Quantum corrections to gravity and implications for cosmology and astrophysics

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- Quantum corrections to vacuum action for massive fields.
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Effective Action of vacuum for QFEXT / gravity.

Independent on whether gravity should be should not be quantized, we know that the matter fields should.

Therefore, it is reasonable to ask whether the quantum effects of matter fields are capable to produce significant effects on the astrophysical or even cosmological scale.

At quantum level the dynamics of gravity is governed by the Effective Action of vacuum $\Gamma[g_{\mu\nu}]$.

$$e^{i\Gamma[g_{\mu\nu}]} = \int d\Phi e^{iS[\Phi,\, g_{\mu\nu}]}\,, \quad \Phi = \{\text{matter fields}\}\,.$$

In case of renormalizable theory

$$S[\Phi,\, g_{\mu\nu}] = S_{vac}[g_{\mu\nu}] + S_m[\Phi,\, g_{\mu\nu}] \quad \Longrightarrow \quad \Gamma[g_{\mu\nu}] = S_{vac}[g_{\mu\nu}] + \bar{\Gamma}[g_{\mu\nu}]\,.$$

In case of renormalizable theory

$$S_{vac} = S_{EH} + S_{HD}, \quad S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R+2\Lambda),$$

and S_{HD} includes higher derivative terms.

$$S_{HD} = \int d^4x\sqrt{-g}\,\left\{a_1 C^2 + a_2 E + a_3 \Box R + a_4 R^2\right\}.$$

Here

$$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3\,R^2$$

is the square of the Weyl tensor and

$$E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4\,R_{\alpha\beta}R^{\alpha\beta} + R^2$$

the integrand of the Gauss-Bonnet topological invariant.

The main problem is to evaluate $\bar{\Gamma}[g_{\mu\nu}]$, at least at 1-loop.

The case of massless conformal fields.

$\bar{\Gamma}[g_{\mu\nu}]$ **can be obtained, e.g., by integrating conformal anomaly.**
Riegert, Fradkin & Tseytlin, (1984).

$$\bar{\Gamma}_{ind} = S_c[g_{\mu\nu}] + \frac{\beta_1}{4}\int_x\int_y \left(E - \frac{2}{3}\Box R\right)_x G(x,y)\,\left(C^2\right)_y$$

$$-\frac{\beta_2}{8}\int_x\int_y \left(E - \frac{2}{3}\Box R\right)_x G(x,y)\,\left(E - \frac{2}{3}\Box R\right)_y + \frac{3\beta_3 - 2\beta_2}{6}\int_x R^2\,.$$

Here $\int_x = \int d^4x\sqrt{g}$ **and** $\Delta_x G(x,x') = \delta(x,x')\,.$

$$\Delta = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}\,R\Box + \frac{1}{3}\,(\nabla^\mu R)\nabla_\mu\,.$$

$S_c[g_{\mu\nu}]$ **is an arbitrary conformal functional. The** β**-functions depend on the number of fields,** $N_0,\ N_{1/2},\ N_1$,

$$\begin{pmatrix} \beta_1 \\ -\beta_2 \\ \beta_3 \end{pmatrix} = \frac{1}{360(4\pi)^2}\begin{pmatrix} 3N_0 + 18N_{1/2} + 36N_1 \\ N_0 + 11N_{1/2} + 62N_1 \\ 2N_0 + 12N_{1/2} - 36N_1 \end{pmatrix}$$

An important application: Starobinsky model.

Massive fields are more complicated and interesting.

And especially if we are interested in the low-energy effects, decause one has to account for the decoupling phenomenon.

High energy QED:

$$-\frac{e^2}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^4}{3(4\pi)^2}\,F_{\mu\nu}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,F^{\mu\nu}\,.$$

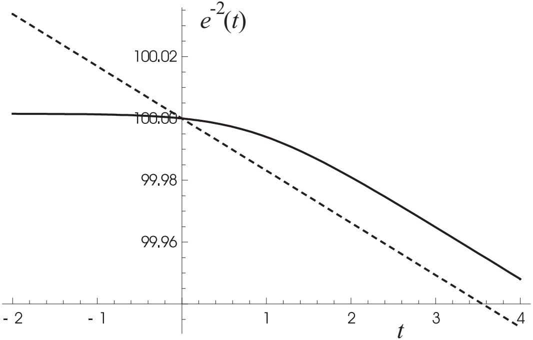
At high energy limit we meet a standard (MS) β-function and at low energies there is quadratic decoupling.

