



# Heavy Quark Spectroscopy

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# Elementary Particles in 2011

Three Generations of Matter [Fermions]			
	I	II	III
mass →	2.4 MeV	1.27 GeV	171.2 GeV
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
name →	u up	c charm	t top
Quarks	u	c	t
	4.8 MeV	104 MeV	4.2 GeV
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	d down	s strange	b bottom
	d	s	b
Lepons	e electron neutrino	$\nu_\mu$ muon neutrino	$\nu_\tau$ tau neutrino
	$< 2.2 \text{ eV}$	$< 0.17 \text{ MeV}$	$< 15.5 \text{ MeV}$
	0	0	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	$\nu_e$ electron neutrino	$\nu_\mu$ muon neutrino	$\nu_\tau$ tau neutrino
Bosons (Forces)	Z weak force		
	91.2 GeV	0	0
	0	0	1
	$\pm 1$		
	W weak force		
	80.4 GeV		
	$\pm 1$		
	W		
	0.511 MeV	105.7 MeV	1.777 GeV
	-1	-1	-1
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	e electron	$\mu$ muon	$\tau$ tau
	e	$\mu$	$\tau$



# The SM at low energies

For  $p \ll m_{W,Z}$  the Standard Model (SM) reduces to

$$\begin{aligned}\mathcal{L}_{QCD+QED} = & -\frac{1}{2} \text{tr} (F_{\mu\nu} F^{\mu\nu}) - \frac{1}{4} F_{\mu\nu}^{em} F^{em\ \mu\nu} + \\ & + \sum_{q=u,d,s} \bar{q} (i \not{D}^q - m_q) q + \sum_{Q=c,b} \bar{Q} (i \not{D}^Q - m_Q) Q + \\ & + \sum_{l=e,\mu,\tau} \bar{l} (i \not{D}^l - m_l) l + \bar{\nu}_l i \not{\partial} \nu_l + \mathcal{O}(1/m_{W,Z})\end{aligned}$$

- ➊  $-igF_{\mu\nu} - iee_q F_{\mu\nu}^{em} = [D_\mu^q, D_\nu^q], D_\mu^{q,Q} = \partial_\mu - igA_\mu - iee_{q,Q} A_\mu^{em}, D_\mu^l = \partial_\mu - iee_l A_\mu^{em}$
- ➋  $e$ , charge of the positron
- ⌋  $e_{q,Q,l}$ , the fraction of the charge of the electron that the corresponding particles have.





# QCD

QCD (**early 70's**) is the current theory of the strong interactions.

$$\mathcal{L}_{QCD} = \mathcal{L}_g + \sum_{q=u,d,s} \mathcal{L}_q + \sum_{Q=c,b,t} \mathcal{L}_Q$$

$$\mathcal{L}_g = -\frac{1}{2} \text{tr} (F_{\mu\nu} F^{\mu\nu})$$

$$\mathcal{L}_q = \bar{q} (iD^\mu - m_q) q \quad , \quad \mathcal{L}_Q = \bar{Q} (iD^\mu - m_Q) Q$$

- $-igF_{\mu\nu} = [D_\mu, D_\nu]$ ,  $D_\mu = \partial_\mu - igA_\mu$  ,  $A_\mu = T^a A_\mu^a$ ,
- $A_\mu^a \in 8$  of  $SU(3)$ , the gluon fields
- $q$  and  $Q \in 3$  of  $SU(3)$ , the light and heavy quark fields
- $T^a$  are the generators of 3 of  $SU(3)$
- $g$  is the QCD coupling constant

# QCD



- $\mathcal{L}_{QCD}$  is invariant under local color  $SU(3)$
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$$m_u = 1.5 - 3.0 \text{ MeV.}, \quad m_d = 3.5 - 6.0 \text{ MeV}, \quad m_s = 70 - 130 \text{ MeV}$$

$$m_c = 1.25 \pm 0.09 \text{ GeV.}, \quad m_b = 4.20 \pm 0.07 \text{ GeV.}, \quad m_t = 171.2 \pm 2.1 \text{ GeV}$$





