

# Phase diagram and thermodynamics for asymmetric planar fermionic systems in the presence of external fields

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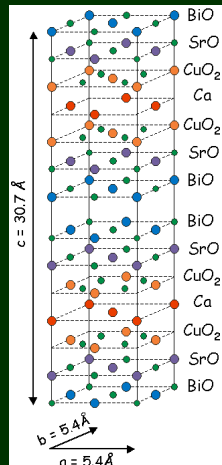
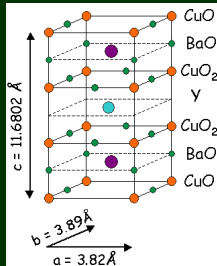
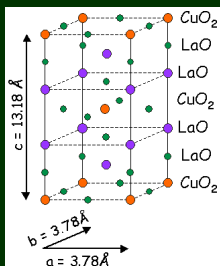
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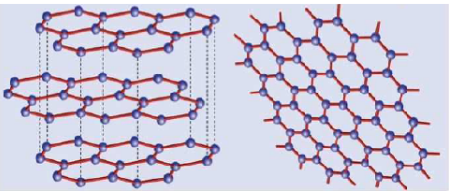
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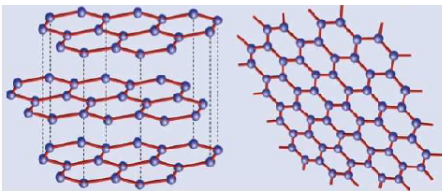
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# Planar Systems in Physics

Many condensate matter systems can be approximated as two-dimensional, like metal and conducting organic (polyacetylene) films, graphene and high-T superconductors (  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_2$ , YBCO, BSCCO ):







It is possible to describe these planar systems in condensed-matter physics through quantum field theory models and techniques for fermions in  $2 + 1$  dimensions [Nambu-Jona-Lasinio (NJL) or Gross-Neveu (GN) type of models are simple examples]

# Describing interacting two-dimensional fermionic systems in quantum field theory

$$\mathcal{L}[\bar{\Psi}, \Psi] = \sum_{s=\uparrow, \downarrow} \bar{\Psi}_s (i\hbar\partial_t - i\hbar v_F \vec{\gamma} \cdot \vec{\nabla}) \Psi_s + \sum_{s=\uparrow, \downarrow} \frac{\lambda}{2N} \hbar v_F (\bar{\Psi}_s \Psi_s)^2$$

Properties:

- describes self-interacting fermions with N flavors
- It is asymptotically free
- Exactly soluble model (mean-field)
- In 2+1d it is renormalizable in the 1/N expansion
- Mass terms (which violate chiral symmetry explicitly) can be included as well without loss of solvability (at large N)
- guide to thermodynamics of chiral symmetry restoration in QCD<sup>1</sup>

<sup>1</sup>Kneur, Pinto and ROR, PRD74, 1252020 (2006), Kneur, Pinto, ROR, Staudt, PRD76, 045020 (2007), Kneur, Pinto and ROR, PRC81, 065205 (2010)

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- At finite  $T$  only version of the model (in  $2+1d$ , massless) with discrete chiral symmetry undergoes  $PT$  (no continuous  $PT$  in 2 space dim)
- Both chiral and superconducting gaps can be implemented
- Useful model to describe low energy (condensed matter) systems as well<sup>2</sup> (continuum version of the Su-Shrieffer-Heeger model for polyacetylene in  $1d$ )

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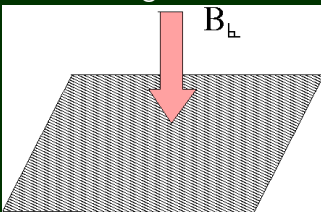
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# Applying an external magnetic field $\vec{B}$

The application of an external magnetic field,  $\vec{A} = (0, x B_{\perp}, 0)$ :

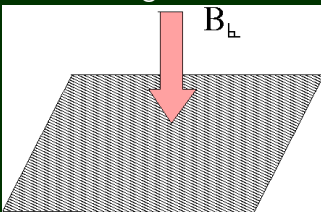


- The perpendicular B field couples to the fermions orbital motion – Landau levels for fermions in a magnetic field;
- Most well known case, studied by many authors<sup>3</sup>
- Chiral symmetry breaking for both positive coupling and negative coupling cases (for the positive coupling case, chiral symmetry breaking for any finite B field – magnetic catalysis);
- B field tends to strengthen the symmetry broken phase, e.g. higher critical temperature in the presence of perpendicular B.