UV limit $\quad p^2 \gg m^2 \;\Longrightarrow\; \beta_e^{1\;UV} = \frac{4\,e^3}{3\,(4\pi)^2} + \mathcal{O}\Big(\frac{m^2}{p^2}\Big)\,.$

IR limit $\quad p^2 \ll m^2 \;\Longrightarrow\; \beta_e^{1\;IR} = \frac{e^3}{(4\pi)^2}\cdot\frac{4\,p^2}{15\,m^2} + \mathcal{O}\Big(\frac{p^4}{m^4}\Big)\,.$

Appelquist and Carazzone decoupling theorem (PRD, 1977).

General expression interpolates between UV and IR.



These plots show the effective electron charge as a function of $\log(\mu/\mu_0)$ in the case of the MS-scheme, and for the momentum-subtraction scheme, with $\ln(p/\mu_0)$.

An interesting high-energy effect is a small apparent shift of the initial value of the effective charge.

Similar results can be obtained for gravity.

E.g., for a massive scalar field (*Gorbar & I.Sh., JHEP, 2003*).

$$\beta_1 \,=\, -\,\frac{1}{(4\pi)^2}\,\left(\frac{1}{18a^2}\,-\,\frac{1}{180}\,-\,\frac{a^2-4}{6a^4}\,A\right)\,.$$

Then

$$\beta_1^{UV} = -\frac{1}{(4\pi)^2}\frac{1}{120} + \mathcal{O}\left(\frac{m^2}{p^2}\right) = \beta_1^{\overline{MS}} + \mathcal{O}\left(\frac{m^2}{p^2}\right),$$

$$\beta_1^{IR} \,=\, -\,\frac{1}{1680\,(4\pi)^2}\,\cdot\,\frac{p^2}{m^2}\,+\,\mathcal{O}\left(\frac{p^4}{m^4}\right)\,,$$

This is the Appelquist & Carazzone Theorem for gravity

However, in the momentum-subtraction scheme $\beta_G = \beta_\Lambda = 0\,.$

In the gravitational sector we meet Appelquist and Carazzone - like decoupling, but only in the higher derivative sectors.
In the perturbative approach, with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we do not see running for the cosmological and inverse Newton constants.

Why do we get $\beta_\Lambda = \beta_{1/G} = 0$ **?**

Momentum subtraction running corresponds to the insertion of, e.g., $\ln(\Box/\mu^2)$ formfactors into effective action.

Say, in QED: $$-\frac{e^2}{4} F_{\mu\nu} F^{\mu\nu} + \frac{e^4}{3(4\pi)^2} F_{\mu\nu} \ln\Big(-\frac{\Box}{\mu^2}\Big) F^{\mu\nu} \,.$$

Similarly, one can insert formfactors into

$$C_{\mu\nu\alpha\beta} \ln\Big(-\frac{\Box}{\mu^2}\Big) C_{\mu\nu\alpha\beta} \,.$$

However, such insertion is impossible for Λ and for $1/G$, because $\Box\Lambda \equiv 0$ and $\Box R$ is a full derivative.

Further discussion:
Ed. Gorbar & I.Sh., JHEP (2003); J. Solà & I.Sh., PLB (2010).

Is it true that physical $\beta_\Lambda = \beta_{1/G} = 0$ **?**

Probably not. Perhaps the linearized gravity approach is simply not an appropriate tool for the CC and Einstein terms.

Let us use the covariance arguments. The EA can not include odd terms in metric derivatives. In the cosmological setting this means no $\mathcal{O}(H)$ **and also no** $\mathcal{O}(H^3)$ **terms, etc. Hence**

$$\rho_\Lambda(H) = \frac{\Lambda(H)}{16\pi G(H)} = \rho_\Lambda(H_0) + C\left(H^2 - H_0^2\right).$$

Then the conservation law for $G(H;\nu)$ **gives**

$$G(H;\nu) = \frac{G_0}{1 + \nu\,\ln\left(H^2/H_0^2\right)}, \quad \textbf{where} \quad G(H_0) = G_0 = \frac{1}{M_P^2}.$$

Here we used the identification

$$\mu \sim H \qquad \textbf{in the cosmological setting.}$$

A small note on the Cosmological Constant (CC) Problem.