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- There is also  $\Lambda_{QCD} \sim 300 - 700 \text{ MeV}$ , dynamically generated scale. Heavy quarks have  $m_Q \gg \Lambda_{QCD}$
- Asymptotic freedom (Gross, Wilczek; Politzer, 1971) implies  $\alpha_s(m_Q) \ll 1$ ,  $\alpha_s = g^2/4\pi$



# QCD



The light quarks fulfil  $m_q \ll \Lambda_{QCD}$   $\rightarrow$  there is an approximate  $U(3) \times U(3)$  chiral symmetry,

- It is explicitly broken in the quantum theory down to  $U(1) \times SU(3) \times SU(3)$
- And spontaneously broken down to flavor  $SU(3)$  producing pseudo-Goldstone bosons,  $G = \{\pi^0, K^0, \eta\}$ ,  $m_G \ll \Lambda_{QCD}$





# Lattice QCD

Put (Euclidean) QCD on a lattice of  $N^4$  points separated a distance  $a$

- $a$  is an UV regulator, hence  $1/a \gg m_h$ ,  $m_h$  being the typical scale of the process we wish to study (e.g. a hadron mass).
- $Na$  is an IR cut-off, hence  $1/Na \ll m_G$  due to the existence of Goldstone bosons, otherwise  $1/Na \ll \Lambda_{QCD}$  would be enough.

The second constrain holds for any system we wish to study. The first one changes depending on the system





# Lattice QCD

- For hadrons involving light quarks only, we have  
 $1/a \gg \Lambda_{QCD}$
- For hadrons involving heavy quarks, we have  
 $1/a \gg m_Q$

QCD lattice calculations involving heavy quarks are very costly. A wise way to proceed is to calculate (factorize) the effects at the scale  $m_Q$  using perturbation theory, and leave for a less costly lattice calculation the effects at lower scales.



# Tools:





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Effective Field Theories (**EFTs**)





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- Construct a new theory (the EFT) involving only the relevant degrees of freedom for the energy region of interest
  - Identify relevant degrees of freedom
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  - Identify relevant degrees of freedom
  - Implement the symmetries of QCD
  - Exploit the hierarchy of energy scales
- The EFT gives equivalent physical results in the region where it holds
  - It may make apparent accidental symmetries in that region, which help constraining the physics.
  - Calculations are usually simpler.





# Heavy-light systems

$Q\bar{q}$  bound state ,  $m_Q \gg \Lambda_{QCD}$  ,  $\alpha_s(m_Q) \ll 1$

- The heavy quark is essentially at rest in the rest frame of the heavy meson
- If the heavy meson moves with velocity  $v$ ,  $v^2 = 1$ , the heavy quark will also move with velocity  $v$  up to fluctuations  $\sim \Lambda_{QCD}$ , namely its momentum can be written as  $p = m_Q v + k$ ,  $k \sim \Lambda_{QCD} \ll m_Q$
- Naïvely, we can write  $Q(x) = e^{-im_Q vx} p_+ h_v^{(Q)}(x)$ ,  $p_\pm = (1 \pm \not{v})/2$  projects onto the particle subspace, and  $h_v^{(Q)}(x)$  contains the momentum fluctuations  $k \sim \Lambda_{QCD}$





