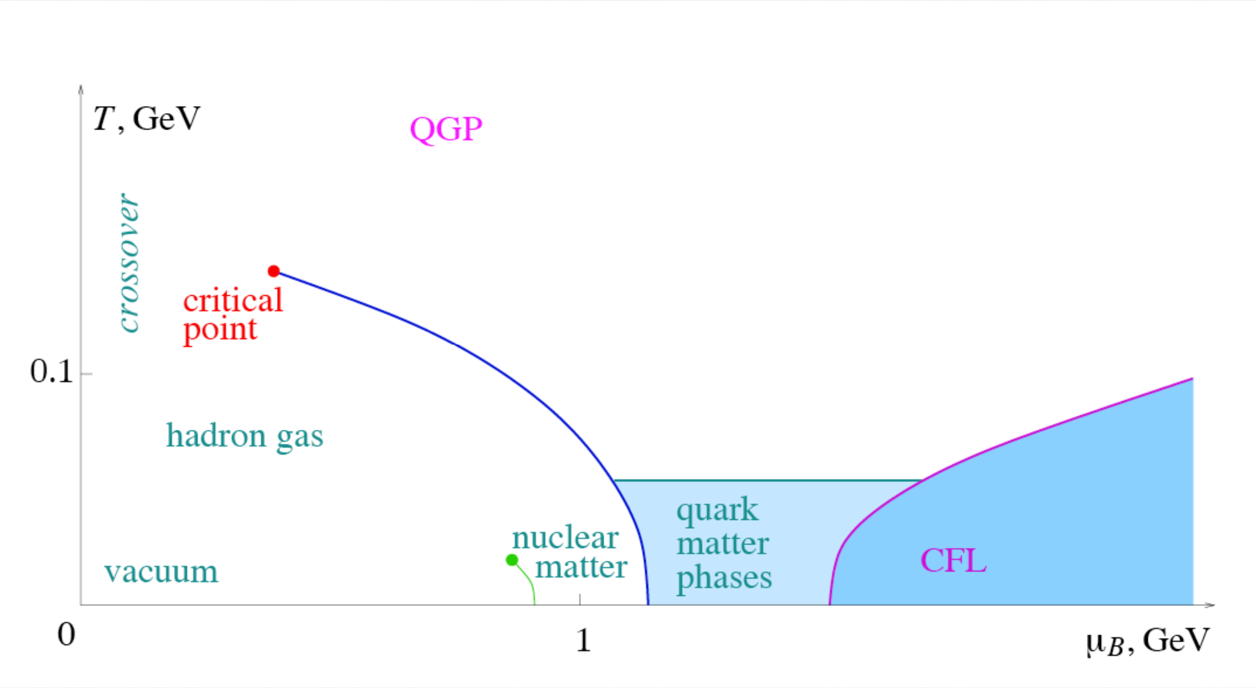
Gross-Neveu Condensates: Integrability at Work

Gerald Dunne
University of Connecticut

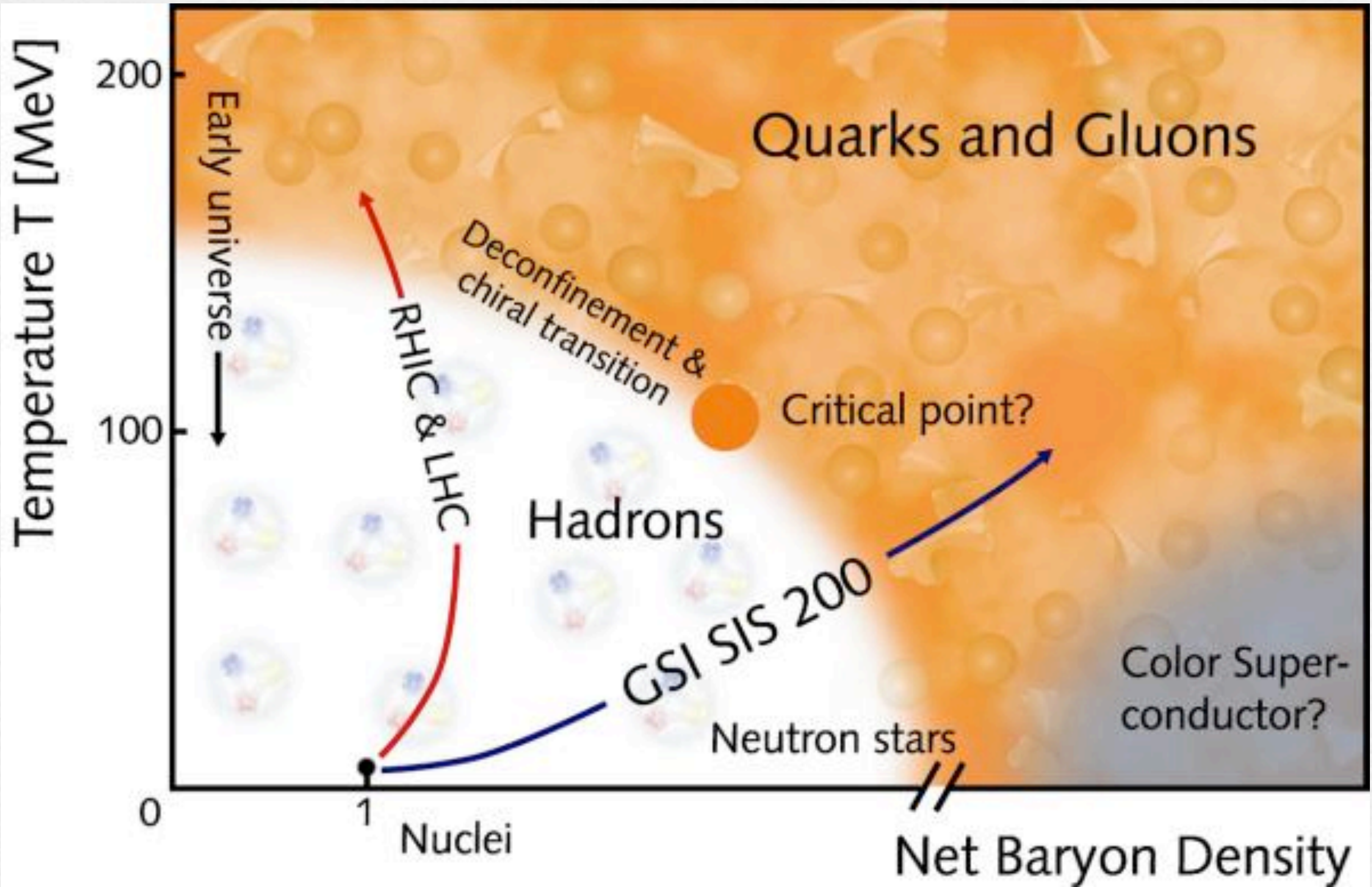
"What is Quantum Field Theory"
Asorey-Fest, Benasque, 2011

phase diagram of quantum chromodynamics (QCD)



important symmetries:

- chiral symmetry
- confinement/deconfinement



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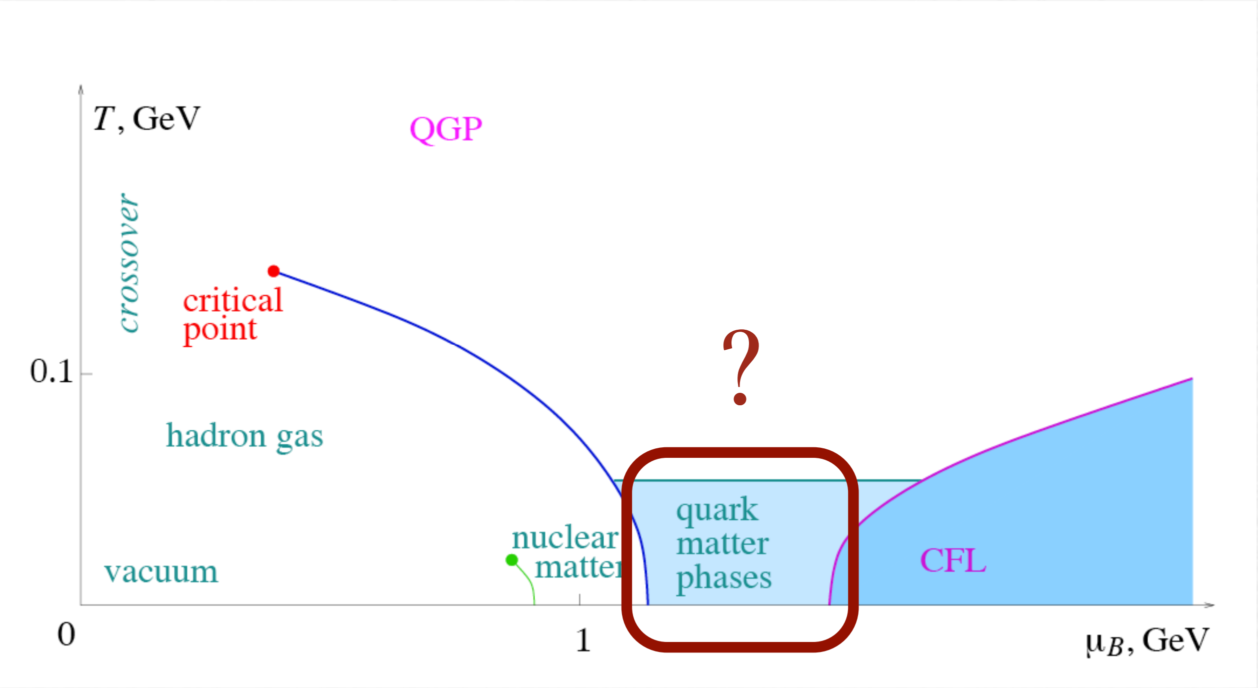

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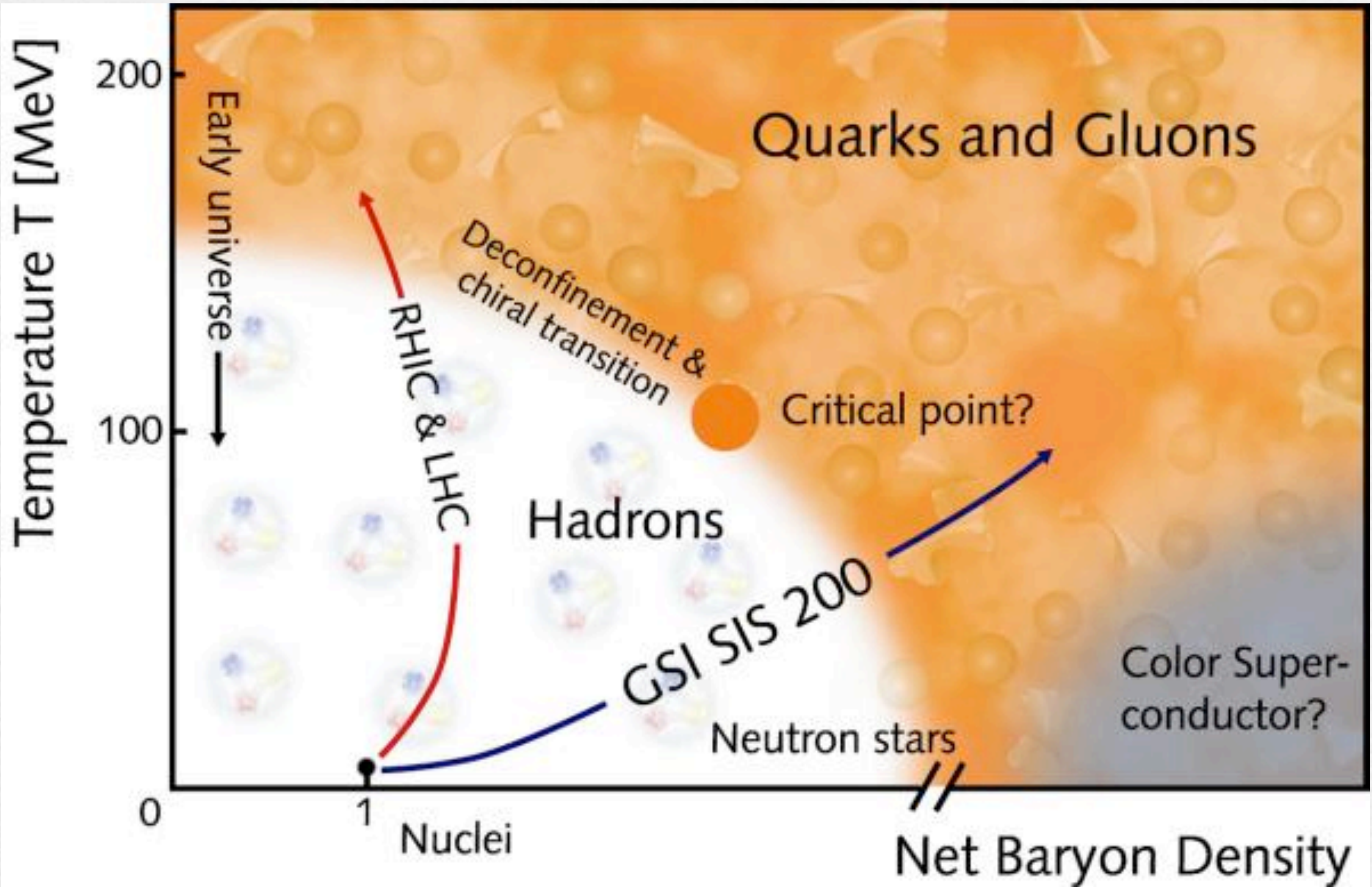

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

“NJL model”

PHYSICAL REVIEW D VOLUME 10, NUMBER 10 15 NOVEMBER 1974

Dynamical symmetry breaking in asymptotically free field theories*

David J. Gross† and André Neveu
Institute for Advanced Study, Princeton, New Jersey 08540

(Received 21 March 1974)

“GN model”

describe chiral symmetry breaking

specific problem:

what is the phase diagram of GN/NJL models as function of temperature T and chemical potential μ ?

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what is [a nontrivial fermionic] QFT [at finite T and μ] ?

Gross-Neveu Models

Gross/Neveu, 1974
Nambu/Jona-Lasinio, 1961

GN$_2$ $$\mathcal{L}_{\mathrm{GN}} = \bar{\psi}\, i\, \partial\!\!\!/ \psi + \frac{g^2}{2}\left(\bar{\psi}\psi\right)^2$$ $\psi \to \gamma^5 \psi$

χGN$_2$
NJL$_2$ $$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\, i\, \partial\!\!\!/ \psi + \frac{g^2}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i \gamma^5 \psi\right)^2\right]$$ $\psi \to e^{i\alpha\,\gamma^5}\,\psi$

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- renormalizable
- asymptotically free
- chiral symmetry breaking
- large N_f limit

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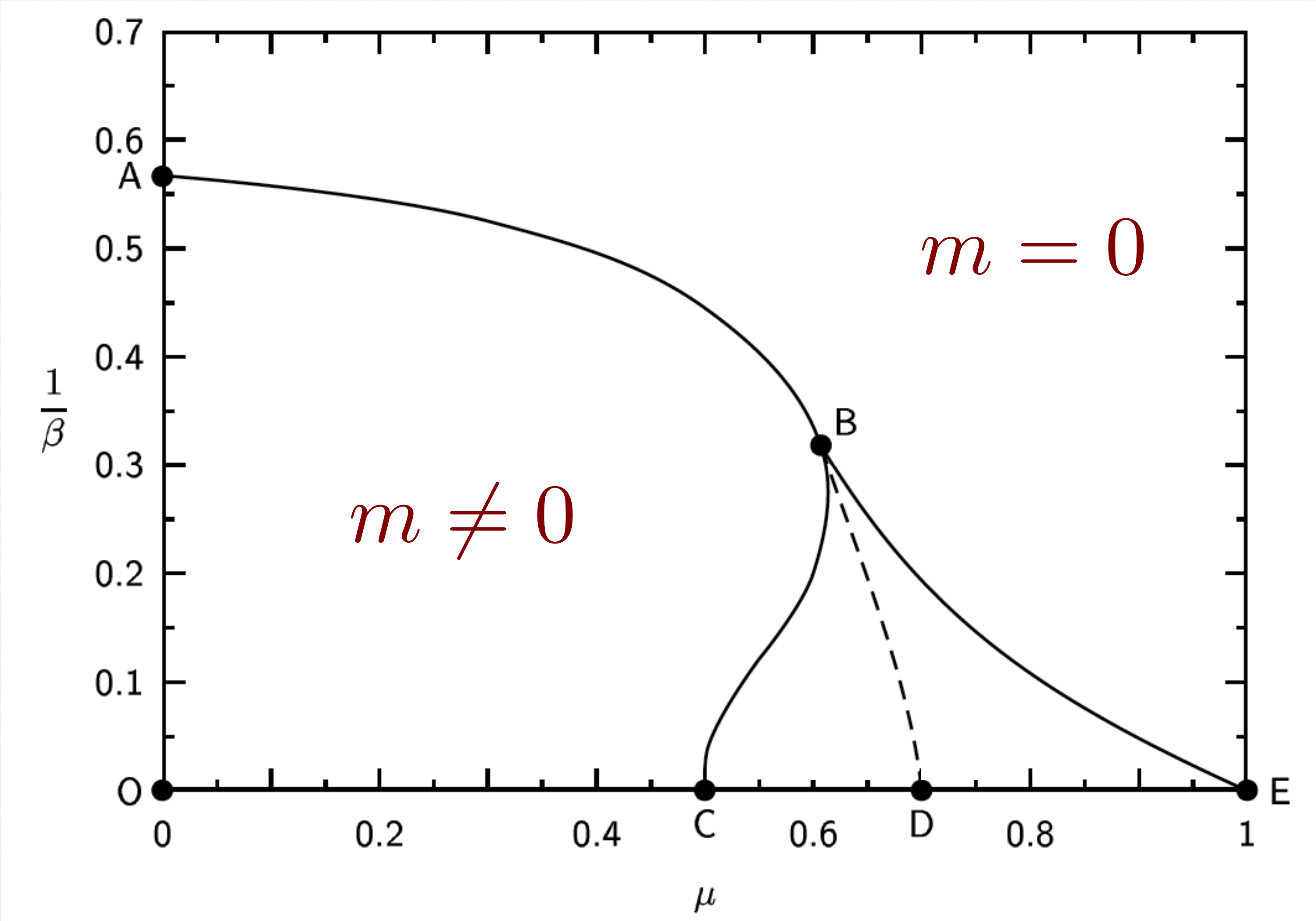
(T, μ) phase diagram?



Phase diagram of Gross-Neveu model

uniform condensate

Wolff, 1985

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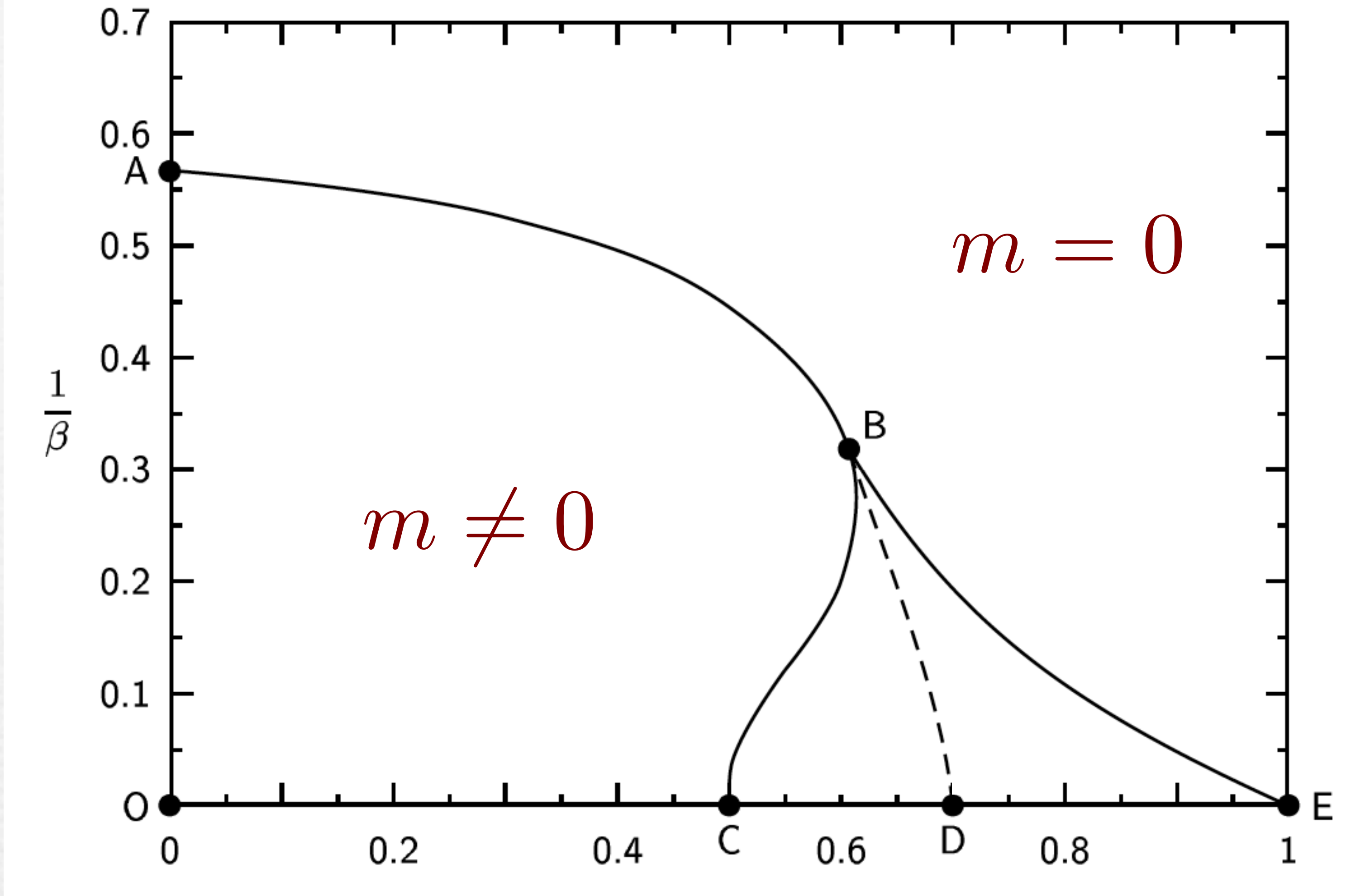

