

Spontaneous electromagnetic superconductivity **of vacuum** in (very) strong magnetic field

M. N. Chernodub

CNRS, University of Tours, France

Based on:

Phys. Rev. D 82, 085011 (2010) [arXiv:1008.1055]

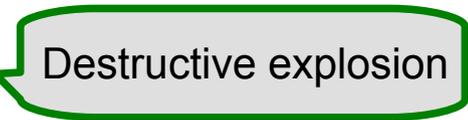
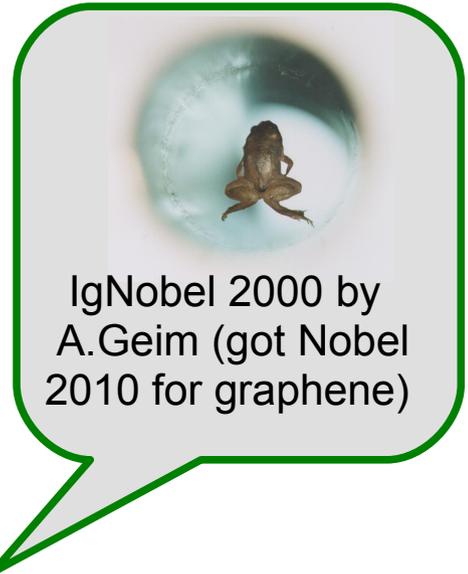
Phys. Rev. Lett. 106, 142003 (2011) [arXiv:1101.0117]

+ arXiv:1104.3767 and arXiv:1104.4404

Published on April 8, 2011 (symbolic?...)

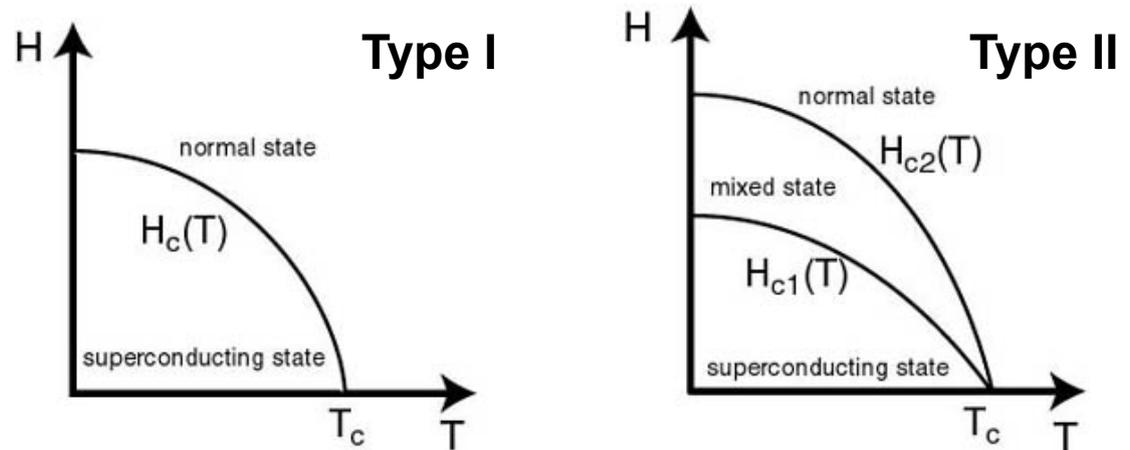
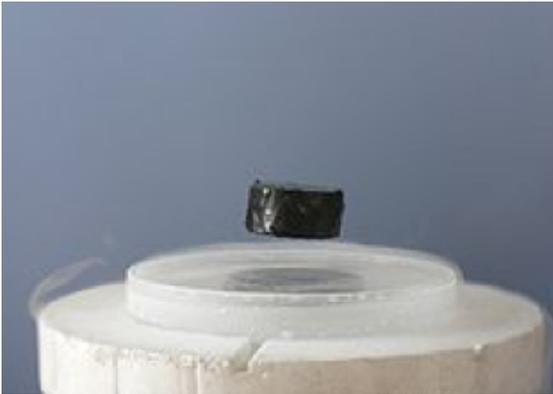
What is «very strong» field? Typical values:

- Thinking — human brain: 10^{-12} Tesla
- Earth's magnetic field: 10^{-5} Tesla
- Refrigerator magnet: 10^{-3} Tesla
- Loudspeaker magnet: 1 Tesla
- Levitating frogs: 10 Tesla
- Strongest field in Lab: 10^3 Tesla
- Typical neutron star: 10^6 Tesla
- Magnetar: 10^9 Tesla
- Heavy-ion collisions: 10^{15} Tesla (and higher)
- Early Universe: even (much) higher



Superconductivity

Discovered by Kamerlingh Onnes at the Leiden University
100 years ago, at 4:00 p.m. April 8, 1911 (Saturday).



- I. Any superconductor has zero electrical DC resistance
- II. Any superconductor is an enemy of the magnetic field:
 - 1) weak magnetic fields are expelled by all superconductors (the Meissner effect)
 - 2) strong enough magnetic field always kills superconductivity

Our claim:

In a background of strong enough magnetic field the vacuum becomes a superconductor.

The superconductivity emerges in empty space. Literally, “nothing becomes a superconductor”.

