Counting in the Landscape

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1-Introduction

- Two types of parameters
  1-Fixed by physical arguments (uniqueness)
  2-Environamental (diversity)

It is difficult to distinguish!

- Which is the case for
  1-The Standard Model (Quantum ,G_N =0)
  2-The Cosmological Model (class., h=0)

  ?
- Both incomplete. A candidate: String Theory
- An observable for both: The angular stone

Vacuum energy The CC

QFT GR

String Theory!

The CC problem (the greatest crisis)

Theoretical framework to address the problem

i) Eternal Inflation (generic)

- Relaxation of vacuum energy
- Coleman-de Luccia instanton

ii) The string landscape

$-10=(3+1)+6 \quad CY_3 \quad 10^{500}$ vacua

i) + ii) → Multiverse to solve the CC problem (environmentally)

Other ways to attack the CC problem:

1- Dynamical (minimum E)

2- Entropic (maximum S)

3- Symmetric

4- Environmental

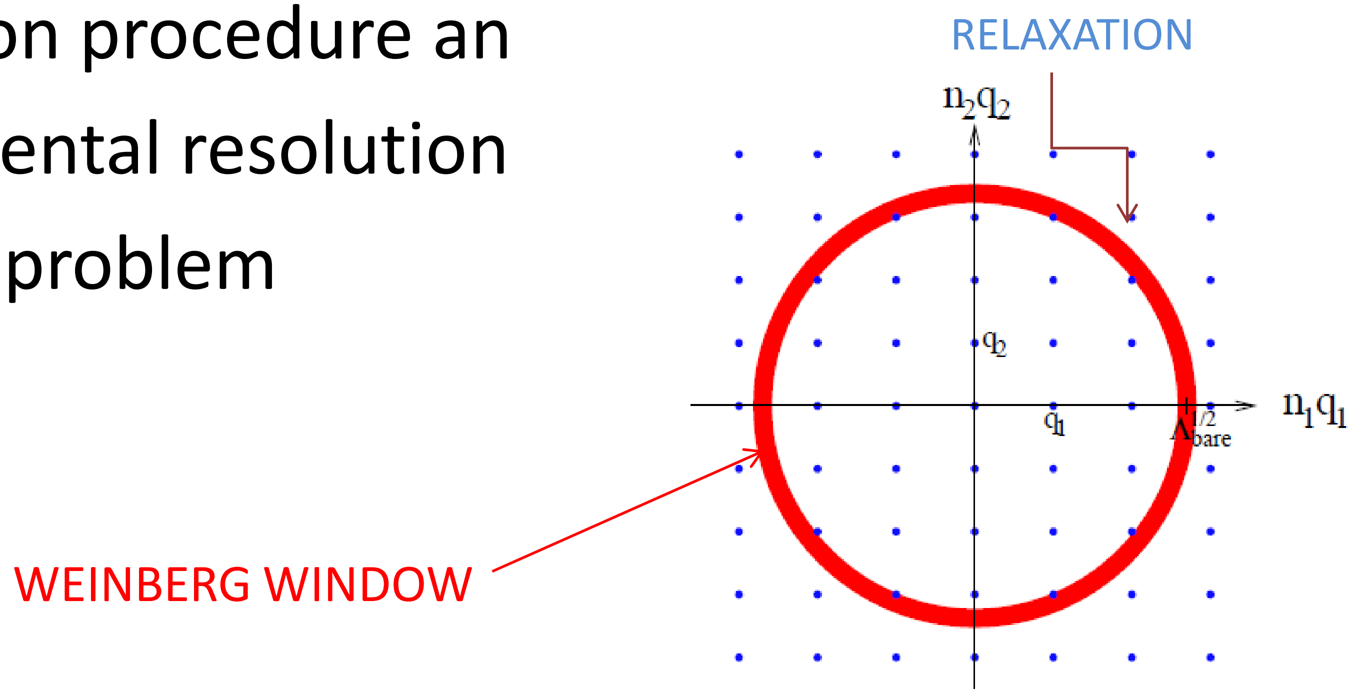
(See Nobbenhuis,S.)

The BP Landscape (generic)

-A large dimensional (J) lattice

-We have a large number of vacua with the desired properties

-By relaxation procedure an environmental resolution of the CC problem



Statistical Description

- Requires a counting procedure
- The naive way to count nodes (BP) has limited validity
- We make an exact count N(h) depending on a ‘t Hooft-like parameter $h = J\, q^2$
- Two asymptotic regimes are obtained

  h-> 0 (BP)

  h-> ∞ (new)

- Using the exact result we obtain a distribution of occupied fluxes

  $\alpha^*(h) = J_{occup} / J_{tot}$

  J small $\alpha^*(h) \approx 1$

  J large $\alpha^*(h) \approx 1/h$

- If the moduli are stabilized by fluxes we have a potential problem. We study a toy model (work in progress)

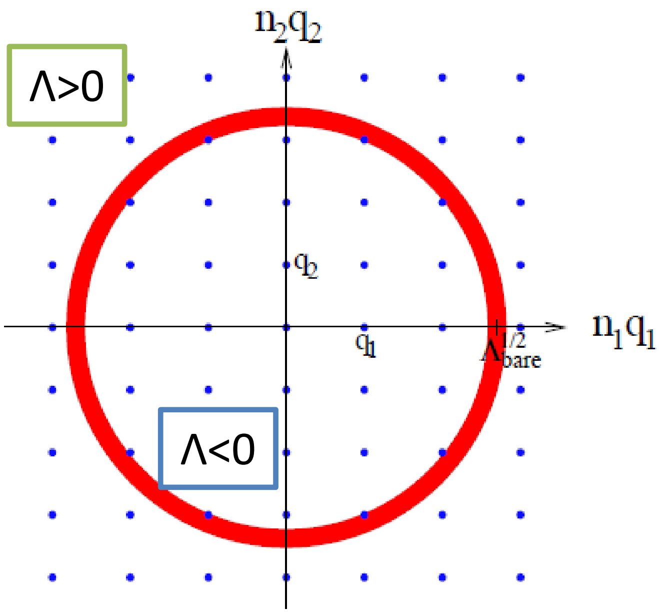
Plan of the talk

1. Introduction
2. Counting in the landscape
3. Typical number of occupied fluxes
4. BP versus KKLT
5. Conclusions and future work