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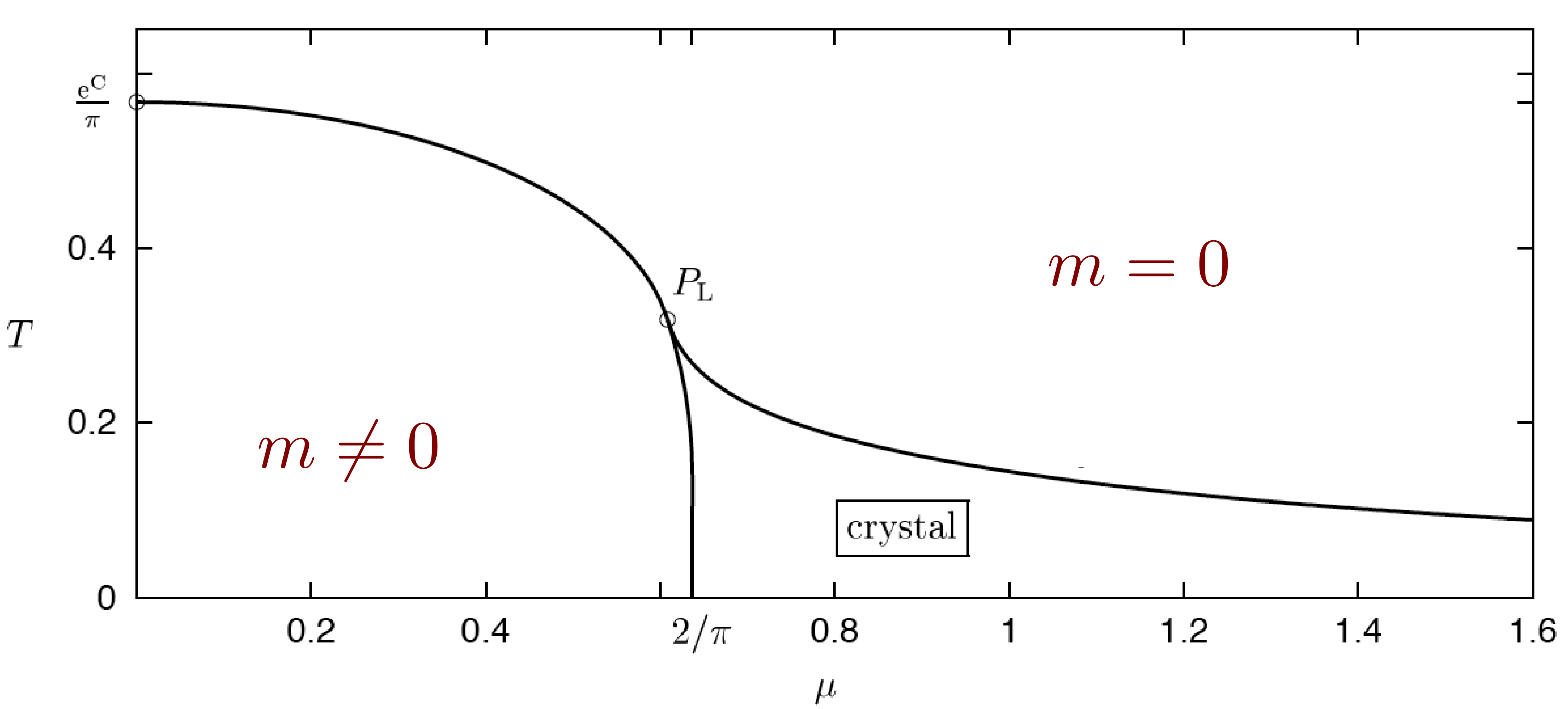

Thies & Urlichs, 2005

Basar, GD, Thies, 2009

periodic, crystalline, phase

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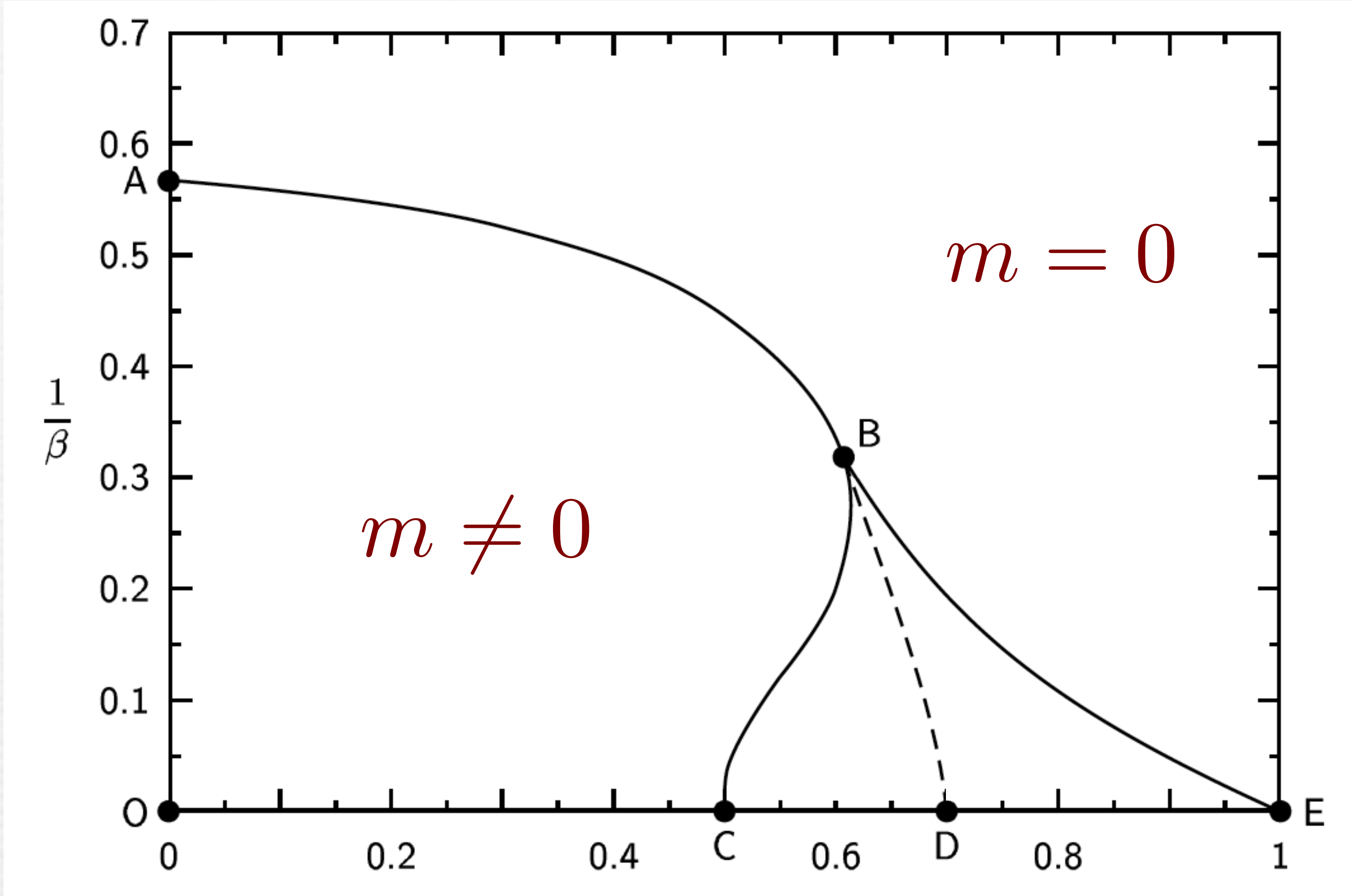

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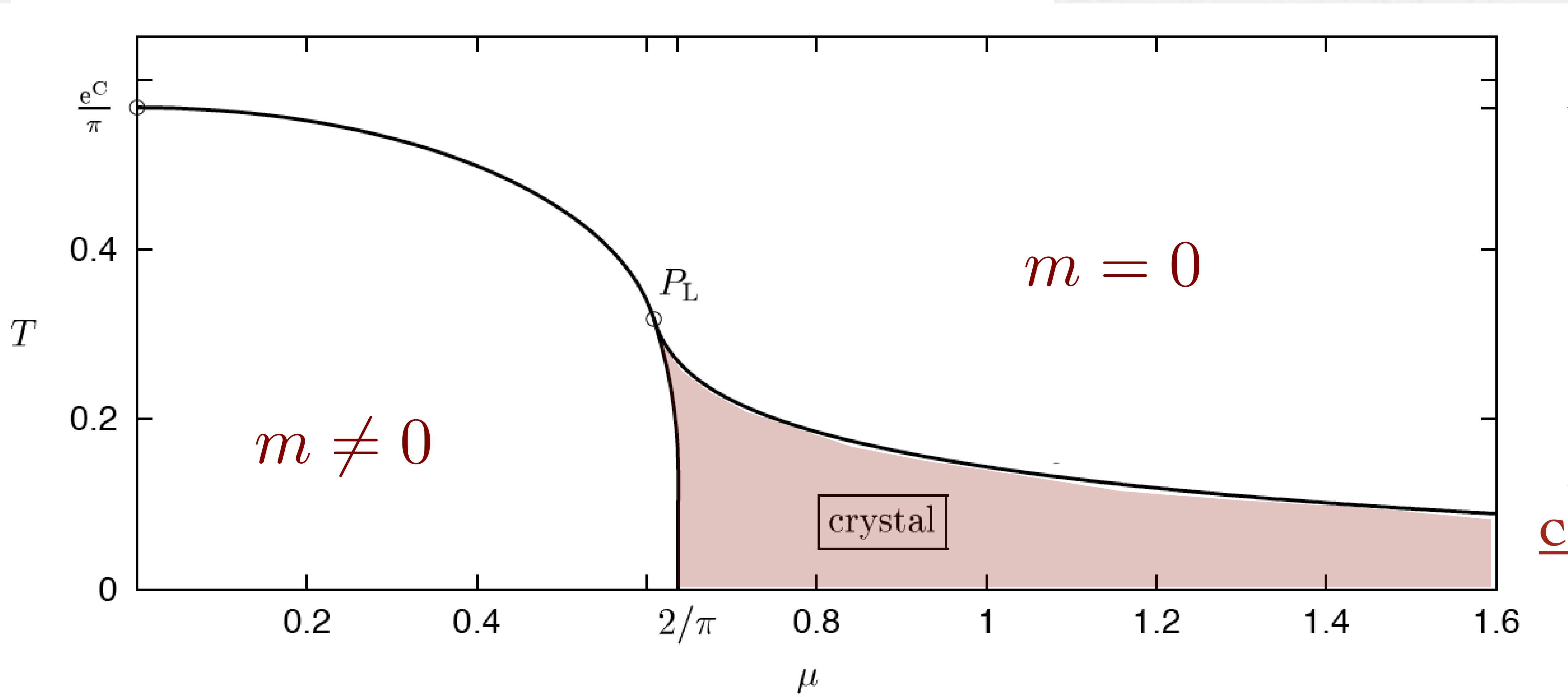

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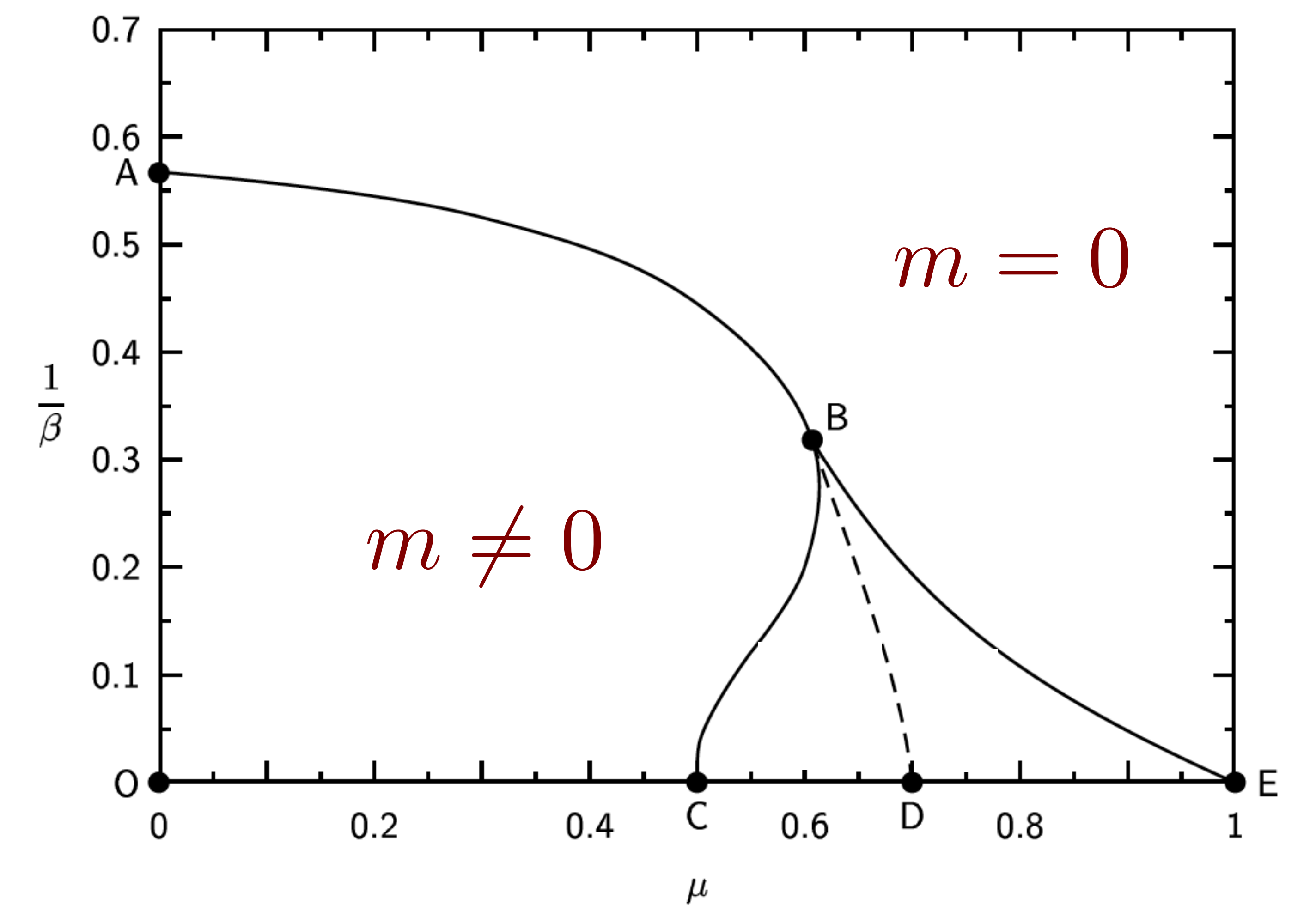
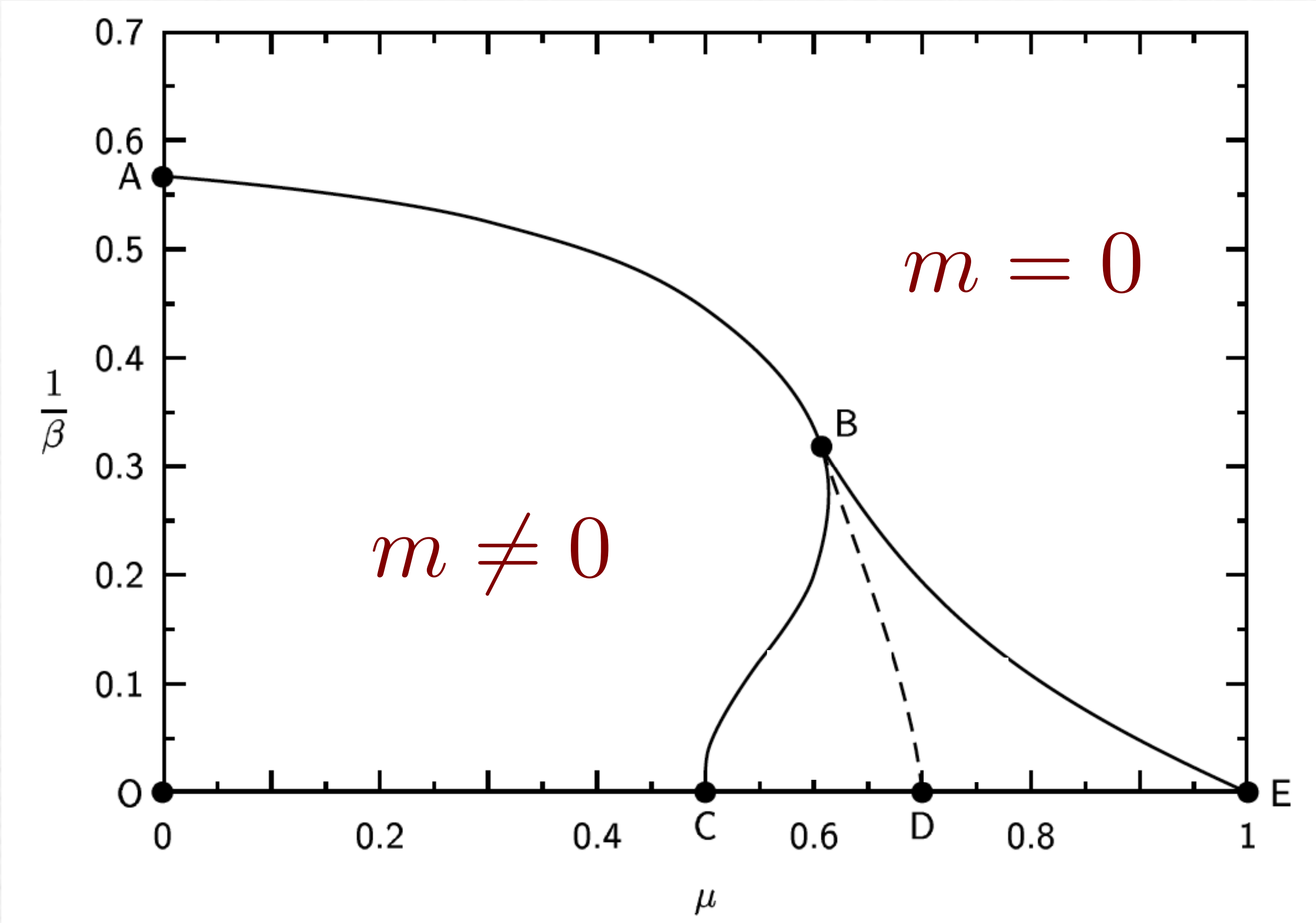


Phase diagram of NJL$_2$ model

uniform condensate
(same as GN$_2$)

Wolff, 1985
Barducci et al, 1995



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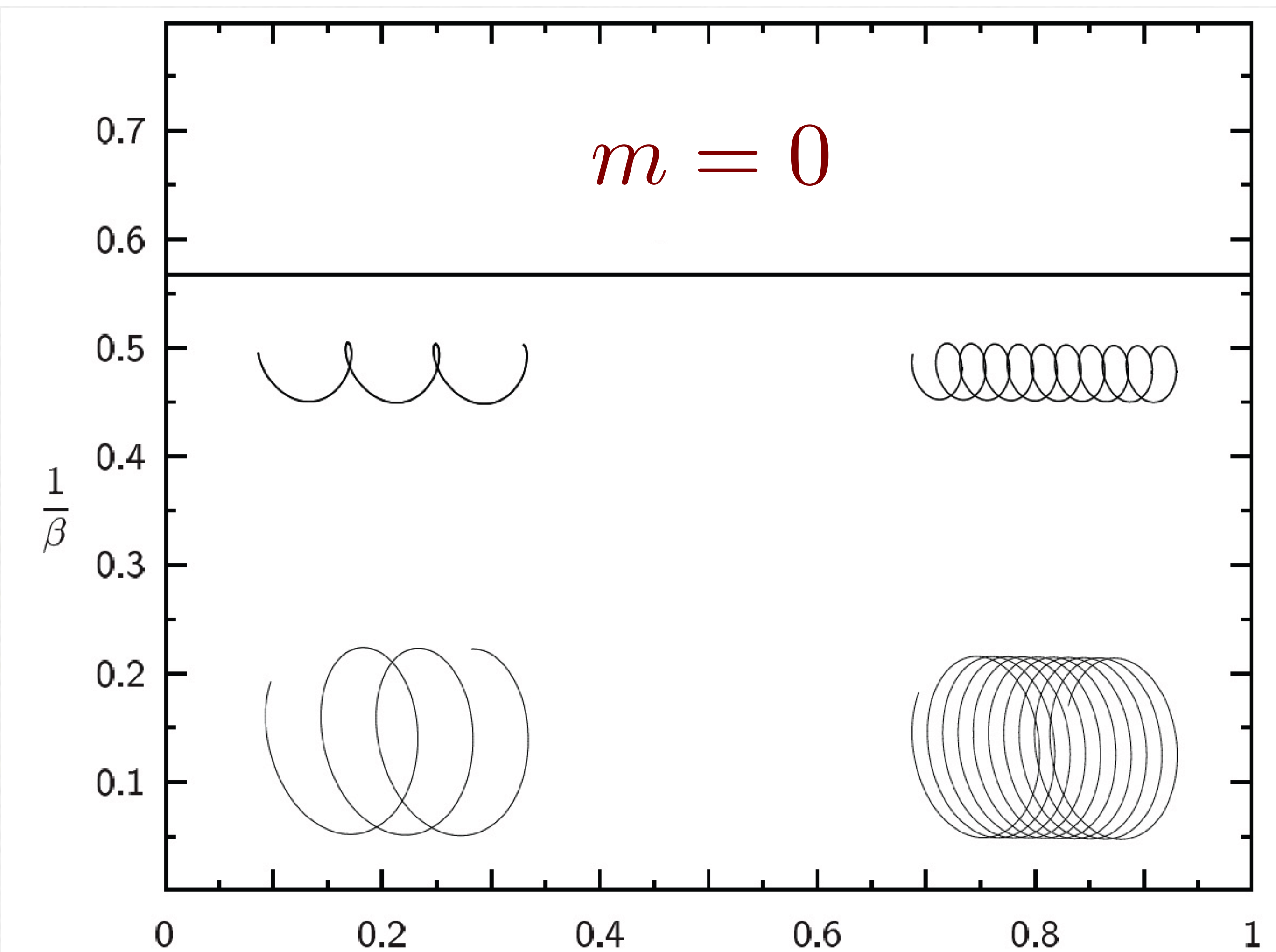