The main relation is $\Lambda_{obs} = \Lambda_{vac}(\mu_c) + \Lambda_{ind}(\mu_c)\,.$

Λ_{obs} **which is likely observed in SN-Ia, LSS, CMB etc is**

$$\Lambda_{obs}(\mu_c) \;\approx\; 0.7\,\rho_c^0 \;\propto\; 10^{-47}\,GeV^4\,.$$

The CC Problem is that the magnitudes of $\Lambda_{vac}(\mu_c)$ **and** $\Lambda_{ind}(\mu_c)$ **are a huge 55 orders of magnitude greater than the sum!**

Obviously, these two huge terms do cancel.
"Why they cancel so nicely" is the CC Problem (Weinberg, 1989).

We take a phenomenological point of view and don't try solving CC problems. Instead we consider whether CC may vary due to IR quantum effects, e.g., the ones of matter fields.

The same $\rho_\Lambda(\mu)$ immediately follows from the assumption of the Appelquist and Carazzone - like decoupling for CC.

A.Babic, B.Guberina, R.Horvat, H.Štefančić, PRD 65 (2002); I.Sh., J.Solà, JHEP 02 (2002).

We know that for a single particle

$$\beta_\Lambda^{MS}(m) \sim m^4 \,,$$

hence the quadratic decoupling gives

$$\beta_\Lambda^{IR}(m) = \frac{\mu^2}{m^2}\,\beta_\Lambda^{MS}(m) \sim \mu^2 m^2 \,.$$

The total beta-function will be given by algebraic sum

$$\beta_\Lambda^{IR} = \sum k_i \mu^2 m_i^2 = \sigma M^2 \mu^2 \propto \frac{3\nu}{8\pi} M_P^2 H^2 \,.$$

This leads to the same result in the cosmological setting,

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi} M_p^2 \left(H^2 - H_0^2\right) .$$

One can also obtain the same $G(\mu)$ in a different way.

I.Sh., J. Solà, JHEP (2002); C. Farina, I.Sh. et al, PRD (2011).

Consider $\overline{\rm MS}$-based renormalization group equation for $G(\mu)$:

$$\mu\frac{dG^{-1}}{d\mu} = \sum_{particles} A_{ij}\, m_i\, m_j = 2\nu\, M_P^2\,, \qquad G^{-1}(\mu_0) = G_0^{-1} = M_P^2\,.$$

Here the coefficients A_{ij} depend on the coupling constants, m_i are masses of all particles. In particular, at one loop,

$$\sum_{particles} A_{ij}\, m_i\, m_j = \sum_{fermions} \frac{m_f^2}{3(4\pi)^2} - \sum_{scalars} \frac{m_s^2}{(4\pi)^2}\Big(\xi_s - \frac{1}{6}\Big)\,.$$

One can rewrite it as

$$\mu\frac{d(G/G_0)}{d\mu} = -\,2\nu\,(G/G_0)^2\,, \quad \Longrightarrow \quad G(\mu) = \frac{G_0}{1+\nu\,\ln\big(\mu^2/\mu_0^2\big)}\,. \quad (*)$$

It is the same formula which results from covariance and/or from AC-like quadratic decoupling for the CC plus conservation law. All in all, (*) seems to a unique possible form of a relevant $G(\mu)$.

All in all, it is not a surprise that the eq.

$$G(\mu) = \frac{G_0}{1 + \nu \ln \left(\mu^2 / \mu_0^2 \right)} \, .$$

emerges in different approaches to renorm. group in gravity:

- **Higher derivative quantum gravity.**

A. Salam and J. Strathdee, PRD (1978);
E.S. Fradkin and A. Tseytlin, NPB (1982).

- **Non-perturbative quantum gravity with (hipothetic) UV-stable fixed point.**

A. Bonanno and M. Reuter, PRD (2002).

- **Semiclassical gravity.**

B.L. Nelson and P. Panangaden, PRD (1982).