1. Counting in the landscape

$$\Lambda = \Lambda_0 + \frac{1}{2}\sum_{j=1}^{J} n_j^2 q_j^2$$

$(\Lambda \approx - E_0 + \frac{1}{2}\ |E|^2)$

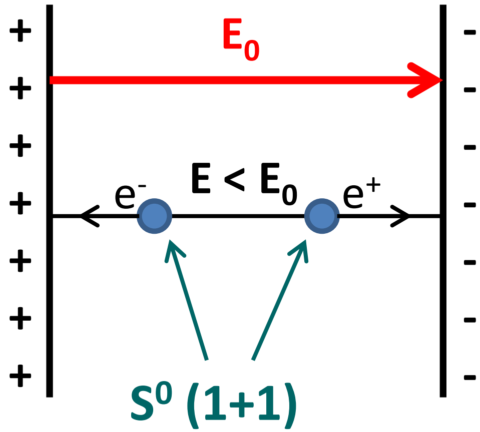


- $\Lambda_0 \approx -1$ $(8\pi G_N = h = c = 1)$
- $(n_1, \ldots n_J)$ integers
- $\{ q_i \}$ i=1,…J J quantized fluxes

An analogy

- 1+1 constant electric field: capacitor

  The production of a pair ($e^+ e^-$),an S^0, reduces the electric field



•M-theory/ 7 dimensional manifold -> 4 dim= 3+1 The reduction of the CC is due to the formation of an S^2 sphere (two legs of the M5 brane, the other three wrapping a 3-cycle of the 7-dim part)

The discretum

- Fluxes quantized => Finely spaced levels=> $\Lambda \neq 0$
- Chances for non environmental mechanisms:

i) Entropic (A. Linde). Use the event horizon + holographic principle

$S \approx \#$ of d.o.f.'s = Area Hor. /4 $\approx \Lambda^{-1}$

$P(\Lambda) \approx \exp S \approx \exp \Lambda^{-1}$

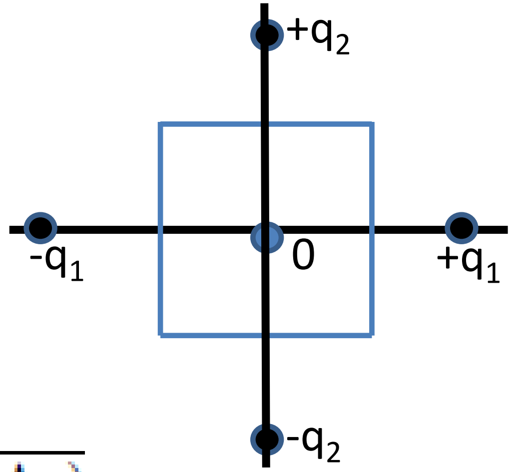
-In the continuum $P(\Lambda)$ not well behaved

-In the discretum the smallest value Λ_0 is strongly peaked

-The next to the smallest $P(\Lambda_1) \approx P(\Lambda_0) \exp(-\Lambda_0)$

ii) Symmetric. Broken by quantum effects $(\Lambda = \Lambda_0)$

The simplest count



- BP count: Divide volumes
- Ball: $B^J(R_\Lambda)$ in flux space ($\Lambda_0 \leq \Lambda \leq \Lambda_1 \approx 1$)
- Voronoi cell Q

$$\mathrm{vol}\, \mathcal{B}^J(R_\Lambda) = \frac{R_\Lambda^J}{J} \mathrm{vol}\, S^{J-1} \qquad R_\Lambda = \sqrt{2(\Lambda - \Lambda_0)}$$

$$\mathrm{vol}\, Q = \prod_{i=1}^J q_i. \qquad \mathrm{vol}\, S^{J-1} = \frac{2\pi^{\frac{J}{2}}}{\Gamma\left(\frac{J}{2}\right)}$$

$$\boxed{\Omega_J(r) = \frac{\mathrm{vol}\, \mathcal{B}^J(r)}{\mathrm{vol}\, Q}}$$

•BP establish the range of validity for its count:
$q_i < R/ J^{1/2}$,for all charges

The large J limit

- In this limit strange things happens
- Whatever the charges, if $J>J_c$:

$$\text{vol } Q > \text{vol } B^J(R) \approx 1/\Gamma(J/2)$$

- This behavior is controlled by the dimensionless 't Hooft-like parameter $h=J(q/R_0)^2$ where $R_0^2=2|\Lambda_0|$ and $q^J=$ vol Q. This strange behavior occurs when:

$$\frac{Jq^2}{R_0^2} > 2\pi e > 17$$

• Assuming a common charge $q_i = q$ there is a value where the semi-diagonal of the Voronoi cell exceeds the radius of the sphere

$$d = q\, J^{1/2} / 2 > R_0 \;\Rightarrow\; h > 4$$

The region near the corner is devoid of states (no isotropy)