"chiral spiral"

$$\sigma(x) - i\pi(x) = A\, e^{2i\mu x}$$

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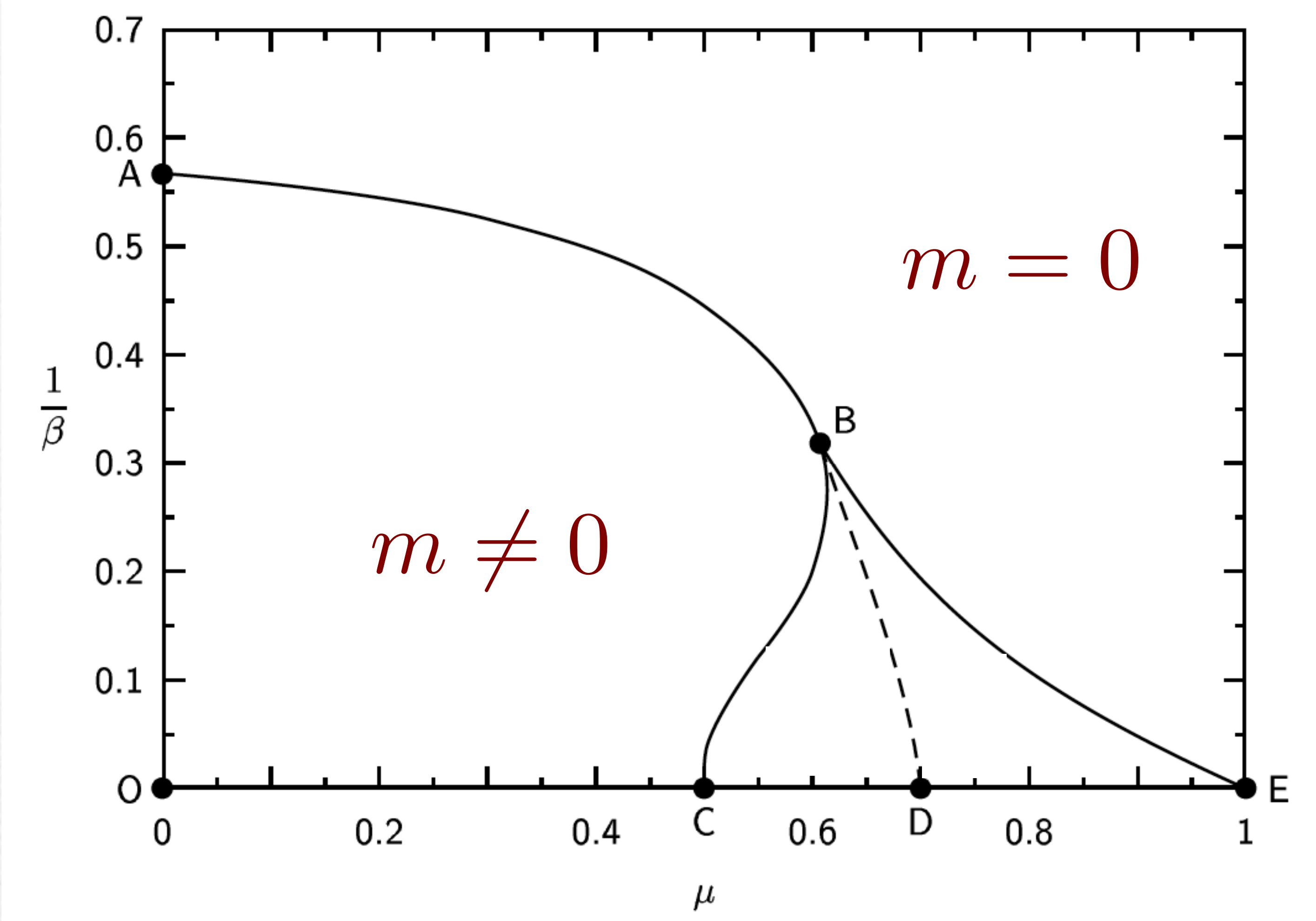

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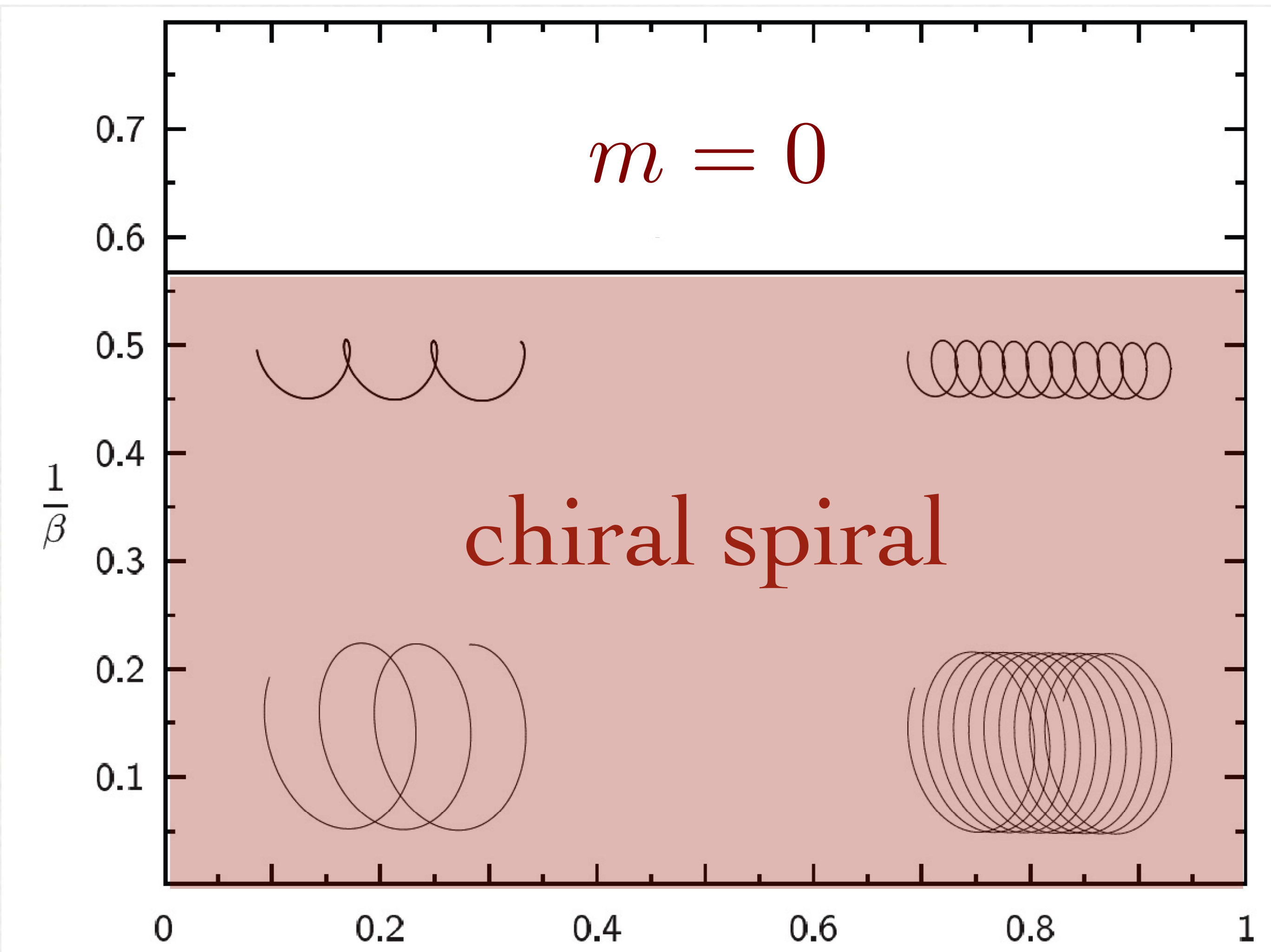

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scalar condensate σ

$$\mathcal{L}_{\rm eff} = \bar{\psi} i \partial\!\!\!/ \psi - \sigma\, \bar{\psi}\psi + \frac{1}{2}\sigma^2$$

gap equation:

$$\sigma(x) = \frac{\delta}{\delta\sigma(x)} \ln\det\left(i\partial\!\!\!/ - \sigma(x)\right)$$

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at finite T, μ

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chiral Gross-Neveu or NJL_2

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scalar condensate σ

pseudoscalar condensate π

$$\mathcal{L}_{\rm eff} = \bar{\psi} i \partial\!\!\!/ \psi - \bar{\psi}\left(\sigma - i\,\gamma^5\,\pi\right)\psi + \frac{1}{2}\left(\sigma^2 + \pi^2\right)$$

$$\Delta = \sigma - i\pi$$

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inhomogeneous phase?

gap equation at zero temperature and density:

Dashen-Hasslacher-Neveu (1975): inverse scattering

$V_\pm = \sigma^2 \pm \sigma'$ "reflectionless" potentials

single kink: $\sigma(x) = m \tanh(m\, x)$

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Shei (1976): inverse scattering

"reflectionless" Dirac operator

twisted kink: $$\Delta(x) = m\, \frac{\cosh\left(m\, \sin(\frac{\theta}{2})\, x - i\, \frac{\theta}{2}\right)}{\cosh\left(m\, \sin(\frac{\theta}{2})\, x\right)}$$

gap equation at nonzero temperature and density:

GD & Basar, PRL, PRD, 2008

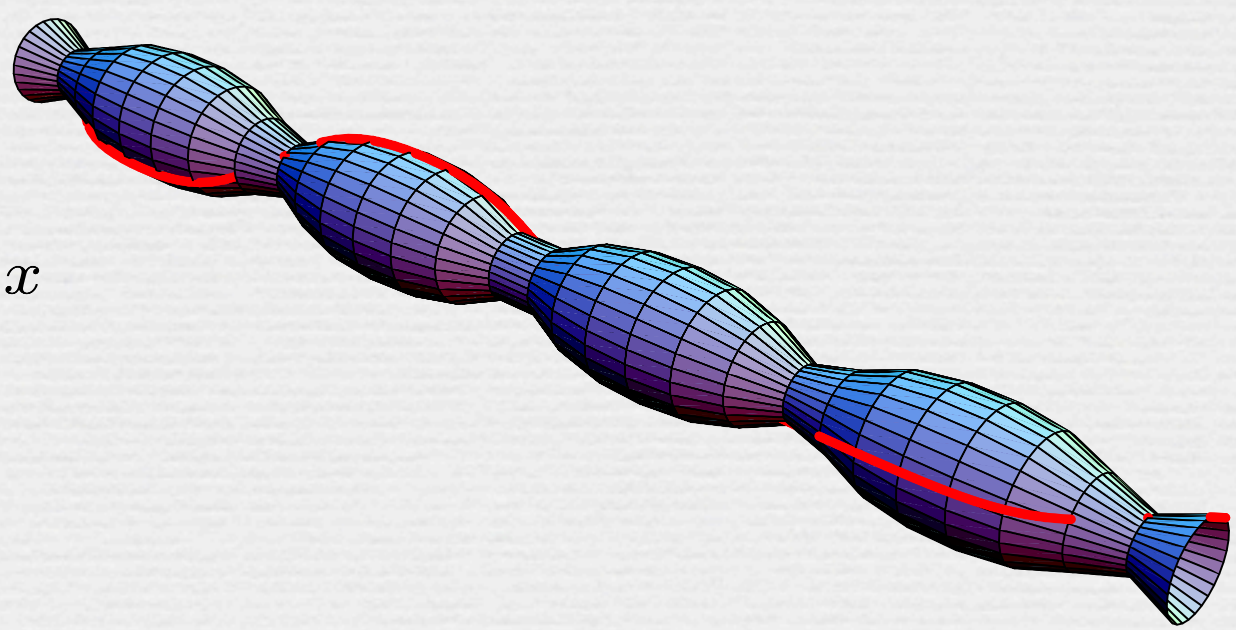
1. gap equation in terms of Gorkov resolvent $R(x;E) = \langle x|\frac{1}{E-H}|x\rangle$

2. ansatz reduces gap eqn. to NLSE, a soluble nonlinear ODE !

general bounded solution = twisted kink crystal

"finite-gap" Dirac system

$$\Delta(x) = A\,\frac{\sigma\left(A\,x + i\mathbf{K}' - i\frac{\theta}{2}\right)}{\sigma\left(A\,x + i\mathbf{K}'\right)\sigma\left(i\frac{\theta}{2}\right)}\,e^{iQx}$$



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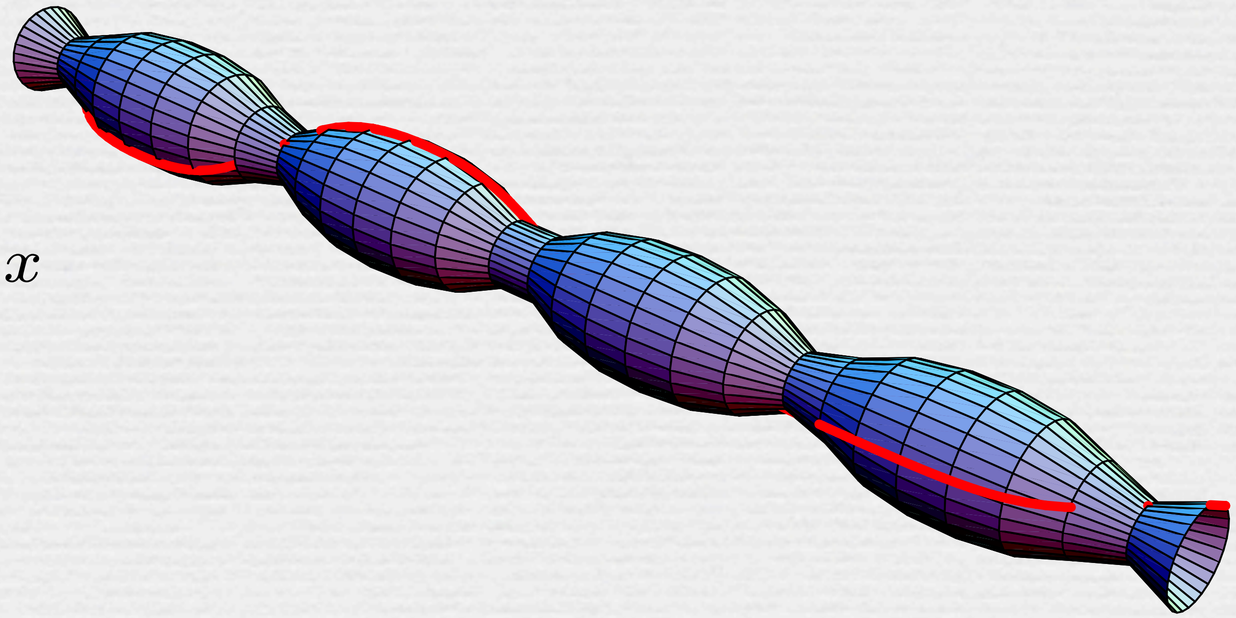

four parameters

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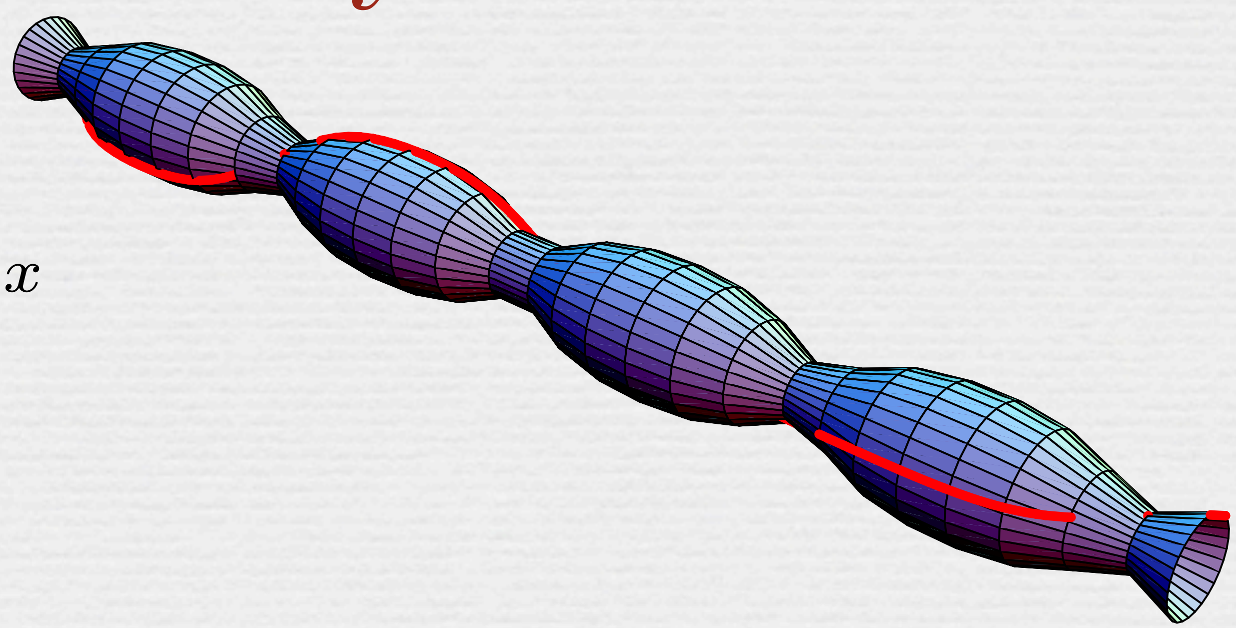
$\Rightarrow$ exact spectral function/density of states



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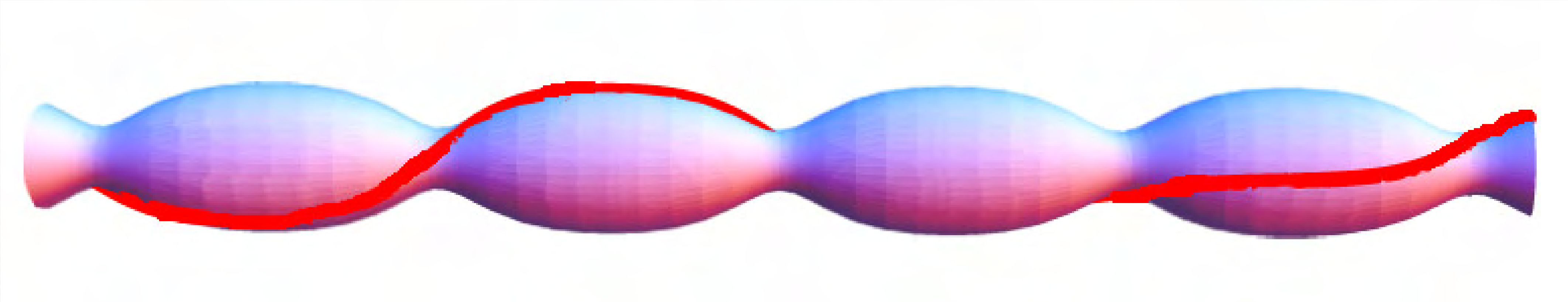
twisted kink crystal: general solution of NJL_2 gap equation



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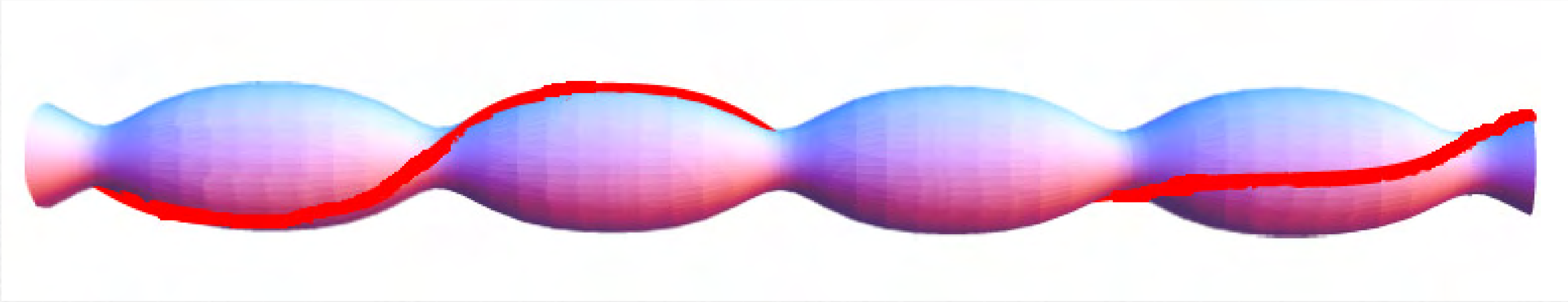
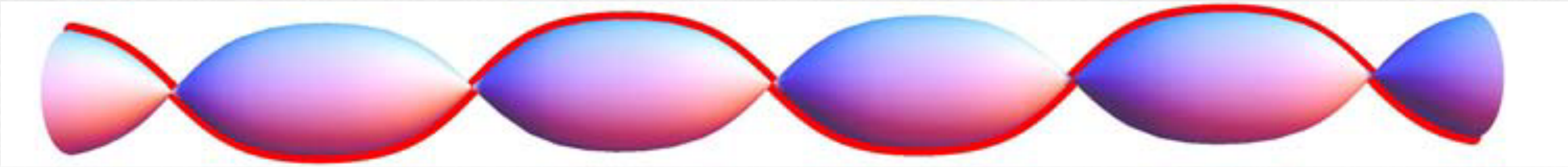


real kink crystal



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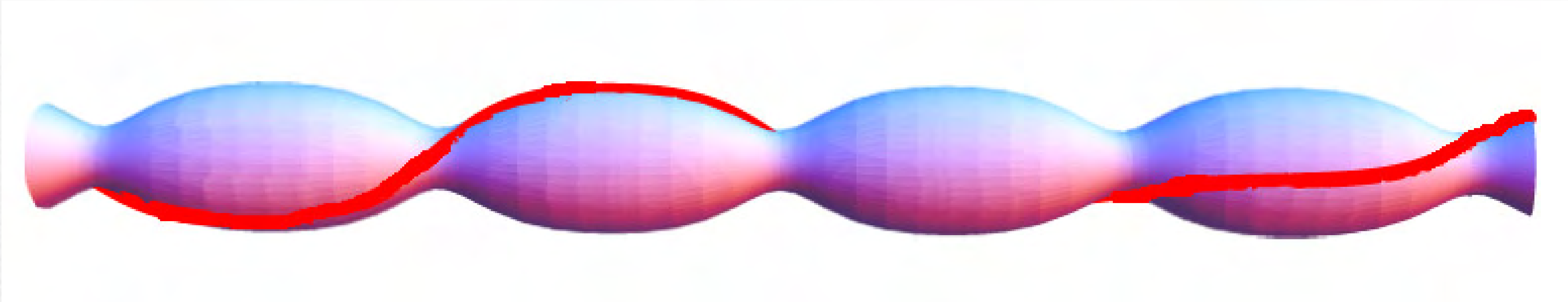

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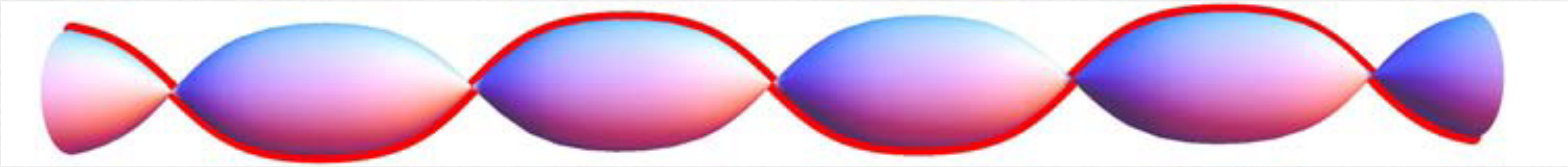
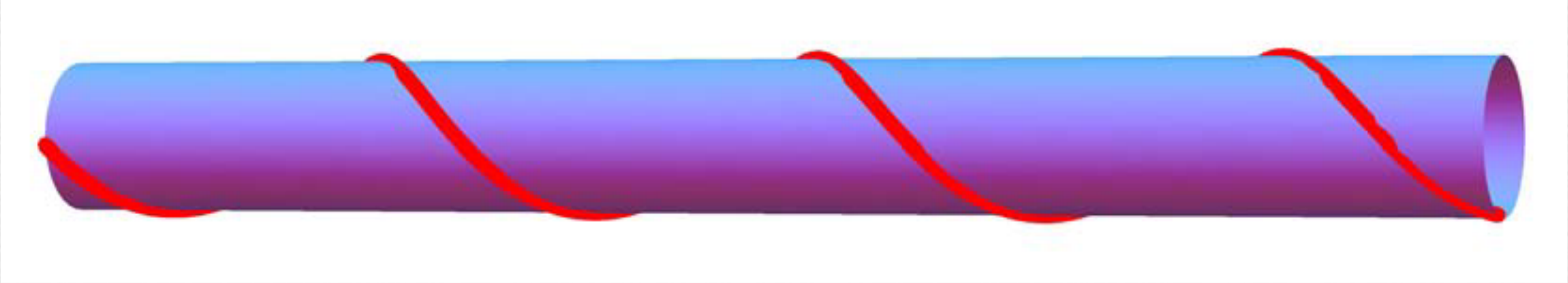