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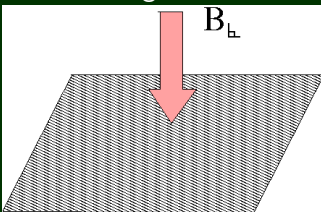


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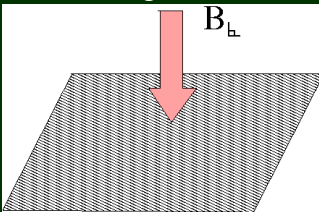


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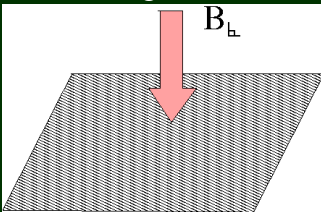


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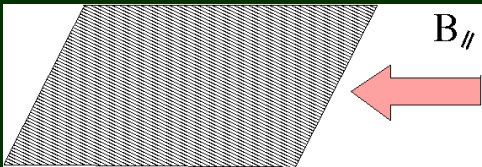


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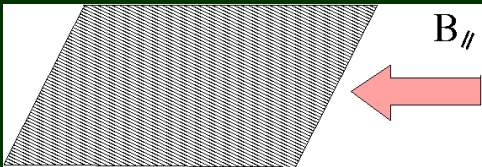


- The parallel (in-plane) B field couples only to the spins of fermions – intrinsic Zeeman effect;
- The least studied case in field theory<sup>4</sup>, but common in condensed matter systems;
- Chiral symmetry breaking only for attractive interaction;
- parallel B field tends to weaken the symmetry broken phase (e.g. smaller critical temperature in the presence of in-plane B).

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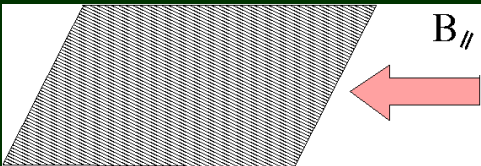
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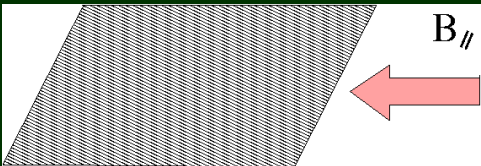


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# Effective Potential

Grand canonical partition function:

$$Z = \int D\Delta \prod_s D\psi^\dagger D\psi \exp \{ -S_E[\bar{\psi}, \psi, \Delta] \},$$

$$S_E[\bar{\psi}, \psi, \Delta] = \int_0^{\hbar\beta} d\tau \int d^2x \left\{ \sum_{s=\uparrow, \downarrow} \bar{\psi}^s [\hbar\partial_\tau + i\hbar v_F \gamma_1 \left( \partial_x + i\frac{e}{c} A_x \right) + i\hbar v_F \gamma_2 \left( \partial_y + i\frac{e}{c} A_y \right) + \Delta + \gamma_0 \mu + \frac{\sigma_s}{2} \gamma_0 g_{\text{Lande}} \mu_{\text{Bohr}} B_{\parallel}] \psi^s + \frac{N}{2\hbar v_F \lambda} \Delta^2 \right\},$$

$$A_x = 0, A_y = x B_{\perp}$$

# In-plane magnetic field $B_{\parallel} \neq 0$

The Zeeman energy term is like an effective chemical potential:

$$\begin{aligned} \sum_{s=\uparrow,\downarrow} \mu_s \bar{\psi}^s \gamma_0 \psi^s &= \sum_{s=\uparrow,\downarrow} \left( \mu + \frac{\sigma_s}{2} g_{\text{Lande}} \mu_{\text{Bohr}} B_{\parallel} \right) \bar{\psi}^s \gamma_0 \psi^s \\ &= \mu_{\uparrow} \psi^{\uparrow\dagger} \psi^{\uparrow} + \mu_{\downarrow} \psi^{\downarrow\dagger} \psi^{\downarrow}, \end{aligned}$$

