To be announced

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1

1

Hommage to Manolo Asorey

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Considerations on Non-perturbative Quantum Field Theory

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HOWEVER

- Original formalism is somewhat ad hoc, involving the **Regularization and Renormalization** procedure.
- Most of the tests involve perturbative calculations.
- Gravity is not included

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- Strong coupling is limited to a few parameters and constants of the theory
  GOOD: One can do Physics at low energies
  BAD: Information about high energies is hidden
- Problem: High energy parameters take absurd or extremely constrained values (Standard Model).
- Different ideas (SUSY, little Higgs, extra dimensions, technicolor)
  $\Rightarrow$ NEW PHYSICS is around the corner (LHC)

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- There are genuine non-perturbative phenomena
- How can we compute at strong coupling? (Resummations?)
- Path integrals in field theory poorly understood mathematically

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- How can we compute at strong coupling? (Resummations?)
- Path integrals in field theory poorly understood mathematically
  $\Rightarrow$ Do we understand quantum field theory?
  (Example: Technicolor)

The lattice approach

Wilson gave a formulation of quantum field theory which has many advantages:

- It is a First Principles formulation (Definition of QFT)
- It is not tied to any particular calculational technique (PT, semiclassical approach, strong coupling, etc)
- All approximations involved are quantifiable and systematically improvable
- Compatible with the gauge principle
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It has also disadvantages:

- Cumbersome (breaks symmetries)
- Frequently a **BLACK BOX**

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  My personal opinion is:

YES

- A good candidate is Yang-Mills theories (A big challenge)
  Presumably many others like N=4 SUSY Yang-Mills
- It is not so clear about the Standard Model. Perhaps the theory is effective and something else is needed at high energies (STRING THEORY?)

Non-perturbative methods

Several non-perturbative methods have been devised and applied successfully to several theories.

- **Monte Carlo methods**: Efficient importance sampling numerical techniques to deal with lattice path integrals.
- The semiclassical approach: A weak coupling approach which goes beyond perturbation theory. Based on non-trivial local minima of the action.
- **1/N expansion**: Expansion in the the number of internal degrees of freedom. Leading order (Large N).
- Strong coupling expansions
- Extrapolations in the space-time volume: Appropriate in AF gauge theories.
- Strong Extra symmetries: Supersymmetry, Conformal Field Theories.
- Ads/CFT: Combines many of the previous ideas.

Combining LGT with other non-perturbative methods

- The previously mentioned non-perturbative methods are not mutually exclusive.
- Interesting to combine different pieces of information.
- Unfortunately conformal invariance and supersymmetry are broken by the lattice approach.
- Topological features are also broken. Semiclassical approach is viable.
- Large N obtained via a limiting procedure.
- Formulation on manifolds constrained to the torus.

Our group in Madrid has been involved in some aspects of this program.
Here I report here on an ongoing programme to understand certain aspects of large N gauge theories on the lattice.

Large N on the lattice

- $1/N$ expansion is originally based upon perturbation theory. Suggests a string picture. Large N still unsolved for gauge theories.
- The Large N limit introduces important simplifications in several approaches.
- On the lattice large N results are obtained extrapolating from finite N (Teper et al)

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- On the lattice large N results are obtained extrapolating from finite N (Teper et al)
- Important simplification found in the 80's

Reduction (Eguchi-Kawai)

Schwinger-Dyson equations for Wilson loops adopt a simple form on the lattice
Expectation values of Wilson loops become volume independent in the large N limit (Assuming Z_N^4 symmetry is unbroken) $\Rightarrow$
Eguchi-Kawai model: LGT in one point

Reduced models: Early history

- Reduction breaks down at weak coupling because the classical vaccuum breaks the symmetry.

Possible solutions were presented

- **Quenched EK model (Bhanot, Heller, Neuberger)** The eigenvalues of the link variables are quenched. At weak coupling they play the role of momenta ($N = L$).
- **Twisted EK model (A.G-A, Okawa)** Use twisted boundary conditions (magnetic flux).

$$S = bN \sum_{\mu,\nu} z_{\mu\nu} \mathrm{Tr}(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger)$$

No breaking of the symmetry found at all values of the coupling
This a 4-matrix model which is proposed to be equivalent to Yang-Mills in euclidean space-time (Not solved)

TEK at weak coupling (AGA-Okawa 1983)

- At weak coupling $Z_N^4 \longrightarrow Z_N^2 = Z_{\sqrt{N}}^4$ with appropriate choices of $z_{\mu\nu} \in \mathcal{Z}_N$.

$$U_\mu = \Gamma_\mu$$

- Space-time is embedded in the group:

$$\lambda^a \longrightarrow \lambda(\vec{p})$$

Finite N is like $(\sqrt{N})^4$ lattice

- Feynman rules can be computed. The vertices contain momentum dependent phases of the form

$$\exp\{iN\theta_{\mu\nu}p_\mu p_\nu\}$$

- Phases cancel for planar diagrams. Produce oscillatory cancelations for non-planar ones.
- Continuum version (A.G-A-Korthals Altes) ⇒ QFT on non-commutative space