We need an exact formula valid for any h

The exact count

- "Brute force"

$$\Omega_J(r) = \left|\{\lambda \in \mathcal{L} \colon \|\lambda\| \leq r\}\right|$$

$$\Omega_J(r) = \sum_{\lambda \in \mathcal{L}} \chi_{[0,r]}(\|\lambda\|)$$

$$\chi_I(t) = \begin{cases} 1 & \text{if } t \in I \\ 0 & \text{if } t \notin I \end{cases}$$

- The density of states is $\omega_J(r) = \dfrac{\partial \Omega_J(r)}{\partial r}$

using

$$\chi_{[0,r]}(\|\lambda\|) = \theta(\|\lambda\|) - \theta\left(\|\lambda\|^2 - r^2\right)$$

we obtain

$$\omega_J(r) = 2r \sum_{\lambda \in \mathcal{L}} \delta\left(r^2 - \|\lambda\|^2\right)$$

Substituting the delta representation

$$\delta\left(r^2 - \|\lambda\|^2\right) = \frac{1}{2\pi i} \int_\gamma e^{s(r^2 - \|\lambda\|^2)} \, \mathrm{d}s$$

$$\gamma = \{c + i\tau : \tau \in \mathbb{R}, c > 0\}$$

Some manipulation

$$\omega_J(r) = \frac{2r}{2\pi i}\int_\gamma e^{sr^2}\left[\sum_{\lambda\in\mathcal{L}} e^{-s\|\lambda\|^2}\right]\mathrm{d}s$$

$$\omega_J(r) = \frac{2r}{2\pi i}\int_\gamma e^{sr^2}\left[\sum_{n_1\in\mathbb{Z}}\cdots\sum_{n_J\in\mathbb{Z}}\prod_{j=1}^{J} e^{-sq_j^2n_j^2}\right]\mathrm{d}s$$

$$= \frac{2r}{2\pi i}\int_\gamma e^{sr^2}\left[\prod_{j=1}^{J}\sum_{n_j\in\mathbb{Z}} e^{-sq_j^2n_j^2}\right]\mathrm{d}s$$

$$= \frac{2r}{2\pi i}\int_\gamma e^{sr^2}\left[\prod_{j=1}^{J}\vartheta(sq_j^2)\right]\mathrm{d}s\,.$$

An old result!

$$\vartheta(s) = \sum_{n\in\mathbb{Z}} e^{-sn^2} \equiv \theta_3(0;e^{-s})$$

A particular case of the Jacobi theta function

$$\theta_3(z;q) = \sum_{n\in\mathbb{Z}} q^{n^2} e^{2\pi inz}$$

Two asymptotic regimes

- S -> 0; ϑ (s) -> (π/s)$^{1/2}$: We reproduces the BP count

$$\omega_J(r) \approx \frac{2r}{2\pi i}\int_\gamma e^{sr^2}\left[\prod_{j=1}^{J}\sqrt{\frac{\pi}{q_j^2 s}}\right] \mathrm{d}s$$

An Inverse Laplace Transform

$$\omega_J(r) \approx \frac{\pi^{\frac{J}{2}}}{\mathrm{vol}\,Q}\frac{2r}{2\pi i}\int_\gamma e^{sr^2}\frac{\mathrm{d}s}{s^{\frac{J}{2}}} = \frac{2\pi^{\frac{J}{2}}}{\Gamma(\frac{J}{2})}\frac{r^{J-1}}{\mathrm{vol}\,Q}$$