Some features of the superconducting state of vacuum:

1. spontaneously emerges above the critical magnetic field

$$B_c \approx 10^{16} \text{ Tesla} = 10^{20} \text{ Gauss} \leftarrow \text{can be reached in experiments!}$$

or

$$eB_c \approx m_\rho^2 \approx 31 m_\pi^2 \approx 0.6 \text{ GeV}^2$$

2. conventional Meissner effect does not exist

The claim seemingly contradicts textbooks which state that:

1. Superconductor is a material (= a form of matter, not an empty space)
2. Weak magnetic fields are suppressed by superconductivity
3. Strong magnetic fields destroy superconductivity

1+4 approaches to the problem:

0. General arguments; [\(this talk\)](#)
1. Effective bosonic model for electrodynamics of ρ mesons based on vector meson dominance
[M.Ch., PRD 2010; arXiv:1008.1055] [\(this talk\)](#)
2. Effective fermionic model (the Nambu-Jona-Lasinio model)
[M.Ch., PRL 2011; arXiv:1101.0117] ~~[\(this talk\)](#)~~
3. Nonperturbative effective models based on gauge/gravity duality (utilizing AdS/CFT duality)
[Callebaut, Dudal, Verschelde (Gent U., Belgium), arXiv:1105.2217];
[Erdmenger, Kerner, Strydom (Munich, Germany), arXiv:1106.4551]
~~[\(this talk\)](#)~~
5. First-principle numerical simulation of vacuum
[ITEP Lattice Group, Moscow, Russia, arXiv:1104.3767] [\(this talk\)](#)

Key players: ρ mesons and vacuum

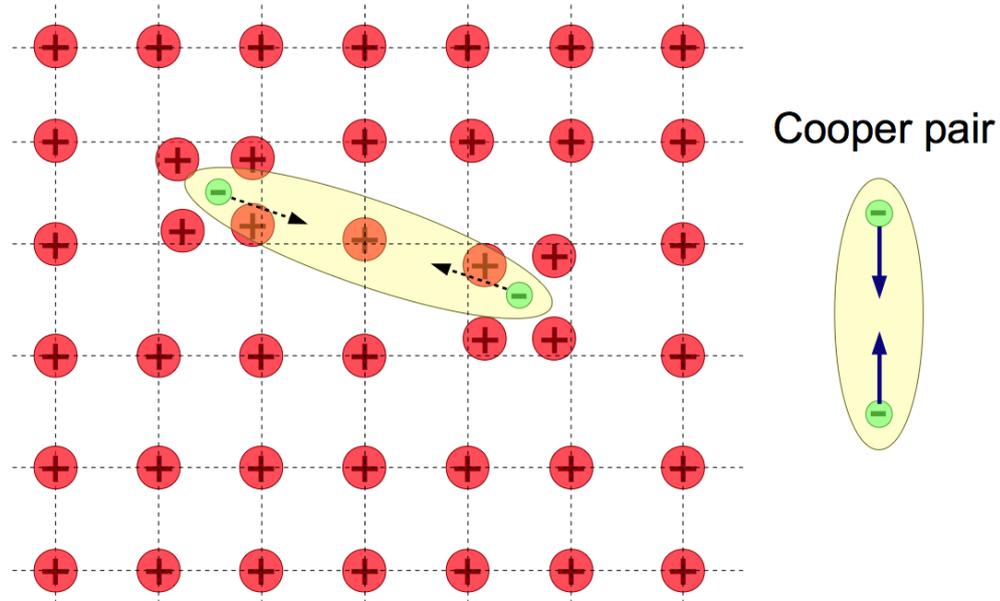
- ρ mesons:

- electrically charged ($q = \pm e$) and neutral ($q = 0$) particles
- spin: $s=1$, vector particles
- quark contents: $\rho^+ = u\bar{d}$, $\rho^- = d\bar{u}$, $\rho^0 = (u\bar{u} - d\bar{d})/2^{1/2}$
- mass: $m_\rho = 775.5$ MeV (approximately 1550 electron masses)
- lifetime: $\tau_\rho = 1.35$ fm/ c (very short: size of the ρ meson is 0.5 fm)

- vacuum: QED+QCD, zero temperature and density

Conventional BCS superconductivity

- 1) The Cooper pair is the relevant degree of freedom!
- 2) The electrons are bounded into the Cooper pairs by the (attractive) phonon exchange.



Three basic ingredients:

- A) the presence of carriers of electric charge (of electric current);
- B) the reduction of physics from (3+1) to (1+1) dimensions;
- C) the attractive interaction between the like-charged particles.

Real vacuum, no magnetic field

1) Boiling soup of everything.

Virtual particles and antiparticles (electrons, positrons, photons, gluons, quarks, antiquarks ...) are created and annihilated every moment.

2) Net electric charge is zero.

An insulator, obviously.

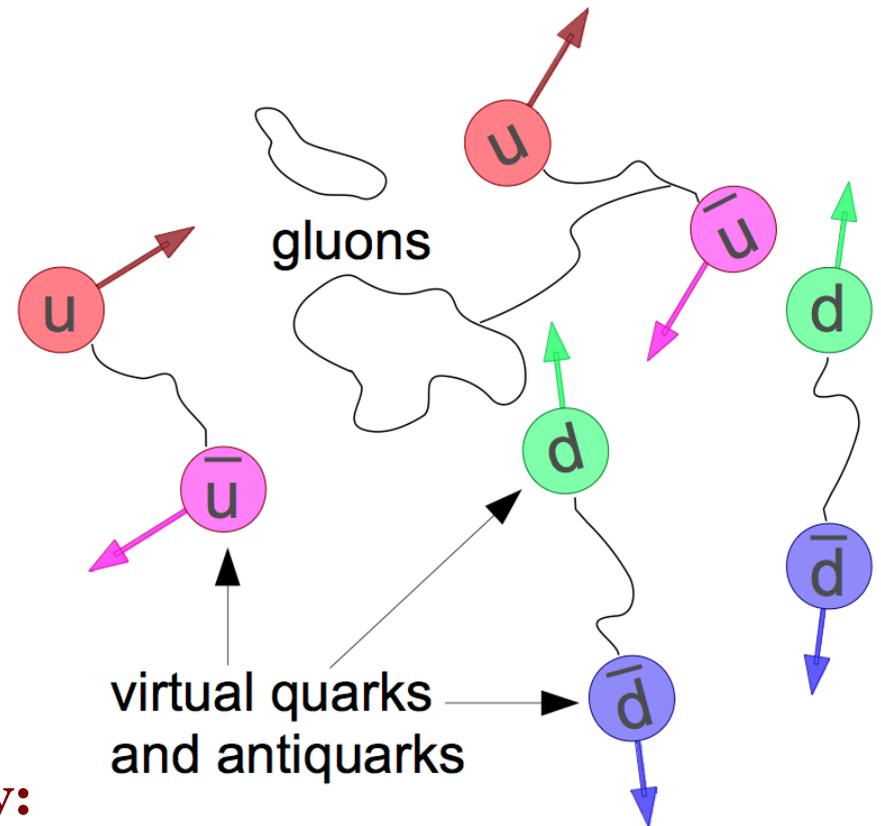
3) We are interested in “strongly interacting” sector of the theory:

a) quarks and antiquarks,

i) u quark has electric charge $q_u = +2 e/3$

ii) d quark has electric charge $q_d = - e/3$

b) gluons (an analogue of photons, no electric charge) “glue” quarks into bound states, “hadrons” (neutrons, protons, etc).



The vacuum in strong magnetic field

Ingredients needed for possible superconductivity:

A. **Presence of electric charges?**

Yes, we have them: there are virtual particles which may potentially become “real” (= pop up from the vacuum) and make the vacuum (super)conducting.

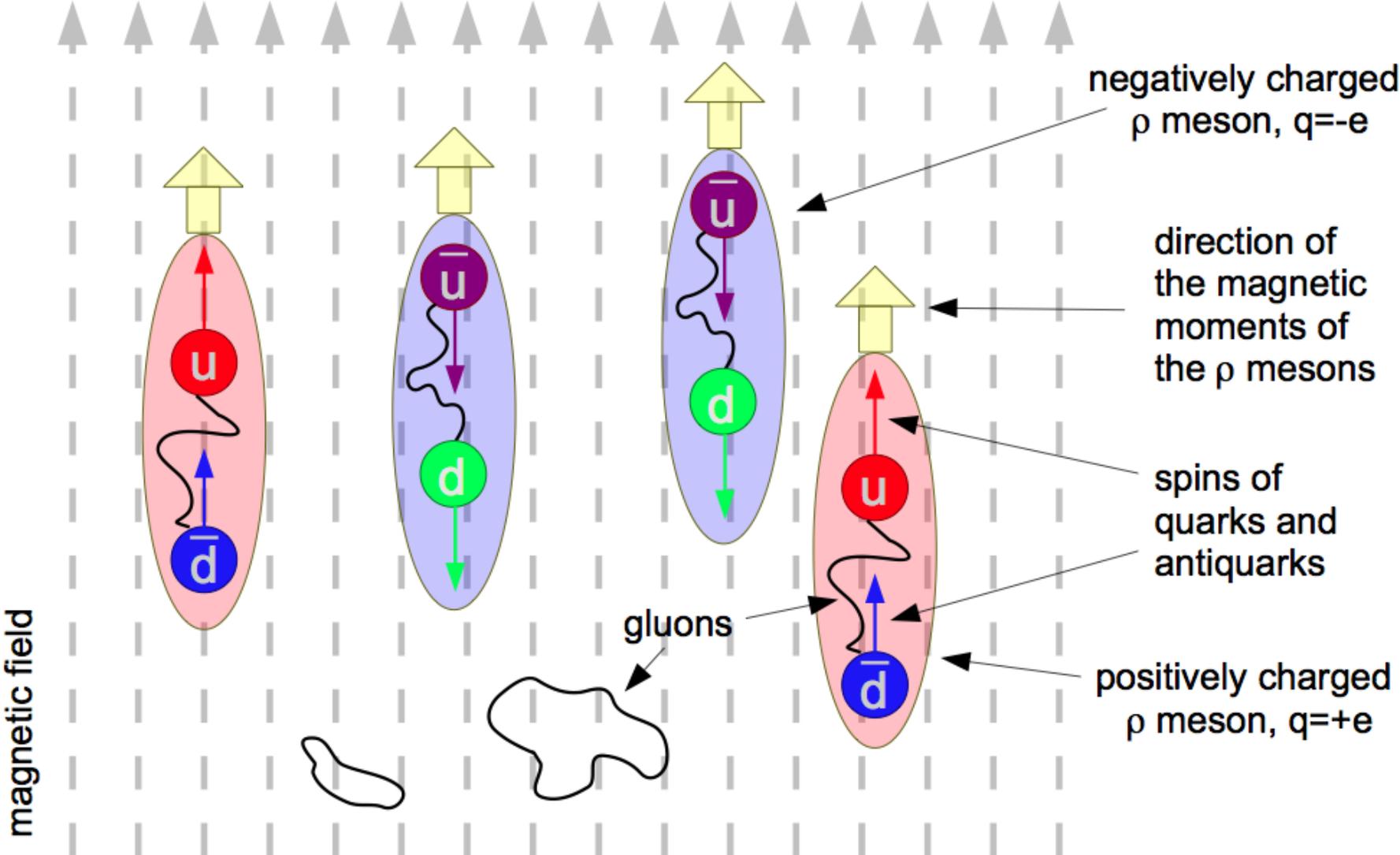
B. **Reduction to 1+1 dimensions?**

Yes, we have this phenomenon: in a very strong magnetic field the dynamics of electrically charged particles (quarks, in our case) becomes effectively one-dimensional, because the particles tend to move along the magnetic field only.