# Heavy-light systems

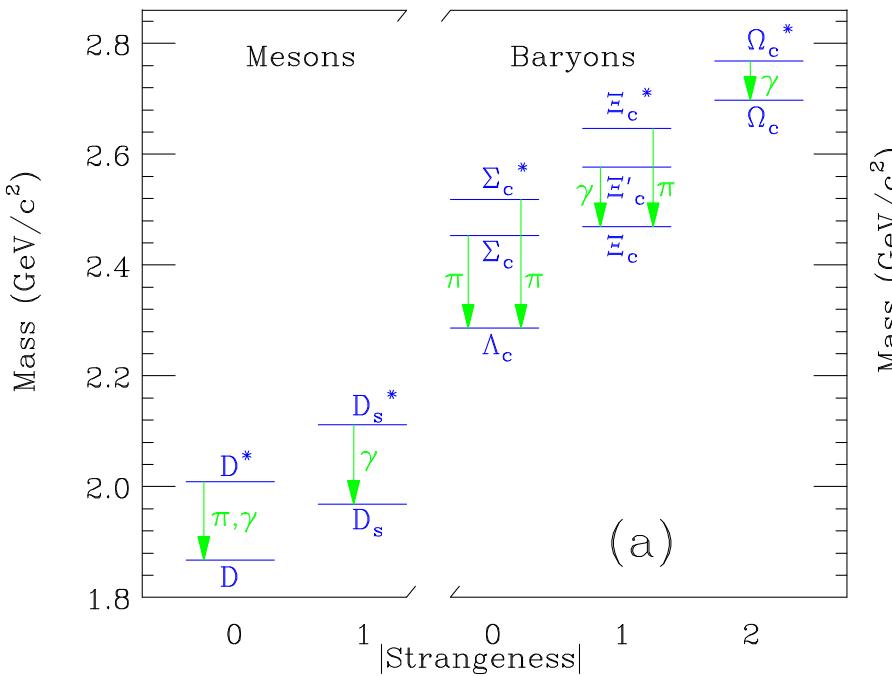
If this is done for all heavy quarks in the heavy quark part of the QCD lagrangian

$$\sum_{Q=c,b} \mathcal{L}_Q \rightarrow \sum_{Q=c,b} \mathcal{L}_{HQET}^{(Q)} = \sum_{Q=c,b} \bar{h}_v^{(Q)} i v.D h_v^{(Q)}$$

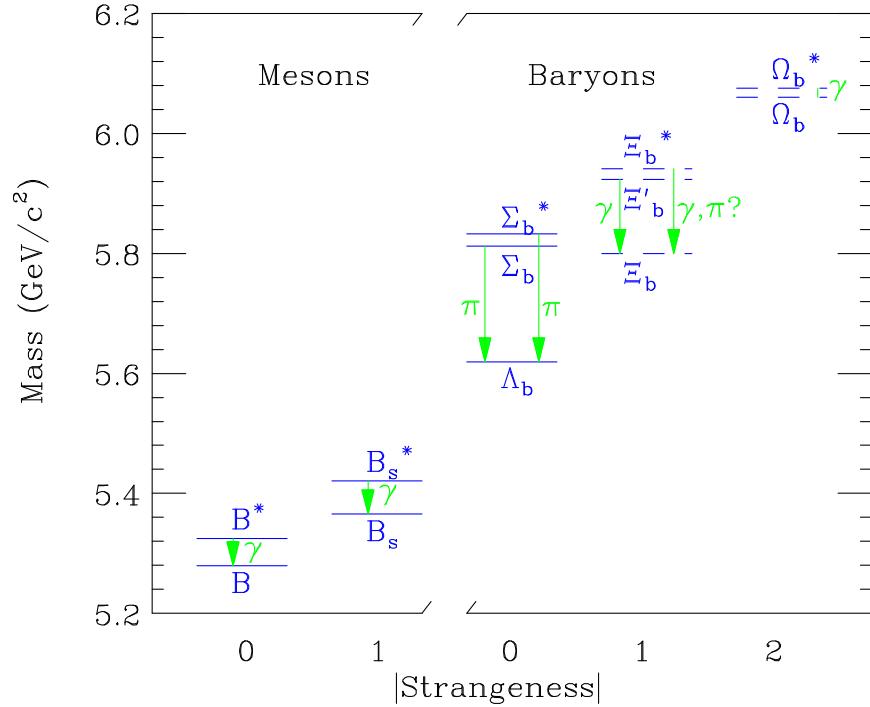
- It enjoys an accidental  $SU(2N_Q)$  spin-flavor symmetry
  - $m_{b\bar{q}} - m_b \sim m_{c\bar{q}} - m_c$ . E.g.  $m_{D_1^0} - m_{D^0} \sim m_{B_1^0} - m_{B^0}$
  - For each  $Q$ , spin doublets  $(S, S+1)$ . E.g.  $m_B \sim m_{B^*}$ ,  
 $m_D \sim m_{D^*}$
- One expects the naïve picture to be modified by
  - Power corrections:  $\sim 1/m_Q$  suppressed terms
  - Radiative corrections:  $\sim \alpha_s(m_Q)$  suppressed terms



