 306

HW and M. Quandt, Phys. Lett. **B690** (2010) 514

HW, N. Graham, and M. Quandt, Phys. Rev. Lett. **106** (2011) 101601

N. Graham, M. Quandt, and HW, Phys. Rev. **D84** (2011) 025017


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- ⋆ fermion bound state and vacuum energies are same $\mathcal{O}(\hbar)$
- ⋆ non–trivial structure at spatial infinity
- ⋆ gauge invariant field combinations are smooth at spatial infinity
- ⋆ individual Feynman diagrams are not gauge invariant

⋆ static string profiles in a $SU_L(2)$ gauge theory

- vector meson (temporal gauge):

$$\vec{W} = n\sin(\xi_1)\frac{f_G(\rho)}{g\rho}\hat{\varphi}\begin{pmatrix} \sin(\xi_1) & i\cos(\xi_1)\,\mathrm{e}^{-in\varphi} \\ -i\cos(\xi_1)\,\mathrm{e}^{in\varphi} & -\sin(\xi_1) \end{pmatrix}$$

- Higgs meson:

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- ξ_1 parameterizes orientation in iso–space
- n counts winding of the string (usually $n = 1$)
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- non–trivial structure at spatial infinity:

neither Born nor Feynman series well defined

II) Spectral (Phase Shift) Method

⋆ phase shift approach: summing the changes of zero–point energies

$$E_{\rm vac} = \frac{\hbar}{2}\sum_n \left(\omega_n - \omega_n^{(0)}\right)\Big|_{\rm ren.} = \frac{\hbar}{2}\sum_j \epsilon_j + \hbar \int dk\, \omega_k\, \Delta\, \rho_{\rm ren.}(k)$$

ϵ_j : true bound state energies

$\omega_k = \sqrt{k^2+m^2}$ energy of continuum states

$\Delta\rho(k)$: change in density of continuum states (in momentum space)

$= \frac{1}{\pi}\frac{d}{dk}\delta(k)$

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$$\begin{aligned}
\epsilon_j \quad &: \quad \text{true bound state energies} \\
\omega_k \quad &= \quad \sqrt{k^2+m^2}\ \text{energy of continuum states} \\
\Delta\rho(k) \quad &: \quad \text{change in density of continuum states} \quad \text{(in momentum space)} \\
&= \quad \frac{1}{\pi}\frac{d}{dk}\delta(k) \\
\delta(k) \quad &: \quad \text{phase shift}
\end{aligned}$$

⋆ requires angular momentum decomposition

⋆ interface formalism (energy per unit length)

- start from energy momentum tensor in field theory
- integrate over momentum of translationally invariant subspace
- dimensional regularization for associated UV divergence (*)
- scattering problem in orthogonal two–dim. subspace
- cancel divergence (*) by sum rules for scattering data

generalization to Levison's theorem

$$E_\delta^{(N)} = \frac{1}{4\pi}\sum_\ell \Bigg\{ D_\ell \int_0^\infty \frac{dk}{\pi}\left[\omega_k^2 \ln\left(\frac{\omega_k^2}{\mu^2}\right) - k^2\right]\frac{d}{dk}\left[\delta_\ell(k)\right]_N + \sum_j \left[(\epsilon_{j,\ell})^2 \ln\frac{(\epsilon_{j,\ell})^2}{\mu^2} - (\epsilon_{j,\ell})^2 + m^2\right]\Bigg\}$$

integration before summation!

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- exchange of sum and integral only possible after rotating to imaginary momentum axis, $t = ik$.

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$$\left. + \sum_j \left[\left(\epsilon_{j,\ell}\right)^2 \ln \frac{\left(\epsilon_{j,\ell}\right)^2}{\mu^2} - \left(\epsilon_{j,\ell}\right)^2 + m^2 \right] \right\}$$

- δ_ℓ: *total* phase shift in (angular momentum) channel ℓ

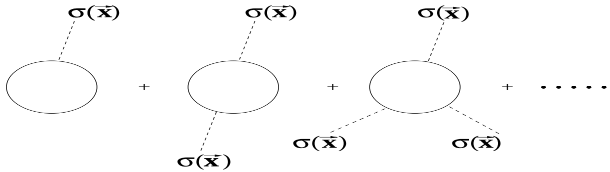
(phase of $det\mathcal{S}$)

- $\omega_k = \sqrt{k^2 + m^2}$: energy of continuum mode
- D_ℓ: degeneracy in (angular momentum) channel ℓ
- N: number of Born subtractions to render integral finite
- $\epsilon_{j,\ell}$: bound state energies
- μ: redundant renormalization scale

(sum rules for scattering data)

⋆ Feynman diagrams (add back in subtractions)

$E_{\mathrm{FD}}^{(N)} =$



(up to order N)