C. **Attractive interaction between the like-charged particles?**

Yes, we have it: the gluons provide attractive interaction between the quarks and antiquarks ($q_u = +2 e/3$ and $q_{\bar{d}} = +e/3$)

Strong magnetic field, picture



Charged relativistic particles in magnetic field

- Energy of a relativistic particle in the external magnetic field B_{ext} :

$$\varepsilon_{n,s_z}^2(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\text{ext}} + m^2$$

\nearrow
 momentum along
 the magnetic field axis

 \nearrow
 nonnegative integer number

 \nwarrow
 projection of spin on
 the magnetic field axis

(the external magnetic field is directed along the z-axis)

- Masses of ρ mesons and pions in the external magnetic field

Scalar particle: $m_{\pi^\pm}^2(B_{\text{ext}}) = m_{\pi^\pm}^2 + eB_{\text{ext}}$ becomes heavier

Vector particle: $m_{\rho^\pm}^2(B_{\text{ext}}) = m_{\rho^\pm}^2 - eB_{\text{ext}}$ becomes lighter
 $\rho^\pm \rightarrow \pi^\pm \pi^0$

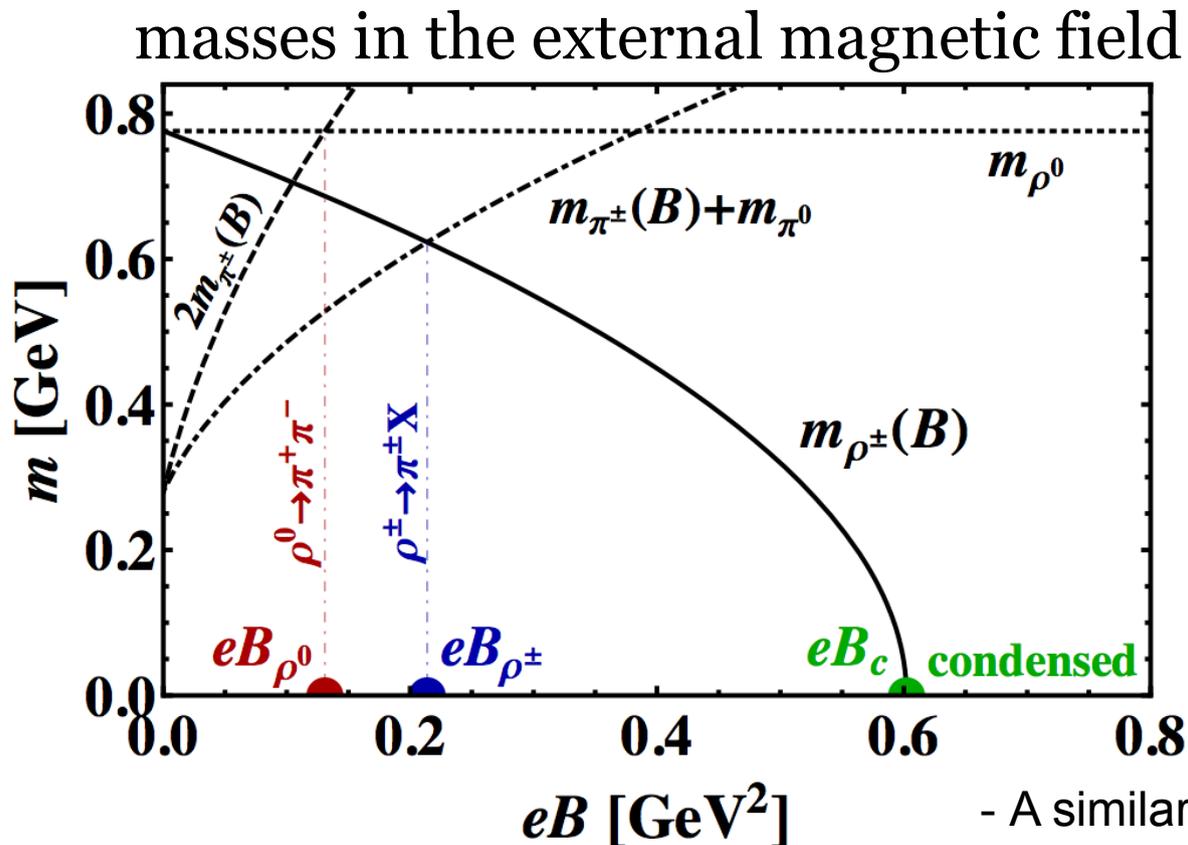
- Masses of ρ mesons and pions:

$$m_\pi = 139.6 \text{ MeV}, \quad m_\rho = 775.5 \text{ MeV}$$

Condensation of ρ mesons

The ρ^\pm mesons become massless and condense at the critical value of the external magnetic field

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16} \text{ Tesla}$$



Kinematical impossibility of dominant decay modes

The pion becomes heavier while the rho meson becomes lighter

- The decay $\rho^\pm \rightarrow \pi^\pm \pi^0$ stops at certain value of the magnetic field

$$m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}$$

- A similar statement is true for $\rho^0 \rightarrow \pi^+ \pi^-$

Electrodynamics of ρ mesons

- Lagrangian (based on vector dominance models):

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \rho_{\mu\nu}^\dagger \rho^{\mu\nu} + m_\rho^2 \rho_\mu^\dagger \rho^\mu - \frac{1}{4} \rho_{\mu\nu}^{(0)} \rho^{(0)\mu\nu} + \frac{m_\rho^2}{2} \rho_\mu^{(0)} \rho^{(0)\mu} + \frac{e}{2g_s} F^{\mu\nu} \rho_{\mu\nu}^{(0)}$$