# Heavy-light systems



(a)



Rosner, hep-ph/0612332



# Power + Radiative Corrections

- For  $v = (1, 0)$ , we have

$$\mathcal{L} = \bar{h}_v \left\{ iD^0 + c_2 \frac{\mathbf{D}^2}{2m} + c_4 \frac{\mathbf{D}^4}{8m^3} + c_F g \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2m} + c_D g \frac{[\mathbf{D} \cdot \mathbf{E}]}{8m^2} + i c_S g \frac{\boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8m^2} + \mathcal{O}(1/m^3) \right\} h_v,$$

- $v$  arbitrary is recovered by  $D^0 \rightarrow v.D$ ,  $\mathbf{D} \rightarrow D^\mu - v^\mu v.D$  and  
 $\boldsymbol{\sigma} \rightarrow i p_+ \gamma^\nu p_- \gamma^\rho p_+ \epsilon_{\mu\nu\rho\sigma} v^\sigma / 2$





# Matching

$$c_2 = c_4 = 1$$

$$c_F = 1 + \frac{\alpha}{\pi} \left[ \frac{1}{2} C_f + \left( \frac{1}{2} - \frac{1}{2} \log \frac{m}{\mu} \right) C_A \right]$$

$$c_D = 1 + \frac{\alpha}{\pi} \left[ \left( \frac{8}{3} \log \frac{m}{\mu} \right) C_f + \left( \frac{1}{2} + \frac{2}{3} \log \frac{m}{\mu} \right) C_A \right]$$

$$c_S = 1 + \frac{\alpha}{\pi} \left[ C_f + \left( 1 - \log \frac{m}{\mu} \right) C_A \right] , \quad C_A = N_c$$

$$\left[ \begin{aligned} \mathcal{L} = & \bar{h}_v \left\{ iD^0 + c_2 \frac{\mathbf{D}^2}{2m} + c_4 \frac{\mathbf{D}^4}{8m^3} + c_F g \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2m} + c_D g \frac{[\mathbf{D} \cdot \mathbf{E}]}{8m^2} \right. \\ & \left. + i c_S g \frac{\boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8m^2} + \mathcal{O}(1/m^3) \right\} h_v \end{aligned} \right]$$





# Renormalization Group

The solution of the RG equations at one loop reads

$$c_F(\mu) = z^{-C_A}$$

$$c_D(\mu) = z^{-2C_A} + \left( \frac{20}{13} + \frac{32}{13} \frac{C_F}{C_A} \right) \left[ 1 - z^{-13C_A/6} \right]$$