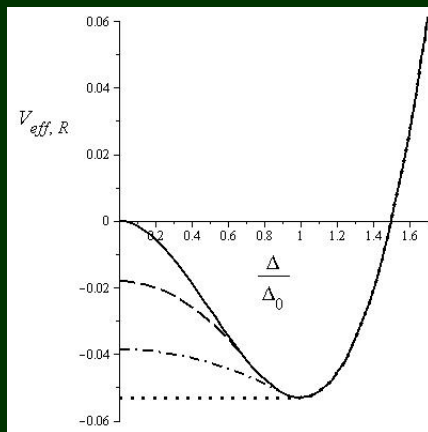
$\mu_{\uparrow} = \mu + \delta\mu$ , and  $\mu_{\downarrow} = \mu - \delta\mu$ , with  $\delta\mu = g_{\text{Lande}} \mu_{\text{Bohr}} B_{\parallel} / 2$ .

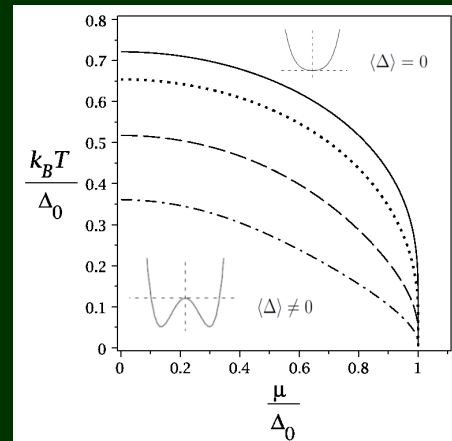
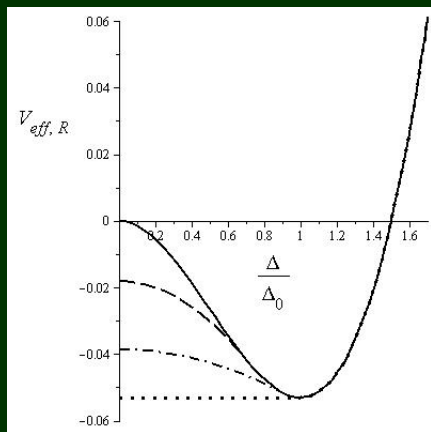
$\mu$  can be interpreted as to account for the extra density of electrons that is supplied to the system by the dopants, while  $\delta\mu$  measures the amount of asymmetry introduced and it is directly proportional to the in-plane applied external magnetic field.

# The effective potential for $B_{\parallel} \neq 0$ , $B_{\perp} = 0$

$$\begin{aligned} \frac{1}{N} V_{\text{eff,R}}(\Delta_c, T, \mu_{\uparrow}, \mu_{\downarrow}) &= \frac{1}{2\hbar v_F \lambda_R} \Delta_c^2 + \frac{1}{\pi(\hbar v_F)^2} \left( \frac{|\Delta_c|^3}{3} - \Delta_0 \Delta_c^2 \right) \\ &+ \frac{|\Delta_c|}{2\pi\beta^2} \left\{ \text{Li}_2[-e^{-\beta(\Delta_c - \mu_{\uparrow})}] + \text{Li}_2[-e^{-\beta(\Delta_c + \mu_{\uparrow})}] \right\} \\ &+ \frac{1}{2\pi\beta^3} \left\{ \text{Li}_3[-e^{-\beta(\Delta_c - \mu_{\uparrow})}] + \text{Li}_3[-e^{-\beta(\Delta_c + \mu_{\uparrow})}] \right\} + (\mu_{\uparrow} \rightarrow |\mu_{\downarrow}|) \end{aligned}$$

$$\Delta_0 = \frac{\hbar v_F \pi}{\lambda_R}, \quad \frac{1}{\lambda_R} = \frac{\hbar v_F}{N} \left. \frac{d^2 V_{\text{eff}}(\Delta_c)}{d\Delta_c^2} \right|_{\Delta_c = \Delta_0}$$