The validity of the BP count is given by

$$\boxed{h < \frac{2}{e} \approx 0.736}$$

$$h = \frac{Jq^2}{r^2} \qquad \log q = \frac{1}{J}\sum_{i=1}^{J}\log q_i$$

Two asymptotic regimes

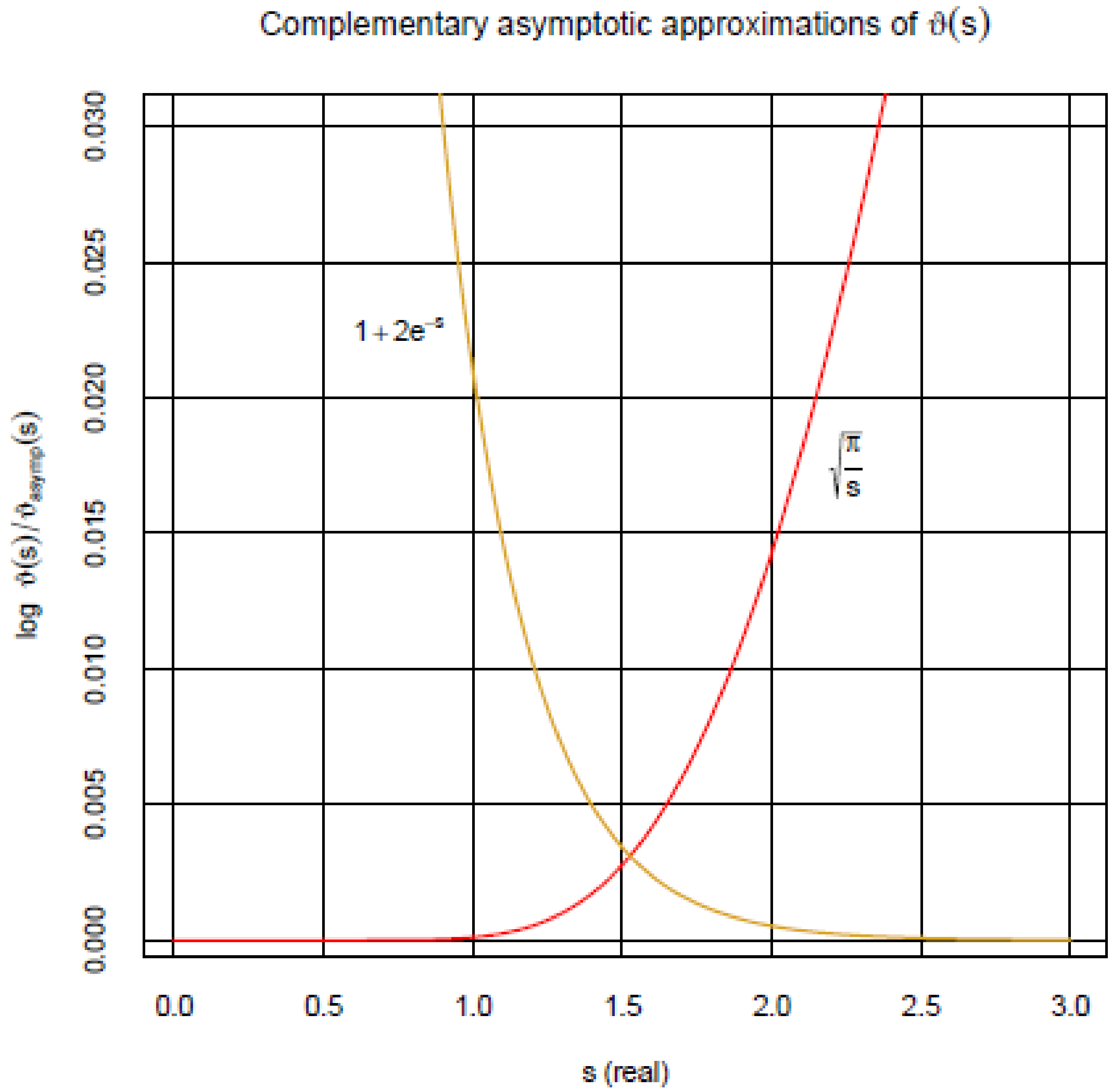
- $\vartheta(s) \xrightarrow{s\to\infty} 1+2e^{-s}$ related with the previous one by Poisson summation formula

$$\vartheta(s) = \sum_{n\in\mathbb{Z}} e^{-sn^2} = \sqrt{\frac{\pi}{s}} \sum_{m\in\mathbb{Z}} e^{-\frac{\pi^2 m^2}{s}} = \sqrt{\frac{\pi}{s}}\, \vartheta\left(\frac{\pi^2}{s}\right)$$

- We cannot solve the saddle point equation in closed form unless $q_i = q$ (i=1,...J)

$$\omega_J(r) = \frac{(2h-2)^{\frac{J}{h}}}{q\sqrt{2\pi h}} \left(\frac{h}{h-1}\right)^{J+\frac{1}{2}}$$

Valid for $\frac{Jq^2}{r^2} > 1 + \frac{e^2}{2} \approx 4.694$



Complementary asymptotic approximations of ϑ(s)
0.000
0.005
0.010
0.015
0.020
0.025
0.030
log ϑ(s) / ϑ_asymp(s)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
s (real)
1+2e^-s
√(π/s)

3-Typical number of occupied fluxes

- Take a shell of width $\varepsilon = R_\varepsilon - R$ with $\varepsilon << q_i$ but with a large number of nodes $N_\varepsilon >> 1$
- We can count the number of states on the shell, e.g. the WW: $R = R_0 = (2|\Lambda_0|)^{1/2}$

$$R_\varepsilon = \sqrt{2(\Lambda_\varepsilon - \Lambda_0)} \approx R_0 + \frac{\Lambda_\varepsilon}{R_0}$$

- The with of the shell is: $\varepsilon = \Lambda_\varepsilon / R_0$

$$0 \leq \Lambda \leq \Lambda_\varepsilon = \Lambda_{ww}$$

- The number of states in the anthropic shell is

$$\mathcal{N}_{\mathrm{WW}} = \frac{\omega_J(R_0)}{R_0} \Lambda_{\mathrm{WW}}$$

Fraction of occupied fluxes

- i) Take a state randomly on the shell

  ii) Find the typical # of non-vanishing components

This number is J for J= 1,2,…

Q: What happens for large J ?

- We compute the fraction of states in the shell having a fraction α of turned on fluxes
- Selecting the state at random with unif. prob. => α a discrete random variable [0,1]
- Assuming equal charges