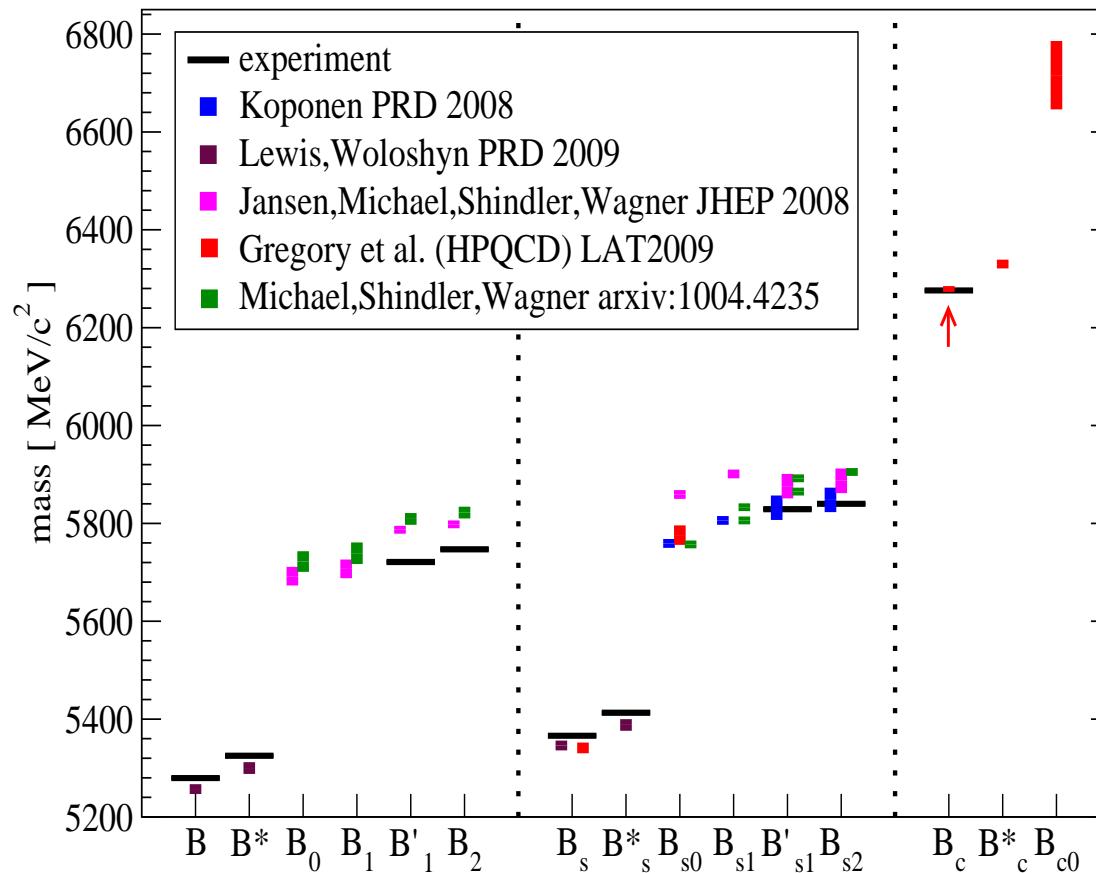
$$z = \left[ \frac{\alpha_s(\mu)}{\alpha_s(m)} \right]^{1/b_0}, \quad b_0 = 11C_A/3 - 4T_F n_f/3$$

## Application:

$$\frac{m_{B^*}^2 - m_B^2}{m_{D^*}^2 - m_D^2} = \left[ \frac{\alpha_s(m_D)}{\alpha_s(m_B)} \right]^{-C_A/b_0} + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_c}, \frac{\Lambda_{QCD}}{m_b}, \alpha_s\right)$$