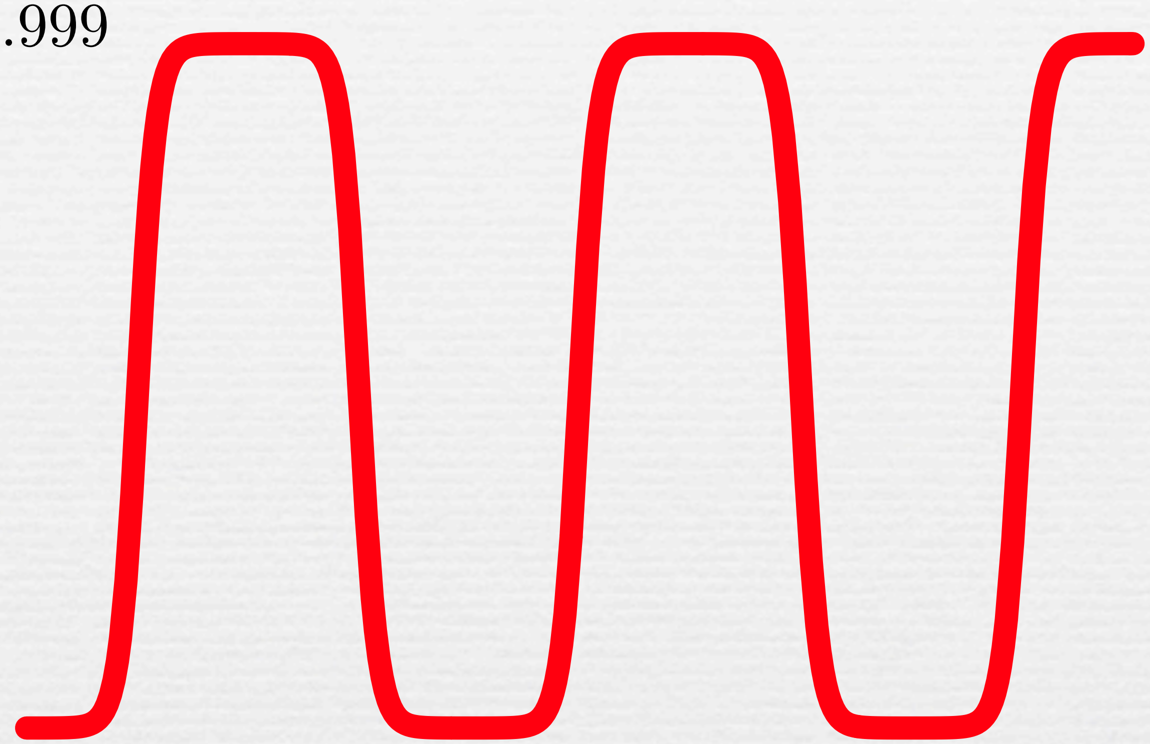
spiral crystal



kink crystal

$$\sigma(x) = m\sqrt{\nu}\,\mathrm{sn}(m\,x|\nu)$$

$\nu = 0.999$



$$H = \begin{pmatrix} -i\partial_x & \sigma(x) \\ \sigma(x) & i\partial_x \end{pmatrix}$$

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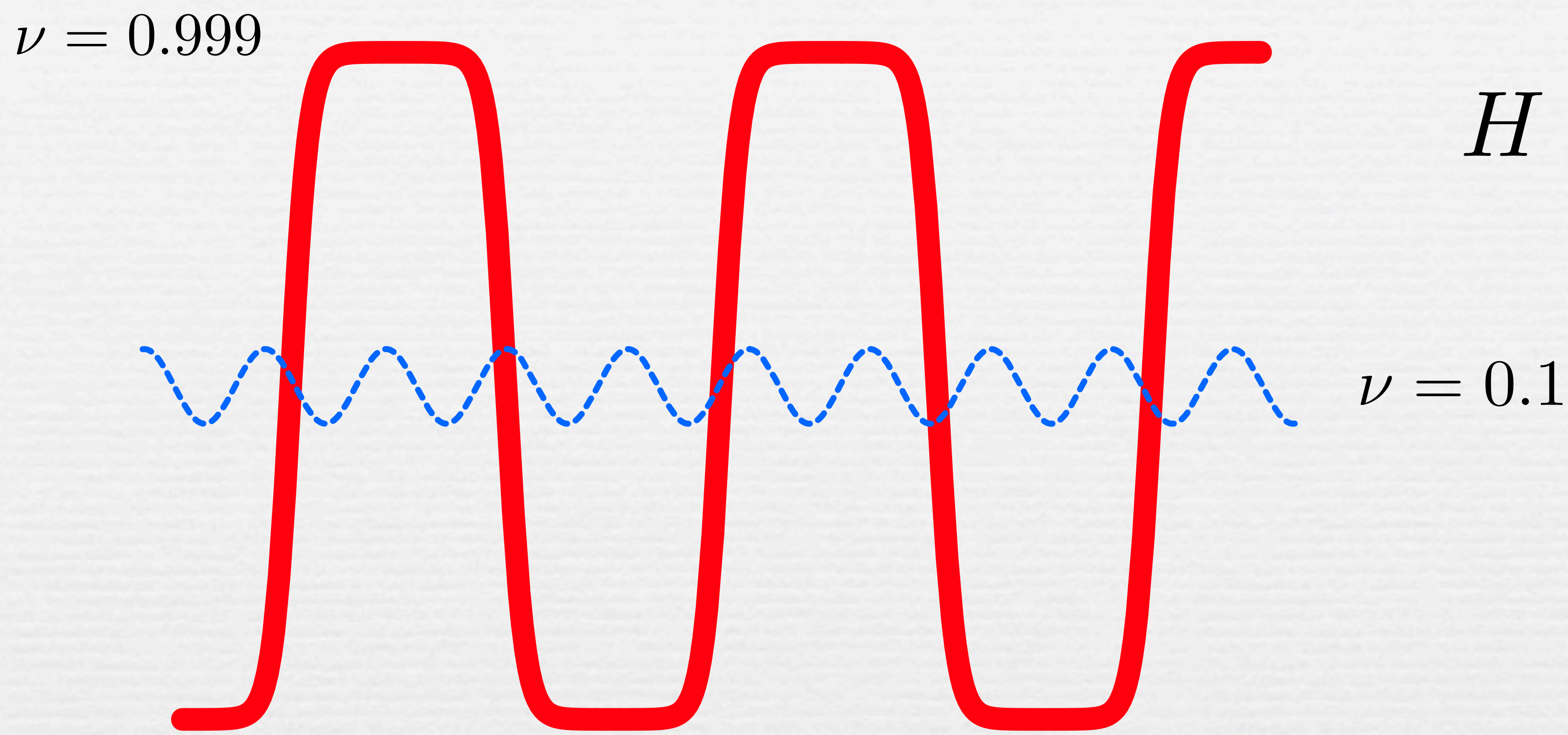

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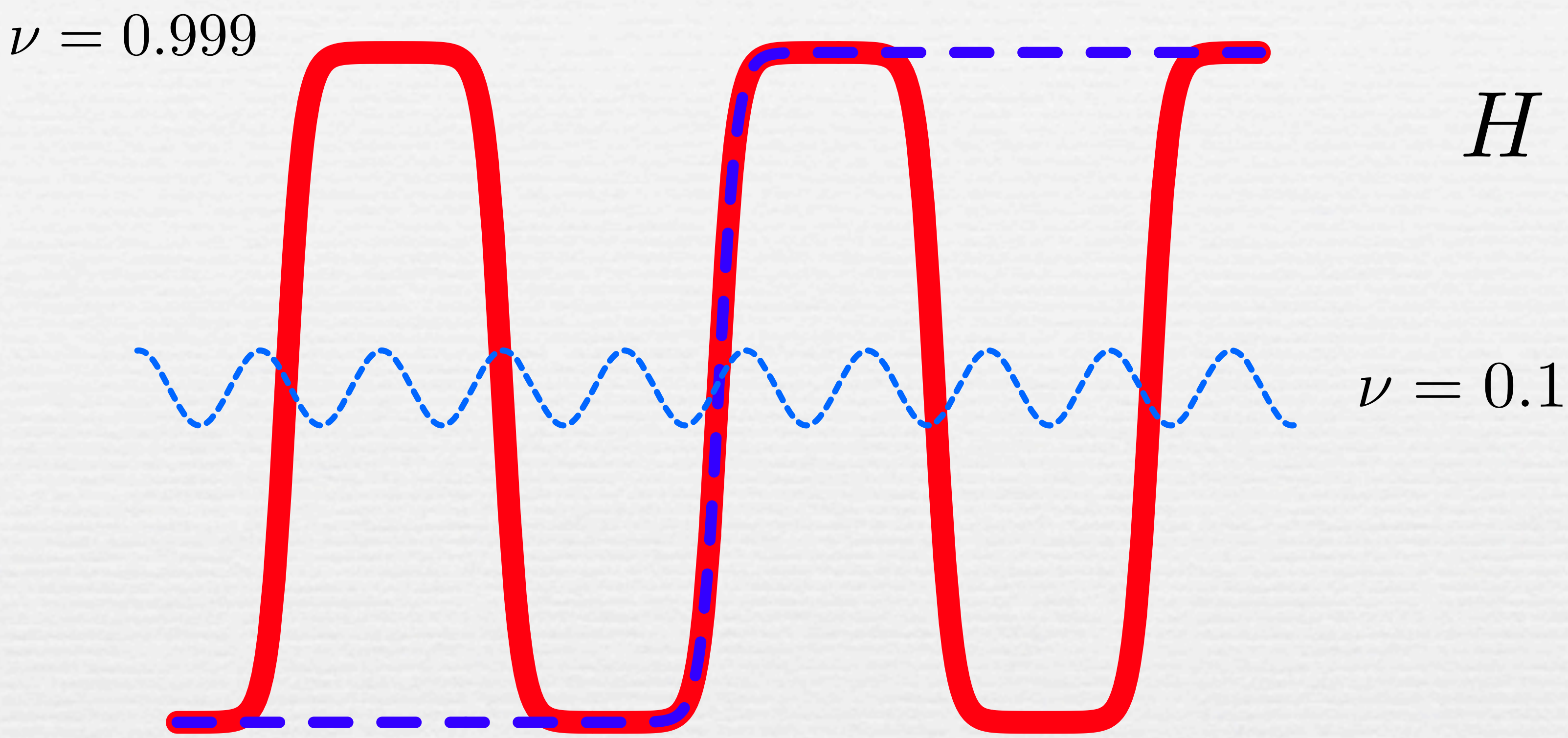

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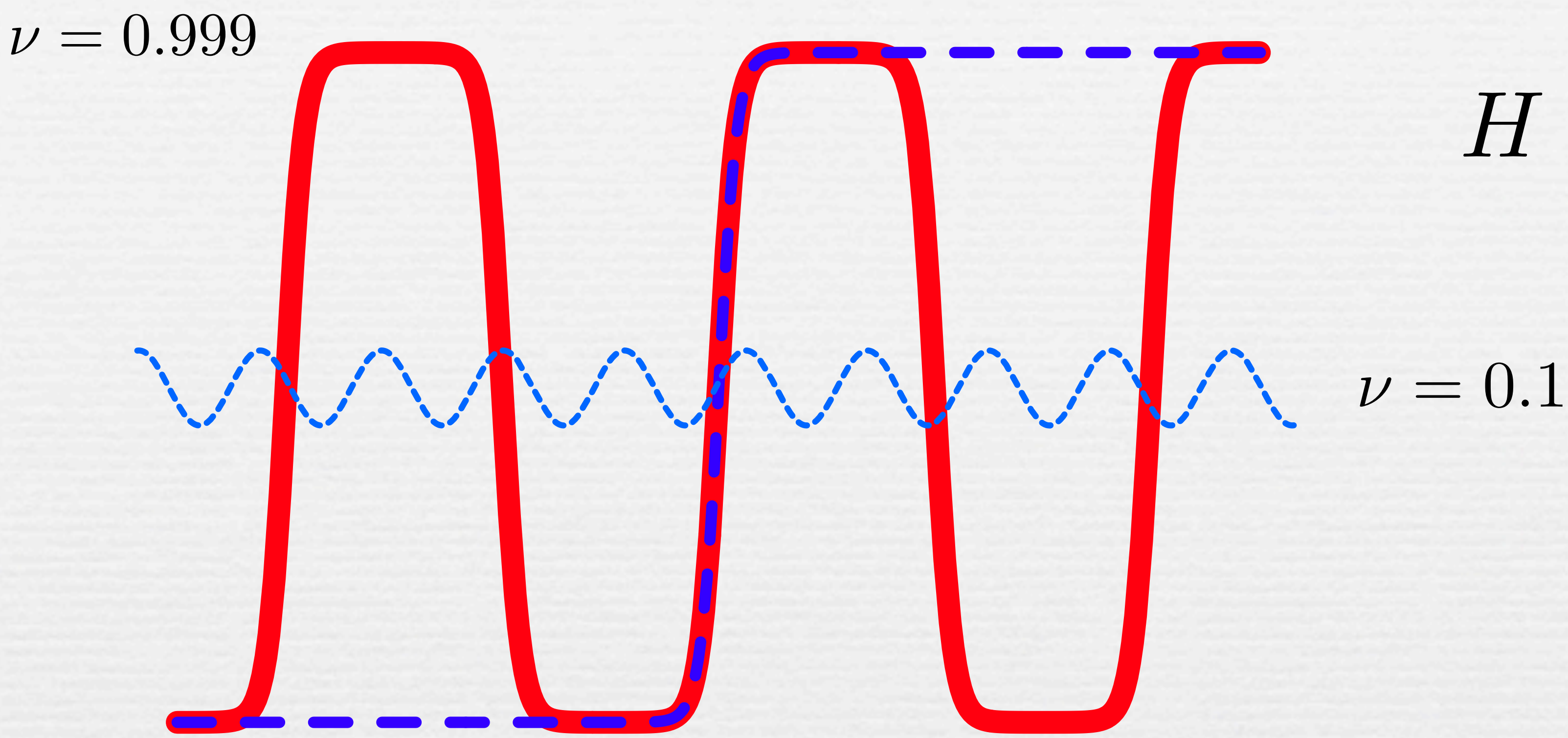

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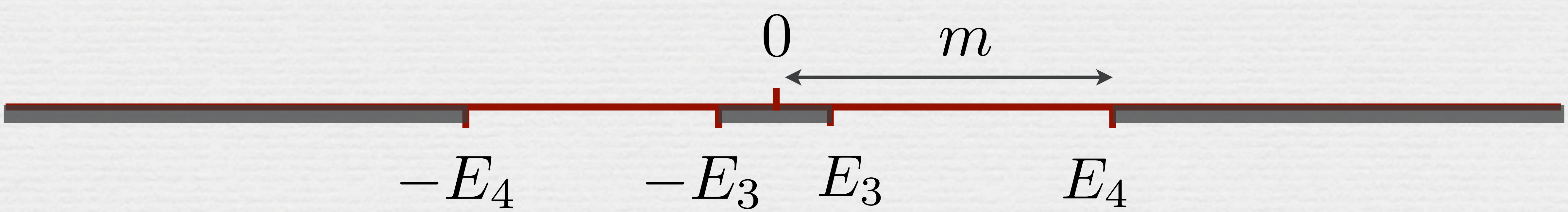
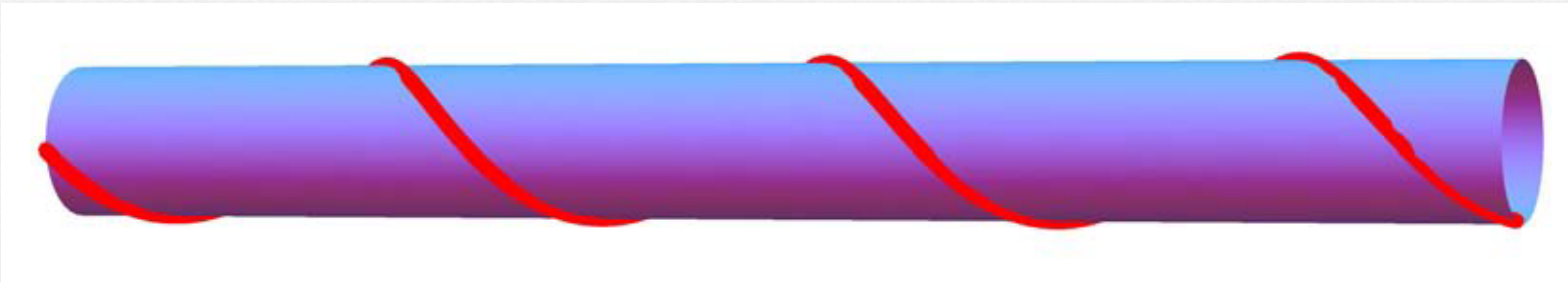
discrete chiral symmetry : charge conjugation symmetry

$$H\,\psi_E = E\,\psi_E \quad \Rightarrow \quad H\,(\gamma^1\psi_E) = -E\,(\gamma^1\psi_E)$$

spectrum is symmetric about 0

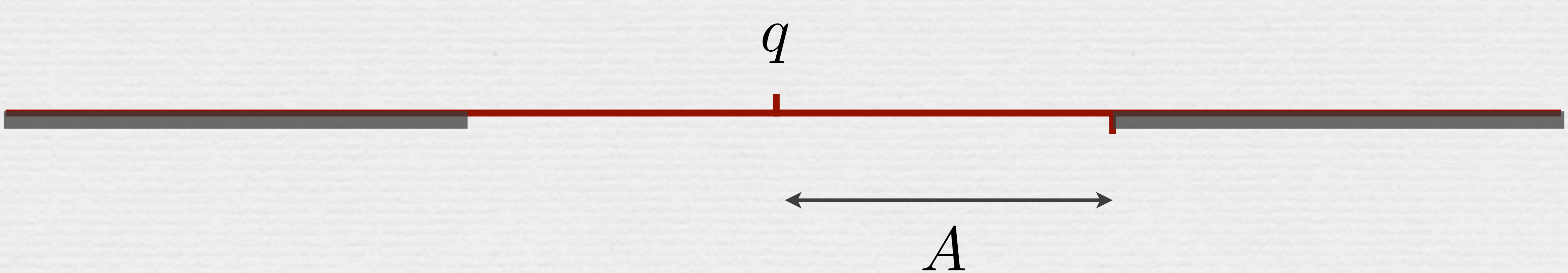


spiral crystal $\Delta(x) = A\, e^{2\,i\,q\,x}$



$$H = \begin{pmatrix} -i\partial_x & \Delta(x) \\ \Delta^*(x) & i\partial_x \end{pmatrix}$$

no charge conjugation symmetry



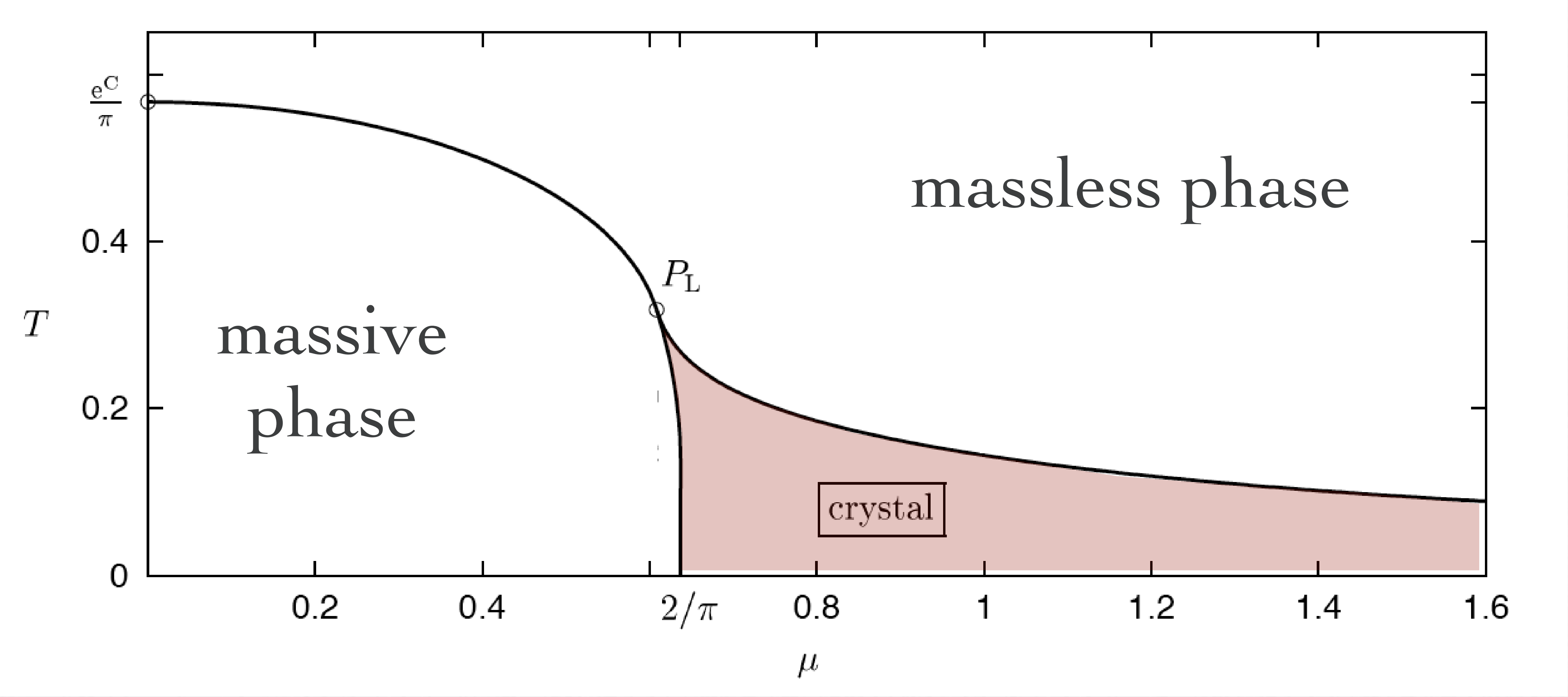
phase diagram of real Gross-Neveu (GN_2)

gap equation solution has 2 parameters

grand potential: $$\Psi = -\frac{1}{\beta}\int dE\,\rho(E)\,\ln\left(1+e^{-\beta(E-\mu)}\right)$$

minimize Ψ w.r.t. parameters, as function of T and μ

$\Longrightarrow$ periodic kink crystal phase $\quad \sigma(x) = m\sqrt{\nu}\,\mathrm{sn}(m\,x|\nu)$