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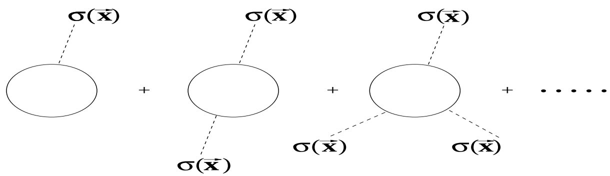
$$E_{\mathrm{FD}}^{(N)} =$$ 

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$$E_{\mathrm{CT}} = \sum_i c_i E^{(i)}[\sigma]$$

- c_i: counterterm coefficients computed from renormalization condition in the perturbative sector

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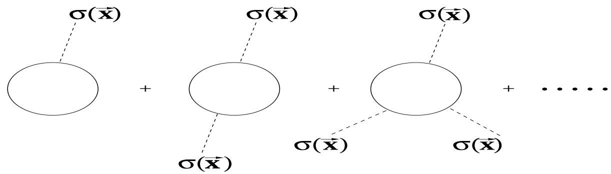
$E_{\text{FD}}^{(N)} =$ 

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⋆ total vacuum energy

$$E_{\text{vac}} = E_\delta^{(N)} + E_{\text{FD}}^{(N)} + E_{\text{CT}}$$

III) Fermions in String Background

- ⋆ fermions dominate vacuum energy at large N_c
- ⋆ pure gauge at spatial infinity

$$U = P_L \exp\left(i\hat{n}\cdot\vec{\tau}\xi_1\right) + P_R \qquad \text{with} \qquad \hat{n} = \begin{pmatrix} \cos(n\varphi) \\ -\sin(n\varphi) \\ 0 \end{pmatrix}$$

so that $\quad g\vec{W} \sim U^\dagger \nabla U \quad$ and $\quad \Phi \sim U\Phi_0$

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⋆ return string at spatial infinity (as for QED flux tubes)

- numerically very costly
- proof–of–principle calculation:

  vacuum polarization energy small in $\overline{\mathrm{MS}}$.

⋆ major differences to QED flux tubes:

- gauge transformation that unwinds the string is unique:

$$U(\varphi) = U(\varphi + 2\pi)$$

  even though there are fractional fluxes

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⋆ radially dependent gauge transformation

$$H \quad \longrightarrow \quad U^{\dagger}(\rho, \varphi)\, H\, U(\rho, \varphi)$$

with

$$U(\rho, \varphi) = P_L \mathrm{exp}\left(i\hat{n} \cdot \vec{\tau}\, \xi(\rho)\right) + P_R$$

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⋆ local gauge invariance: specific shape of ξ irrelevant

$\longrightarrow$ further support for phase shift method

⋆ field parameterization

$$f_H(\rho) = 1 - e^{-\frac{\rho}{w_H}}$$

$$f_G(\rho) = 1 - e^{-\left(\frac{\rho}{w_G}\right)^2}$$

$$\xi(\rho) = \xi_1 \left[1 - e^{-\left(\frac{\rho}{w_\xi}\right)^2}\right]$$

- w_H, w_G, ξ_1: variational parameters for physical string
- w_ξ: gauge parameter, not measurable

- similarly, the scale parameter for the fake boson field is not measurable
  (technicality to simplify 3rd and 4th order FDs)

⋆ data in $\overline{\text{MS}}$ scheme (scale set by fermion mass)

w_ξ	E_{FD}	E_δ	E_{B}	E_{vac}
0.5	-0.2515	0.3489	0.0046	0.1020
1.0	-0.0655	0.1606	0.0032	0.0983
2.0	-0.0358	0.1294	0.0038	0.0974
3.0	-0.0320	0.1235	0.0056	0.0971
4.0	-0.0302	0.1193	0.0080	0.0971

$$w_H = 2.0$$
$$w_G = 2.0$$
$$\xi_1 = 0.4\pi$$

- E_{FD} renormalized first and second order Feynman diagram
- E_δ phase shift contribution, first and second Born order removed
  - ∗ computed by analytic continuation
  - ∗ log. div. of 3rd and 4th order by a fake field
  - ∗ numerically very costly
  - ∗ about 1 . . . 2% numerical error
- E_{B} remnant of fake field

