Infinite Matrix Product States and Conformal Field Theory

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Overview

In recent years low-D quantum lattice systems have been actively investigated using a variety of numerical methods:

- Density Matrix Renormalization Group (DMRG)
- Matrix Product States (MPS)
- Projected Entangled Pair States (PEPS)
- Multiscale Entanglement Renormalization Ansatz (MERA)

The DMRG or the MPS are more appropiate to 1D gapped systems which satisfy the area law: finite subsystem entanglement entropy

However the area law is violated in 1D critical systems which leads to troubles, that can be partially overcome using finite size methods.

In MPS the finite entanglement entropy is related to finite bond dimension, so a possible alternative to describe critical systems is to consider infinite dimensional matrices.

A concrete realization of this idea is provided by Conformal Field Theory.

The Hilbert space of CFT is infinite dimensional and can be used as auxiliary space for MPS, supporting long range entanglement.

Like the standard MPS, the iMPS depends on a set of variational parameters that can be fixed by minimization of the energy for a given model Hamiltonian.

In some cases one can even construct the parent Hamiltonian.
The iMPS constructed from the Wess-Zumino-Witten models
yield parent Hamiltonians related to Haldane-Shastry model.

CFT has also been applied to construct ansatzs for GS and excitations in the Fractional Quantum Hall Effect, like the Laughlin and Moore-Read wave functions.

These leads to many analogies between iMPS and FQHE wave functions, with potential applications to non abelian states and TQC.

Plan of the talk

-- Brief review of MPS

-- Vertex operators and iMPS

-- Applications to spin chains

-- Haldane-Shastry model

-- Generalizations of the Haldane-Shastry model

-- Relation with FQH wave functions

Matrix Product States

Consider a 1D spin 1/2 system with N sites and Hamiltonian

$$H = \sum_{i=1}^{N} h_{i,i+1}$$

The GS wave function is given in a local spin basis by

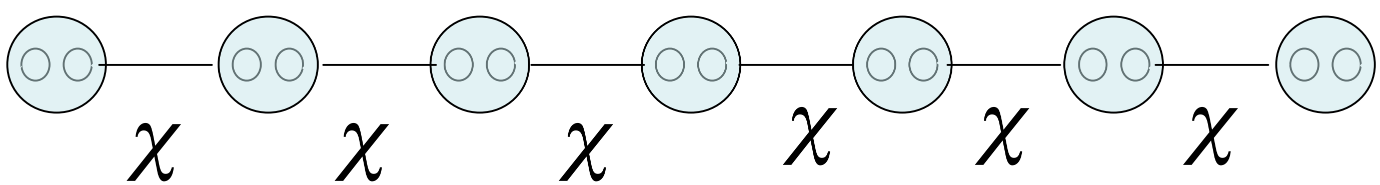
$$|\psi\rangle = \sum_{s_1,\ldots s_N} \psi(s_1, s_2, \ldots, s_N)\,|s_1, s_2, \ldots, s_N\rangle, \quad s_i = \pm 1$$

The MPS is an ansatz of the form

$$\psi(s_1, s_2, \ldots, s_N) = \langle v | A^{(1)}(s_1) \cdots A^{(N)}(s_N) | w \rangle$$

Where $A^{(i)}(s_i), \langle v|, |w\rangle$ are χ dimensional matrices and vectors

$\chi =$ Bond dimension / Schmidt number / m of the DMRG



The entanglement entropy in a bipartition A U B scales as

$$S_A \propto \log \chi \qquad \text{(1D area law)}$$

In a critical system described by a CFT (periodic BCs)

$$\boxed{S_A = \frac{c}{3} \log L + c_1} \qquad \text{c = central charge}$$

hence one needs very large matrices to describe critical systems

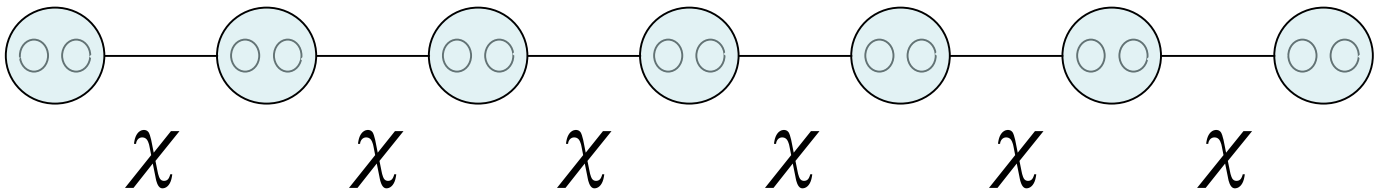
$$N \propto \chi^{\kappa}, \quad \kappa = \kappa(c)$$

Tagliacozzo et al
Pollman et al.

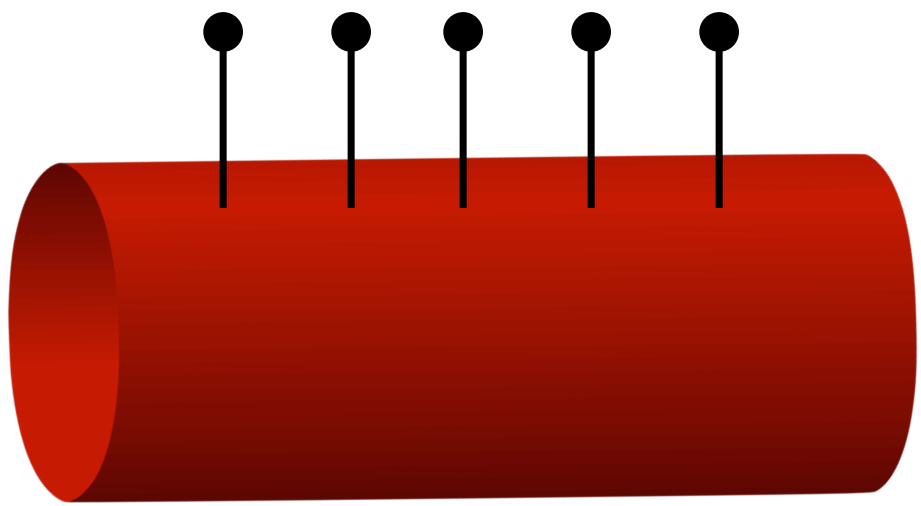
Another alternative is to choose infinite dimensional matrices:

$$\boxed{\chi = \infty}$$

MPS state



iMPS state



$$\boxed{\chi = \infty}$$

The simplest CFT: massless boson (c=1)