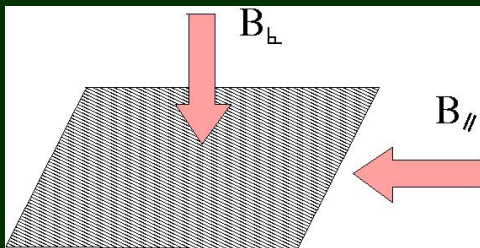


critical chemical potential  $\mu_c = \Delta_0$ . For  $\delta\mu \geq \mu_c$ ,  $\langle \Delta \rangle = 0$ .  
 Magnetic properties for the spin asymmetric system studied in  
 Caldas and ROR, PRB80, 115428 (2009)



$$B_{\parallel} \neq 0 \text{ and } B_{\perp} \neq 0$$

With  $B_{\parallel} \neq 0$  and  $B_{\perp} \neq 0$ ,  $\vec{A} = (0, xB_{\perp}, yB_{\parallel})$ ,



Dispersion energy (in two space dimensions) for fermions with  $\mathbf{p}^2 \rightarrow (2k + 1 - s)eB_{\perp}$ ,  $s = \pm 1$ ,  $k = 0, 1, 2, \dots$  (Landau levels),

$$\int \frac{d^3 q_E}{(2\pi)^3} \rightarrow \frac{eB_{\perp}}{2\pi} \frac{1}{\beta} \sum_n \sum_k$$

$n = 0, \pm 1, \pm 2, \dots$ , sum over Matsubara's frequencies for fermions,  
 $\omega_n = (2n + 1)\pi/\beta$ .

# The effective potential for $B_{\parallel} \neq 0$ , $B_{\perp} \neq 0$

$$\begin{aligned}
 \frac{1}{N} V_{\text{eff,R}}(\beta, \mu_{\uparrow}, \mu_{\downarrow}, \Delta_c) &= \frac{\Delta_c^2}{2\lambda_R} + \Delta_c^2 \frac{|eB_{\perp}|^{1/2}}{2\sqrt{2}\pi} \zeta\left(\frac{1}{2}, \frac{\Delta_0^2}{2|eB_{\perp}|}\right) \\
 &- \frac{\Delta_0^2 \Delta_c^2}{4\sqrt{2}\pi |eB_{\perp}|^{1/2}} \zeta\left(\frac{3}{2}, \frac{\Delta_0^2}{2|eB_{\perp}|}\right) - \frac{\sqrt{2}|eB_{\perp}|^{3/2}}{\pi} \zeta\left(-\frac{1}{2}, \frac{\Delta_0^2}{2|eB_{\perp}|} + 1\right) \\
 &- \frac{|eB_{\perp}|}{4\pi\beta} \left\{ \ln\left(1 + e^{-\beta(\Delta_c - \mu_{\uparrow})}\right) + \ln\left(1 + e^{-\beta(\Delta_c - |\mu_{\downarrow}|)}\right) \right. \\
 &+ 2 \sum_{k=1}^{\infty} \ln\left(1 + e^{-\beta(\sqrt{\Delta_c^2 + 2k|eB_{\perp}|} - \mu_{\uparrow})}\right) \\
 &+ 2 \sum_{k=1}^{\infty} \ln\left(1 + e^{-\beta(\sqrt{\Delta_c^2 + 2k|eB_{\perp}|} - |\mu_{\downarrow}|)}\right) \\
 &\left. + (\mu_{\uparrow} \rightarrow -\mu_{\uparrow}, |\mu_{\downarrow}| \rightarrow -|\mu_{\downarrow}|) \right\}
 \end{aligned}$$

$\mu_{\uparrow} = \mu + \delta\mu$ , and  $\mu_{\downarrow} = \mu - \delta\mu$ , with  $\delta\mu = g_{\text{Lande}} \mu_{\text{Bohr}} B_{\parallel} / 2$ .

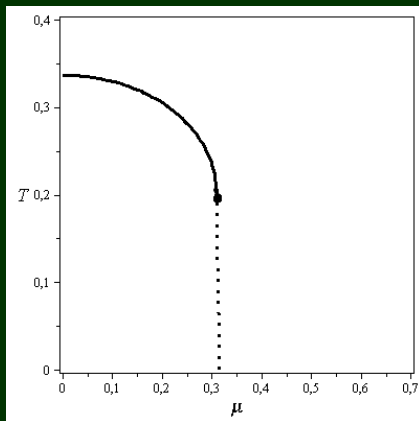


Figure: Phase diagram for fixed  $eB_{\perp} = 0.5\Delta_0^2$ . Black:  $\delta\mu = 0.0$ ; blue:  $\delta\mu = 0.3\Delta_0$ ; and green:  $\delta\mu = 0.5\Delta_0$ .