Nonminimal coupling leads to $g=2$

- Tensor quantities

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu ,$$

$$f_{\mu\nu}^{(0)} = \partial_\mu \rho_\nu^{(0)} - \partial_\nu \rho_\mu^{(0)} ,$$

$$\rho_{\mu\nu}^{(0)} = f_{\mu\nu}^{(0)} - ig_s (\rho_\mu^\dagger \rho_\nu - \rho_\mu \rho_\nu^\dagger)$$

$$\rho_{\mu\nu} = D_\mu \rho_\nu - D_\nu \rho_\mu ,$$

- Gauge invariance

$$U(1) : \begin{cases} \rho_\mu^{(0)}(x) \rightarrow \rho_\mu^{(0)}(x) , \\ \rho_\mu(x) \rightarrow e^{i\omega(x)} \rho_\mu(x) , \\ A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \omega(x) \end{cases}$$

- Covariant derivative

$$D_\mu = \partial_\mu + ig_s \rho_\mu^{(0)} - ie A_\mu$$

- Kawarabayashi-Suzuki-Riadzuddin-Fayyazuddin relation

$$g_s \equiv g_{\rho\pi\pi} = \frac{m_\rho}{\sqrt{2}f_\pi} = 5.88$$

$$g_s \gg e \equiv \sqrt{4\pi\alpha_{\text{e.m.}}} \approx 0.303$$

Homogeneous approximation

- Energy density: $\epsilon \equiv T_{00} = \frac{1}{2}F_{0i}^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{2}(\rho_{0i}^{(0)})^2 + \frac{1}{4}(\rho_{ij}^{(0)})^2$
 $+ \frac{m_\rho^2}{2} [(\rho_0^{(0)})^2 + (\rho_i^{(0)})^2] + \rho_{0i}^\dagger \rho_{0i} + \frac{1}{2}\rho_{ij}^\dagger \rho_{ij}$
 $+ m_\rho^2(\rho_0^\dagger \rho_0 + \rho_i^\dagger \rho_i) - \frac{e}{g_s} F_{0i} \rho_{0i}^{(0)} - \frac{e}{2g_s} F_{ij} \rho_{ij}^{(0)}$

- Disregard kinetic terms (for a moment) and apply B_{ext} :

$$\begin{aligned} \epsilon_0^{(2)}(\rho_\mu) &= ieB_{\text{ext}}(\rho_1^\dagger \rho_2 - \rho_2^\dagger \rho_1) + m_\rho^2 \rho_\mu^\dagger \rho_\mu \\ &= \sum_{a,b=1}^2 \rho_a^\dagger \mathcal{M}_{ab} \rho_b + m_\rho^2(\rho_0^\dagger \rho_0 + \rho_3^\dagger \rho_3) \end{aligned}$$

mass matrix \nearrow

$$\mathcal{M} = \begin{pmatrix} m_\rho^2 & ieB_{\text{ext}} \\ -ieB_{\text{ext}} & m_\rho^2 \end{pmatrix}$$

$$\vec{B} = (0, 0, B)$$

- Eigenvalues and eigenvectors of the mass matrix:

$$\mu_\pm^2 = m_\rho^2 \pm eB_{\text{ext}}, \quad \rho_\pm = \frac{1}{\sqrt{2}}(\rho_1 \pm i\rho_2)$$

At the critical value of the magnetic field: imaginary mass (=condensation)!

Homogeneous approximation (II)

- The condensate of the rho mesons: $\rho_1 = -i\rho_2 = \rho$
- The energy of the condensed state:

$$\epsilon_0(\rho) = \frac{1}{2}B_{\text{ext}}^2 + 2(m_\rho^2 - eB_{\text{ext}}) |\rho|^2 + 2g_s^2 |\rho|^4$$

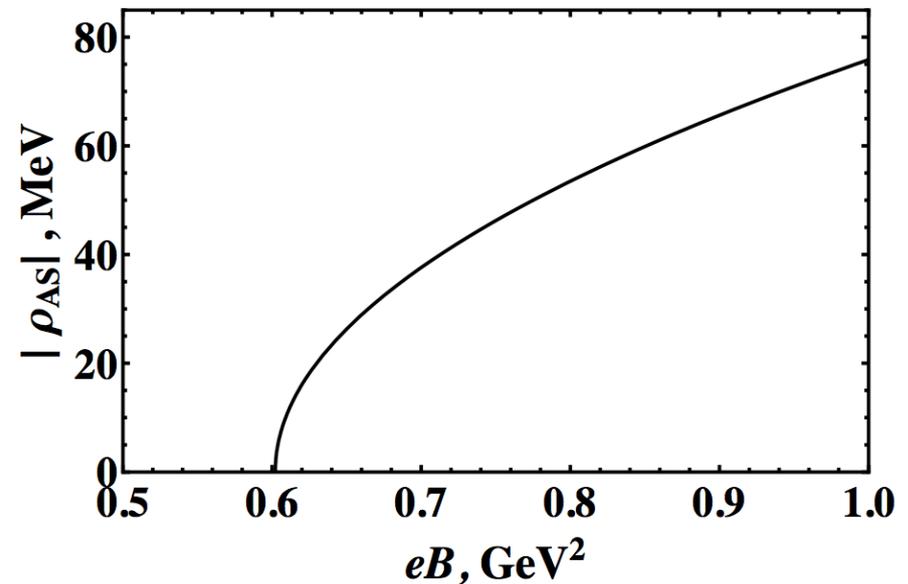
(basically, a Ginzburg-Landau potential for an s-wave superconductivity!)