Recent results

Many new results have been obtained in recent years

- Narajanan and Neuberger claimed that volume independence at large N still holds in EK for $l > l_c$. On the lattice both L and N are kept finite.
- Bringoltz and Sharpe showed the for the QEK model there is symmetry breaking associated with non-straight Polyakov loops.
- Vairinhos and Teper showed numerically that for $N > 100$ Z_N symmetry is broken at intermediate couplings β. Prevents continuum limit of TEK $a(\beta_c)\sqrt{N} \longrightarrow 0$.
- Kovtun, Unsal Jaffe argued for volume independence in YM with massless fermions in the adjoint rep. For ex. $S_1 \times R^3$
- large N gauge theory with Adjoint quarks is another large N limit: QCD(AS) $\equiv$ QCD (for SU(3))
  QCD(AS) $\equiv$ Adj QCD (for SU(∞)): Orientifold Planar equivalence

TEK revisited

The reason why TEK fails was analysed by Teper: There are several local mimima of the action. Some break the symmetry and some don't. Although, the ground state respects the $Z^4_{\sqrt{N}}$ symmetry, other local minima don't. First order phase transitions might occur if

$$b\Delta E - \Delta S = 0$$

One expects $\Delta S \approx \mathcal{O}(N^2)$, while in some cases $\Delta E \approx \mathcal{O}(1)$.

SOLUTION

By changing $z_{\mu\nu}$ (increasing the flux ($m \propto N$) through each face) one gets $\Delta E \approx \mathcal{O}(N^2)$, while keeping the perturbative reduction unchanged.

Numerical Results show no symmetry breaking up to N=841 starting from the ordered state.

How can we make sure there is no breaking at higher N?

Testing the idea

In the absence of a proof we adopted a pragmatic view: Check for the behaviour of physical quantities ▸ Click Work in progress (Okawa and A.G-A)

Hamiltonian analysis in 2+1 dimensions

- A simpler starting point: $L^2 \times \mathcal{R}$.
- We aim at understanding the spectrum of YM theory as function of L, N, magnetic flux m and electric flux $\vec{e}$.
- For small spatial size L one can compute the spectrum in perturbation theory. Dependence goes through LN and $0 < \theta(k, N) < 1$ (REDUCTION)
- The model has also non-trivial local extrema of the energy. These minima do not show REDUCTION and might spoil it.

Work in collaboration with M. Garcia Perez and Okawa

Future projects

The next step would be that of exploring the Hamiltonian description in 3+1 at large N.
It is very appealing to consider the study of adjoint quarks, which have a number of nice properties:

- For $N_f = 1$ they are supersymmetric
- At large N we expect *Volume independence* (Narayanan + Hietanen)
- Are candidates for walking technicolor theories
- Are compatible with twisted boundary conditions which should improve approach to volume independence.

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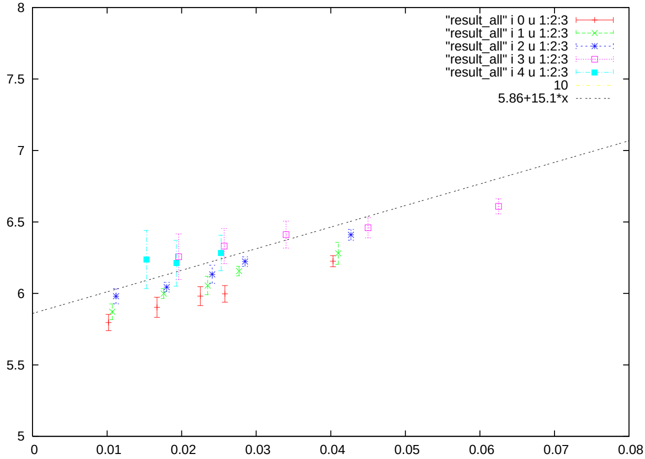
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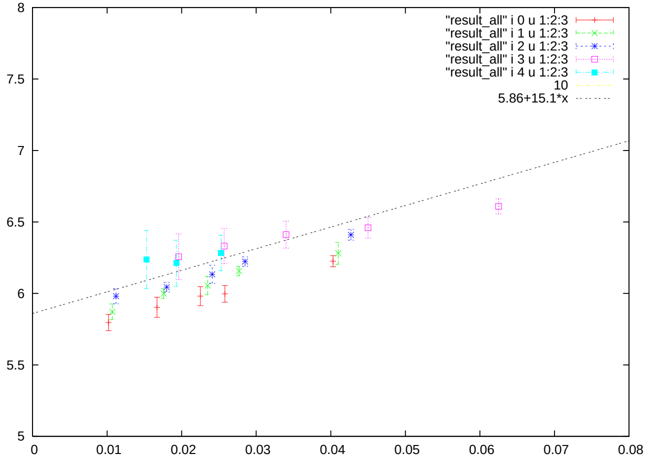

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- No experimental evidence. Hard to test
- Harder to compute.
- Does not solve the main hierarchy problem: Cosmological Constant

Anthropic principle

The proposed solution to the Cosmological Constant problem.

- Is it Philosophy or Physics? Does it have any precise (not a posteriori) predictions?
- What is its precise formulation? Are humans necessary?
- What are its limits? Can it be applied to the mass of the electron, the fine structure constant or the mass of the Higgs? Was there a problem in the first place with our fine-tuned field theoretical parameters?