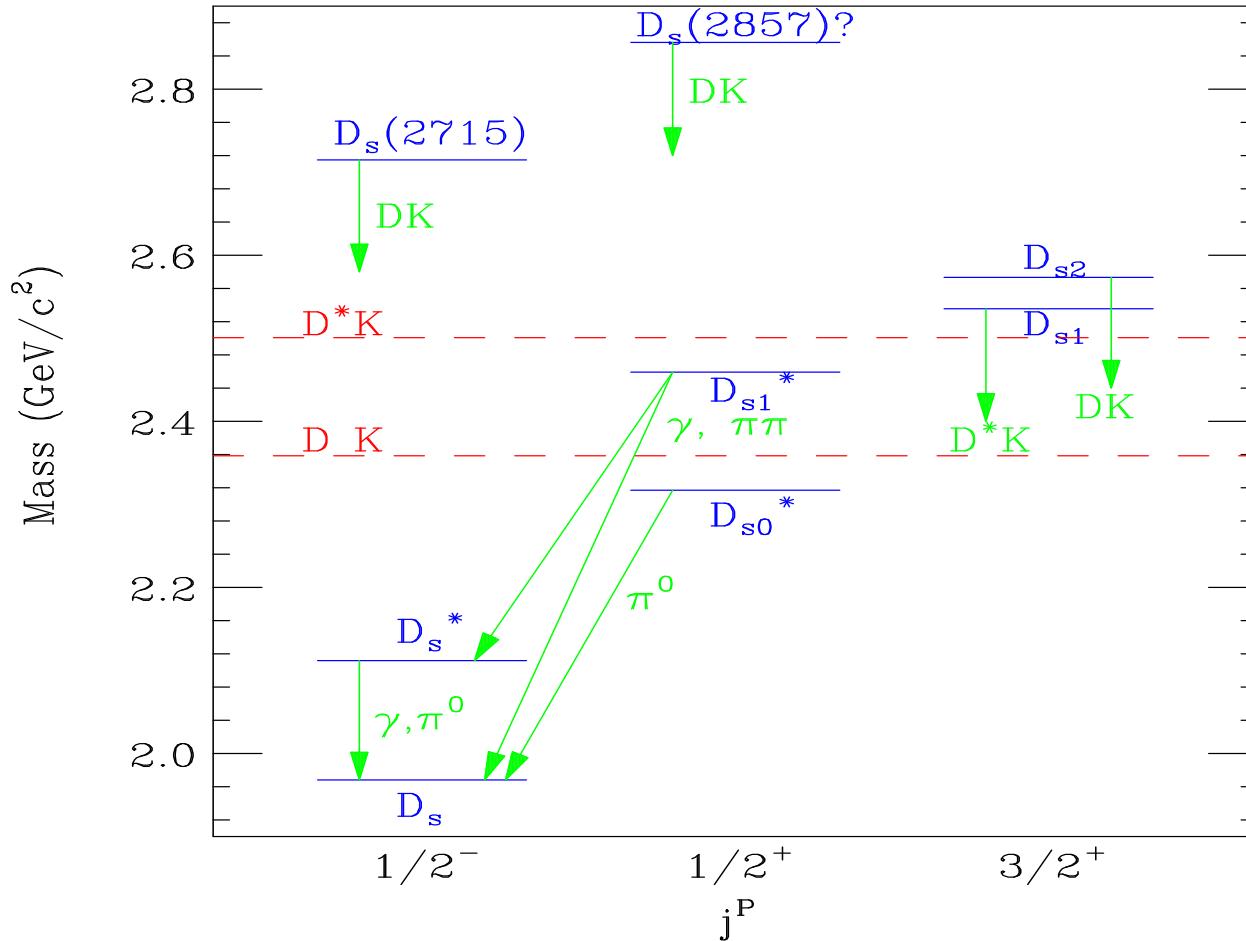


# Heavy-light systems



L Lewis, 1010.0889

# Heavy-light systems



Rosner, hep-ph/0612332



# Heavy Quarkonium



# Heavy Quarkonium

Heavy quark-antiquark system:



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Heavy quark-antiquark system:

- Charmonium ( $c\bar{c}$ ,  $J/\psi$  family)
- Bottomonium ( $b\bar{b}$ ,  $\Upsilon$  family)





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Heavy quark-antiquark system:

- Charmonium ( $c\bar{c}$ ,  $J/\psi$  family)
- Bottomonium ( $b\bar{b}$ ,  $\Upsilon$  family)
- Also:
  - $B_c$  ( $b\bar{c}$ )
  - $t\text{-}\bar{t}$
  - $\tilde{q}\bar{\tilde{q}}, \tilde{g}\bar{\tilde{g}}, \dots$
  - Double-heavy baryons ( $ccq$ ,  $q = u, d, s$ )