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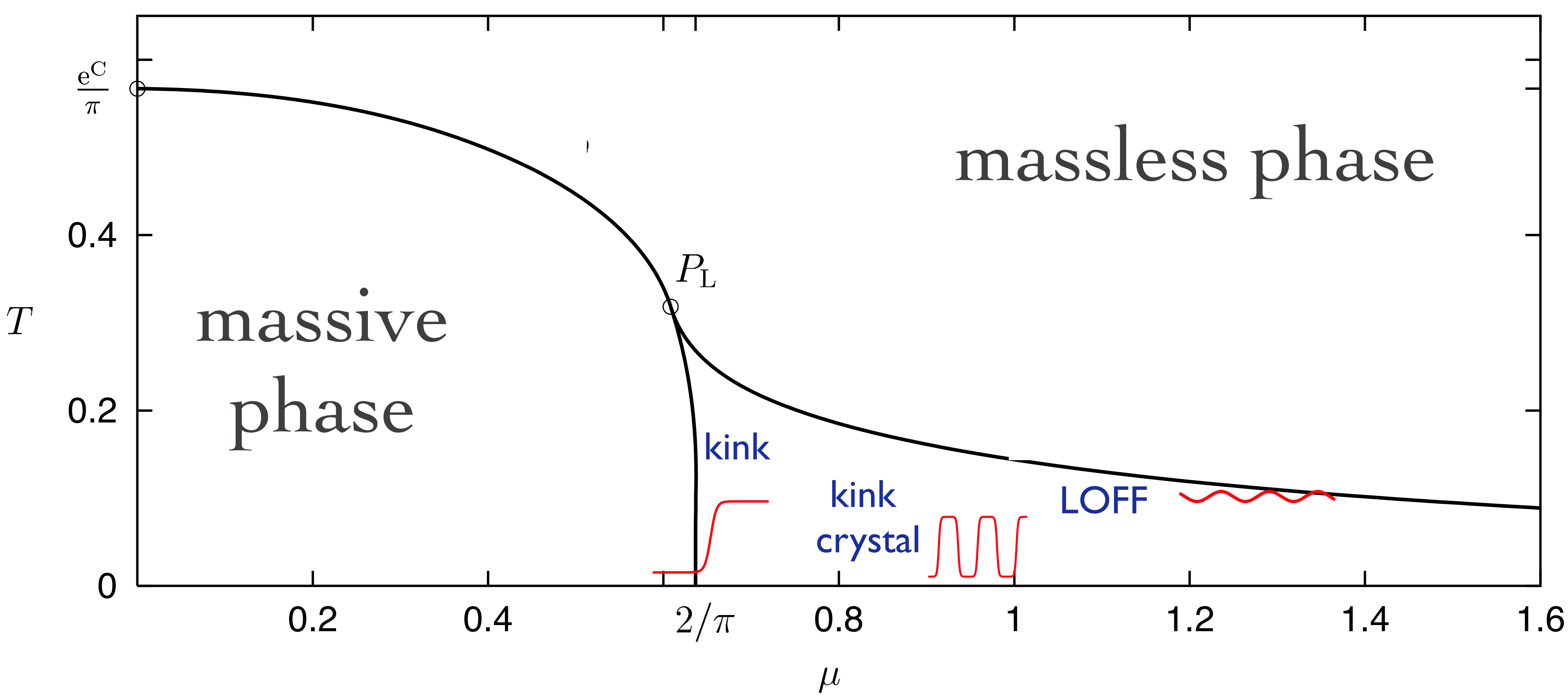
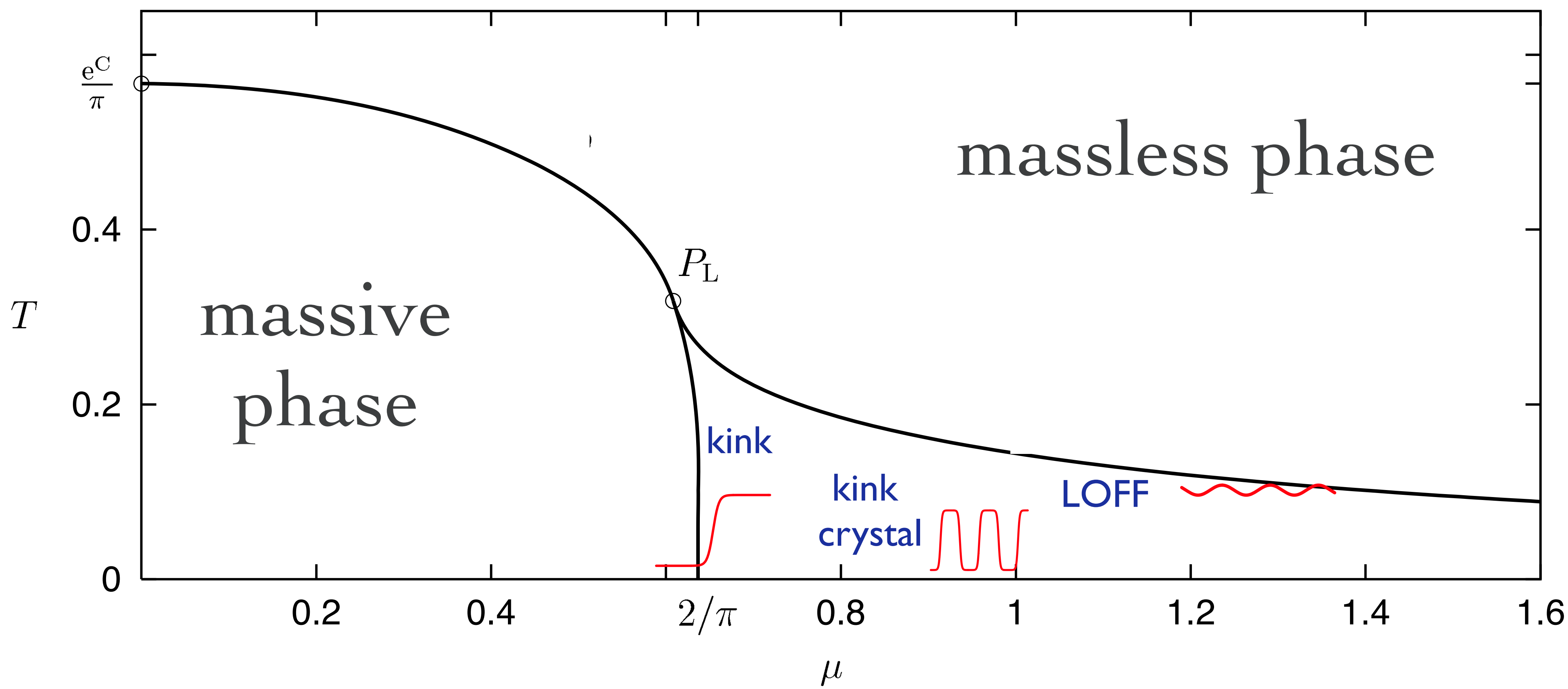


$\frac{e^C}{\pi}$
0.4
0.2
0
T
massive phase
P_L
massless phase
kink
kink crystal
LOFF
0.2
0.4
$2/\pi$
0.8
1
1.2
1.4
1.6
μ


all gap
no gap


massless phase
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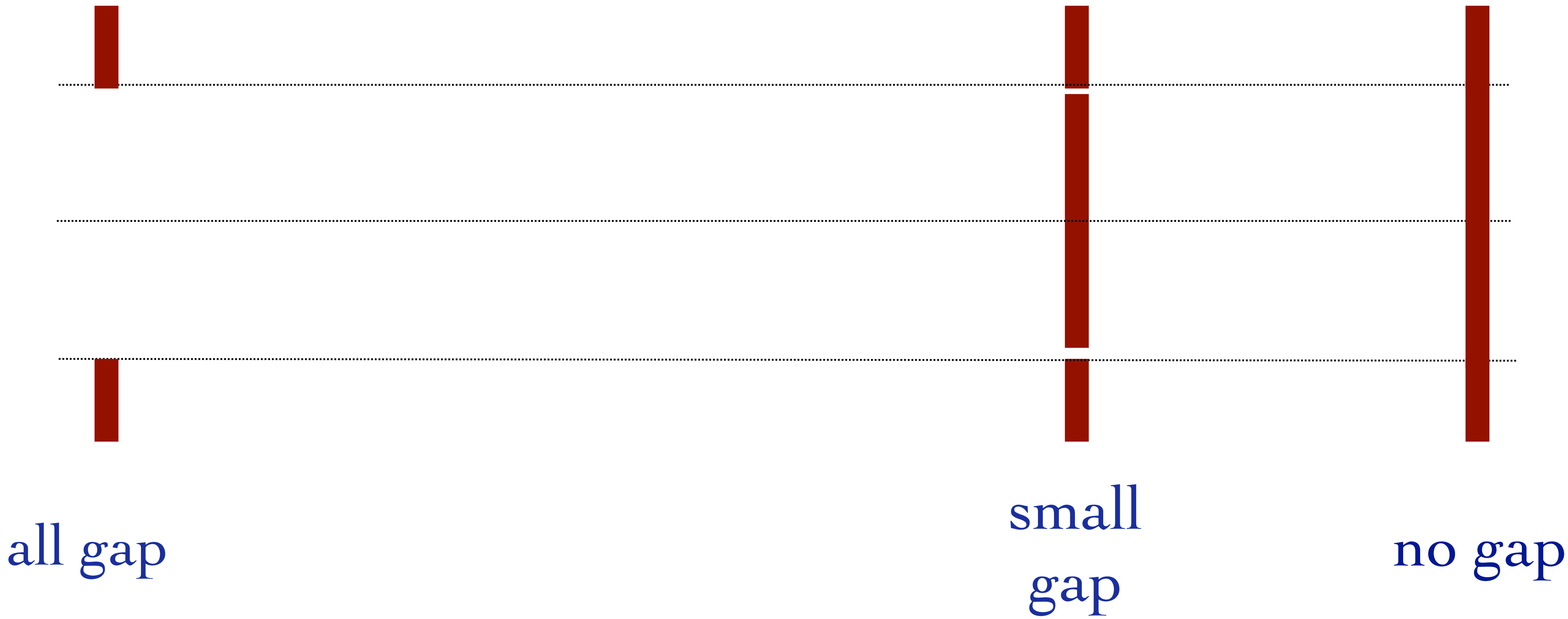
T
μ
$\frac{e^C}{\pi}$
0.4
0.2
0
0.2
0.4
$2/\pi$
0.8
1
1.2
1.4
1.6


all gap
small gap
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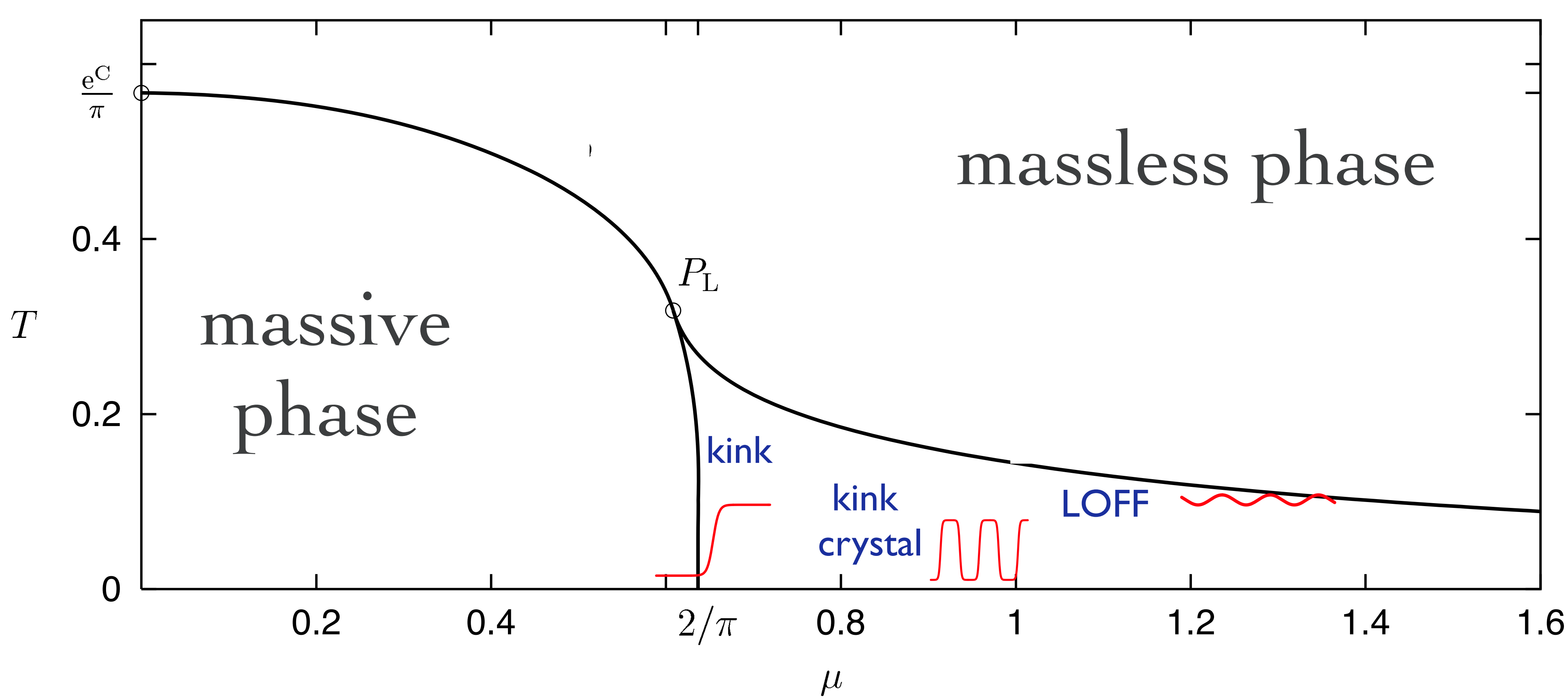
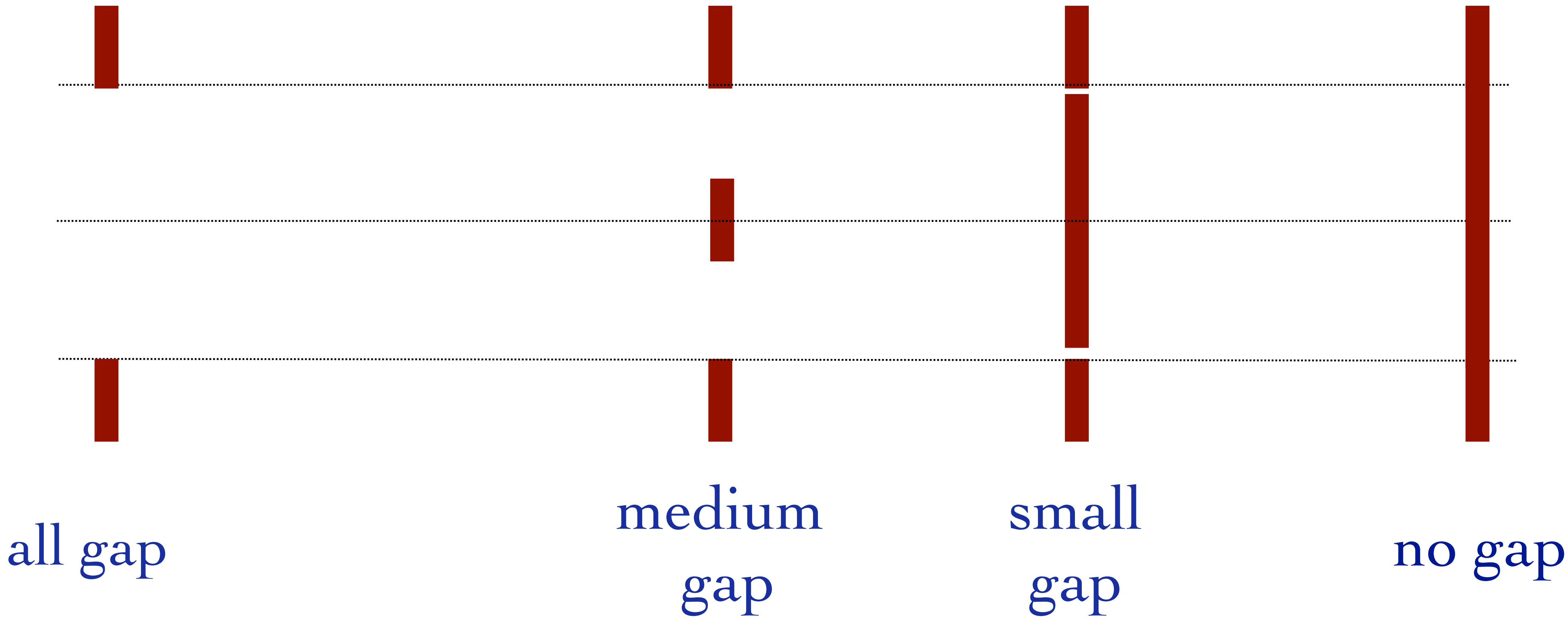

massless phase
massive phase
kink
kink crystal
LOFF
P_{L}
T
μ
$\frac{\mathrm{e}^{\mathrm{C}}}{\pi}$
0.4
0.2
0
0.2
0.4
$2/\pi$
0.8
1
1.2
1.4
1.6