(qualitatively the same picture in NJL)

- The amplitude of the condensate:

$$|\rho|_0 = \begin{cases} \sqrt{\frac{e(B_{\text{ext}} - B_c)}{2g_s^2}}, & B_{\text{ext}} \geq B_c \\ 0, & B_{\text{ext}} < B_c \end{cases}$$

Second order (quantum) phase transition, critical exponent = 1/2



Structure of the condensates

In terms of quarks, the state $\rho_1 = -i\rho_2 = \rho$ implies

$$\langle \bar{u}\gamma_1 d \rangle = \rho(x_\perp), \quad \langle \bar{u}\gamma_2 d \rangle = i\rho(x_\perp)$$

↑ Depend on transverse coordinates only

(the same results in different models,
for example, in Nambu-Jona-Lasinio)

$$\vec{B} = (0, 0, B)$$

$$U(1)_{\text{e.m.}} : \quad \rho(x) \rightarrow e^{i\omega(x)} \rho(x) \quad \text{Abelian gauge symmetry}$$

$$O(2)_{\text{rot}} : \quad \rho(x) \rightarrow e^{i\varphi} \rho(x) \quad \text{Rotations around B-axis}$$

- The condensate “locks” rotations around field axis and gauge transformations:

$$U(1)_{\text{e.m.}} \times O(2)_{\text{rot}} \rightarrow U(1)_{\text{locked}}$$

Basic features of ρ meson condensation, results

(now we are solving the full set of equations of motion)

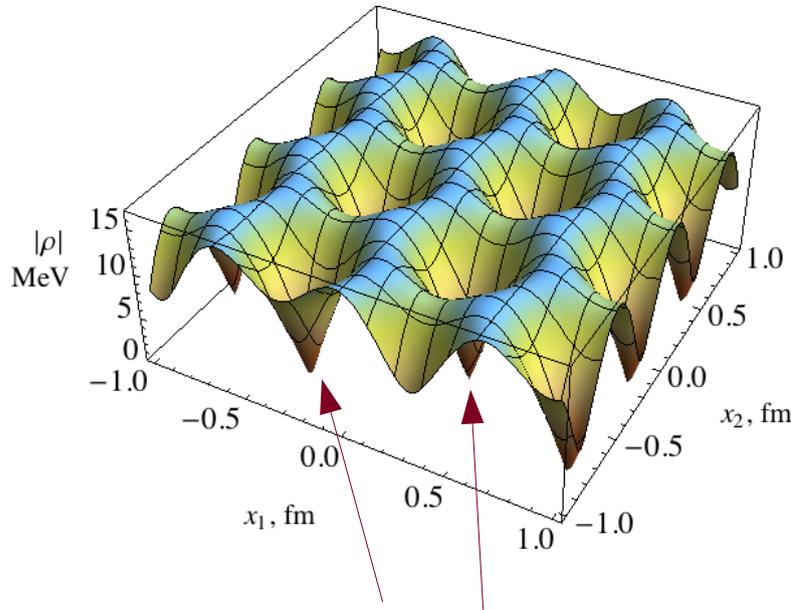
- The condensate of the ρ mesons appears in a form of an inhomogeneous state, analogous to the Abrikosov lattice in the mixed state of type-II superconductors.

A similar state, the vortex state of W bosons, may appear in Electroweak model in the strong external magnetic field [Ambjorn, Olesen (1989)]

- The condensate forms a lattice, which is made of the new type of topological defects, the ρ vortices.
- The emergence of the condensate of the charged ρ mesons induces spontaneous condensation of the neutral ρ mesons.
- The condensate of charged ρ mesons implies superconductivity.
- The condensate of neutral ρ mesons implies superfluidity.

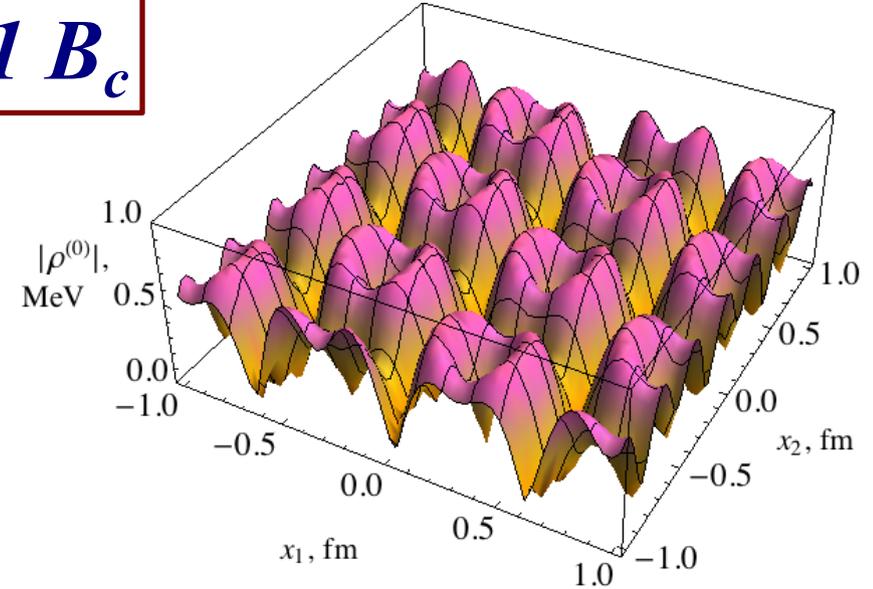
Solution for condensates of ρ mesons

Superconducting condensate
(charged rho mesons)