Consider a chiral free boson field $\varphi(z)$

$$\varphi(z) = \varphi_0 - i\pi_0 - i\sum_{m=1}^{\infty} \frac{1}{\sqrt{m}} a_m^* z^m + i\sum_{m=1}^{\infty} \frac{1}{\sqrt{m}} a_m z^{-m}$$

Zero modes, φ_0, π_0 and infinite number of oscillators $a_m \ (m = 1, \ldots, \infty)$

$$[\varphi_0, \pi_0] = i \qquad [a_m, a_n^*] = \delta_{n,m} \quad (n, m = 1, \ldots, \infty)$$

The Hilbert space is generated by all the bosonic oscillators (string)

$$\exp\left(i\sqrt{\alpha}\varphi_0\right) a_{m_1}^* \cdots a_{m_N}^* |0\rangle, \quad a_m |0\rangle = 0$$

$\sqrt{\alpha}$: momentum of the state (eigenvalue of π_0)

Two point correlator: $\langle \varphi(z_1)\varphi(z_2) \rangle = -\log(z_1 - z_2)$

Vertex operators

Normal ordered exponentials of the boson field

$$V_\alpha(z) = :\exp\left(i\sqrt{\alpha}\,\varphi(z)\right):=$$

$$\exp\left(i\sqrt{\alpha}\,\varphi_0 + \sqrt{\alpha}\sum_{m=1}^{\infty}\frac{1}{\sqrt{m}}a_m^* z^m\right)\exp\left(\sqrt{\alpha}\,\pi_0 \log z - \sqrt{\alpha}\sum_{m=1}^{\infty}\frac{1}{\sqrt{m}}a_m z^{-m}\right)$$

Multipoint correlator of Jastrow type

$$\left\langle V_{\alpha_1}(z_1)\cdots V_{\alpha_N}(z_N)\right\rangle = \prod_{i<j}\left(z_i - z_j\right)^{\sqrt{\alpha_i\,\alpha_j}}, \quad \sum_i \sqrt{\alpha_i} = 0$$

Application: Laughlin wave function

$$\psi\left(z_1,\ldots z_N\right) = \left\langle V_m(z_1)\cdots V_m(z_N)\right\rangle e^{-\sum |z_n|^2/4} = \prod_{i<j}\left(z_i - z_j\right)^m e^{-\sum |z_n|^2/4}$$

Infinite Matrix Product States

Idea: use the vertex operators as MPS "matrices"

For a spin system (i.e. Heisenberg spin chain)

MPS: $$s_i \to A_i(s_i) : \chi \times \chi \; matrix$$

iMPS: $$s_i \to A_i(s_i) : vertex \; operator$$

$$A_i(s_i) = \chi_{s_i} \; :e^{i s_i \sqrt{\alpha}\, \varphi(z_i)}:$$

Variational parameters

$$z_i \; , i = 1, \ldots N$$

$$\sqrt{\alpha}, \quad \chi_{s_i} = \pm 1$$

The "momentum" of vertex is proportional to the spin of each site

Wave function generated by the iMPS:

$$\psi\left(s_1, s_2, \ldots, s_N\right) = \langle 0 | A_{z_1}(s_1) A_{z_2}(s_2) \cdots A_{z_N}(s_N) | 0 \rangle$$

Using the vertex correlators one gets

$$\psi\left(s_1, s_2, \ldots, s_N\right) = \chi_{s_1, \ldots s_N} \prod_{i<j} (z_i - z_j)^{\alpha s_i s_j} \times \delta(\sum_i s_i)$$

Spin version of the Laughlin wave function

Conservation of "momentum" $\quad S^z_{tot} = \frac{1}{2} \sum_{i=1}^{N} s_i = 0, \quad N : even$

$\alpha, z_1, \ldots, z_N$ minimization of the energy of the ansatz

If the GS state is translationally invariant one can choose

$z_n = e^{2\pi i n/N}, \quad n = 1, \ldots, N$ (N spins on a circle)

$$\psi\left(s_1, s_2, \ldots, s_N\right) = \chi_{s_1, \ldots s_N} \prod_{i>j} \left(\sin \frac{\pi (i-j)}{N} \right)^{\alpha s_i s_j} \times \delta(\sum_i s_i)$$

The sign factors given by the Marshall rule of antiferromagnets (Perron-Frobenius theorem)

$$\chi_{s_1,\ldots,s_N} = e^{i\pi/2\sum_{i:odd}(s_i-1)}$$

Applications to spin 1/2 Heisenberg like chains:

- Anisotropic (XXZ)
- $J_1 - J_2$
- Random bond

Determining the parameters $\alpha, z_1,\ldots,z_N$ in terms of the couplings

- Overlaps with exact wave functions up to N=20 sites
- Spin-spin correlators
- Renyi entropy

Anisotropic spin 1/2 Heisenberg model

Hamiltonian periodic BCs

$$H = \sum_{1=1}^{N} S_i^x \, S_{i+1}^x + S_i^y \, S_{i+1}^y + \Delta \, S_i^z \, S_{i+1}^z$$