all gap
medium gap
small gap
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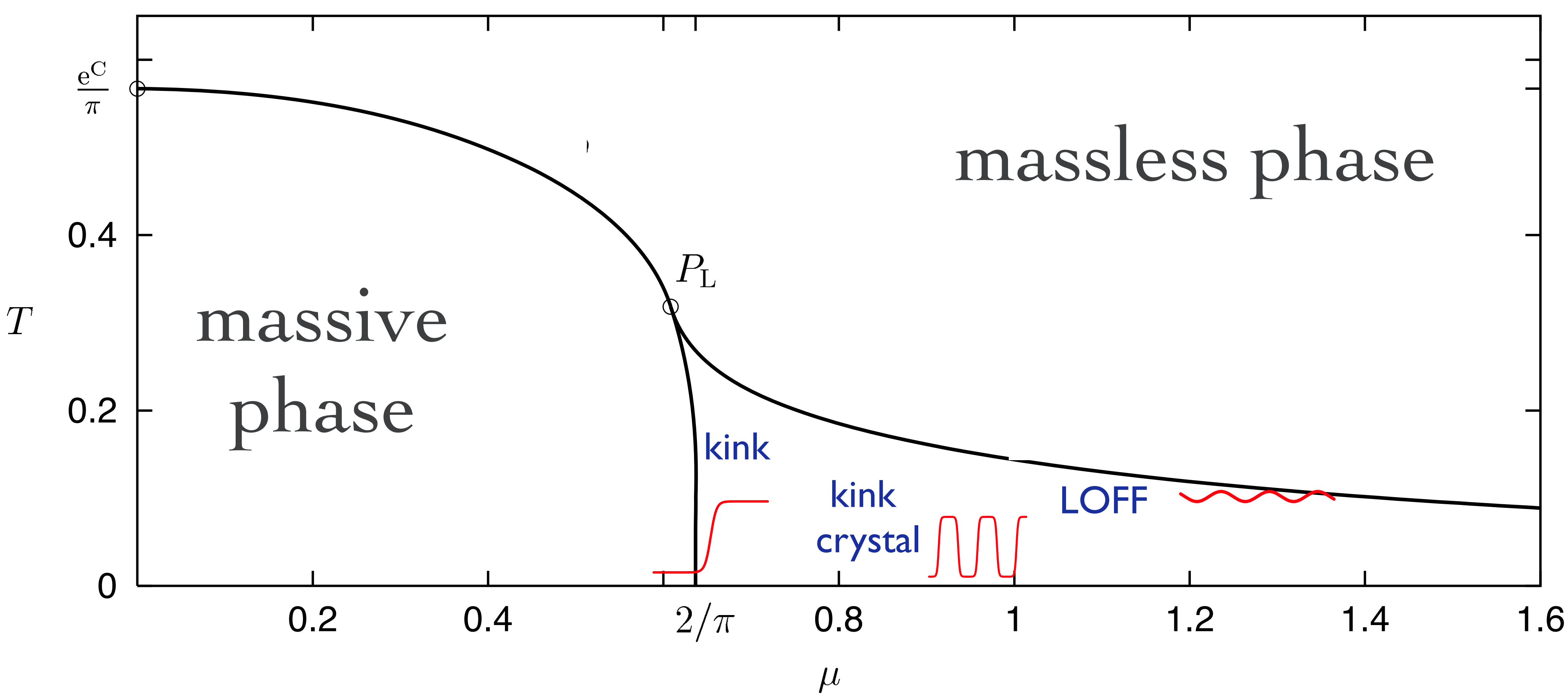
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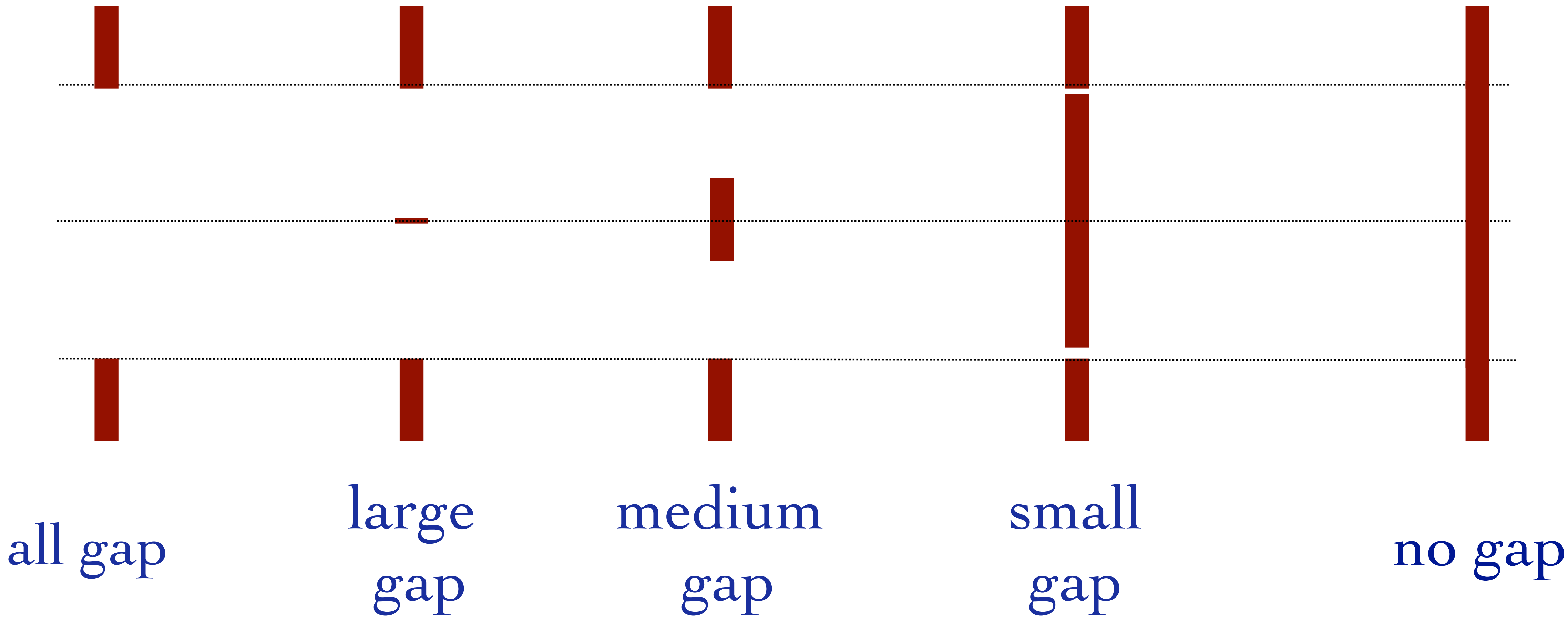
all gap
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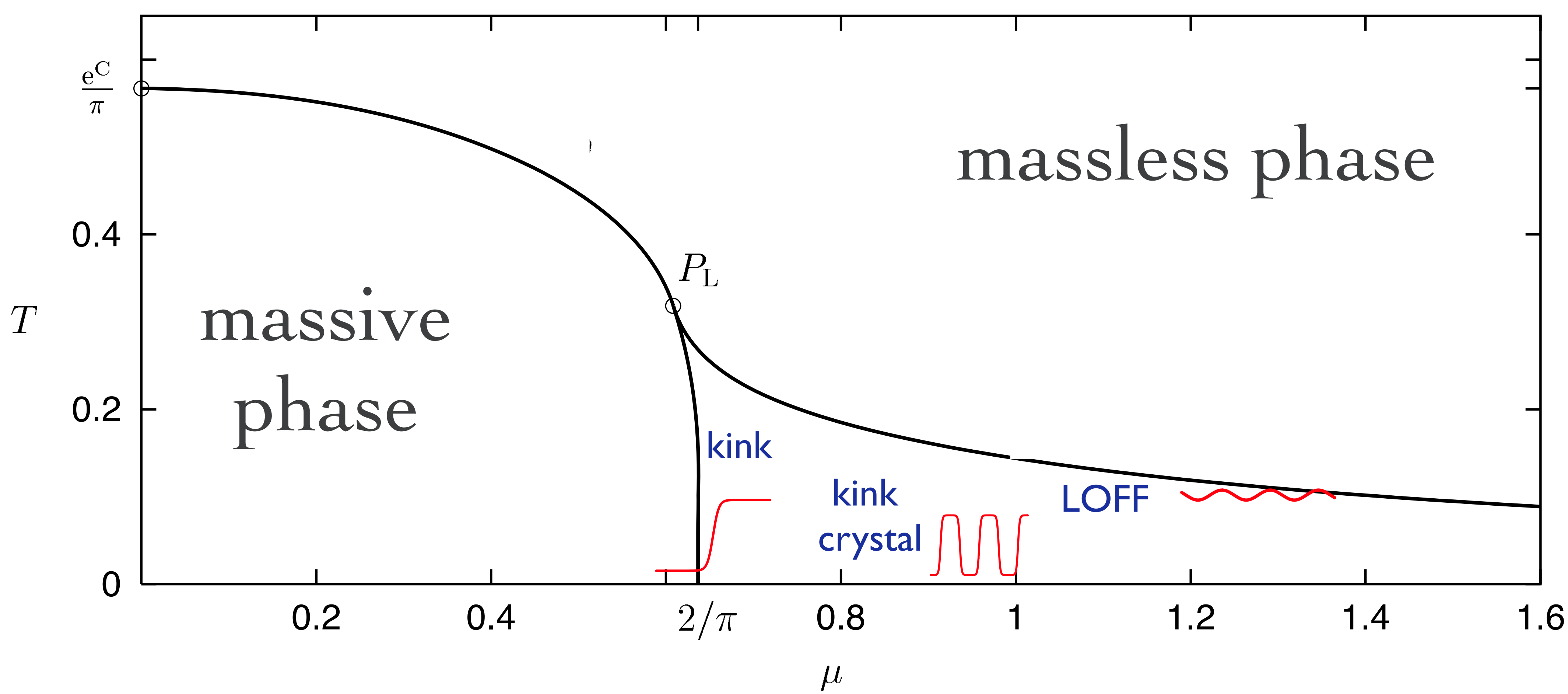
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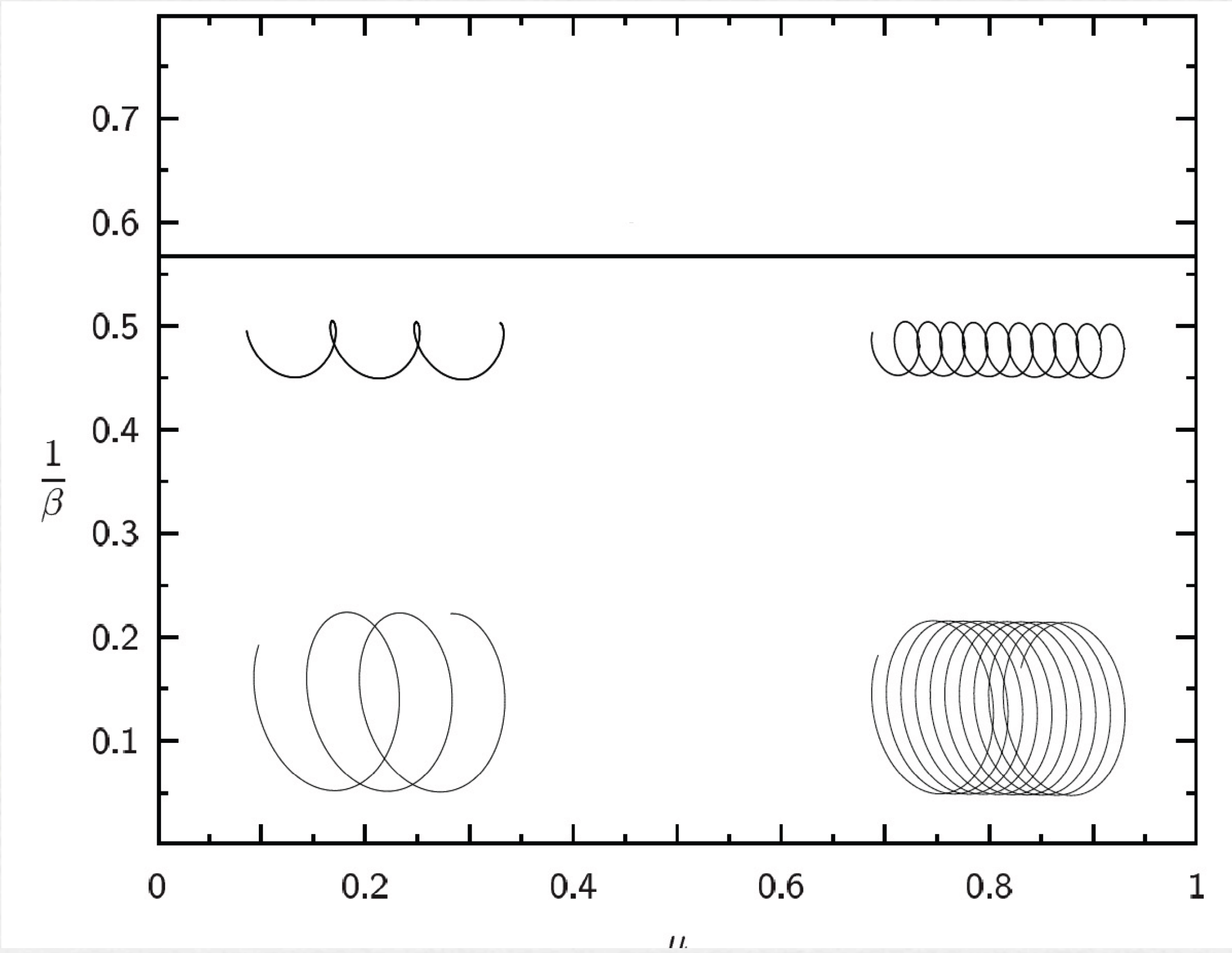
phase diagram of chiral Gross-Neveu (NJL_2)

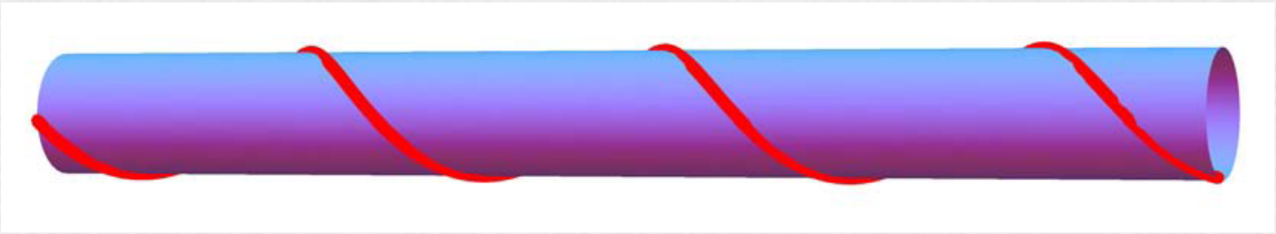
gap equation solution has 4 parameters

grand potential: $$\Psi = -\frac{1}{\beta}\int dE\,\rho(E)\,\ln\left(1+e^{-\beta(E-\mu)}\right)$$

minimize Ψ w.r.t. parameters, as function of T and μ



"chiral spiral"



$$\sigma(x) - i\,\pi(x) = A(T)\,e^{2i\mu\,x}$$

Schön, Thies, 2000

Basar, GD, Thies, 2009

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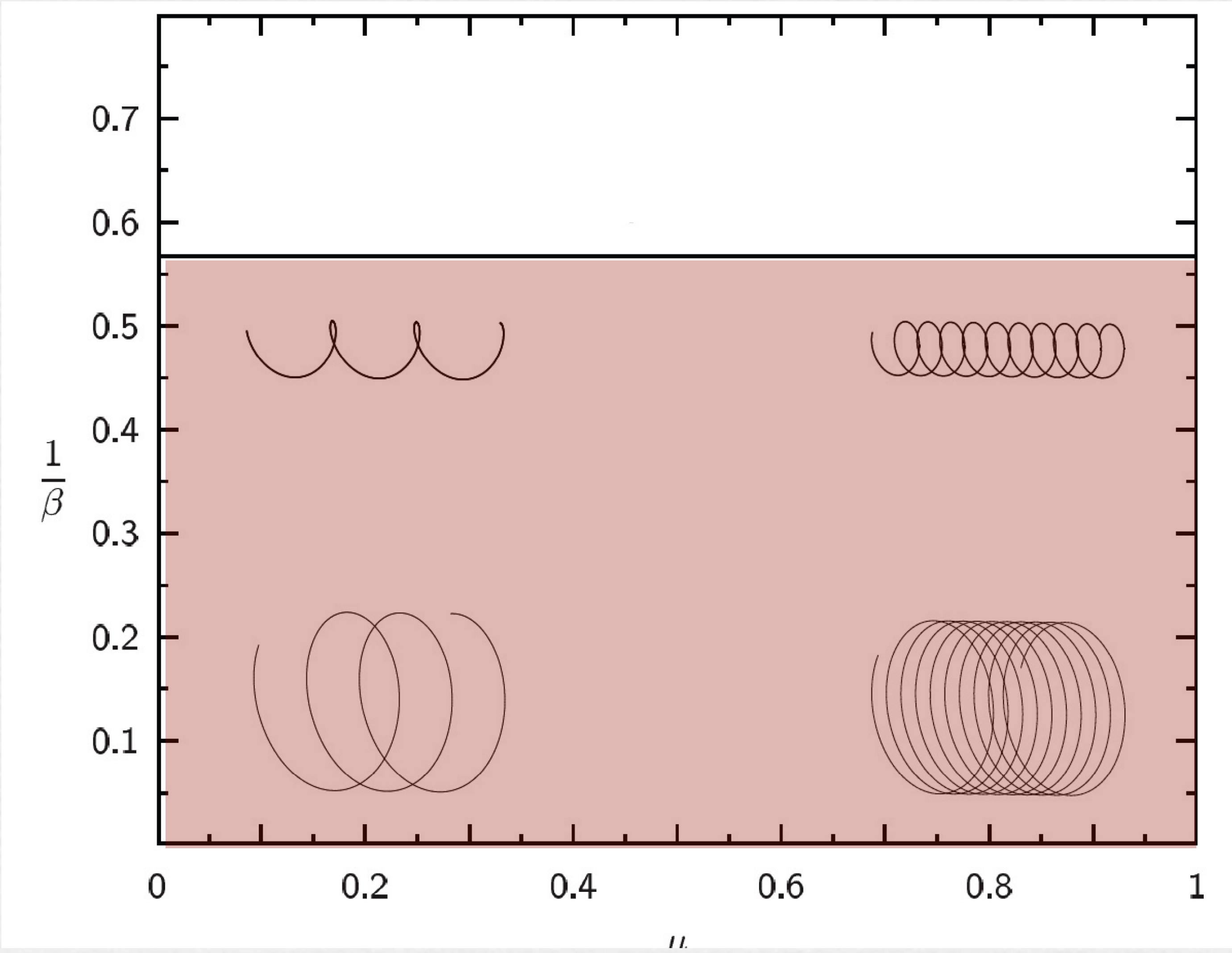

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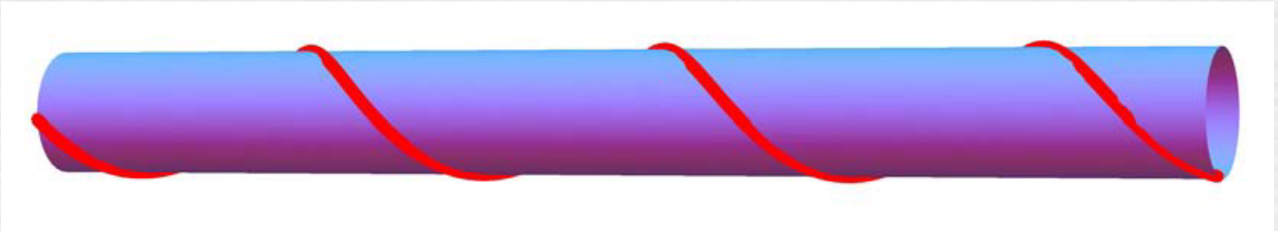

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Schön, Thies, 2000

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why can these gap equations be solved?

Ginzburg-Landau approach

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$$\Psi_{\mathrm{GL}} = \sum_n \alpha_n(T,\mu)\int \hat{g}_n(x)$$

Ginzburg-Landau expansion for GN$_2$ (real condensate)

$$\begin{aligned}\Psi &= \alpha_2 \int \sigma^2 + \alpha_4 \int \left[\sigma^4 + (\sigma')^2\right] \\ &\quad +\alpha_6 \int \left[2\sigma^6 + 10\sigma^2(\sigma')^2 + (\sigma'')^2\right] + \dots\end{aligned}$$

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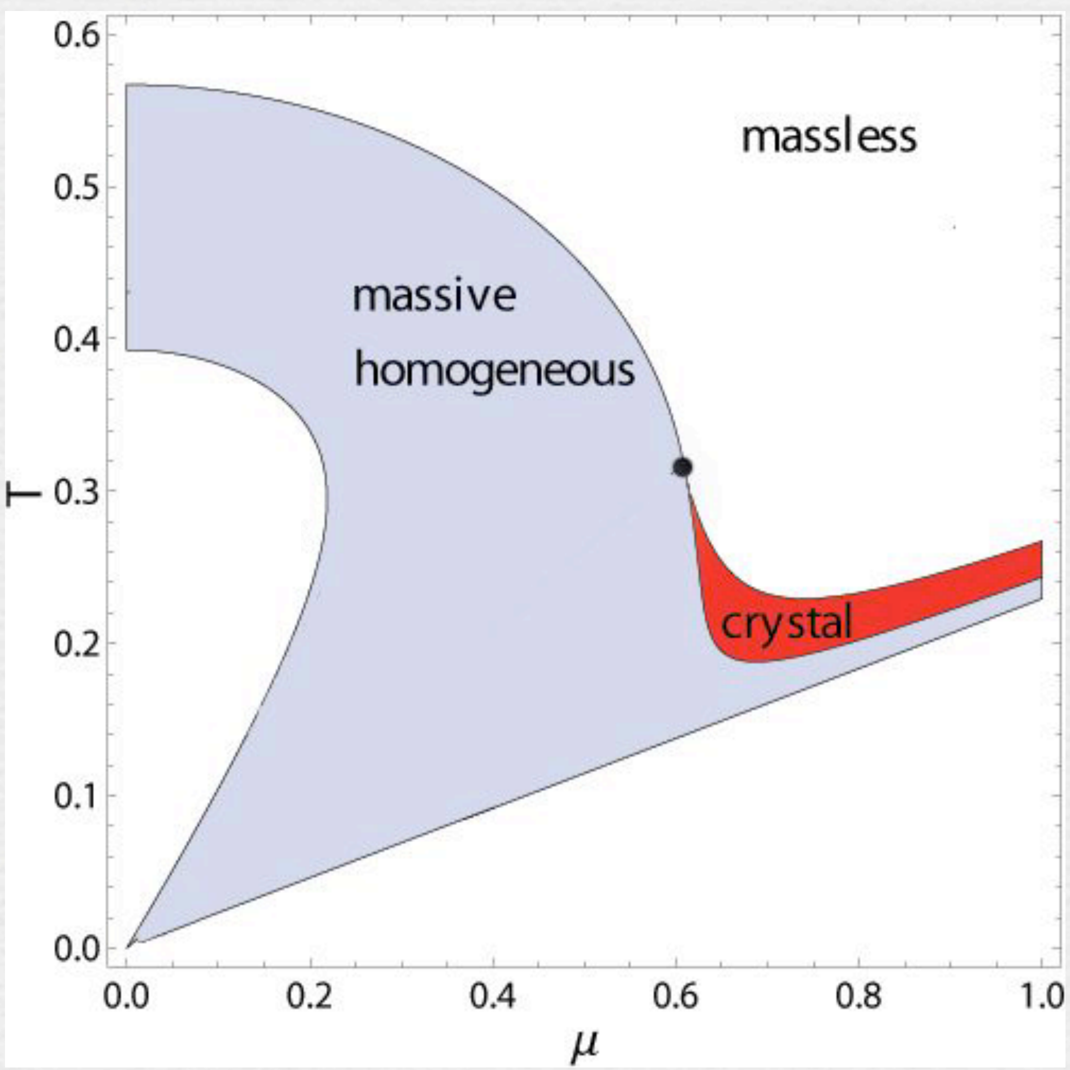

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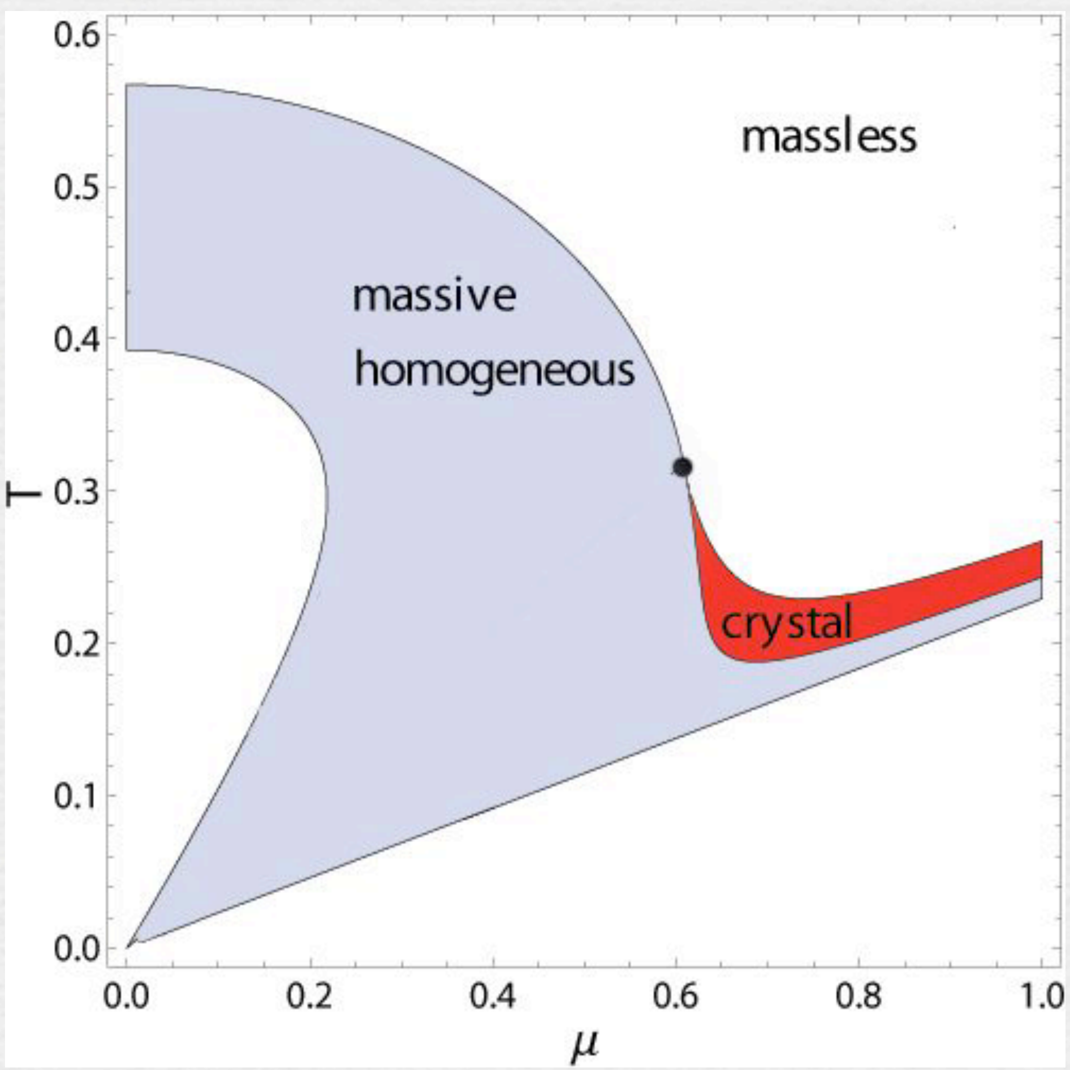