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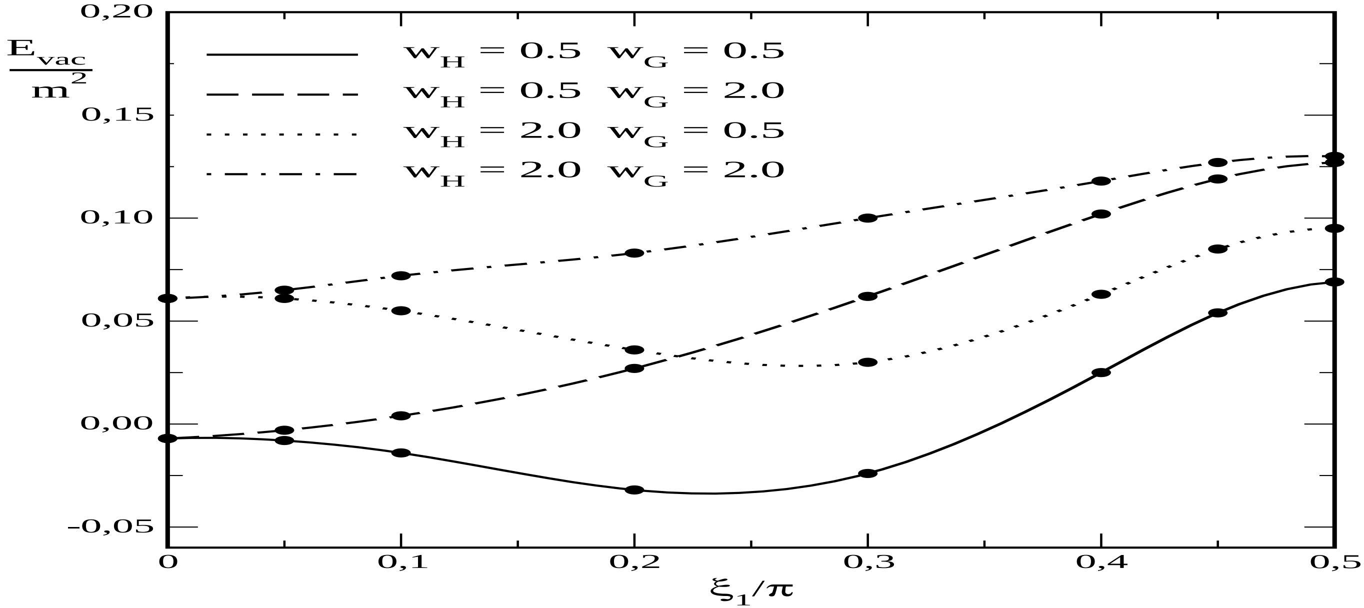
spectral methods to compute vacuum energies verified

⋆ on–shell scheme for E_{CT}

add (maximally four) finite local and gauge–invariant counterterms:

- no–tadpole interaction
- Higgs mass unchanged
- normalization of Higgs particle unchanged
- normalization of vector meson unchanged
- vector meson mass will be a prediction,
  $\longrightarrow$ tune gauge coupling to reproduce M_W

⋆ results in on–shell scheme



- finite counterterm contribution adds positively
- quantum effects provide some binding for thin strings (Landau ghost ?)
- competitive with classical mass only for

(i) fermion masses about 1TeV
(ii) many internal degrees of freedom (color)

⋆ total energy: $E = E_{\rm cl} + 3E_{\rm vac}$ $(N_C = 3)$

⋆ classical contribution:

$$\frac{E_{\rm cl}}{m^2} = 2\pi \int_0^\infty \rho d\rho \left\{ n^2 \sin^2 \xi_1 \left[\frac{2}{g^2} \left(\frac{f_G'}{\rho} \right)^2 + \frac{f_H^2}{f^2 \rho^2} \left(1 - f_G\right)^2 \right] + \frac{f_H'^2}{f^2} + \frac{\mu_h^2}{4 f^2} \left(1 - f_H^2\right)^2 \right\}$$

(quantities under the integral are scaled to be dimensionless, $\mu_h = \frac{m_H}{vf}$)

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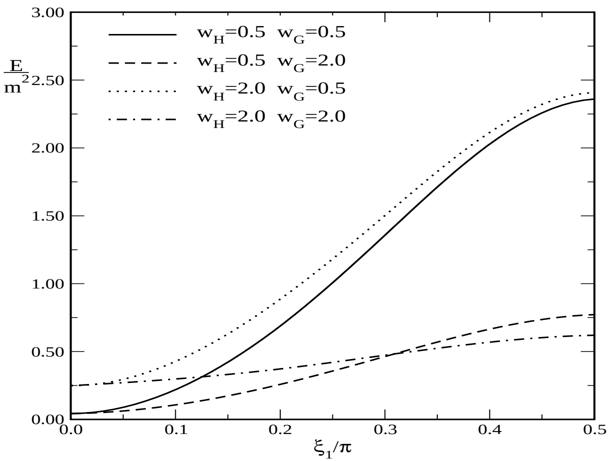
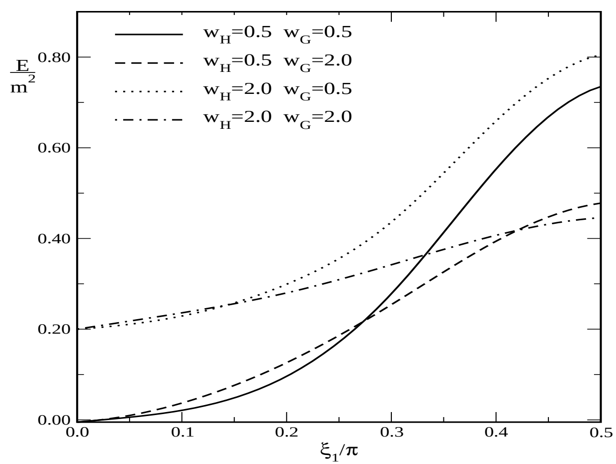
⋆ numerical results:



$f = g = 5.0$



$f = g = 10.0$

IV) Charged Cosmic Strings

⋆ Cosmic strings induce fermionic bound state levels: $\epsilon_i < m$

- ϵ_i eigenvalues of two–dimensional Hamiltonian
- for $\xi_1 = \pi/2$ an exact zero mode exists
- for wide strings, $w_{\rm H} \gtrsim 4$ many levels emerge (100)

⋆ bound states carry longitudinal momenta:

$$E_i(p_n) = \sqrt{\epsilon_i^2 + p_n^2} \qquad p_n = \frac{n\pi}{L} \qquad \text{(length of the string)}$$

⋆ populating these levels may form a charged string with energy less than equally many free fermions

⋆ binding energy is of same order as $E_{\rm vac}$, both in $N_{\rm C}$ and $\hbar$.

⋆ require leading contribution to the total energy as $L \to \infty$

$$\sum_n \longrightarrow \frac{L}{\pi} \int dp$$

⋆ search for the minimum of bound state contribution

- introduce chemical potential $\mu \leq m$
- populate all levels with $E_i(p) \leq \mu$
- Fermi momentum for each populated level $p_i^F(\mu) = \sqrt{\mu^2 - \epsilon_i^2}$

  (states with $\epsilon_i > \mu$ are not populated)

⋆ charge density (per unit length)

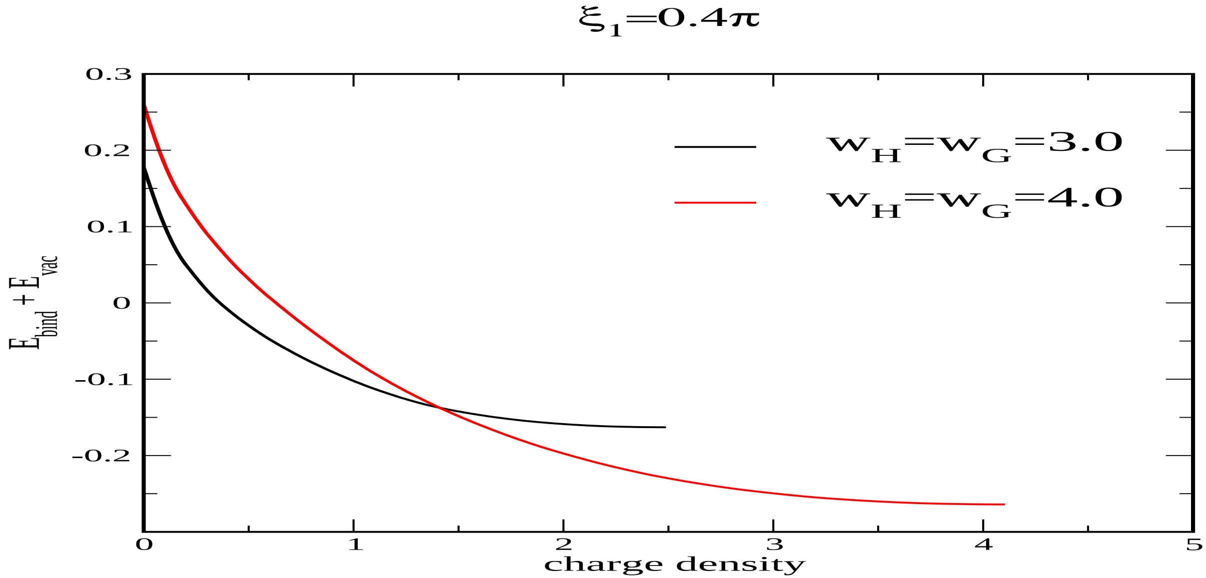
$$Q(\mu) = \sum_i \frac{p_i^F(\mu)}{\pi}$$