So, we arrived at the two relations:

$$\rho_\Lambda(H) \,=\, \rho_\Lambda(H_0) \,+\, \frac{3\nu}{8\pi}\, M_p^2 \left(\mu^2 - \mu_0^2\right) \qquad (1)$$

and $$G(\mu) = \frac{G_0}{1+\nu\,\ln\left(\mu^2/\mu_0^2\right)}\,. \qquad (2)$$

Remember the standard identification

$\mu \,\sim\, H$ **in the cosmological setting.**

A. Babic, B. Guberina, R. Horvat, H. Štefančić, PRD (2005).

Cosmological models based on the assumption of the standard AC-like decoupling for the cosmological constant:

Models with (1) and energy matter-vacuum exchange:
I.Sh., J.Solà, Nucl.Phys. (PS), IRGA-2003;
I.Sh., J.Solà, C.España-Bonet, P.Ruiz-Lapuente, PLB (2003).

Models with (1), (2) and without matter-vacuum exchange:
I.Sh., J.Solà, H.Štefančić, JCAP (2005).

- **Models with constant $G \equiv G_0$ and permitted energy exchange between vacuum and matter sectors.**

For the equation of state $P = \alpha\rho$ the solution is analytical,

$$\rho(z;\nu) = \rho_0 \, (1+z)^r \,,$$

$$\rho^\Lambda(z;\nu) = \rho_0^\Lambda + \frac{\nu}{1-\nu} \, [\, \rho(z;\nu) - \rho_0 \,] \,,$$

and

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi} \, M_p^2 \left(H^2 - H_0^2\right) .$$

The limits from density perturbations / LSS data $|\nu| < 10^{-6}$.

J. Fabris, I.Sh., J. Solà, JCAP 0702 (2007).

Also analog models:
R. Opher, A. Pelinson, PRD 70 (2004);
P. Wang, X.H. Meng, Cl.Q.Gr. 22 (2005).

How to perturb a model with variable $\rho_\Lambda = \rho_\Lambda(H)$?

$$\rho_t = \rho_m + \rho_\Lambda\,, \qquad P_\Lambda = -\rho_\Lambda\,, \qquad P_m = 0\,.$$

Introduce U^α. In the co-moving coordinates $U^0 = 1$ and $U^i = U_i = 0$.

$$T_\mu^\nu = \big(\rho_t + P_t\big)\, U^\nu U_\mu - P_t\,\delta_\mu^\nu\,,$$

such that $T_0^0 = \rho_t$ and $T_i^j = -P_t\,\delta_i^j$.

Let us derive the covariant derivative of U^μ

$$\nabla_\mu U^\mu = 3H\,.$$

The last equation enables one to perturb the running CC.

$$\rho_\Lambda = A + B\,\big(\nabla_\mu U^\mu\big)^2\,,$$

where

$$A = \rho_\Lambda^0 - \frac{3\,\nu}{8\pi}\,M_P^2\,H_0^2\,, \qquad B = \frac{\nu\,M_P^2}{24\pi}\,.$$

Final form of the perturbations equations

$$v' + \frac{(3f_1 - 5)}{1+z}\,v - \frac{k^2\varrho(1+z)}{Hf_1}\,v = -\frac{k^2\varrho(1+z)}{2H}\,\hat{h};$$

$$\hat{h}' + \frac{2(\nu-1)}{1+z}\,\hat{h} = \frac{2\,\nu}{(1+z)}\left(\frac{2\,v}{f_1} - \frac{f_1\,\delta_m}{\varrho}\right);$$

$$\delta_m' + \left(\frac{f_1'}{f_1} - \frac{3f_2}{1+z}\right)\delta_m = \frac{1}{f_1}\left(\frac{\varrho\,\hat{h}}{2} - \frac{\varrho\,v}{f_1}\right)' + \frac{1}{1+z}\left(3\varrho - \frac{1}{H}\right)\left(\frac{\hat{h}}{2} - \frac{v}{f_1}\right),$$

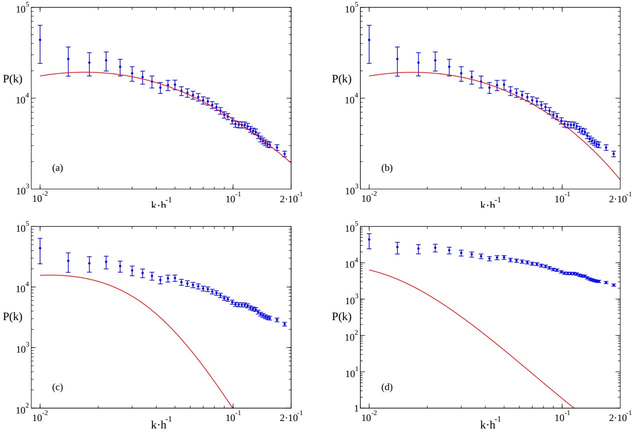
where $(f_1, f_2) = \left(\frac{\rho_m}{\rho_t}, \frac{\rho_\Lambda}{\rho_t}\right), \quad f(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3}\,\tilde{f}(\vec{k}, t)\,e^{i\vec{k}\cdot\vec{x}}.$