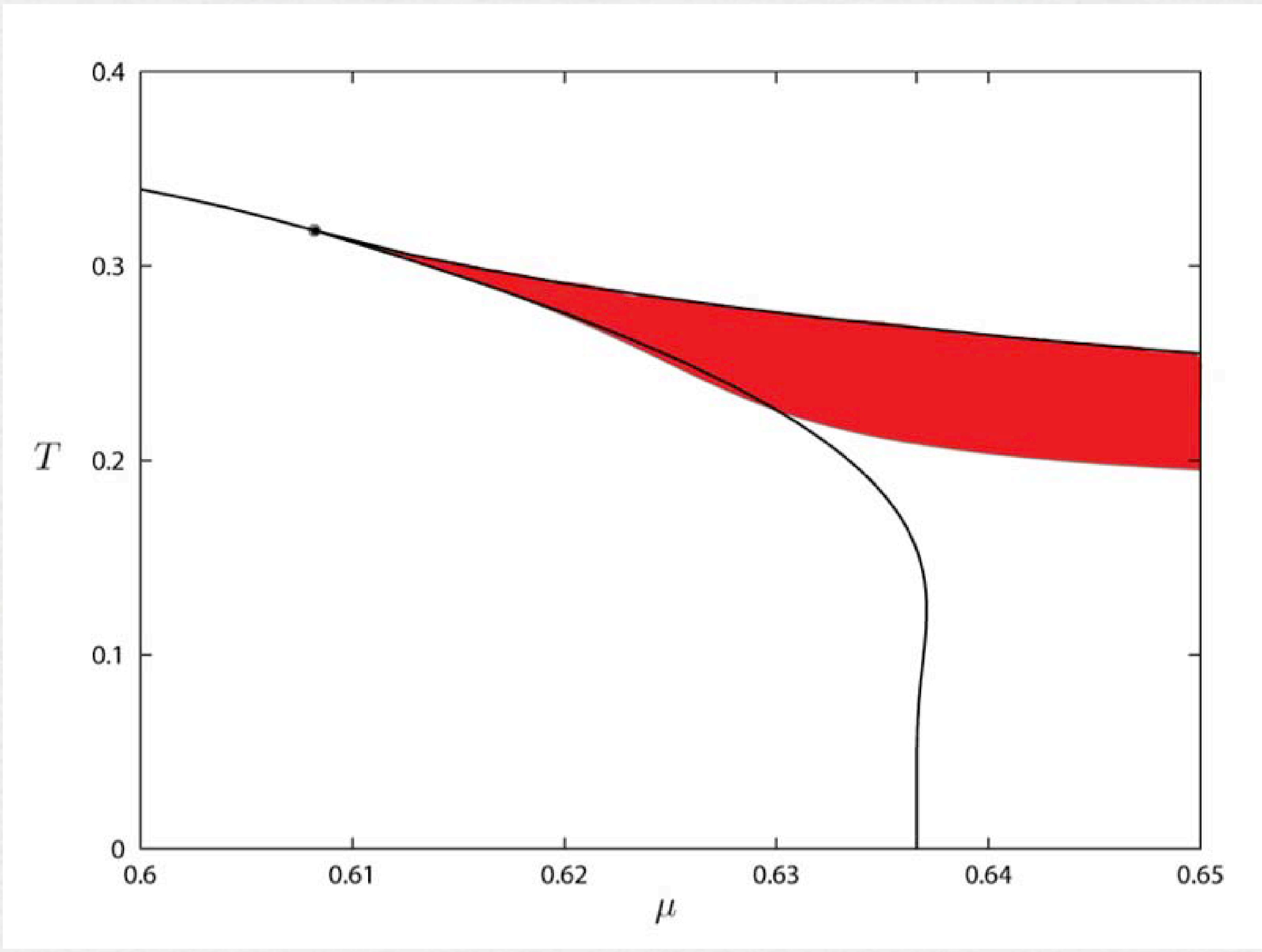

Ginzburg-Landau for NJL$_2$ (complex condensate)

$$\begin{aligned}\Psi &= \alpha_2 \int |\Delta|^2 + \alpha_3 \int \mathrm{Im}[\Delta(\Delta')^*] + \alpha_4 \int \left[|\Delta|^4 + |\Delta'|^2\right] \\ &\quad + \alpha_5 \int \mathrm{Im}\left[(\Delta'' - 3|\Delta|^2\Delta)(\Delta')^*\right] \\ &\quad + \alpha_6 \int \left[2\Delta|^6 + 8|\Delta|^2|\Delta'|^2 + 2\mathrm{Re}\left((\Delta')^2(\Delta^*)^2\right) + |\Delta''|^2\right] + \ldots\end{aligned}$$

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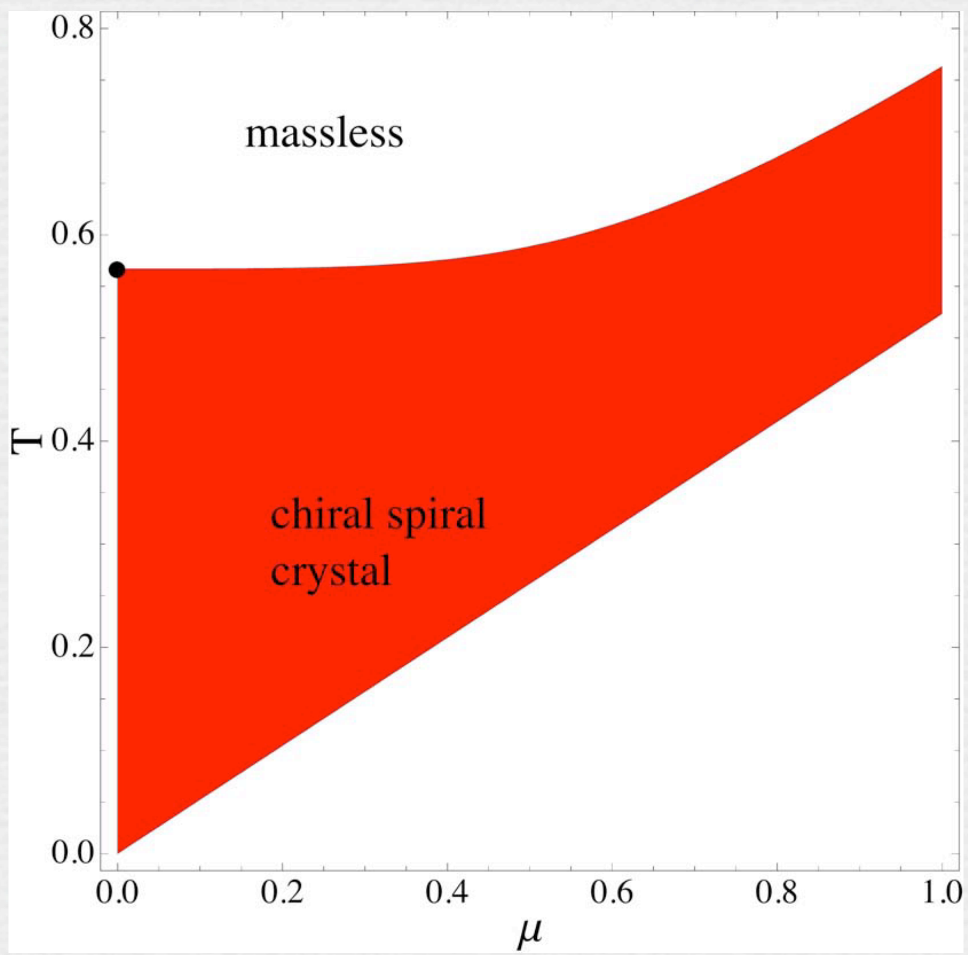

for Gross-Neveu and NJL we can solve the Ginzburg-Landau expansion to all orders!

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[integrable hierarchies]

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conserved quantities of mKdV/AKNS

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conserved quantities of mKdV/AKNS

mKdV: modified Korteweg-de Vries

Miura transformation: $V_\pm = \sigma^2 \pm \sigma'$

AKNS: Ablowitz, Kaup, Newell, Segur

Ablowitz-Kaup-Newell-Segur integrable hierarchy

$$
\begin{aligned}
\hat{g}_0 &= 1 \quad , \\
\hat{g}_1 &= 0 \quad , \\
\hat{g}_2 &= \frac{1}{2}|\Delta|^2 \quad , \\
\hat{g}_3 &= \frac{i}{4}\left(\Delta\Delta'^{*} - \Delta'\Delta^{*}\right) \quad , \\
\hat{g}_4 &= \frac{1}{8}\Big(3|\Delta|^4 + 3|\Delta'|^2 - (|\Delta|^2)''\Big) \quad , \\
\hat{g}_5 &= \frac{i}{16}\left(\Delta'''\Delta^* - \Delta\Delta^{*'''} + \Delta'\Delta^{*''} - \Delta''\Delta^{*'} + 6|\Delta|^2(\Delta^{*'}\Delta - \Delta'\Delta^*)\right) \\
\hat{g}_6 &= \frac{1}{32}\Big(\Delta^{(\mathrm{iv})}\Delta^* + \Delta^{*(\mathrm{iv})}\Delta - (|\Delta'|^2)'' + 3|\Delta''|^2 - 10|\Delta|^2(\Delta''\Delta^* + \Delta^{*''}\Delta) - 5(\Delta^{*2}\Delta'^2 + \Delta^2\Delta^{*'2}) + 10|\Delta|^6 \\
&\vdots
\end{aligned}
$$

real condensate -> mKdV integrable hierarchy

for (real) condensate of Gross-Neveu model

$$\boxed{\frac{\delta}{\delta\sigma(x)}\int \hat{g}_n(x) = \gamma_n\,\sigma(x) \qquad \forall \quad n}$$

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solves the gap equation !

implication:

Gross-Neveu:

Ginzburg-Landau expansion = mKdV hierarchy

NJL:

Ginzburg-Landau expansion = AKNS hierarchy

Correa, GD, Plyushchay, 2009

Başar, GD, Thies, 2009

Q: is this just “magic” of 1+1 dimensions,

or

could there be some integrable structure
in 2+1 dimensions?

integrability in 2+1 dimensions?

Gross-Neveu Models, Nonlinear Dirac Equations, Surfaces and Strings

Gökçe Başar and Gerald V. Dunne

JHEP, 2011

Gross-Neveu models and string theory

Kink dynamics, sinh-Gordon solitons and strings in AdS$_3$ from the Gross-Neveu model

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Andreas Klotzek* and Michael Thies†

Hartree-Fock: $$(i\partial\!\!\!/ - \sigma(x))\,\psi_k = 0$$

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$$\Leftrightarrow$$

$\sigma'' - 2\sigma^3 + \sigma = 0$ NLSE

even more amazingly ...
exact solutions to time-dependent Hartree-Fock

Hartree-Fock: $\left(i\partial\!\!\!/ - \sigma(x,t)\right)\psi_k = 0$

$$\sigma(x,t) = \sum_k \bar{\psi}_k(x,t)\psi_k(x,t)$$

amazingly, "mode-by-mode" :

$$\bar{\psi}_k(x,t)\psi_k(x,t) = f(k)\,\sigma(x,t) \qquad \forall\, k$$

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nonlinear Dirac equation :

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$\sigma = \bar{\psi}\psi \equiv e^{\theta/2}$ $\quad \Rightarrow$ 2 dim. Sinh-Gordon

$$\partial_\mu \partial^\mu \theta + 4\sinh\theta = 0$$

boosted solution: $$\sigma(x) \to \sigma\left(\frac{x - vt}{\sqrt{1 - v^2}}\right)$$

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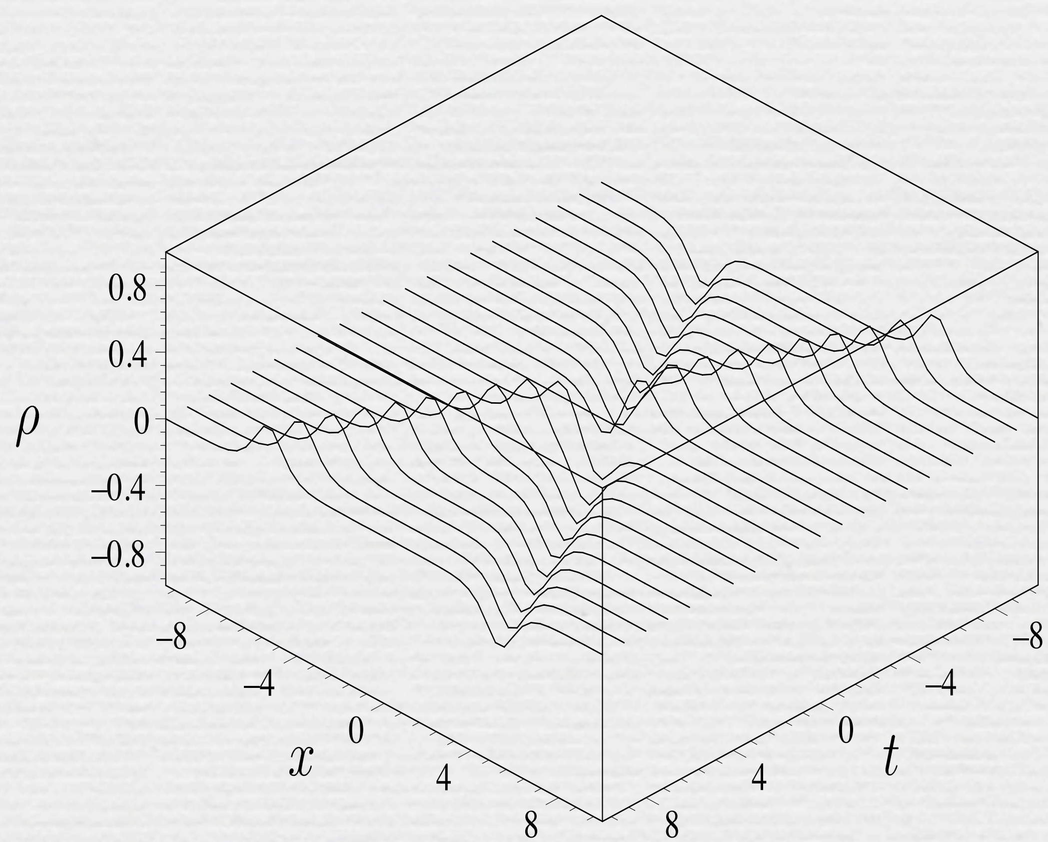
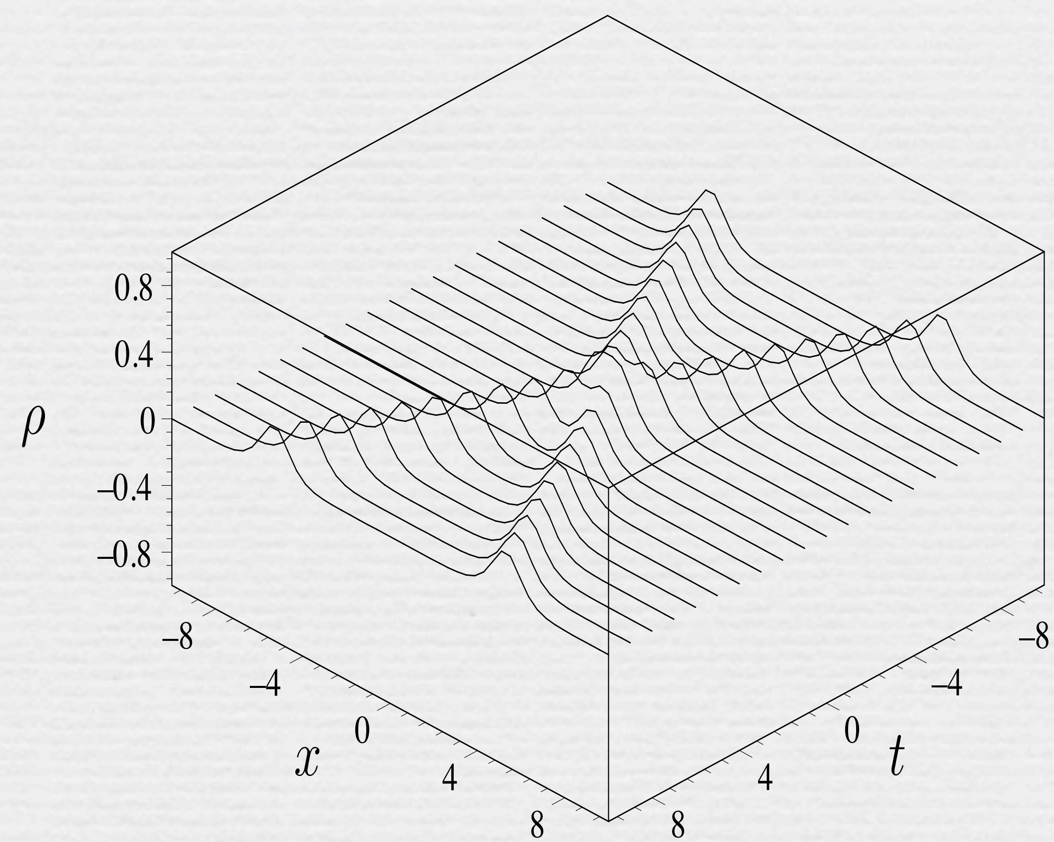
scattering solution :

$$\sigma(x,t) = \frac{v\cosh(2x/\sqrt{1-v^2}) - \cosh(2vt/\sqrt{1-v^2})}{v\cosh(2x/\sqrt{1-v^2}) + \cosh(2vt/\sqrt{1-v^2})}$$



baryon-antibaryon



baryon-baryon

so we have a solution to the gap equation:

$$\sigma(x,t) = \frac{\delta}{\delta\sigma(x,t)} \ln\det\left(i\partial\!\!\!/ - \sigma(x,t)\right)$$

perhaps we can find a solution to :

$$\sigma(x,y) = \frac{\delta}{\delta\sigma(x,y)} \ln\det\left(i\partial\!\!\!/ - \sigma(x,y)\right)$$

this could represent a static crystalline phase of the 2+1 dimensional Gross-Neveu model a geometric (and integrable models) perspective ...

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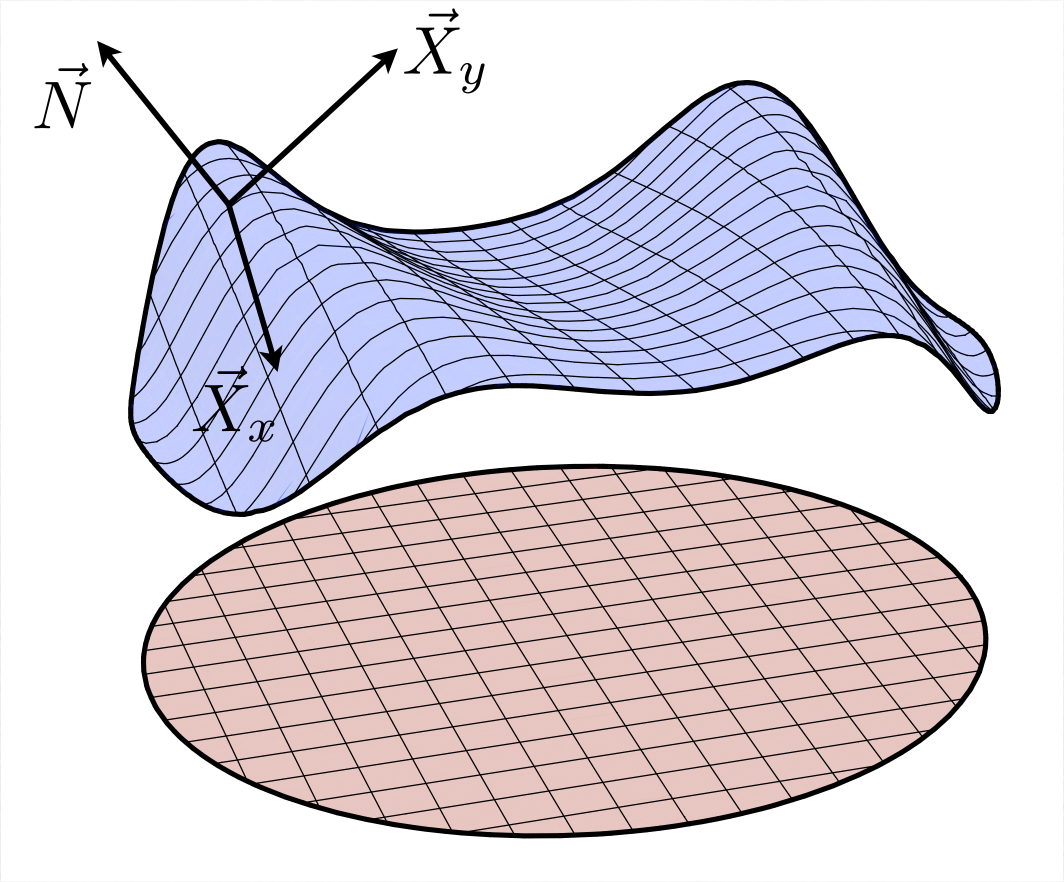
immersion of a surface in 3 dimensions



$$ds^2 = f^2(x_+, x_-)\, dx_+\, dx_-$$

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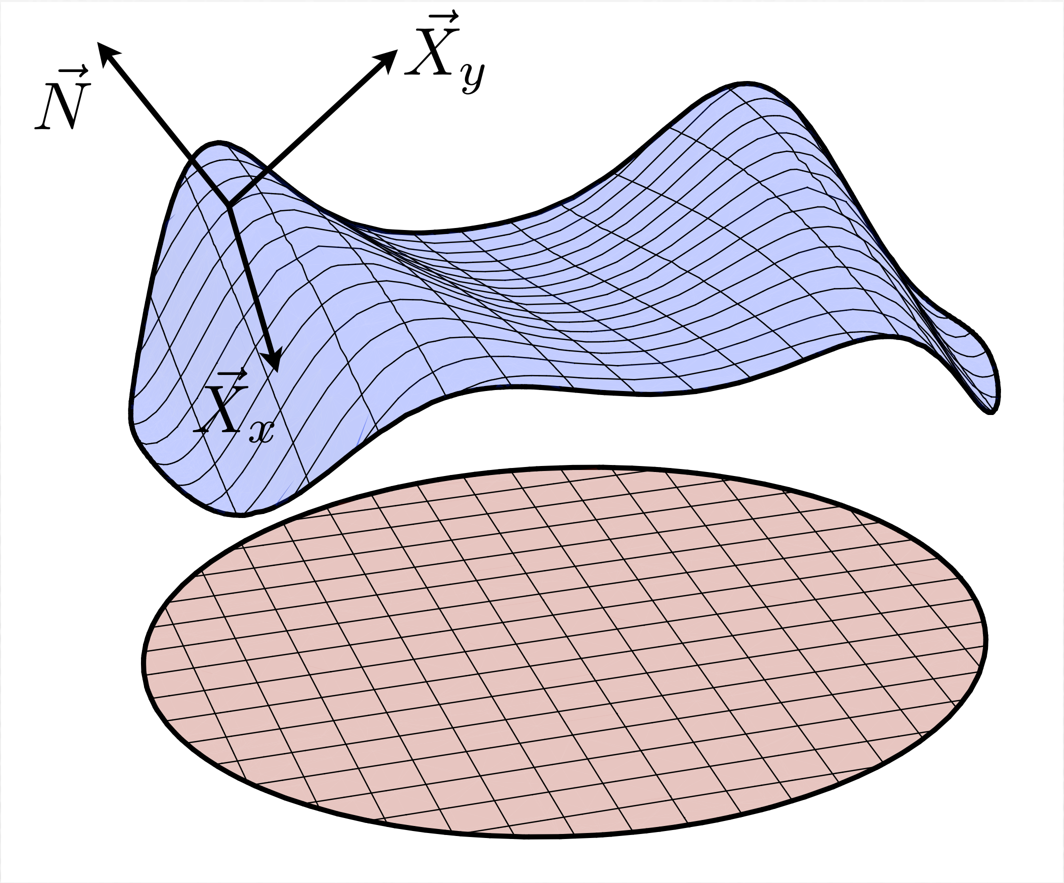

$$ds^2 = f^2(x_+, x_-)\, dx_+\, dx_-$$

H: mean curvature

Gauss-Codazzi equations

$$\begin{aligned}
f f_{+-} - f_+ f_- - \frac{1}{4} H^2 f^4 &= -Q^{(+)} Q^{(-)} \\
Q_-^{(+)} &= \frac{1}{2} f^2 H_+ \\
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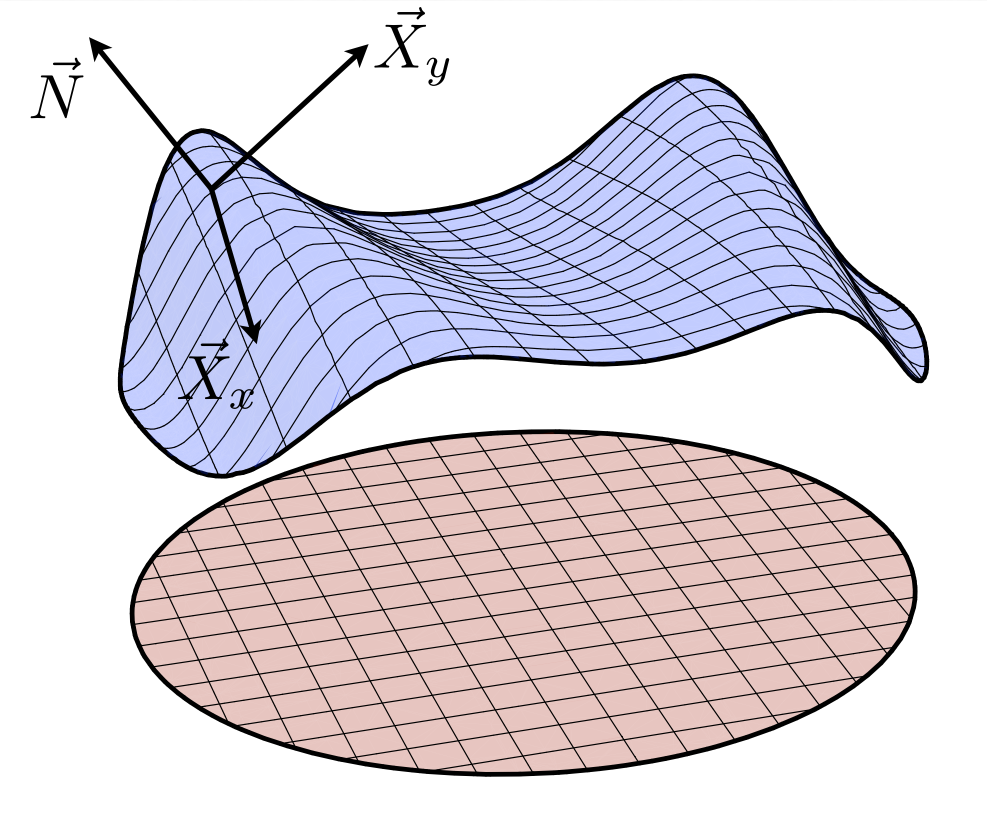
\end{aligned}$$

spinor representation of surfaces



(Weierstrass, Enneper, Eisenhart, Hopf, Bobenko, Konopelchenko, ...)