monotonously rising, can be inverted

$$\mu = \mu(Q) \qquad \text{and} \qquad p_i^F = p_i^F(Q)$$

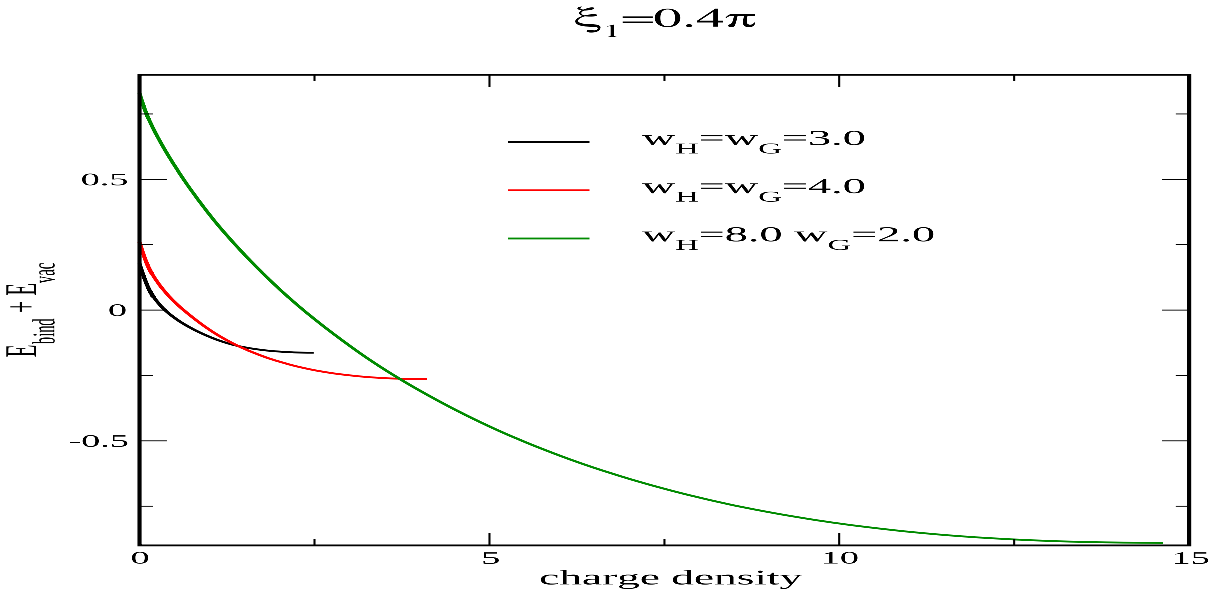
⋆ binding energy (density) at prescribed charge (density)

$$E_{\text{bind}}(Q) = \frac{1}{\pi} \sum_i \int_0^{p_i^F(Q)} dp \left[\sqrt{\epsilon_i^2 + p^2} - m \right] \qquad \left(\begin{array}{l} \text{relative to} \\ \text{free fermions} \end{array} \right)$$



- endpoint: all available levels populated
- small charge: narrow string preferred
- large charge: wide string preferred

⋆ adding more configurations



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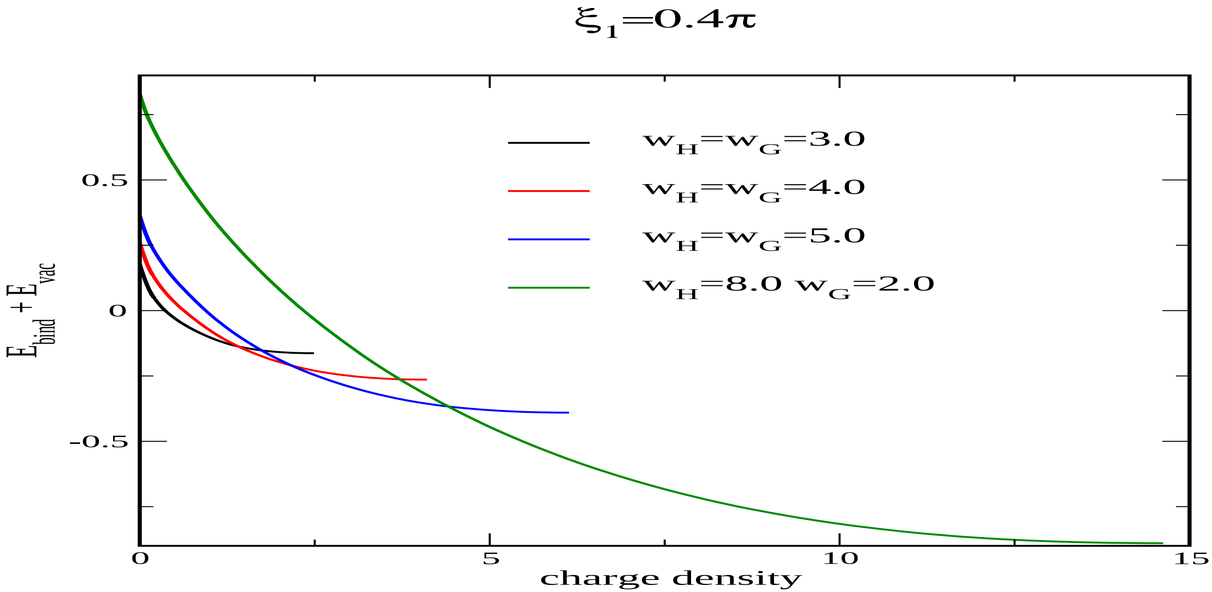