$$v = f_1\nabla_i(\delta U^i), \quad \hat{h} \equiv \frac{\partial}{\partial t}\left(\frac{h_{ii}}{a^2}\right), \quad \delta\rho_m = \rho_m\delta_m$$

Given the Harrison-Zeldovich initial spectrum, the power spectrum today can be obtained by integrating this system.

Initial data based on $w(z)$ **from** *J.M. Bardeen et al, Astr.J. (1986).*

Results of numerical analysis:



The ν-dependent power spectrum vs the LSS data from the 2dfFGRS. The ordinate axis represents $P(k) = |\delta_m(k)|^2$ where $\delta_m(k)$ is the solution at $z = 0$. In all cases $(\Omega_B^0, \Omega_{DM}^0, \Omega_\Lambda^0) = 0.04, 0.21, 0.75)$ & $\nu = 10^{-8},\ 10^{-6},\ 10^{-4},\ 10^{-3}$.

●● **Models with variable $G = G(H)$ but without energy exchange between vacuum and matter sectors.**

Theoretically this looks better!

$$\rho_\Lambda(H) \,=\, \rho_\Lambda(H_0) \,+\, \frac{3\nu}{8\pi}\, M_p^2 \left(H^2 - H_0^2\right).$$

By using the energy-momentum tensor conservation we find

$$G(H;\nu) = \frac{G_0}{1 + \nu\,\ln\left(H^2/H_0^2\right)}\,, \quad \textbf{where} \quad G(H_0) = \frac{1}{M_P^2}\,.$$

These relations exactly correspond to the RG approach discussed above, with $\mu = H$.

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

The limits on ν from density perturbations

J.Grande, J.Solà, J.Fabris & I.Sh., Cl. Q. Grav. 27 (2010) .

An important general result is: In the models with variable Λ and G in which matter is covariantly conserved, the solutions of perturbation equations *do not* depend on the wavenumber k.

$$\dot{\hat{h}} + 2H\hat{h} = 8\pi\left[\rho_m - 2\rho_\Lambda\right]\delta G + 8\pi G\left[\delta\rho_m - 2\delta\rho_\Lambda\right]\,;$$

$$\delta\dot{\rho}_m + \rho_m\left(\theta - \frac{\hat{h}}{2}\right) + 3H\delta\rho_m = 0\,;$$

$$\dot{\theta} + 2H\theta = 0\,;$$

$$\delta\dot{G}(\rho_m + \rho_\Lambda) + \delta G\dot{\rho}_\Lambda + \dot{G}(\delta\rho_m + \delta\rho_\Lambda) + G\delta\dot{\rho}_\Lambda = 0\,;$$

$$k^2\left[G\delta\rho_\Lambda + \rho_\Lambda\delta G\right] + a^2\rho_m\dot{G}\theta = 0\,.$$

As a consequence we meet relatively weak modifications of the spectrum compared to ΛCDM.

In our case the bound $\nu < 10^{-3}$ comes just from the "F-test"

R. Opher & A. Pelinson, astro-ph/0703779.
J.Grande, R.Opher, A.Pelinson, J.Solà, JCAP 0712 (2007)

It is related only to the modification of the function $H(z)$.

One can obtain the same restriction for ν also from the primordial nucleosynthesis (BBN).

Can we apply the running $G(\mu)$ to other physical problems?

In the renormalization group framework the relation

$$G(\mu) = \frac{G_0}{1 + \nu \ln\left(\mu^2/\mu_0^2\right)}\,, \quad \textbf{where} \quad \mu = H$$

in the cosmological setting.

What could be an interpretation of μ in astrophysics?

Consider the rotation curves of galaxies. The simplest assumption is $\mu \propto 1/r$.

Applications for the point-like model of galaxy:

J.T.Goldman, J.Perez-Mercader, F.Cooper & M.M.Nieto, PLB (1992).
O. Bertolami, J.M. Mourao & J. Perez-Mercader, PLB 311 (1993).
M. Reuter & H. Weyer, PRD **70** *(2004); JCAP* **0412** *(2004).*
I.Sh., J.Solà, H.Štefančić, JCAP (2005).