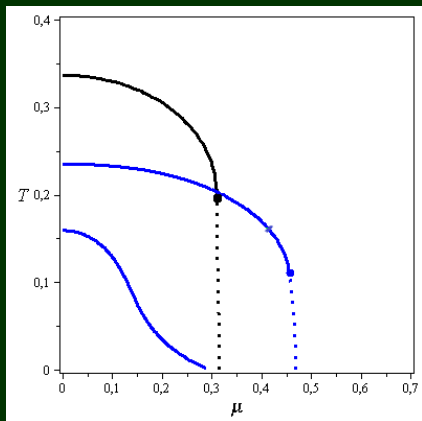


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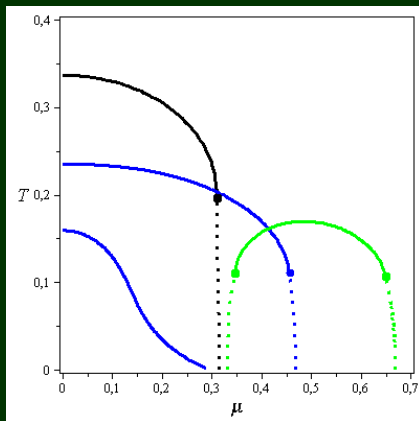
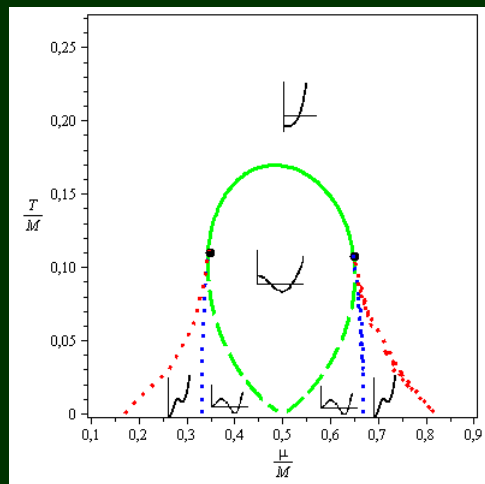
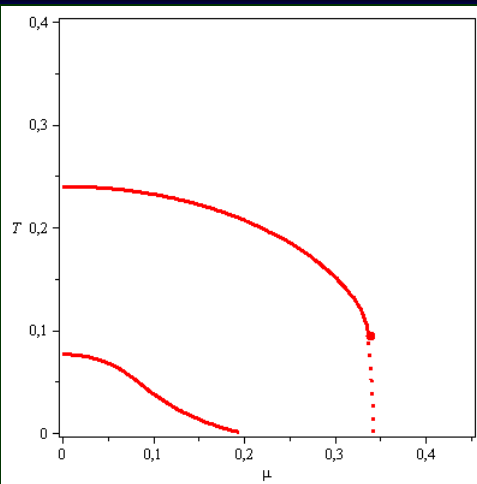
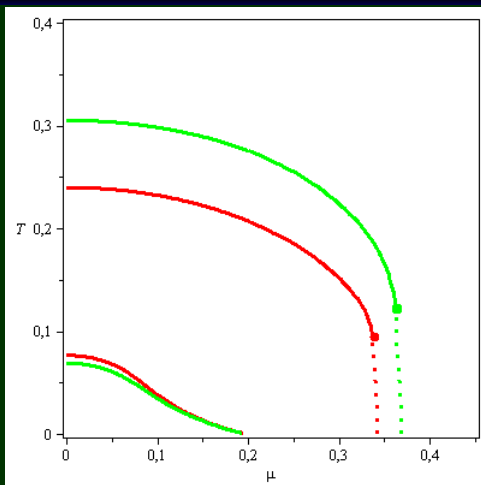