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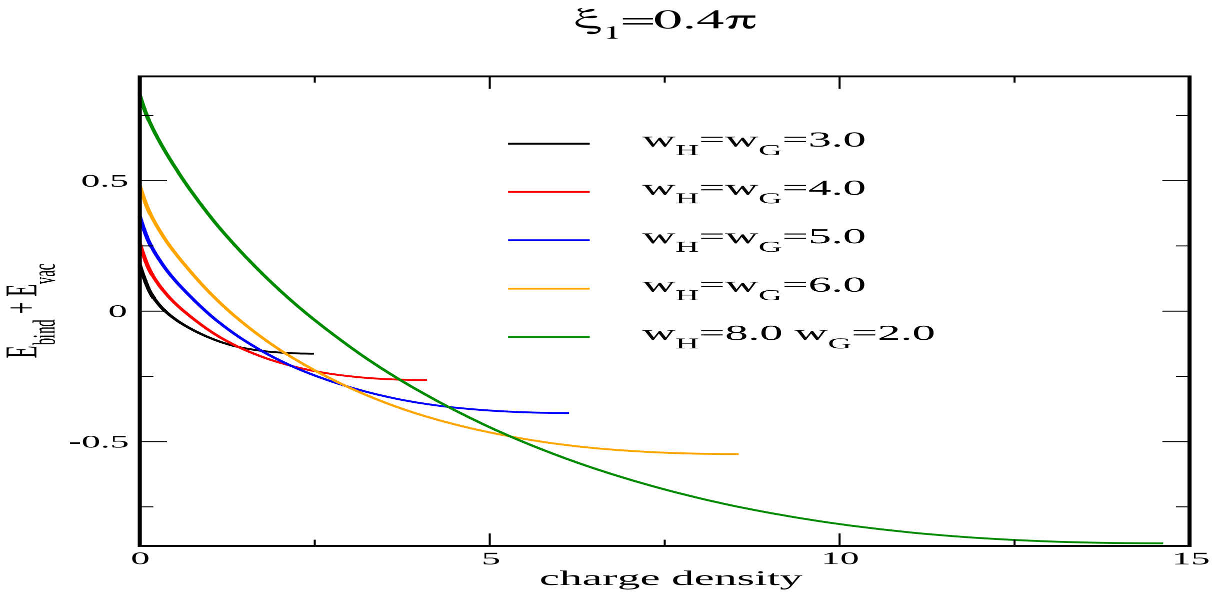

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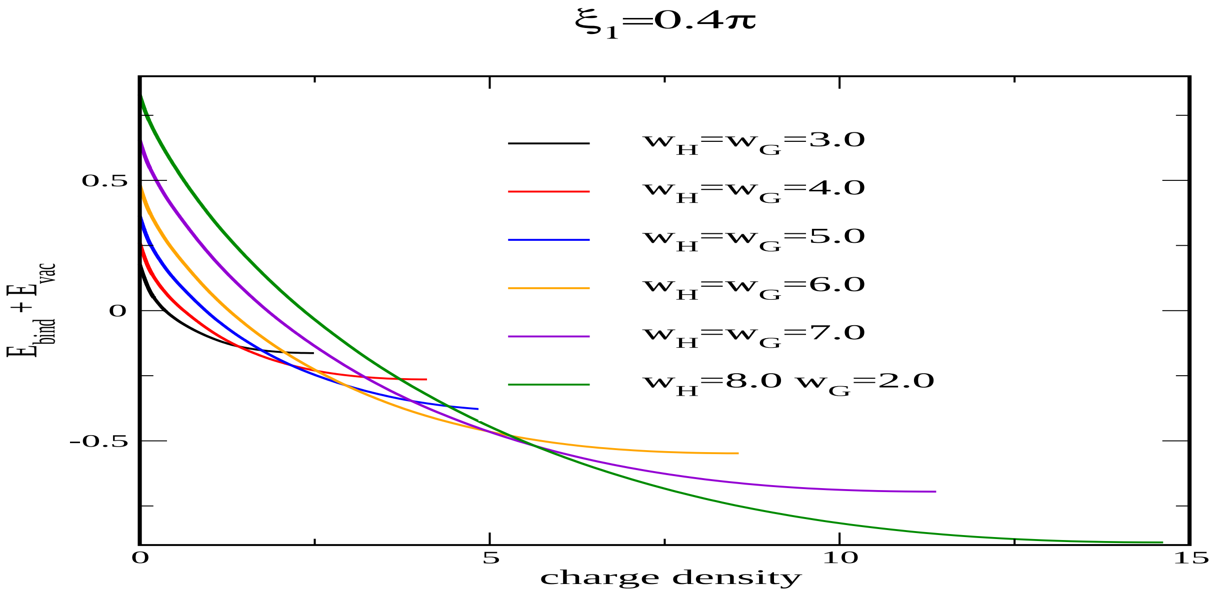