We can safely restrict the consideration by a weakly varying G,

$$G = G_0 + \delta G = G_0(1+\kappa)\,, \qquad |\kappa| \ll 1\,.$$

We already know that the appropriate value of the parameter ν is small, the same should be with $\kappa = \delta G/G_0$.

In order to link the metric in the variable G case with the standard one, perform a conformal transformation

$$\bar{g}_{\mu\nu} = \frac{G_0}{G}\, g_{\mu\nu} = (1-\kappa) g_{\mu\nu}\,.$$

Up to the higher orders in κ, the metric $\bar{g}_{\mu\nu}$ satisfies usual Einstein equations with constant G_0.

The nonrelativistic limits of the two metrics

$$g_{00} = -1 - \frac{2\Phi}{c^2} \quad \text{and} \quad \bar{g}_{00} = -1 - \frac{2\Phi_{\text{Newt}}}{c^2} \quad ,$$

where Φ_{Newt} is the usual Newton potential and Φ is a potential of the modifies gravitational theory.

We have

$$g_{00} \,=\, -1 - \frac{2\Phi}{c^2} \,=\, (1+\kappa)\bar{g}_{00}$$

$$= \,(1+\kappa)(-1 - \frac{2\Phi_{\rm Newt}}{c^2}) \approx -1 \,-\, \frac{2\Phi_{\rm Newt}}{c^2} \,-\, \kappa$$

and, hence,

$$\Phi = \Phi_{\rm Newt} + \frac{c^2}{2}\,\kappa = \Phi_{\rm Newt} + \frac{c^2\,\delta G}{2\,G_0}\,.$$

For the nonrelativistic limit of the modified gravitational force we obtain, therefore,

$$-\Phi^{,i} \,=\, -\Phi_{\rm Newt}^{,i} \,-\, \frac{c^2\,G^{,i}}{2\,G_0}\,,$$

where we used the relation $G^{,i} = (\delta G)^{,i}$.

The last formula

$$-\Phi^{,i} = -\Phi^{,i}_{\text{Newt}} - \frac{c^2\, G^{,i}}{2\, G_0} = -\Phi^{,i}_{\text{Newt}} - \frac{c^2}{2}\,\kappa^{,i}\,,$$

is indeed very instructive.

● **Quantum correction comes multiplied by $c^2 \Longrightarrow$ it does not need being large to make real effect at the typical galaxy scale.**

E.g., for a point-like model of galaxy and $\mu \propto 1/r$ it is sufficient to have $\nu \approx 10^{-6}$ to provide flat rotation curves.

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

●● **$\mu \propto 1/r$ is, obviously, not a really good choice for a non-point-like model of the galaxy.**

The reason is that this identification produces the "quantum-gravitational" force even if there is no mass at all !!

What would be the “right” identification of the renormalization group scale parameter μ in the quase-Newtonian regime?

Let us come back to the quantum field theory (QFT). We are currently unable to derive quantum corrections to the GR action.

At the same time the QFT gives us a good hint:
μ must be accociated to some parameter which characterizes the energy of the particle corresponding to the external gravitational line in the Feynman diagram.

Of course, $\mu \propto 1/r$ is not not the right choice.

We need a relatively simple parameter which can characterize the energy of the gravitational field in the almost-Newtonian (that means almost static, in particular) regime.

The most natural choice is to associate μ with the Newtonian potential Φ_{Newt} at the given point of space.

The phenomenologically good choice is

$$\frac{\mu}{\mu_0} = \left(\frac{\Phi_{\text{Newt}}}{\Phi_0}\right)^{\alpha},$$

where α is a phenomenological parameter which can be distinct for different spiral galaxies. We have found that α is nonlinearly growing with the mass of the galaxy.

D. Rodrigues, P. Letelier & I.Sh., JCAP (2010).

Definitely, α is a "good guy" in this story.

From the QFT viewpoint the presence of α reflects the fact that the association of μ with Φ_{Newt} is not an ultimate choice. Remember the vacuum EA is a relativistic object and taking Φ_{Newt} as a scale definitely ignores some relevant information.