Phases of the model

$$\Delta > 1 \quad \textit{gapped antiferromagnet}$$

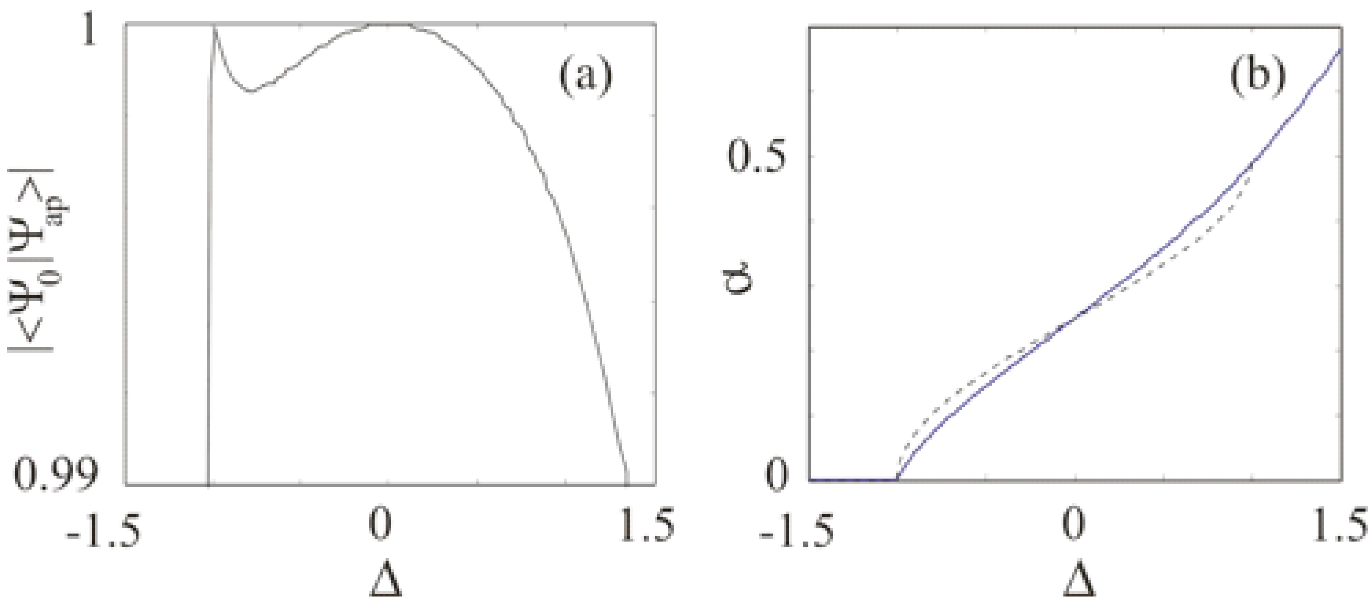
$$-1 < \Delta \le 1 \quad \textit{gapless} \ \ (c = 1 \, CFT)$$

$$\Delta \le -1 \quad \textit{Ferromagnetic}$$

To find α we minimize the energy of the iMPS

Choose $z_n = e^{2\pi i n / N}, \quad n = 1, \ldots, N$

Overlap of exact and the iMPS wave functions (N=20)



$$\Delta = -\cos(2\pi\alpha)$$

The iMPS is exact in two cases

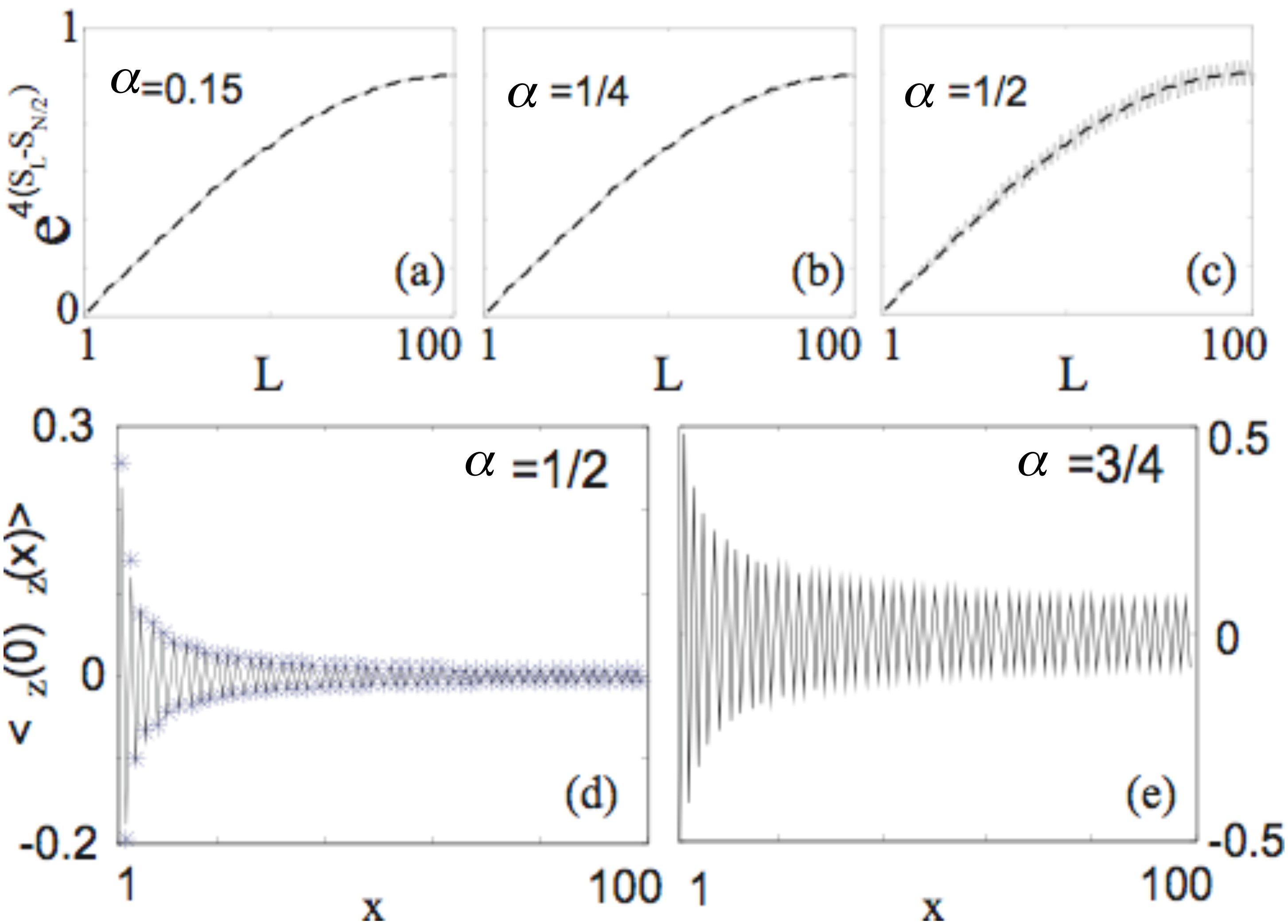
$$\Delta = -1 \rightarrow \alpha = 0$$ isotropic ferromagnetic chain

$$\Delta = 0 \rightarrow \alpha = 1/4$$ XX chain

At the isotropic AFH model

$$\Delta = 1 \rightarrow \alpha = 1/2$$ Haldane-Shastry spin chain

Renyi entropy $S_L = -\log Tr\,\rho_L^2$ and spin correlators (MC method)



In the critical regime it agrees with a c=1 CFT

$J_1 - J_2$ Model (zig-zag chain)

$$H = \sum_{1=1}^{N} J_1 \vec{S}_i \cdot \vec{S}_{i+1} + J_2 \vec{S}_i \cdot \vec{S}_{i+2} \qquad (J_1 = 1)$$