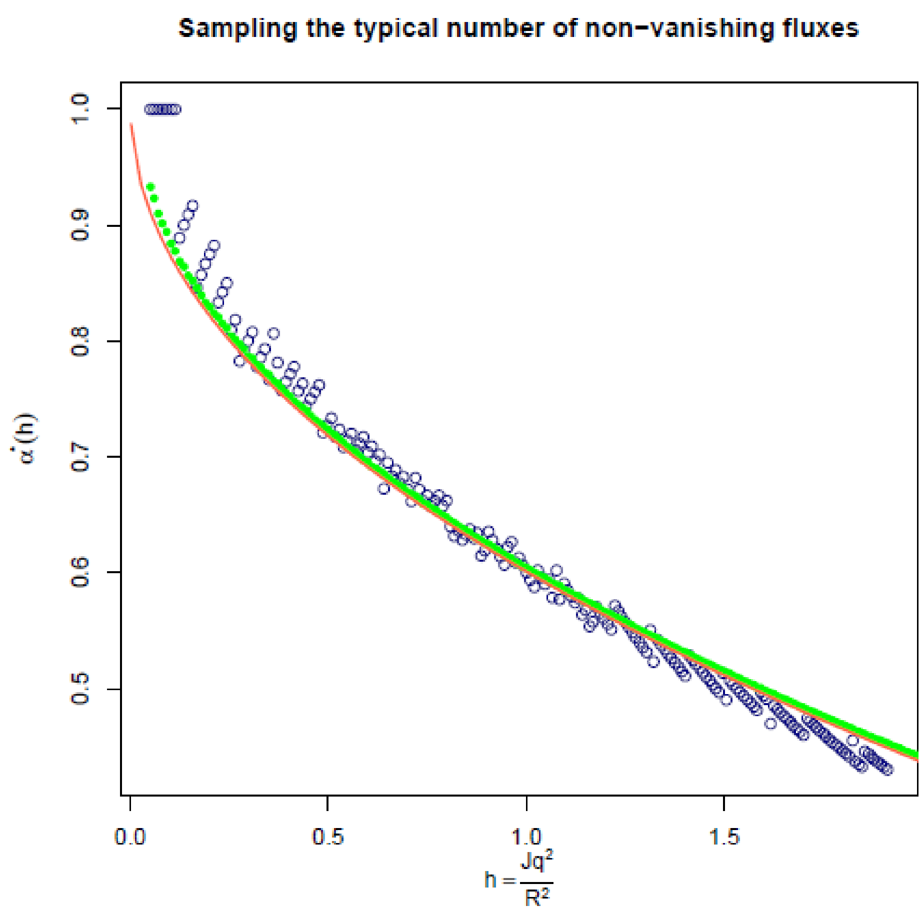
$$P(\alpha) = \frac{2R}{\omega_J(R)} \binom{J}{\alpha J} \frac{1}{2\pi i} \int_\gamma e^{\phi(s,\alpha)}\, \mathrm{d}s$$

with

$$\phi(s,\alpha) = sR^2 + \alpha J \log\left[\vartheta(q^2 s) - 1\right]$$

Results on # of fluxes ≠ 0

- P (α) Gaussian around its peak α^* with standard deviation $1/J^{1/2}$
- $J\,\alpha^*(h)$= typical # of occupied fluxes on the shell essentially also on the whole lattice



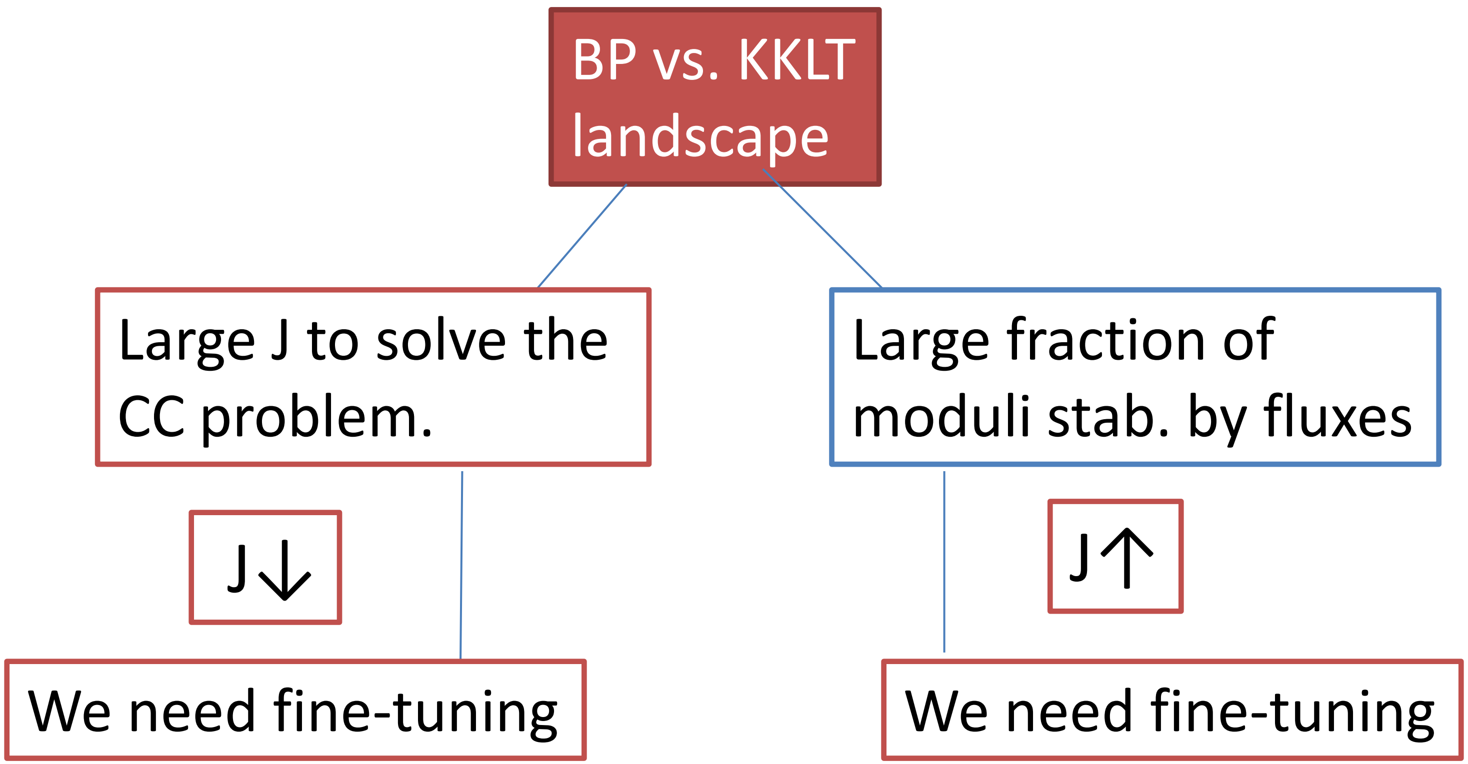
- The eff. dimension of the lattice is $J_{eff} = J\,\alpha^*$.
- When $\alpha^* \neq 1$: a "fractal" lattice!
- For large h, $\alpha^* \approx 1/h$