$$B = 1.01 B_c$$

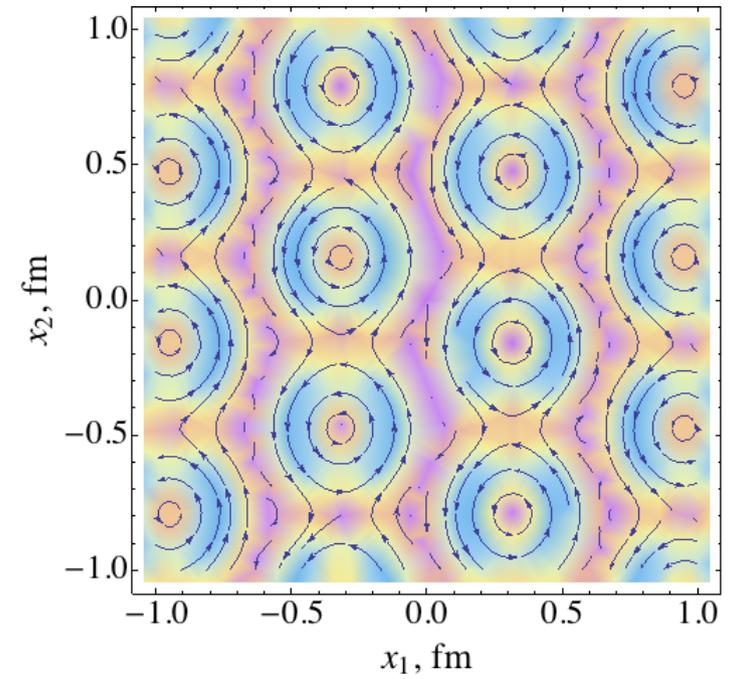
Superfluid condensate
(neutral rho mesons)



New objects, topological vortices,
made of the rho-condensates
(the phase of the rho-field winds around the
rho-vortex center, the rho-condensate vanishes)

Hexagonal or, equilateral
triangular **lattice**. Electric currents:

(similar results in the Nambu-Jona-Lasinio model)



Anisotropic superconductivity

(an analogue of the London equations)

- Apply a weak electric field \mathbf{E} to an ordinary superconductor (described, say, by the Ginzburg-Landau model).
- Then one gets accelerating electric current along the electric field:

$$\frac{\partial \vec{J}_{\text{GL}}}{\partial t} = m_A^2 \vec{E} \quad \text{[London equation]}$$

- In the QCD vacuum, we get an accelerating electric current iff the electric field \mathbf{E} is directed along the magnetic field \mathbf{B} :

$$\frac{\partial}{\partial t} \langle J_3 \rangle = -\frac{2e^3}{g_s^2} (B_{\text{ext}} - B_c) E_3 \quad (\text{for } B \geq B_c)$$

$$\frac{\partial}{\partial t} \langle J_1 \rangle = \frac{\partial}{\partial t} \langle J_2 \rangle = 0$$

Written for an electric current averaged over one elementary (unit) rho-vortex cell

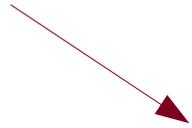
(similar results in NJL)

Anisotropic superconductivity

(Lorentz-covariant form of the London equations)

We are working in the vacuum, thus the transport equations may be rewritten in a Lorentz-covariant form:

Electric current
averaged over
one elementary
rho-vortex cell



$$\partial_{[\mu, j_{\nu]} = \kappa \frac{(F \cdot \tilde{F})}{(F \cdot F)} \tilde{F}_{\mu\nu}$$



A scalar function of Lorentz invariants.
In this particular model:

$$\kappa = (e^3 / g_s^2) (\sqrt{(F \cdot F) / 2} - B_c)$$

(slightly different form of κ function in NJL)

Lorentz invariants:

$$(F \cdot \tilde{F}) = F^{\mu\nu} \tilde{F}_{\mu\nu} \equiv 4(\vec{B} \cdot \vec{E})$$

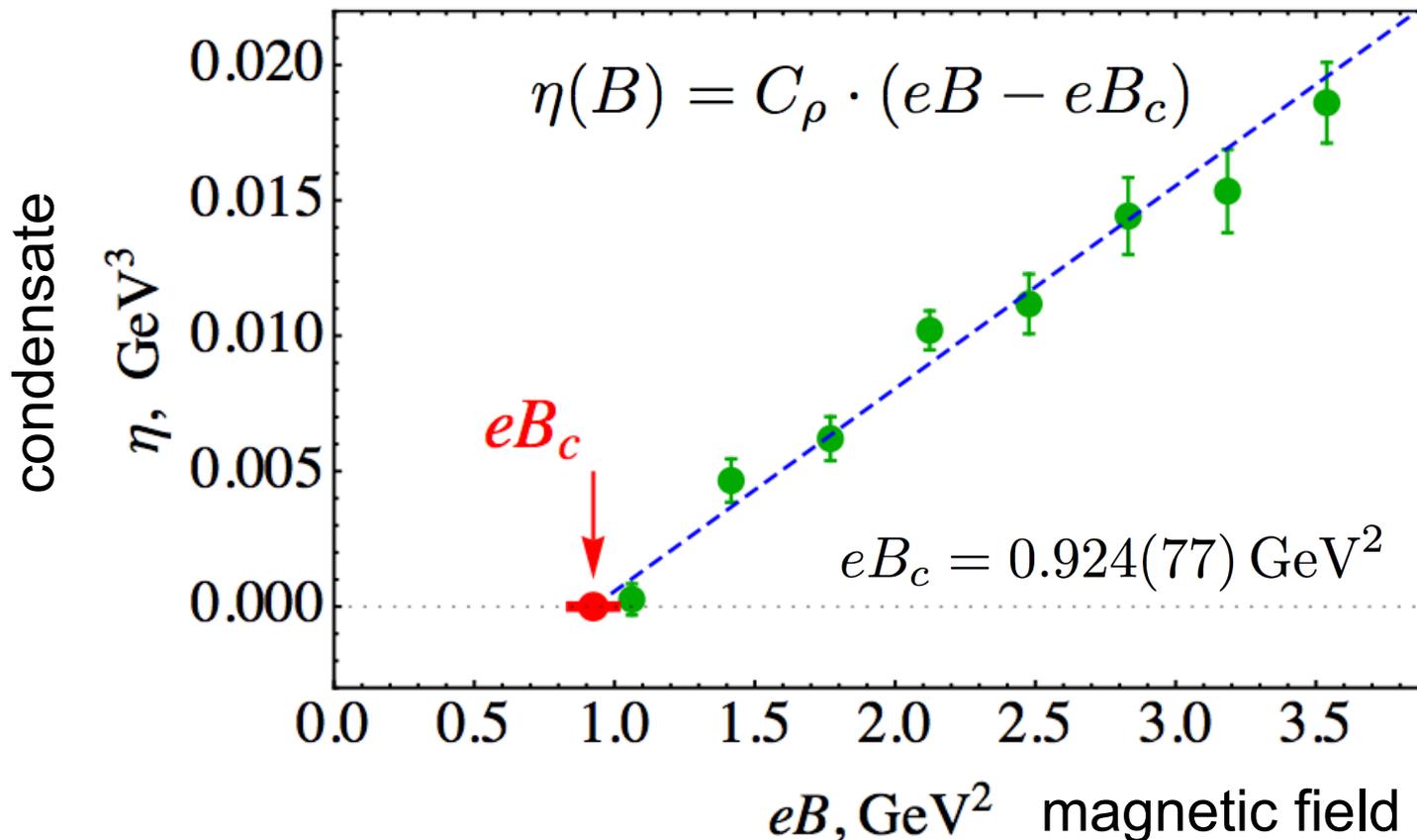
$$(F \cdot F) = F^{\mu\nu} F_{\mu\nu} \equiv 2(\vec{B}^2 - \vec{E}^2)$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$$