# Early approaches



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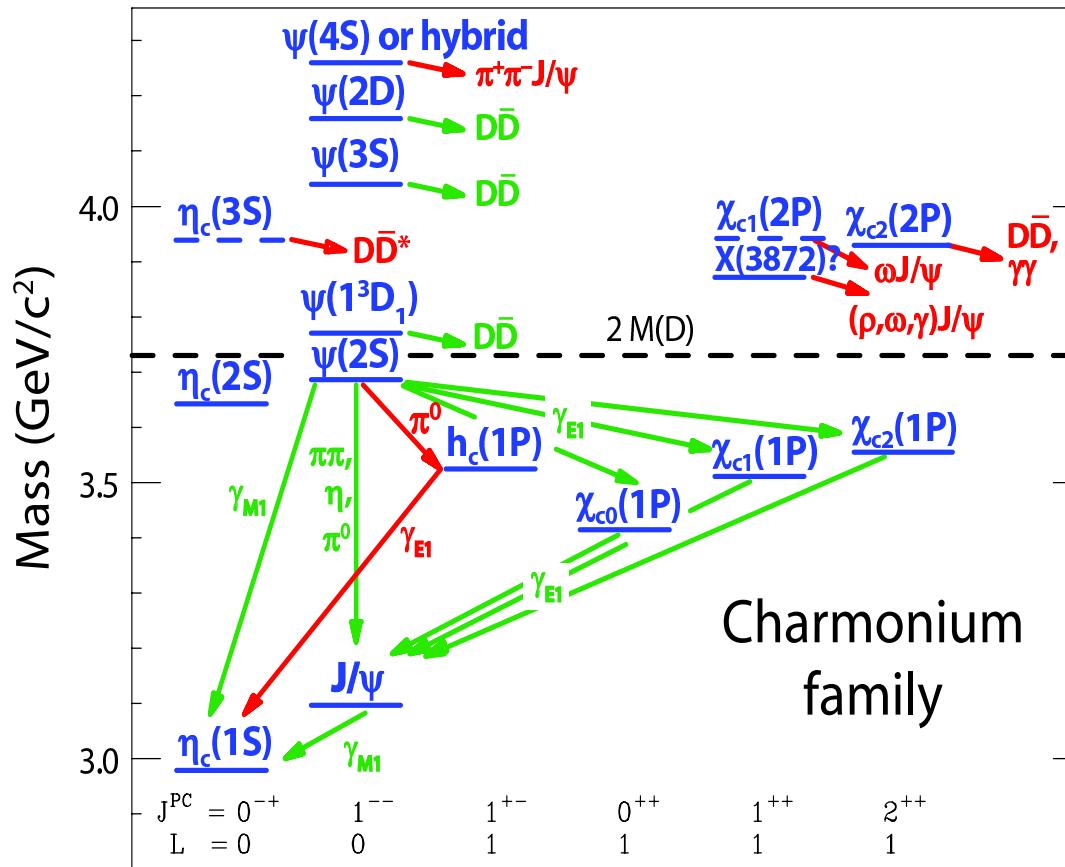
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  - But did not quite follow the pattern of the positronium spectrum. They need not to if  $\Lambda_{QCD}$  is taken into account (Voloshin, 1979; Leutwyler, 1981)
- Potential models: modify the Coulomb-type potential, e.g. Cornell potential (1980):  $V(r) = \sigma r - \frac{b}{r}$