4- BP vs KKLT

- A potential problem (to be tested)

  i) Consider a landscape with a large fraction of moduli stabilized by fluxes

  ii) Use this landscape to address anthropically the CC problem



A Toy Model (caution! Work in progress)

- The simplest case to

  i) count fluxes in the landscape

  ii) study moduli stabilization by fluxes

- 4D Einstein-Maxwell 4=2(1+1) (cosmological)

  (6D see A. Vilenkin) + 2 ($\kappa=S^2$,vol=V)

  Only one modulus ! The S^2 Volume

$$\mathrm{d}s^2 = e^{2\phi(x,t)}\left(-\mathrm{d}t^2 + \mathrm{d}x^2\right) + e^{2\psi(z,w)}\left(\mathrm{d}z^2 + \mathrm{d}w^2\right)$$

- Λ (4D) > 0 (dS)
- Monopole: $\mathbf{F} = \frac{Q}{V} e^{2\psi(z,w)} \mathrm{d}z \wedge \mathrm{d}w$

$$V = \mathrm{vol}\, \mathcal{K} = \int_{\mathcal{K}} e^{2\psi(z,w)} \mathrm{d}z \wedge \mathrm{d}w \qquad \int_{\mathcal{K}} \mathbf{F} = Q$$

The Einstein- Liouville equations

$$(\phi_{tt} - \phi_{xx})e^{-2\phi} = \Lambda - \left(\frac{Q}{V}\right)^2 = \lambda$$

$$-(\psi_{zz} + \psi_{ww})e^{-2\psi} = \Lambda + \left(\frac{Q}{V}\right)^2 = K$$

Gaussian curvatures

- g=0 and *K* constant and Gauss-Bonnet =>

$$\frac{1}{2\pi}\int_K Ke^{2\psi}\mathrm{d}z\mathrm{d}w = 2 \quad \Rightarrow \quad \frac{KV}{2\pi} = 2 \quad \Rightarrow \quad V = \frac{4\pi}{K}$$

- An algebraic relation for *K* with two branches

$$K = \Lambda + \left(\frac{Q}{V}\right)^2 = \Lambda + \left(\frac{QK}{4\pi}\right)^2$$

$$K_\pm = 2\Lambda\left(\frac{Q_{\max}}{Q}\right)^2\left[1 \pm \sqrt{1 - \left(\frac{Q}{Q_{\max}}\right)^2}\right]$$

A maximum charge

$$Q_{\max} = \frac{2\pi}{\sqrt{\Lambda}}$$

- If $Q > Q_{max}$, K is complex, the compact part collapses => a singularity
- Two branches also for the 2D CC $\lambda_{\pm} = 2\Lambda - K_{\mp}$

•The Dirac Q condition: Q e =2πn => $n_{\max} = \left\lfloor \frac{e}{\sqrt{\Lambda}} \right\rfloor$

•Our landscape has $0 \leq |n| \leq n_{max}$

•n=0 only one branch $K=\lambda=\Lambda$. Unstable, not supported by the em field

•If $\Lambda < 0$, $K_- < 0$ and only the K_+ branch with $\lambda_- = 2\Lambda - K_+ < 0$ (always) n not restricted

Modulus stabilization (a minimal charge)

$$ds^2 = e^{2\phi(t,x)-2\xi(t,x)}\left(-dt^2+dx^2\right) + e^{2\psi(z,w)+2\xi(t,x)}\left(dz^2+dw^2\right)$$