$$SO(1,2) \sim SU(1,1)$$

$$\vec{X} = (X_1, X_2, X_3) \qquad \leftrightarrow \qquad X = -i \begin{pmatrix} X_3 & X_1 - iX_2 \\ X_1 + iX_2 & -X_3 \end{pmatrix}$$

$$\vec{X}_+, \quad \vec{X}_-, \quad \vec{N} \qquad \Rightarrow \qquad SU(1,1) \text{ spinors } \psi$$

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spinor representation:

Dirac equation: $(i\partial\!\!\!/ - S)\psi = 0$

induced metric factor: $f = \bar{\psi}\psi$

mean curvature: $S = H\,\bar{\psi}\psi$

Hopf differentials:

$$Q^{(+)} = -i(\psi_1^* \psi_{1,+} - \psi_{1,+}^* \psi_1)$$

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constant mean curvature H=*l* : nonlinear Dirac equation

$$\left(i\partial\!\!\!/ - l\,\bar{\psi}(x)\psi(x)\right)\psi = 0$$

Gauss-Codazzi equations

$$
\begin{aligned}
f f_{+-} - f_+ f_- - \frac{1}{4} H^2 f^4 &= -Q^{(+)} Q^{(-)} \\
Q^{(+)}_- &= \frac{1}{2} f^2 H_+ \\
Q^{(-)}_+ &= \frac{1}{2} f^2 H_-
\end{aligned}
$$

spinor representation:

Dirac equation: $(i\partial\!\!\!/ - S)\psi = 0$

induced metric factor: $f = \bar{\psi}\psi$

mean curvature: $S = H\,\bar{\psi}\psi$

Hopf differentials:

$$Q^{(+)} = -i(\psi_1^* \psi_{1,+} - \psi_{1,+}^* \psi_1)$$

$$Q^{(-)} = i(\psi_2^* \psi_{2,-} - \psi_{2,-}^* \psi_2)$$

constant mean curvature H=*l* : nonlinear Dirac equation

$$\left(i\partial\!\!\!/ - l\,\bar{\psi}(x)\psi(x)\right)\psi = 0$$

$f^2 = e^\theta \quad \Rightarrow \quad$ Sinh-Gordon $\quad \partial_\mu \partial^\mu \theta + 4\sinh\theta = 0$

constant mean curvature in flat space

$$\Leftrightarrow$$

zero mean curvature in AdS_3 space

constant mean curvature in flat space

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explicit map between time-dependent solutions
to Gross-Neveu gap equation
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suggests new geometrical approach to search for
inhomogeneous solutions to Gross-Neveu gap equations

geometric meaning of static inhomogeneous condensates in 1+1 dimensions for GN_2

immersion of curves into 3 dimensional space

Da Rios (1906),
student of Levi-Civita

"vortex filament equations"

potential satisfies NLSE=ShG

SUL MOTO D'UN LIQUIDO INDEFINITO CON UN FILETTO VORTICOSO

che, per le (18) e (21), diventano:

$$-\frac{d\tau}{dt} - \left(\frac{c''}{c} - \tau^2\right)' = cc',$$

$$c'' = c'' - c\tau^2 + c\tau^2,$$

$$\frac{dc}{dt} = c\tau' + 2c'\tau.$$

Abbiamo quindi finalmente le equazioni cercate:

$$\text{(22)} \quad \begin{cases} \dfrac{dc}{dt} = c\tau' + 2c'\tau, \\[2mm] \dfrac{d\tau}{dt} = -cc' + \left(\tau^2 - \dfrac{c''}{c}\right)'. \end{cases}$$

Il teorema di esistenza, applicato a questo sistema di equazioni a
mette di asserire con tutto rigore che le funzioni $c(s, t)$, $\tau(s, t)$
regolarità) univocamente definite dai valori inizi

The intrinsic equations (22) as they were presented by Da Rios in his first paper published in 1906; c and τ stand for curvature and torsion of the vortex filament, respectively.

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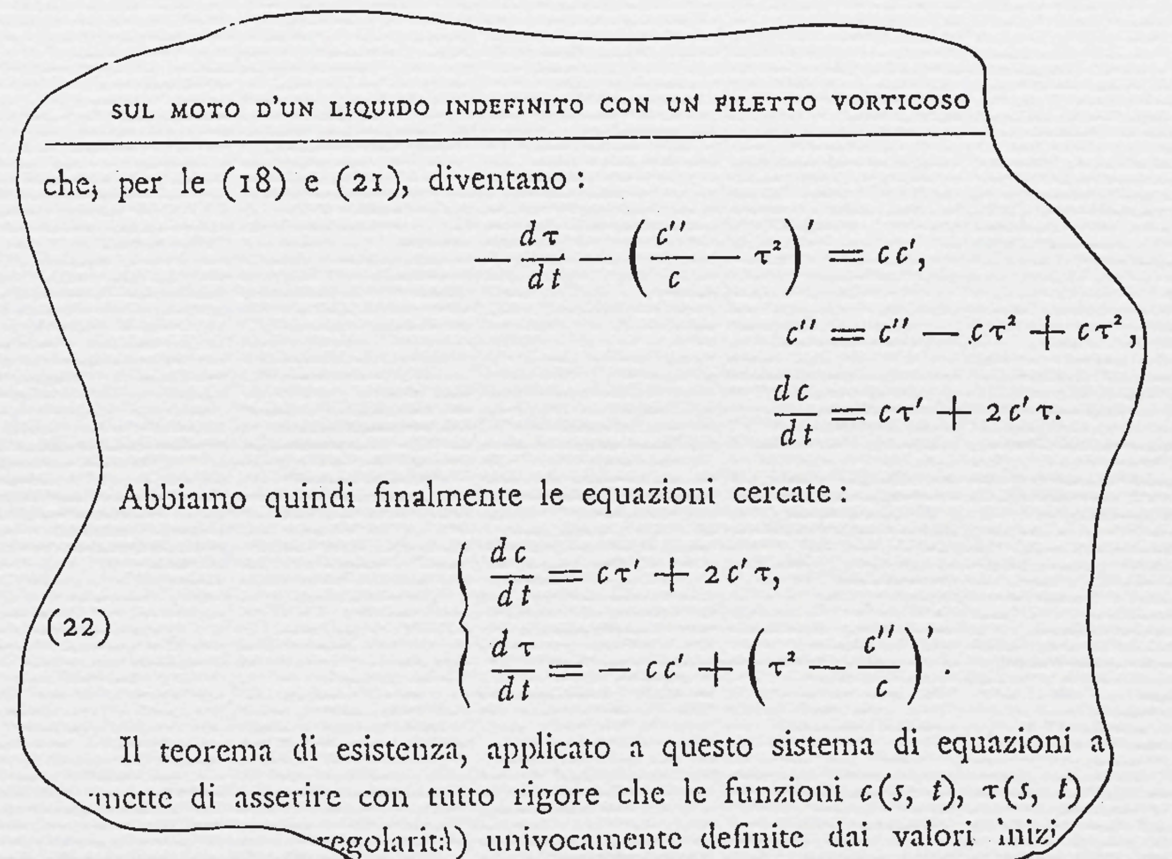
(spectral) deformations:
mKdV hierarchy



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mKdV governs thermodynamics of 1+1 GN model

proposal/conjecture for 2+1 dim GN:

Gauss-Codazzi equations for moving frame of surface embedding can be written as a Dirac equation

solutions satisfy Sinh-Gordon

(spectral) deformations of these surfaces :
(m) Novikov-Veselov hierarchy

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question: does mNV govern the thermodynamics of 2+1 dimensional GN model ?

lowest nontrivial equation of mKdV :

$$\Delta'' - 2|\Delta|^2\Delta = \nu\,\Delta$$

lowest nontrivial equation of mNV :

$$\nabla^2\Delta - \left[\left(\frac{\partial}{\bar{\partial}} + \frac{\bar{\partial}}{\partial}\right)|\Delta|^2\right]\Delta = \nu\,\Delta$$

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but: less is known about solutions ...

Conclusions

- general solution of gap equation for GN_2/NJL_2
- full, exact, thermodynamics & phase diagram
- Ginzburg-Landau expansion = mKdV or AKNS hierarchy
- geometric picture: curve and surface embedding

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- higher dimensional models : Novikov-Veselov hierarchy ?

congratulations Manuel,

and many more!

physics: the Peierls Instability

one dimension: gap formation at the Fermi surface can lead to breakdown of translational symmetry

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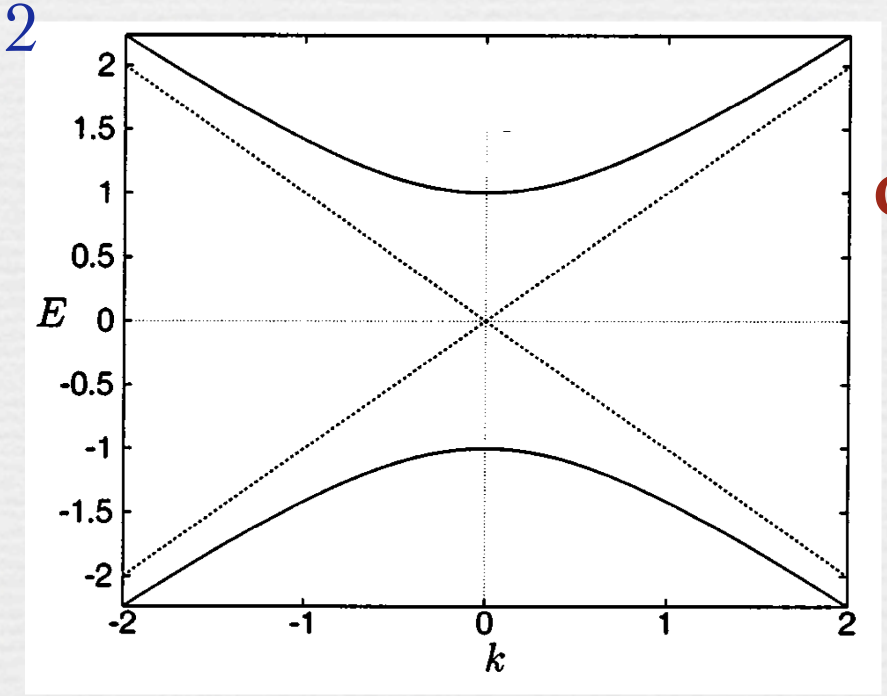
GN_2



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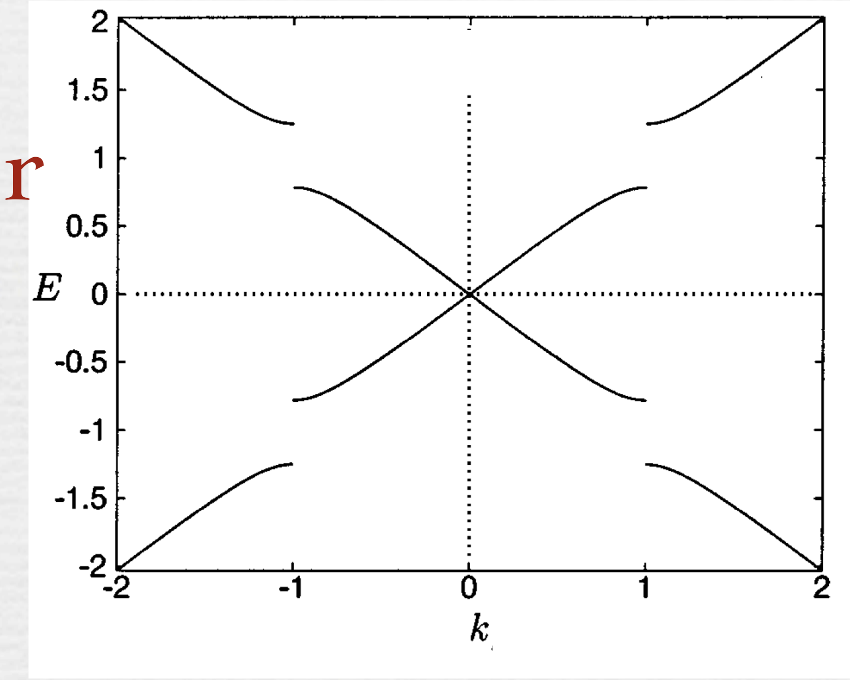

phase diagram of chiral Gross-Neveu (NJL_2)

Peierls instability for NJL model

continuous chiral symmetry : BdG equation

$$\begin{pmatrix} -i\partial_x & \Delta(x) \\ \Delta^*(x) & i\partial_x \end{pmatrix} \psi = E\psi$$

invariant under :

$$\Delta(x) \to e^{2iqx}\Delta(x)$$
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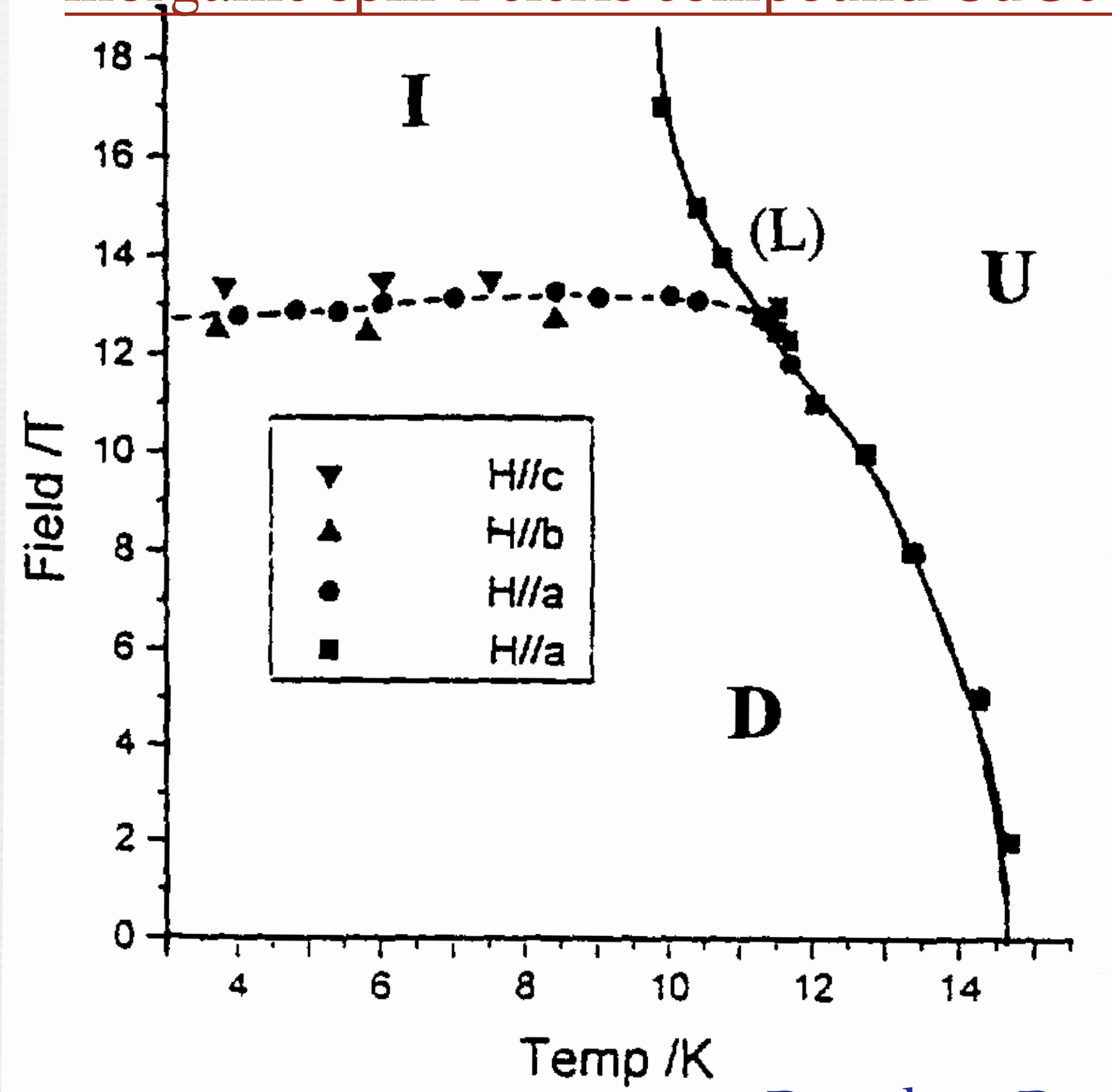
minimizing the thermodynamic potential $\Rightarrow \quad q = \mu$

"system prefers to open a gap at the Fermi level"

inorganic spin-Peierls compound $CuGeO_3$



J. Hofmann:1009.4071

Boucher, Reynault, 1996

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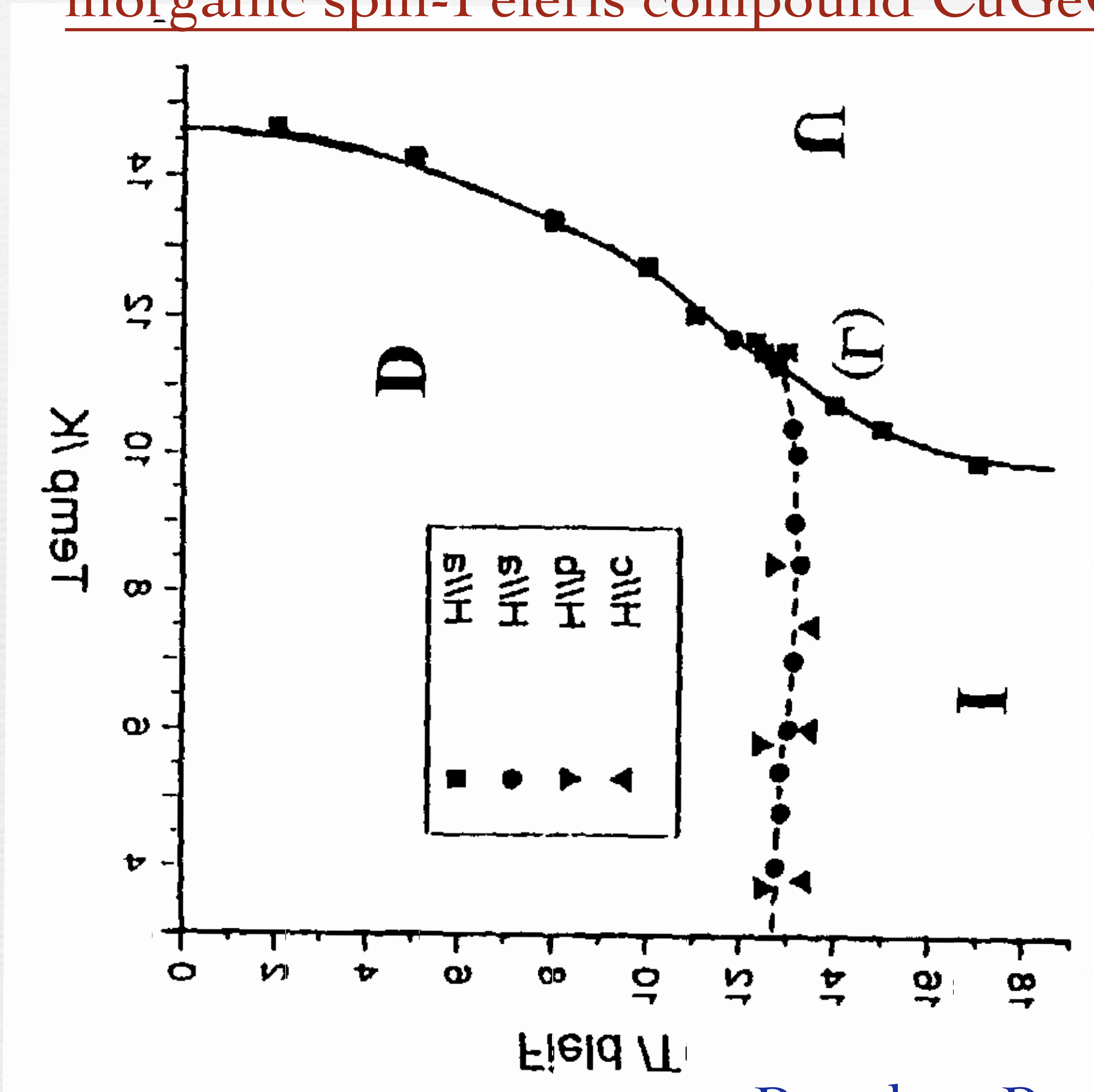

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