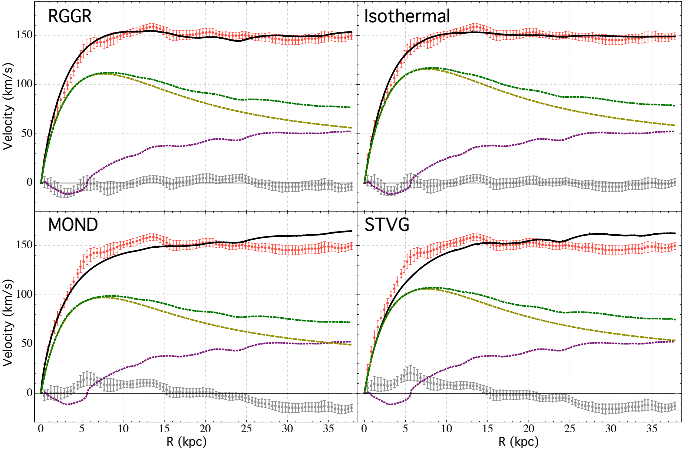
With greater mass of the galaxy the "error" in identification becomes greater too, hence we need a greater α to correct this. Furthermore, if α increases with the mass of the galaxy, it must be very small at the scale of the Sun system and of course at the scale of laboratory, when the Newton law is better verified.

Remarkably, the recently-proposed regular scale-setting procedure gives the very same result:

S. Domazet, H.Štefančić, PLB (2011) - in press.

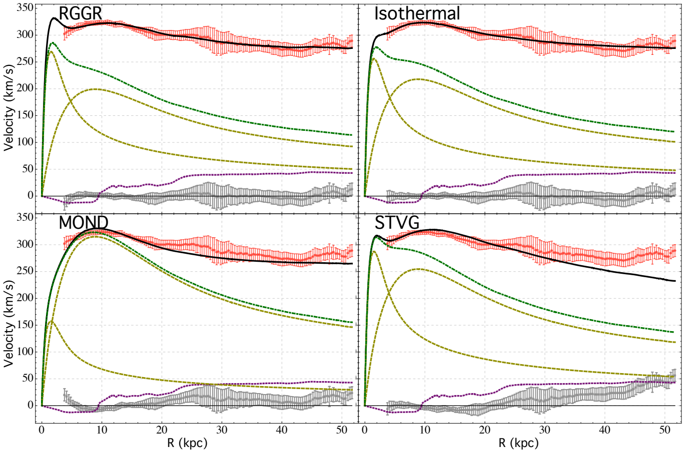
Last, but not least, the astro-ph application is impressively successful

D. Rodrigues, P. Letelier & I.Sh., JCAP (2010). *(9 samples)*



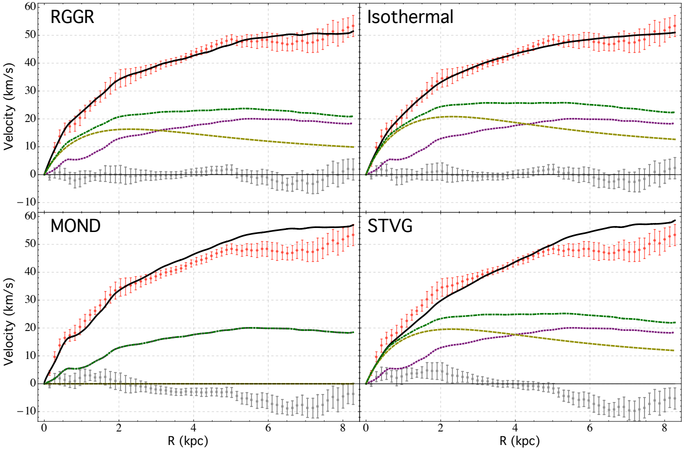
Rotation curve of the spiral galaxy *NGC 3198*
[Collaboration THINGS (2008)].

One more example, this time with descendent rotation curve.



Rotation curve of the galaxy *NGC 2841*. RGGR is based on hypothetical covariant quantum corrections without DM.

One more example, this time with low-surface brightness galaxy with ascendent rotation curve.



Rotation curve of the dwarf galaxy *DDO 154*.

What about the Solar System?

C. Farina, W. Kort-Kamp, S. Mauro, I.Sh., PRD 83 (2011).

We used the dynamics of the Laplace-Runge-Lenz vector in the $G(\mu) = G_0/(1 + \mu \log(\mu/\mu_0))$ - **corrected Newton gravity.**

Upper bound for the Solar System: $\alpha\nu \leq 10^{-17}$.