Numerical simulations of “quenched” vacuum in the magnetic field background

V.Braguta, P. Buividovich, M. Polikarpov, M.Ch., arXiv:1104.3767

Numerical simulation: $B_c = (1.56 \pm 0.13) \cdot 10^{16}$ Tesla



Theory:

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16} \text{ Tesla}$$
$$\eta \sim \sqrt{B - B_c}$$

for $B \geq B_c$

[qualitatively realistic vacuum, quantitative results may receive corrections (20%-50% typically)]

Superconducting metamaterial?

arXiv.org > physics > arXiv:1108.2203

Physics > Optics

Vacuum as a hyperbolic metamaterial

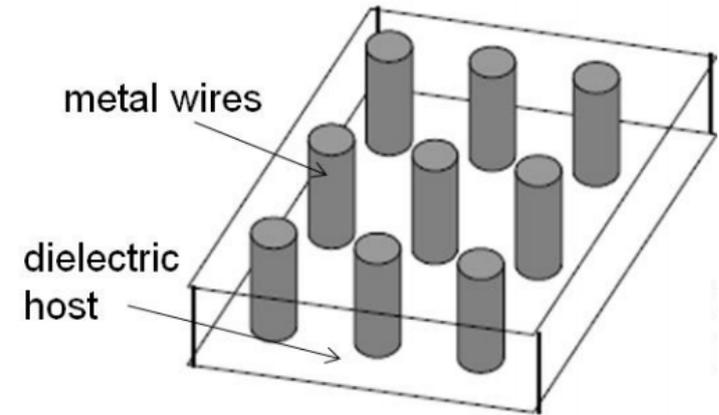
Igor I. Smolyaninov

(Submitted on 10 Aug 2011)

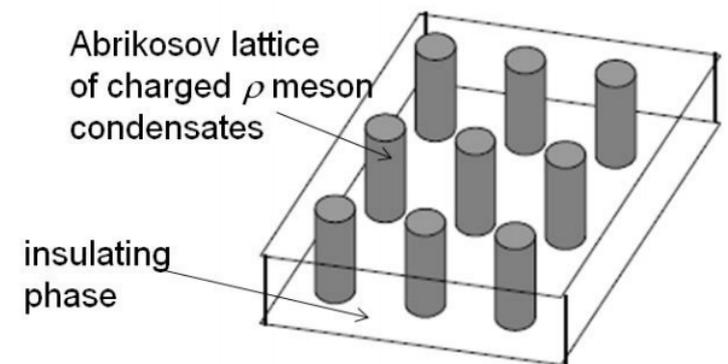
From Abstract:

... vacuum in a strong magnetic field behaves as a periodic Abrikosov vortex lattice in a type-II superconductor. ... Since superconductivity is realized along the axis of magnetic field only, strong anisotropy of the vacuum dielectric tensor is observed. ... As a result, vacuum behaves as a hyperbolic metamaterial medium. ... We also note that hyperbolic metamaterials behave as diffractionless "perfect lenses". ...

"wired" hyperbolic metamaterial



vacuum in strong magnetic field



Too strong magnetic field?

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16} \text{ Tesla}$$

Very strong magnetic fields (with a typical strength of the QCD scale) may be generated in heavy-ion collisions and in Early Universe (duration is short, however – further clarification is needed).

A bit of dreams (in deep verification stage):

Signatures of the superconducting state of the vacuum could possibly be found in ultra-periferal heavy-ion collisions at LHC.

[ultra-periferal: cold vacuum is exposed to strong magnetic field]

Conclusions

- In a sufficiently strong magnetic field condensates with ρ^\pm meson quantum numbers are formed spontaneously via a second order phase transition with the critical exponent $1/2$.
- The vacuum (= no matter present, = empty space, = nothing) becomes electromagnetically superconducting.
- The superfluidity of the neutral ρ^0 mesons emerges as well.
- The superconductivity is anisotropic: the vacuum behaves as a superconductor only along the axis of the magnetic field.
- New type of topological defects, " ρ vortices", emerge.
- The ρ vortices form Abrikosov-type lattice in transverse directions.
- The Meissner effect is absent.