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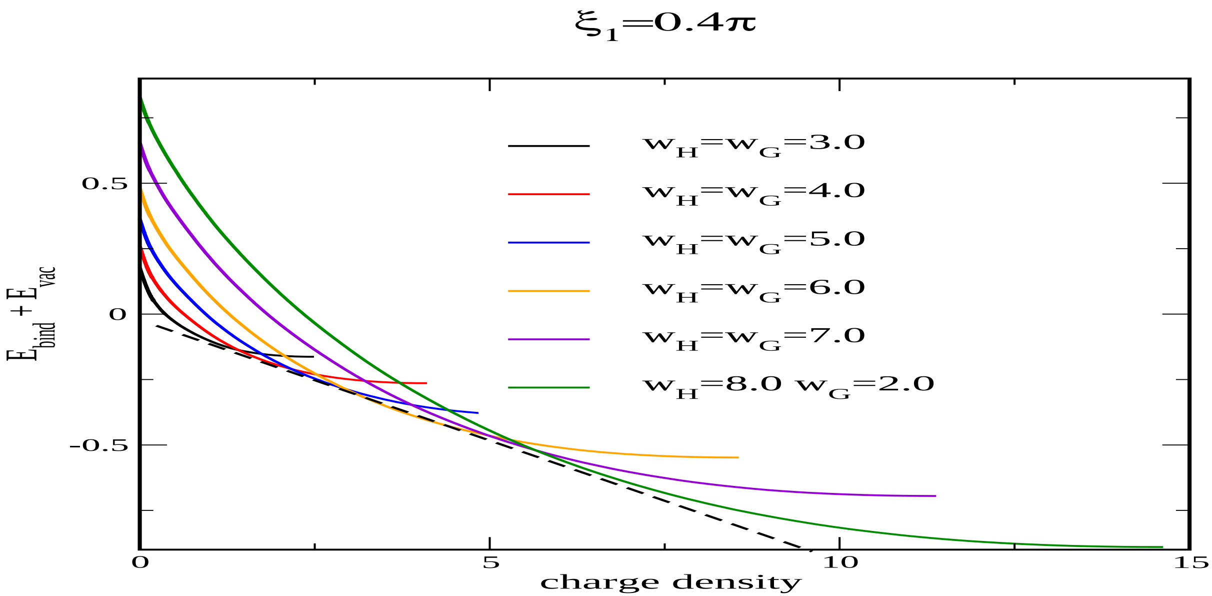

- delicate balance between E_{vac} and ϵ_i:

$$\min\left(E_{\text{bind}}+E_{\text{vac}}\right)\propto Q$$

- strong variation with w_H
- essentially independent of w_G and ξ_1

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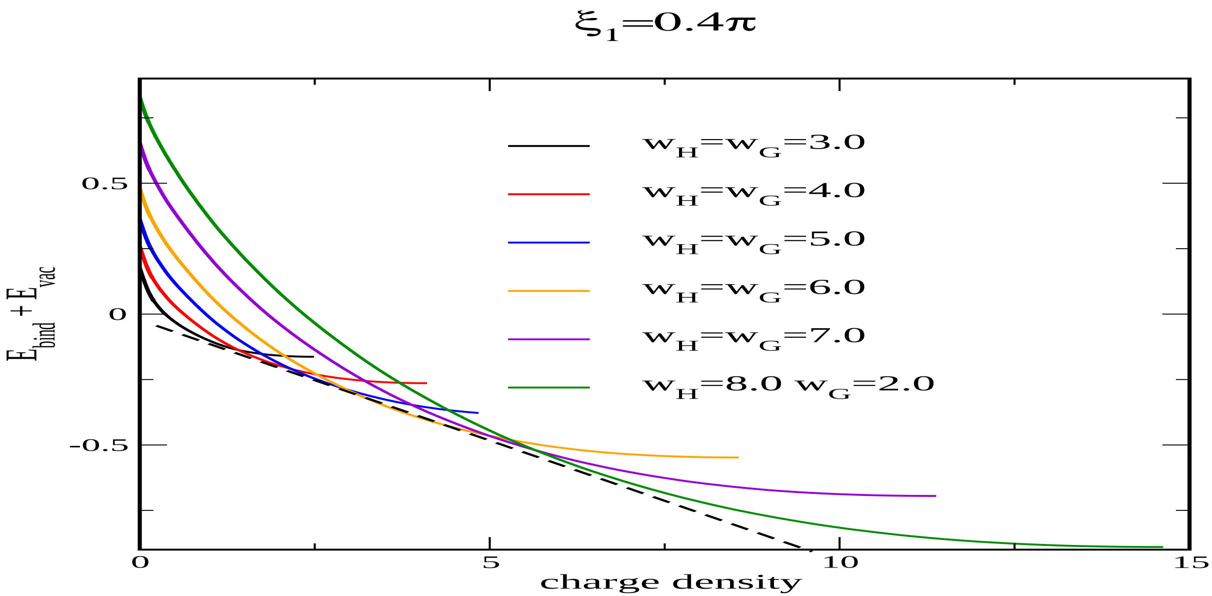