$J_2 > 0$ frustrated spin system

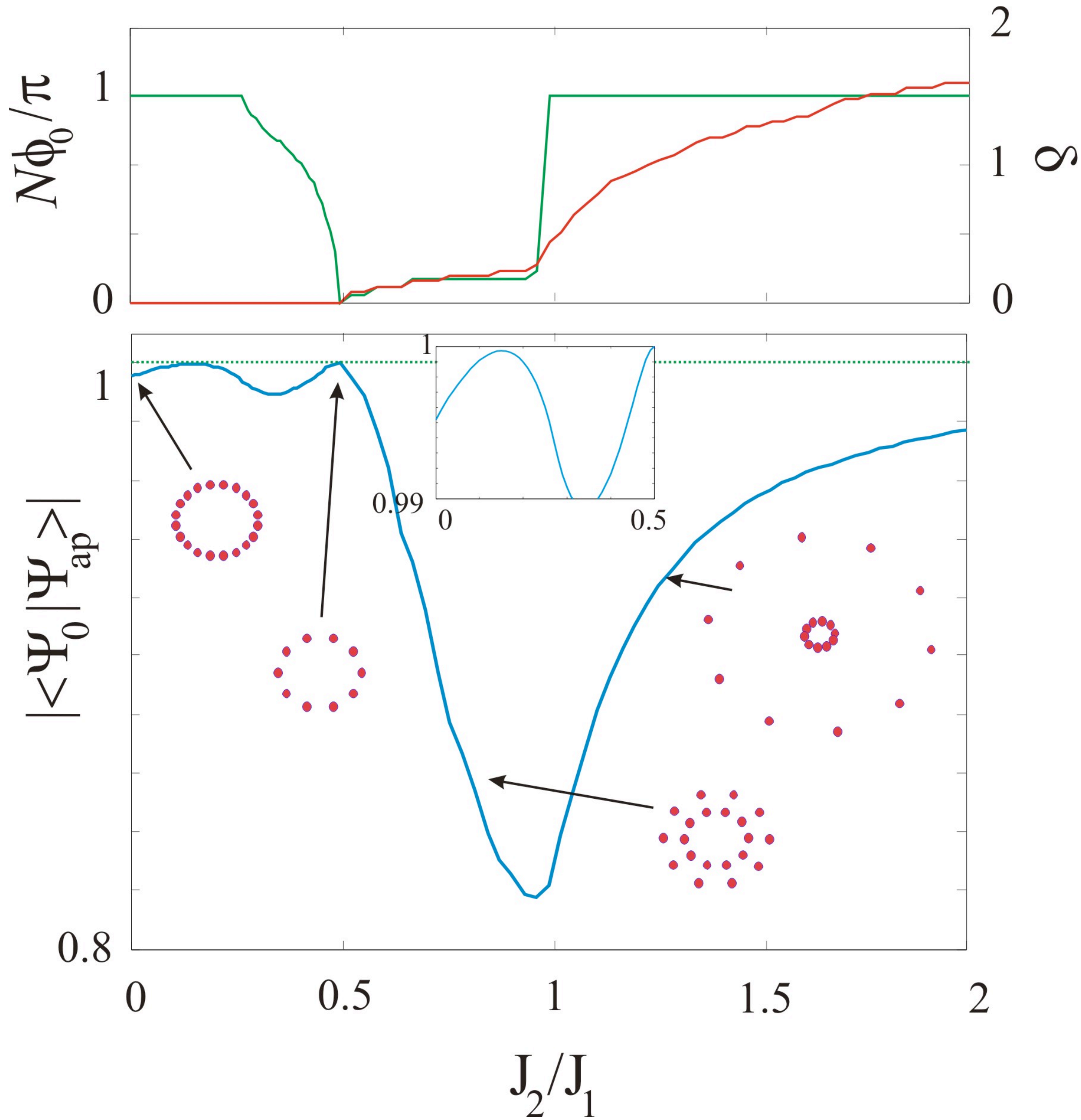
Phases

$0 \leq J_2 < J_{2c} \approx 0.241$	Critical c=1
$J_{2c} < J_2 < J_{MG} = 0.5$	Spontaneously dimerized
$J = J_{MG}$	Majumdar-Gosh point
$J_{MG} < J < \infty$	Dimer spiral phase

Choice of parameters $\alpha = \frac{1}{2}$ -> rotational invariance

$$z_n = \begin{cases} \exp(\delta - i\phi_0)\exp(2\pi i(0,2,4,\ldots)/N) & \textit{even sites} \\ \exp(-\delta + i\phi_0)\exp(2\pi i(0,2,4,\ldots)/N) & \textit{odd sites} \end{cases}$$

ϕ_0 -> dimerization $\qquad$ δ -> “split” of the chain



$N\phi_0/\pi$
ϱ
$|<\Psi_0|\Psi_{ap}>|$
J_2/J_1
0
0.5
1
1.5
2
0.8
0.99

The Haldane-Shastry model (1988)

Fermi state of a spin 1/2 particle on a circle at half filling

$$|FS\rangle = \Pi_{|k|<k_F}\, c^*_{k\uparrow}\, c^*_{k\downarrow}\, |0\rangle \qquad k_F = \frac{\pi}{2}$$

Eliminate the states doubly occupied (Gutzwiller projection)

$$|\psi_G\rangle \propto P_G\,|FS\rangle = \Pi_i\,(1 - n_{i\uparrow}\, n_{i\downarrow})\,|FS\rangle$$

Spin-spin correlator (Gebhard-Vollhardt 1987)

$$\langle S^a_n\, S^a_0 \rangle = (-1)^n\, \frac{3Si(\pi\, n)}{4\,\pi\, n} \approx (-1)^n\, \frac{c_1}{n} + \frac{c_2}{n^2} \;\; (n \to \infty)$$

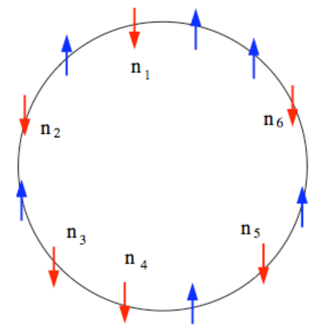
Compare with the correlator in the AF Heisenberg model

$$\langle S^a_n\, S^a_0 \rangle \approx (-1)^n\, \frac{c_1\sqrt{\log n}}{n} + \frac{c_2}{n^2} \;\; (n \to \infty)$$