One of the works now on track: extending the galaxies sample.

P. Louzada, D. Rodrigues, J. Fabris, ..., in work: **50+ disk galaxies.**

D. Rodrigues, N. Napolitano, ..., in progress: **elliptical galaxies.**

The general tendency which we observe so far is greater α **needed to for larger mass of the astrophysical object: from Solar System (upper bound) to biggest tested galaxies.**

It looks like we do not need CDM to explain the rotation curves of the galaxies. However, does it really mean that we can really go on with one less dark component?

Maybe not, but it is worthwhile to check it. It is well known that the main requests for the DM come from the fitting of the LSS, CMB, BAO, lenthing etc.

However there is certain hope to relpace, e.g., ΛCDM by a ΛWDM (e.g. sterile neutrino) with much smaller Ω_{DM}.

The idea to trade $0.04, 0.23, 0.73 \quad \Longrightarrow \quad 0.04,\ 0.0x,\ 0.9(1\text{-}x)$

Such a new concordance model would have less relevant coincidence problem, and in general such a possibility is interesting to verify.

First move:

J. Fabris, A. Toribio & I.Sh., Testing DM warmness and quantity via the RRG model. arXiv:1105.2275 [astro-ph.CO]

We are using "our" Reduced Relativistic Gas model.

The Reduced Relativistic Gas model is a Simple cosmological model with relativistic gas.

G. de Berredo-Peixoto, I.Sh., F. Sobreira, Mod.Ph.Lett. A (2005);
J. Fabris, I.Sh., F.Sobreira, JCAP (2009).

The model describes ideal gas of massive relativistic particles with all of them have the same kinetic energy.

The Equation of State (EOS) of such gas is

$$P = \frac{\rho}{3}\left[1 - \left(\frac{mc^2}{\varepsilon}\right)\right]^2 = \frac{\rho}{3}\left(1 - \frac{\rho_d^2}{\rho^2}\right).$$

In this formula ε is the kinetic energy of the individual particle, $\varepsilon = mc^2/\sqrt{1-\beta^2}$. Furthermore, $\rho_d = \rho_{d0}^2(1+z)^3$ is the mass (static energy) density. One can use one or another form of the equation of state (1), depending on the situation.

The nice thing is that one can solve the Friedmann equation in this model analytically. The deviation from Maxwell or relativistic Fermi-Dirac distribution is less than 2.5%.

The model was sucessfully used to impose the upper bound to the warmness of DM from LSS data, providing the same results as more complicated models.

J. Fabris, I.Sh., F.Sobreira, JCAP (2009). DM particles: how warm they can be?

So, why it is "our" and not just our model?

Because we were not first. The same EOS has been used by A.D. Sakharov in 1965. to predict the oscillations in the CMB spectrum for the first time!!

A.D. Sakharov, Soviet Physics JETP, 49 (1965) , 345.

In the recent preprint (under consideration in PRD)

J. Fabris, A. Toribio & I.Sh., Testing DM warmness and quantity via the RRG model. arXiv:1105.2275 [astro-ph.CO]

we have used RRG without quantum effects to fit
Supernova type Ia (Union2 sample), $H(z)$**, CMB (**R **factor), BAO, LSS (2dfGRS data)**
In this way we confirm that Λ**CDM is the most favored model.**

However, for the LSS data alone we met the possibility of an alternative model with a small quantity of a WDM.

This output is potentially relevant in view of the fact that the LSS is the only test which can not be affected by the possible quantum renormalization-group running in the low-energy gravitational action.

Conclusions

- **The evaluation of quantum corrections from massive fields is, to some extent, reduced to existing-nonexisting paradigm.**

- **In the positive case we arrive at the cosmological and astrophysical model with one free parameter ν plus certain freedom of scale identification.**

- **The rotation curves of all tested galaxies can be described by the $G(\mu)$ formula. The situation with clasters and other tests, especially CMB and lensing, remains unclear.**

- **The power spectrum tests are less sensible to the $G(\mu)$ and exactly in this case we meet an alternative to ΛCDM in the zero-order approximation.**

- **Finally, there is still some (albeit small) chance that the vacuum effects of QFT in an external gravitational field play more significant role in our Universe that we use to imagine.**