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⋆ identify configurations along the envelope to compute associated $E_{\rm cl}$

- ⋆ about $m = 1, \ldots, 500$ configurations for which $E_{\rm vac}$ and ϵ_i have been calculated
- ⋆ prescribe Q and compute

$$E_{\rm tot}^{(m)}(Q) = E_{\rm cl}^{(m)} + N_C \left[E_{\rm vac}^{(m)} + E_{\rm bind}^{(m)}(Q) \right]$$

- ⋆ find minimum by scanning through m

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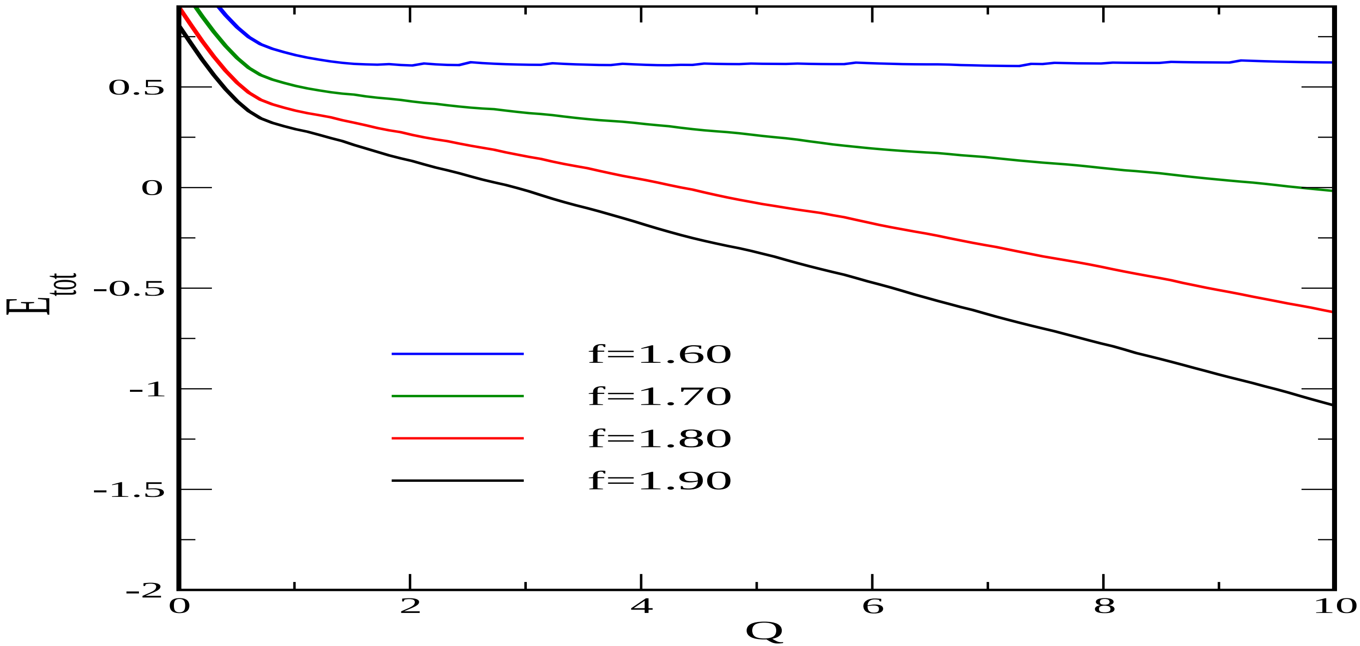
- ⋆ but: $E_{\rm cl}^{(m)}$ decreases rapidly with increasing f



- ⋆ stable configurations for $f \gtrsim 1.7$
- ⋆ safe against Landau ghost problems (widths parameters large)
- ⋆ stable strings should emerge if a strongly interacting fermion doublet with twice the top quark mass existed
- ⋆ stable configurations have $\xi_1 \approx 0$ (since we keep g small)

⋆ total mass

- small to moderate binding: $M \approx LQm$
- typical charge (density) for binding: $Q \approx 5m$
- typical fermion mass at binding: $m \approx 300\mathrm{GeV}$
- cosmological scale: $L \approx R_{\odot}$

$$M \approx 10^{-20} M_{\odot}$$

- too light to have cosmological impact

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  further support for phase shift method

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⋆ future work:

- boson loops
- currents along the string
- closed strings