The Gutzwiller states has only spin degrees of freedom that can be seen as a hardcore boson

$$|\uparrow\rangle \leftrightarrow |0\rangle \qquad \textit{empty}$$

$$|\downarrow\rangle \leftrightarrow a^* |0\rangle \quad \textit{occupied}$$

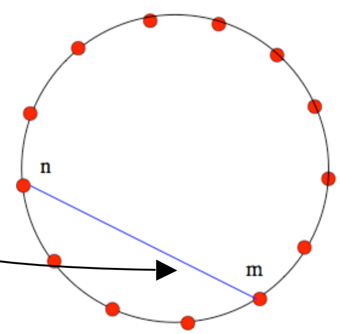


$$|\psi_G\rangle \propto \sum_{n_1,\dots,n_{N/2}} e^{i\pi\sum_i n_i} \prod_{i<j} \left|\sin\frac{\pi(n_i - n_j)}{N}\right|^2 a^*_{n_1}\dots a^*_{n_{N/2}}|0\rangle$$

n_i : position of the i-boson (i.e. spin down)

$|\psi_G\rangle$ ground state of the Hamiltonian (Haldane-Shastry)

$$H = \frac{J\pi^2}{N^2}\sum_{n<m}\frac{\vec{S}_n\cdot\vec{S}_m}{\sin^2(\pi(n-m)/N)}$$



Properties of the HS model

- spin-spin correlation functions decays algebraically

- elementary excitations: spinons (spin 1/2 with fractional statistics)

- degenerate spectrum described by a Quantum Group symmetry

- critical theory at the fixed point of the renormalization group

- this fixed point is described a CFT: $SU(2)_{k=1}$ WZW model

The HS model and the AF Heisenberg model belong to the same universality class described by the WZW model, but the AFH model is a marginal irrelevant perturbation of the WZW which give rise to the log corrections in correlators

HS, iMPS, WZW

The HS wavefunction in spin variables

$$\psi\left(s_1, s_2, \ldots, s_N\right) = \chi_{s_1, \ldots s_N} \prod_{i<j} (z_i - z_j)^{s_i s_j / 2}, \; z_n = e^{2\pi i n / N}$$

This is an iMPS

$$\psi\left(z_1, s_1, \ldots, z_N, s_N\right) = \left\langle A_{z_1}(s_1) A_{z_2}(s_2) \cdots A_{z_N}(s_N) \right\rangle$$

with $\alpha = \frac{1}{2}$ $\quad A_z(s) = \chi_s : e^{i s \varphi(z)/\sqrt{2}}$

$A_z(s) = \phi_{1/2,s}(z)$ are the primary fields of spin 1/2 of the $SU(2)_k$ WZW model at level k=1

Model equivalent to a free boson (c=1) = bosonization

Chiral correlators in CFT (conformal blocks)

A CFT is defined by a collection of primary fields

$$\phi_i(z), \quad i = 1, \ldots, P$$

with two and three point correlators

$$\left\langle \phi_i(z_1)\,\phi_j(z_2) \right\rangle = \frac{\delta_{i,j}}{z_{12}^{2h_i}}$$

$$\left\langle \phi_i(z_1)\,\phi_j(z_2)\,\phi_k(z_3) \right\rangle = \frac{C_{ijk}}{z_{12}^{h_i+h_j-h_k}\, z_{13}^{h_i+h_k-h_j}\, z_{23}^{h_j+h_k-h_i}}$$

h_i Scaling dimensions (conformal weights)

C_{ijk} Structure constants (OPE coefficients)

Fusion rules of the primary fields (skeleton of OPE)

$$If\ \ C_{ijk} = 0 \rightarrow N_{ijk} = 0$$

$$If\ \ C_{ijk} \neq 0 \rightarrow N_{ijk} = 1,2\ldots$$

It is a tensor product decomposition

$$\boxed{\phi_i \times \phi_j = \sum_k N_{ij}^k \, \phi_k}$$

Fusion rules give the number of chiral correlators

$$\left\langle \phi_{i_1}(z_1)\ldots\phi_{i_N}(z_N) \right\rangle_p \qquad p = 1,\ldots,M$$

$$M = \sum_{k_1\ldots k_{N-2}} N_{i_1\,i_2}^{k_1}\, N_{k_1\,i_3}^{k_2} \cdots N_{k_{N-2}\,i_{N-1}}^{i_{N-1}}$$

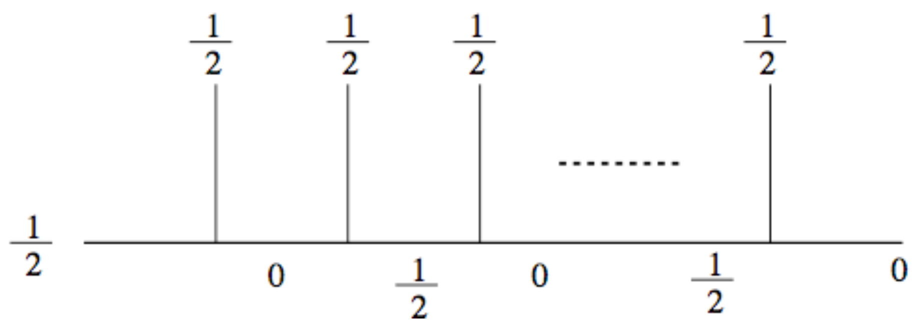
WZW model SU(2)@ k=1

Primary fields $\phi_0 = identity, \quad \phi_{1/2} = spin 1/2$

Fusion rule: $\phi_{1/2} \times \phi_{1/2} = \phi_0$

Unique chiral correlator involving N even primary fields $\phi_{1/2}$

$$\psi\left(z_1, s_1, ,\ldots, z_N, s_N\right) = \left\langle \phi_{1/2,s_1}(z_1) \cdots \phi_{1/2,s_N}(z_N) \right\rangle$$



Haldane- Shastry wave function

$$\psi\left(s_1, s_2, \ldots, s_N\right) = \chi_{s_1, \ldots s_N} \prod_{i<j} (z_i - z_j)^{s_i s_j/2}, \ z_n = e^{2\pi i n/N}$$

is the unique chiral correlator of this model

Notice that the "positions" of the spins are equally spaced

The HS wavefunction above is also good description for rotational invariant <u>critical and non critical</u> systems (recall the J1-J2 model)

Question: is there are Hamiltonian for which this wave function Is the GS for generic values of the parameters $z_n, n = 1, \cdots, N$?

$$H = -\sum_{n \neq m} \left(\frac{z_n z_m}{(z_n - z_m)^2} + \frac{1}{12} w_{n,m} (c_n - c_m) \right) \vec{S}_n \cdot \vec{S}_m$$

$$w_{n,m} = \frac{z_n + z_m}{z_n - z_m}, \quad c_n = \sum_{n \neq m} w_{n,m}, \qquad E_0 = \frac{1}{16} \sum_{n \neq m} w_{n,m}^2 - \frac{N(N+1)}{16}$$