Non compact

Exchange V

Compact

- Stabilization condition => $Q > Q_{\min} = \frac{2\sqrt{2}}{3} Q_{\max}$

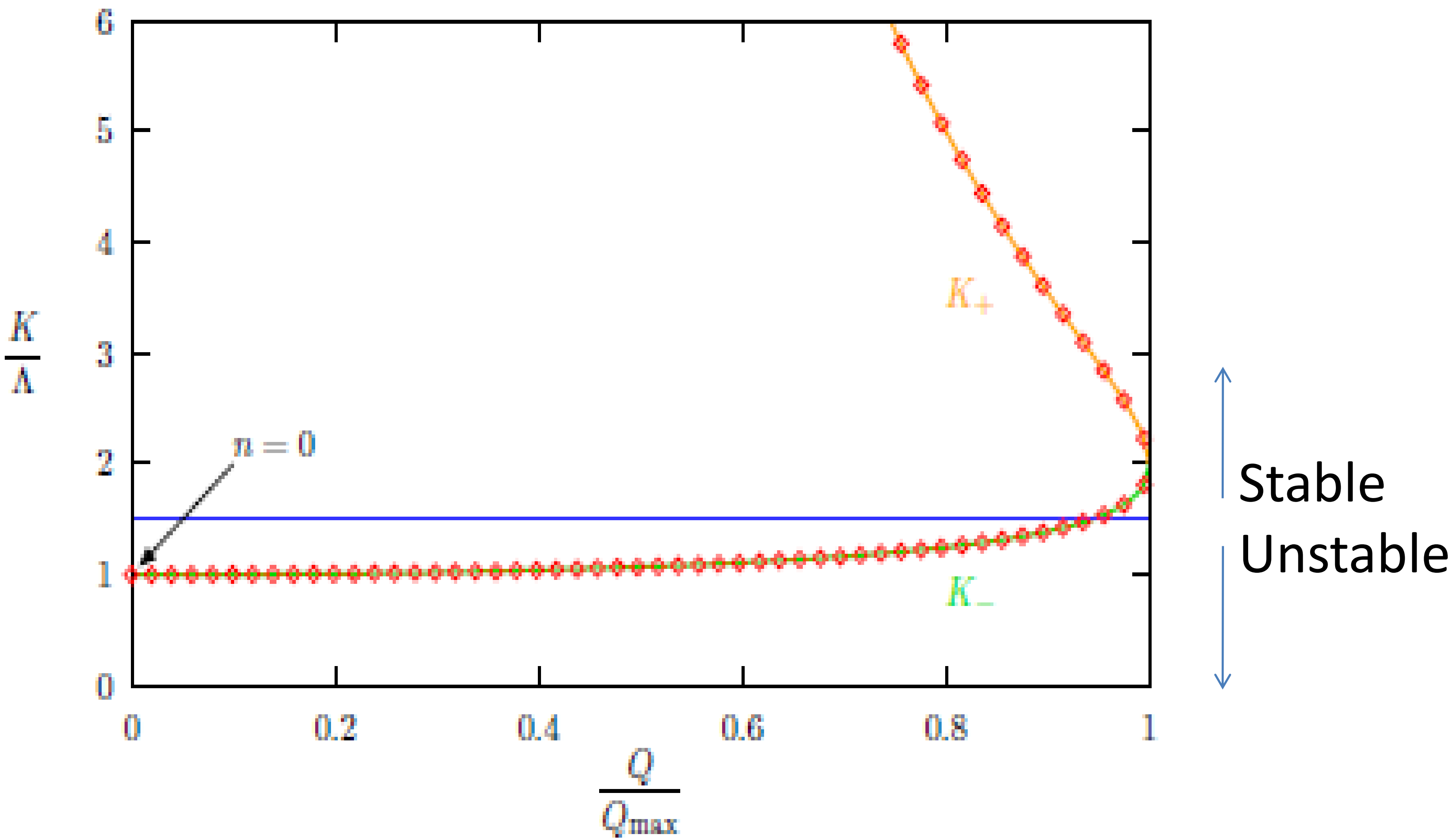
  otherwise decompactification

- All states in the dS_2 branch $n \geq n_{\min} = \left\lceil \frac{Q_{\min} e}{2\pi} \right\rceil$

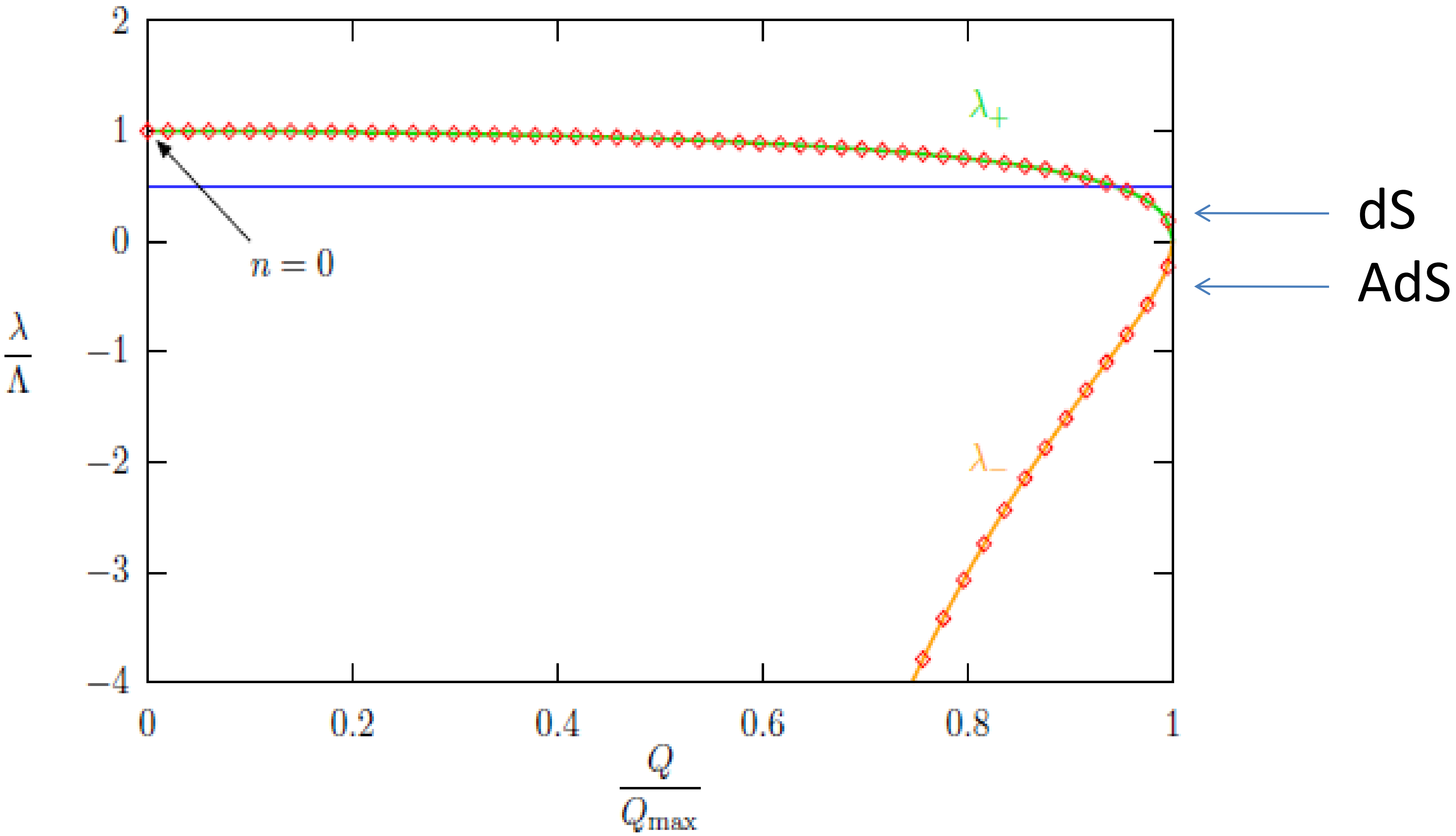
  with $n < n_{min}$ are unstable (including n=0)