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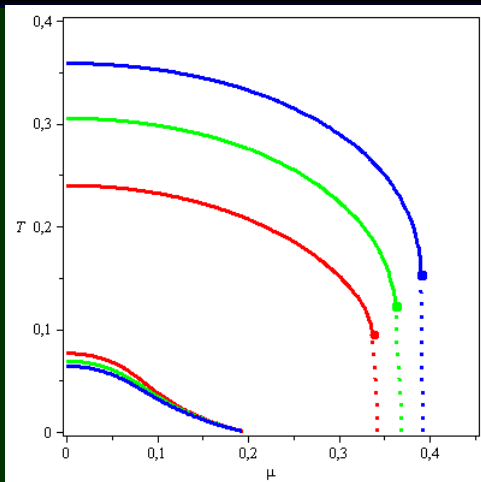


$\delta\mu = 0.3\Delta_0$ ; red:  $eB_{\perp} = 0.5\Delta_0^2$ ;

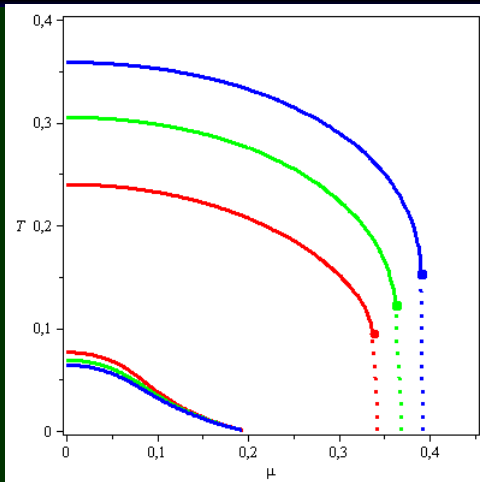


$\delta\mu = 0.3\Delta_0$ ; red:  $eB_{\perp} = 0.5\Delta_0^2$ ; green:  $eB_{\perp} = 1.0\Delta_0^2$ ;





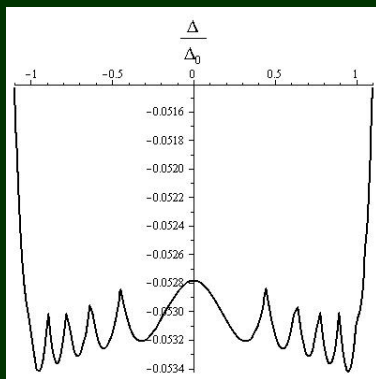
$\delta\mu = 0.3\Delta_0$ ; red:  $eB_{\perp} = 0.5\Delta_0^2$ ; green:  $eB_{\perp} = 1.0\Delta_0^2$ ; and blue:  $eB_{\perp} = 1.5\Delta_0^2$



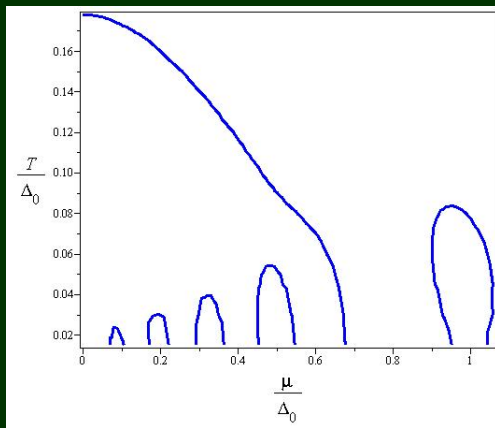
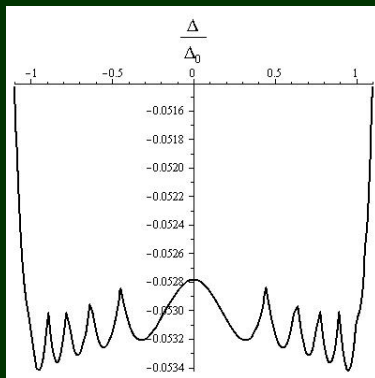
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$B_{\perp}$  enlarges the symmetry broken phase (generic even for charged scalar field systems, Duarte, Farias, ROR, arXiv:1108.4428)

For the critical asymmetry case,  $\delta\mu = \mu_c = \Delta_0$ ,  $eB_{\perp} = 0.1\Delta_0^2$ :



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precise control of the gap in semiconductors devices:  
⇒ use oblique magnetic fields at given doping (chemical potential) and temperature !