$z_n = e^{2\pi i n/N} \to c_n = 0 \quad \forall n$ inhomogenous HS model

Review: Null vectors of the SU(2)@k WZW model

The modes J_n^a of the currents operators satisfy the Kac-Moody algebra

$$[J_n^a, J_m^b] = i\,\varepsilon^{abc}\, J_{n+m}^c + \frac{k}{2}\, n\, \delta^{ab}\, \delta_{n+m}$$

Null vector at the Virasoro level k+1 in the Verma module of the identity

$$\chi_{k+1} = \left(J_{-1}^+\right)^{k+1} \phi_0, \qquad J_n^a\, \chi_{k+1} = 0, \quad \forall n > 0$$

Decoupling of this null vector yields the primary fields (Gepner-Witten)

$$j = 0, \frac{1}{2}, \cdots, \frac{k}{2}$$

and the fusion rules of the model

$$\phi_{j_1} \otimes \phi_{j_2} = \sum_{j=|j_1-j_2|}^{\min(j_1+j_2,\, k-j_1-j_2)} \phi_j$$

The null vector $\chi_{k+1} = \left(J_{-1}^{+}\right)^{k+1} \phi_0$ is the highest weight vector of a multiplet with total spin m = k+1.

$$\boxed{\chi_m^{a_1 \ldots a_m} = C^{(m)}_{a_1 \ldots a_m\, b_1 \ldots b_m} J_{-1}^{b_1} \cdots J_{-1}^{b_m} \phi_0}$$

$\chi_m^{a_1 \ldots a_m}$ is totally symmetric and traceless with the m indices

The tensors $C^{(m)}_{a_1 \ldots a_m b_1 \ldots b_m}$ are proyectors in this space

$$C^{(2)}_{a_1 a_2 b_1 b_2} = \frac{1}{2}\left(\delta_{a_1 b_1}\,\delta_{a_2 b_2} + \delta_{a_1 b_2}\,\delta_{a_2 b_1}\right) - \frac{1}{3}\delta_{a_1 a_2}\,\delta_{b_1 b_2}$$

$$C^{(3)}_{a_1 a_2 a_3 b_1 b_2 b_3} = \frac{1}{6}\left(\delta_{a_1 b_1}\,\delta_{a_2 b_2}\,\delta_{a_3 b_3} + permutations\right) - \frac{1}{15}\left(\delta_{a_1 a_2}\,\delta_{b_1 b_2}\delta_{a_3 b_3} + \cdots\right)$$

Decoupling of null vectors in correlators of primary fields

$$\left\langle \chi_m^{a_1\ldots a_m}(z)\,\phi_1(z_1)\cdots\phi_n(z_n)\right\rangle = 0$$

Using the Ward identity

$$\left\langle (J_{-1}^a\chi)(z)\,\phi_1(z_1)\cdots\phi_n(z_n)\right\rangle = \sum_{j=1}^{n}\frac{t_j^a}{z-z_j}\left\langle \chi(z)\,\phi_1(z_1)\cdots\phi_n(z_n)\right\rangle$$

One finds that the conformal blocks

$$\psi(z_1\cdots z_n) = \left\langle\, \phi_1(z_1)\cdots\phi_n(z_n)\right\rangle$$

satisfies $$R^{(m)}_{a_1\ldots a_m}\left(z,z_1,\ldots z_n\right)\psi(z_1\cdots z_n) = 0, \quad \forall\, z$$

$$R^{(m)}_{a_1\ldots a_m}\left(z,z_1,\ldots z_n\right) = \sum_{j_1\ldots j_m}^{n} C^{(m)}_{a_1\ldots a_m b_1\ldots b_m}\,\frac{(z+z_{j_1})\ldots(z+z_{j_m})}{(z-z_{j_1})\ldots(z-z_{j_m})}\,t_{j_1}^{b_1}\ldots t_{j_m}^{b_m}$$

Taking residues $\left(w_{i\,j} = \frac{z_i + z_j}{z_i - z_j} \right)$

$$R^{(m,i)}_{a_1 \ldots a_m} \equiv \oint_{z_i} \frac{dz}{z} R^{(m)}_{a_1 \ldots a_m}\left(z, z_1, \ldots z_n\right) = \sum_{j_2 \ldots j_m \neq i}^{n} C^{(m)}_{a_1 \ldots a_m b_1 \ldots b_m} w_{i\,j_2} \ldots w_{i\,j_m} t_i^{b_1} \ldots t_{j_m}^{b_m}$$

Define the operators

$$H^{(m,i)} \equiv \sum_{a_1 \ldots a_m} \left(R^{(m,i)}_{a_1 \ldots a_m} \right)^* R^{(m,i)}_{a_1 \ldots a_m}$$

Properties:

1) $\left(H^{(m,i)} \right)^* = H^{(m,i)}$

2) $H^{(m,i)} \geq 0$

3) $\left[H^{(m,i)}, \sum_i t_i^a \right] = 0$

4) $H^{(m,i)} \psi = 0$

Spin 1 version of the Haldane-Shastry model

Take SU(2)@k=2 -> c = 3/2

Primary fields: $\phi_0, \; \phi_{1/2}, \; \phi_1$

Fusion rule of spin 1 field $\phi_1 \times \phi_1 = \phi_0$

This theory is equivalent to three Ising models (c= 3 x 1/2)

Spin 1 field -> triplet of Majorana fermions $\chi_a \; (a = 1,2,3)$

Fusion rules -> a unique chiral correlator for these fields:

$$\psi_{a_1 a_2 \ldots a_n} = \left\langle \chi_{a_1}(z_1) \cdots \chi_{a_n}(z_n) \right\rangle$$

(Pfaffian like)