- Number of stable states in this one-flux landscape

$$\mathcal{N} = \underbrace{n_{\max}}_{\mathrm{AdS}_2} + \underbrace{n_{\max} - n_{\min} + 1}_{\mathrm{dS}_2} = 2\left\lfloor \frac{e}{\sqrt{\Lambda}} \right\rfloor - \left\lceil \frac{2\sqrt{2}}{3} \frac{e}{\sqrt{\Lambda}} \right\rceil + 1 \approx \frac{e}{\sqrt{\Lambda}}\left(2 - \frac{2\sqrt{2}}{3}\right)$$



$U_{eff}''(\xi=0) = 2K - 3\Lambda > 0$



Q_{max} = 50.265 => N =53 (50 AdS, 3 dS)

5- Conclusions

1. We have developed an exact way to count on a BP landscape
2. Two asymptotic regimes, controlled by a ‘t Hooft- like parameter (h), have been studied
3. We have obtained the typical fraction of active fluxes $\alpha^*(h)$. For large h, $\alpha^* \approx 1/h$
4. We speculate on the tension between a large J (to solve the CC problem) and the previous result
5. We have begun to explore the landscape of a toy model. Preliminary results for the one-flux case are presented
6. The extension to a large number of moduli using a $g>0$ surface is under scrutiny

¡FELICIDADES MANOLO!

80’s

90’s

¡FELICIDADES MANOLO!

00’s

GRACIAS

A richer landscape (g>0)

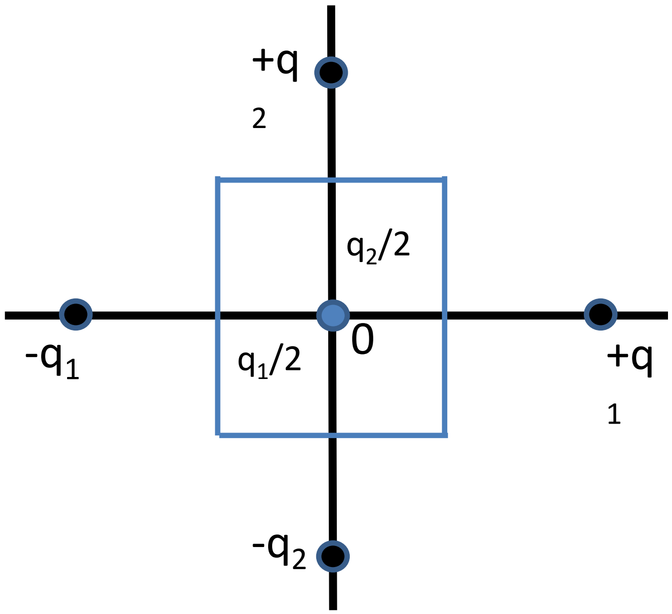
- A complex Riemann curve

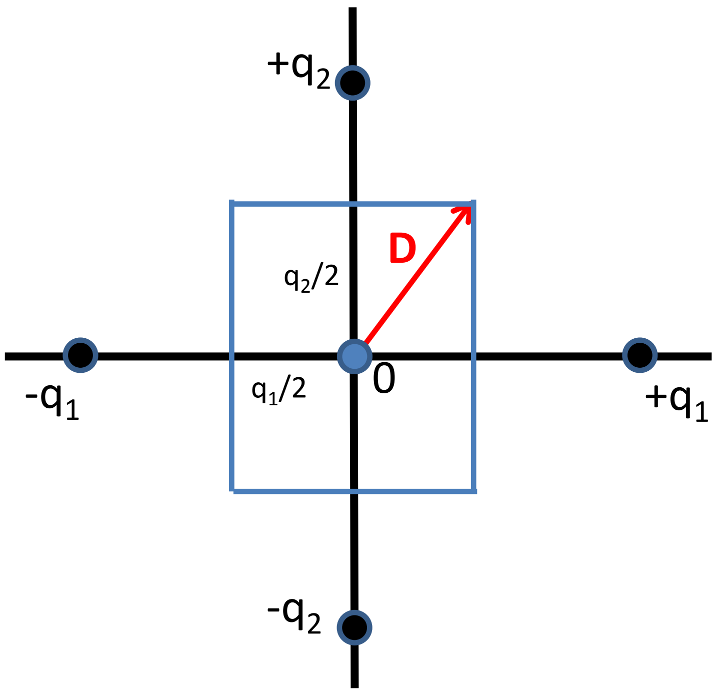
$$y^2 = P_k(u)$$

$$P_k(u) = u^k + a_{k-2}u^{k-2} + \cdots + a_1 u + a_0$$

The genus is related with the polynomial degree by

k=2g+2


+q
2
q2/2
-q1
0
q1/2
+q
1
-q2



+q2
D
q2/2
-q1
q1/2
0
+q1
-q2