The Hamiltonian contains 4 body spin-spin couplings

Renyi entropy and correlators -> this is a critical models SU(2)@k=2

Degenerate spin 1/2 version of the Haldane-Shastry model

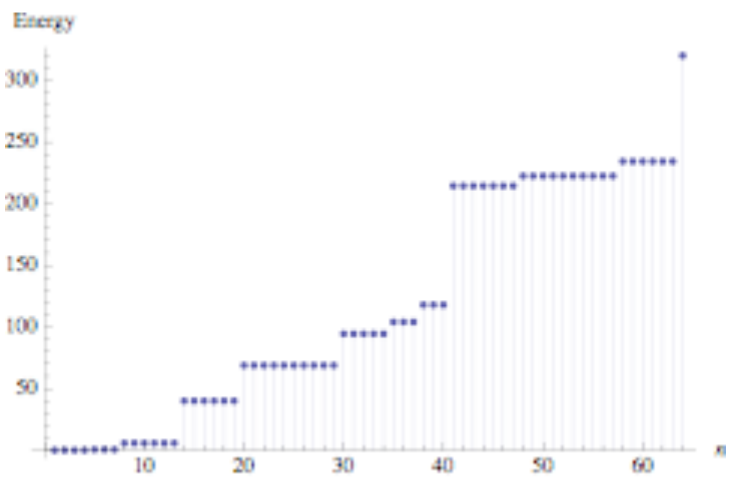
Take again SU(2)@k=2

Fusion rule of spin 1/2 field $\phi_{1/2} \times \phi_{1/2} = \phi_0 + \phi_1$

Number of chiral correlators of N spin 1/2 fields = $2^{N/2-1}$

Now the GS is NOT unique but degenerate !!

Example N = 6 -> 4 GS



The Hamiltonian contains 4 body terms

Mixing spin 1/2 and spin 1 for SU(2)@k=2

$$\left\langle \phi_{1/2} \dots^{N_{1/2}} \dots \phi_{1/2}\, \phi_1 \dots^{N_1} \dots \phi_1 \right\rangle_p \quad \text{->} \quad 2^{\frac{1}{2}N_{1/2}-1}$$

The degeneracy only depends on the number of spin 1/2 fields

SU(2)@2 = Boson + Ising
c =3/2 = 1 + 1/2

Spin 1 field $\qquad \phi_{1,\pm 1}(z) = e^{\pm i\varphi(z)}, \quad \phi_{1,0}(z) = \chi(z)$

Spin 1/2 field $\qquad \phi_{1/2,\pm 1/2}(z) = \sigma(z)\, e^{\pm i\varphi(z)/2}$

$\chi(z)$ is the Majorana field and $\sigma(z)$ is the spin field of the Ising model

Ising fusion rules $\quad \chi \times \chi = id, \quad \chi \times \sigma = \sigma, \quad \sigma \times \sigma = id + \chi$

Laughlin wave function: boson (c=1)

Electron operator: $\psi_e(z) = e^{i\sqrt{m}\,\varphi(z)}$

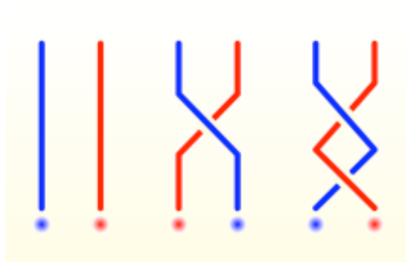
Quasihole operator $\psi_{qh}(z) = e^{\frac{i}{\sqrt{m}}\varphi(z)}$

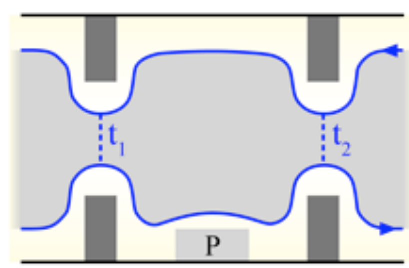
The wave functions with quasiholes are NOT degenerate

Quasiholes have fractional charge 1/m and fractional statistics

Exchange (braiding) of two quasiholes -> $e^{i\pi/m} \neq 1, -1 \quad (m = 3, 5, \ldots)$

Nor bosons nor fermions= abelian anyons





Moore-Read wave function for FQHE @5/2 (1992)

CFT = boson (c=1) + Ising (c=1/2)

Electron operator $\psi_e(z) = \chi(z)\, e^{i\sqrt{2}\varphi(z)}$

Ground state wave function

$$\langle \psi_e(z_1)\ldots\psi_e(z_N)\rangle = \prod_{i<j}(z_i - z_j)^2 \langle \chi(z_1)\ldots\chi(z_N)\rangle$$

$$\langle \chi(z_1)\ldots\chi(z_N)\rangle = Pfaffian\,\frac{1}{z_i - z_j} = \sqrt{\det \frac{1}{z_i - z_j}}$$

Quasihole operator $\psi_{qh}(z) = \sigma(z)\, e^{\frac{i}{2\sqrt{2}}\varphi(z)}$

$\left\langle \psi_{qh}\ldots^{N_{qh}}\ldots\psi_{qh}\,\psi_e\ldots^{N_e}\ldots\psi_e\right\rangle_p$ -> Degeneracy $2^{\frac{1}{2}N_{qh}-1}$

Fusion rules of Ising

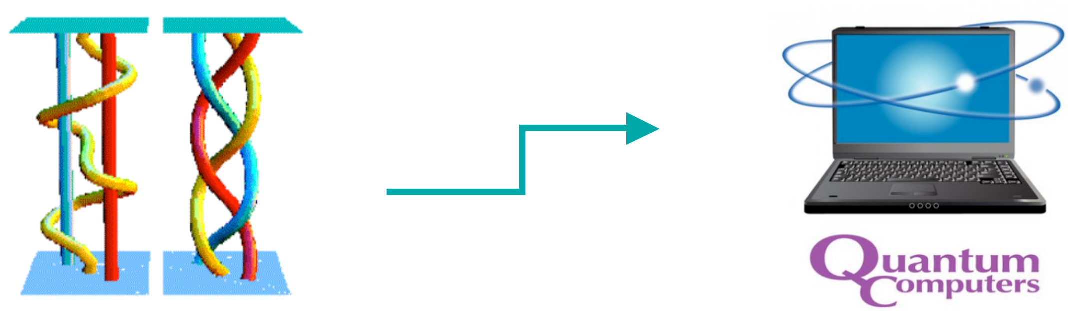
The quasiholes of the Moore-Read state: non abelian anyons

$$\left\langle \psi_{qh}(z_i)\ldots\psi_{qh}(z_j)\ldots\psi_e\ldots\psi_e \right\rangle_p = \sum_q B^{\pm}_{pq} \left\langle \psi_{qh}(z_j)\ldots\psi_{qh}(z_i)\ldots\psi_e\ldots\psi_e \right\rangle_q$$

The degenerate wave functions mix under the braiding operations

Braidring matrices: $B^{\pm}_{pq} : M \times M \;\; matrices, \quad M = 2^{N_{qh}/2-1}$

Basis for Topological Quantum Computation
(braids -> gates)



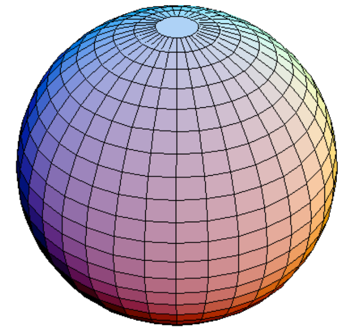
An analogy via CFT

FQHE ⟵	CFT ⟶	Spin Models
Electron	χ field	spin 1
Quasihole	σ field	spin 1/2
Braiding of quasiholes	Monodromy of correlators	Adiabatic change of H

In the FQHE braiding is possible because electrons live effectively in 2 dimensions

To have "braiding" for the spin systems we need to generalize these models to 2D

SU(2)@k=1, spin 1/2, D=2



The wave function is defined in the sphere

$$\psi(s_1,,\ldots,s_N) = \prod_i \chi_{s_i} \prod_{i<j} \left(u_i v_j - u_j v_i\right)^{s_i s_j/4} = \prod_i \chi_{s_i} \prod_{i<j} \left(\rho_{ij}\right)^{-s_i s_j/4}$$

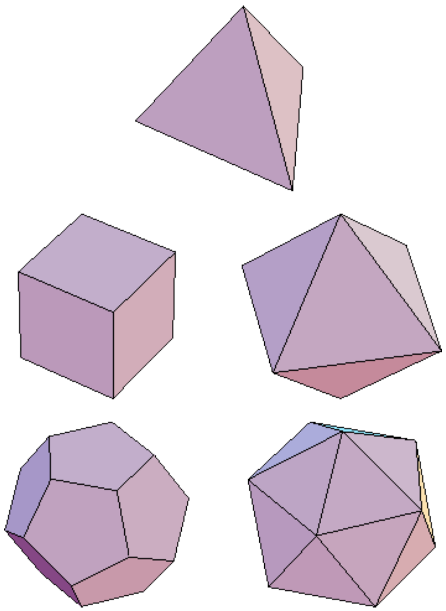
u and v are the spinor coordinates. This is the GS of the Hamiltonian

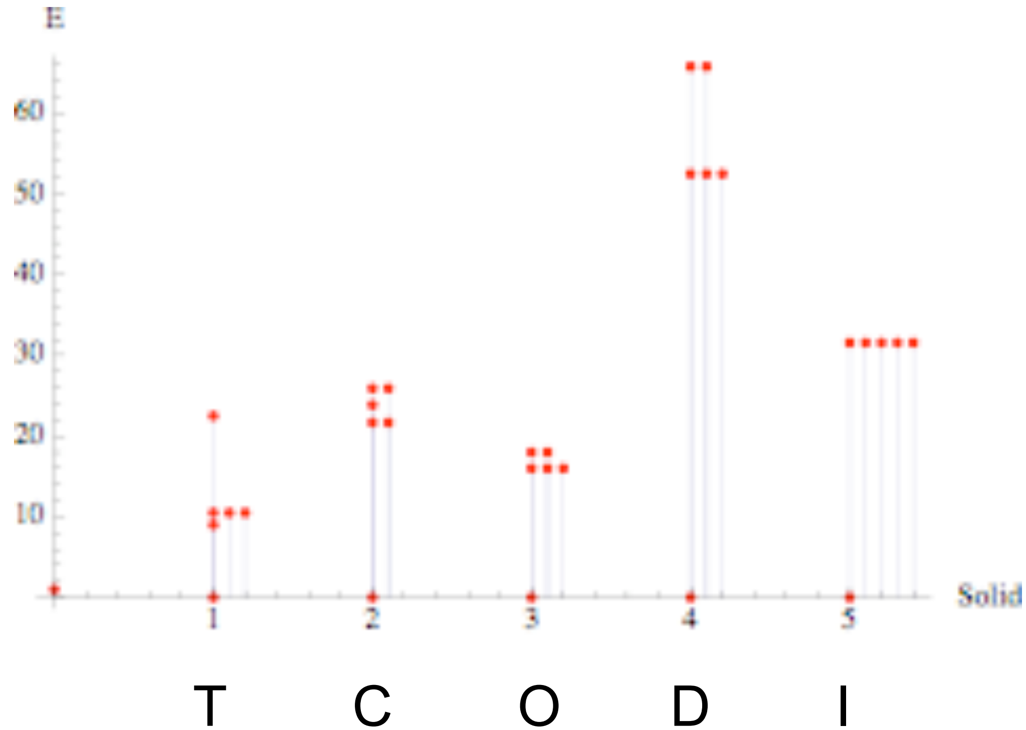
$$H = \frac{3}{4} \sum_{i_1 \neq i_2} \left|\rho_{i_1 i_2}\right|^2 + \sum_{i_1 \neq i_2} [\left|\rho_{i_1 i_2}\right|^2 + \sum_k \bar{\rho}_{k i_1} \rho_{k i_2} (\bar{u}_{i_1} u_{i_2} + \bar{v}_{i_1} v_{i_2})] t^a_{i_1} t^a_{i_2}$$

$$- i \sum_{i_i \neq i_2 \neq i_3} \sum_k \bar{\rho}_{i_1 i_2} \rho_{i_1 i_3} (\bar{u}_{i_2} u_{i_3} + \bar{v}_{i_2} v_{i_3})] \varepsilon^{abc} t^a_{i_1} t^b_{i_2} t^c_{i_3}$$

2D generalization of the Haldane-Shastry model

Low energy spectrum on the Platonic Solids





The SU(2)@k=2 in 2D is the analogue of the Moore-Read state

In the FQHE the z's are the positions of the electrons or quasiholes

In the spin models the z's parametrize the couplings of the Hamiltonian. They are not real positions of the spins.

Braiding amounts to change these couplings is a certain way.

So in principle one can do topological quantum computation in these spin systems.

But one has first to show that Holonomy = Monodromy

This problem has been recently solved for the Moore-Read state (Bonderson, Gurarie, Nayak, 2010)

Conclusions

- Using CFT we extended the MPS to infinite dimensional matrices

- Description of critical and non critical systems

- Generalization of the Haldane-Shastry model in several directions
    1) inhomogenous
    2) higher spin
    3) degenerate ground states
    4) 1D -> 2D
    5) analogues of non abelian FQHE

Prospects

- Physics of the generalized HS Hamiltonians

- Relation between the iMPS and MERA

- TCQ with HS models

Manolo
Happy 60's