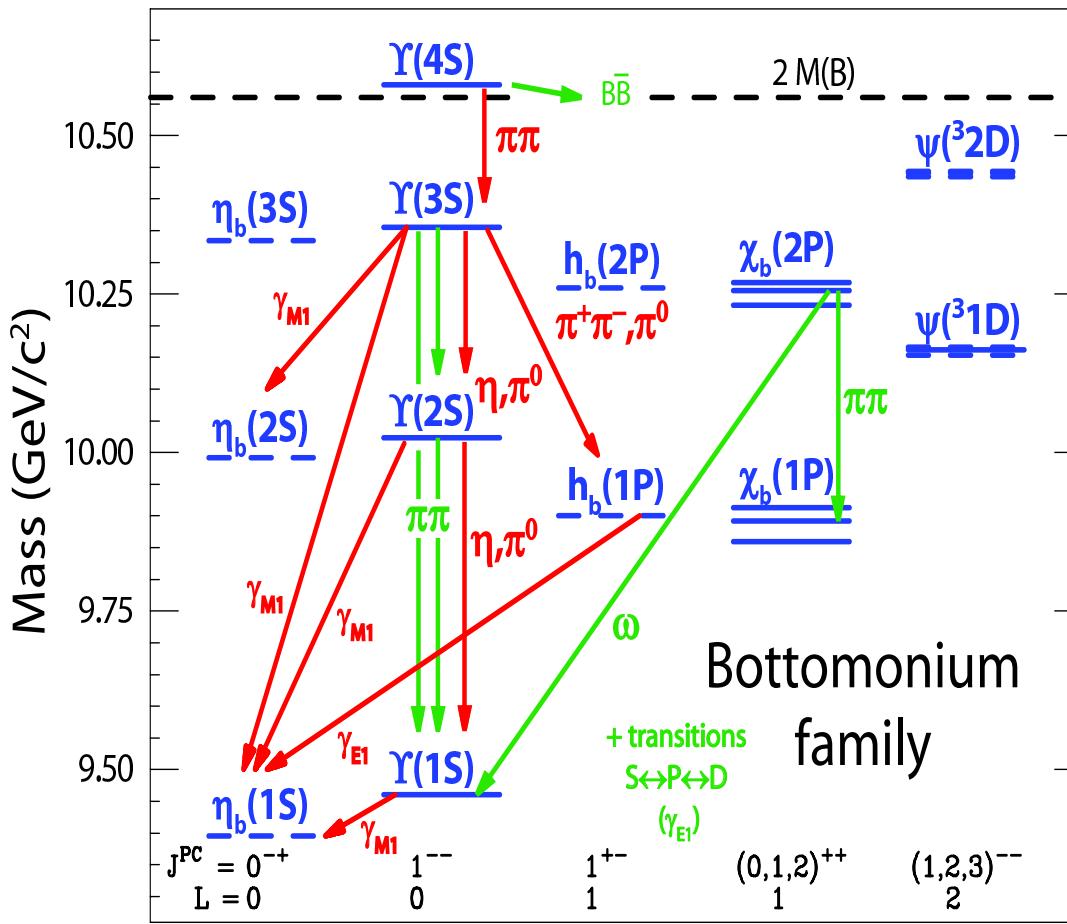
# Charmonium



Eichten, Godfrey, Mahlke, Rosner (2007)



# Bottomonium



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- Non-relativistic system → multiscale problem
  - $m_Q \gg m_Q v \gg m_Q v^2$
  - $m_Q \gg \Lambda_{QCD}$
- EFTs are useful (N. Brambilla, A. Pineda, JS and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005))





# NRQCD

W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986)

G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51**  
(1995) 1125

$$m_Q \quad >> \quad m_Q v \quad , \quad m_Q v^2 \quad , \quad \Lambda_{QCD}$$

$$\begin{aligned} \mathcal{L}_\psi = & \psi^\dagger \left\{ iD_0 + \frac{1}{2m_Q} \mathbf{D}^2 + \frac{1}{8m_Q^3} \mathbf{D}^4 + \frac{c_F}{2m_Q} \boldsymbol{\sigma} \cdot g \mathbf{B} + \right. \\ & \left. + \frac{c_D}{8m_Q^2} (\mathbf{D} \cdot g \mathbf{E} - g \mathbf{E} \cdot \mathbf{D}) + i \frac{c_S}{8m_Q^2} \boldsymbol{\sigma} \cdot (\mathbf{D} \times g \mathbf{E} - g \mathbf{E} \times \mathbf{D}) \right\} \psi \end{aligned}$$

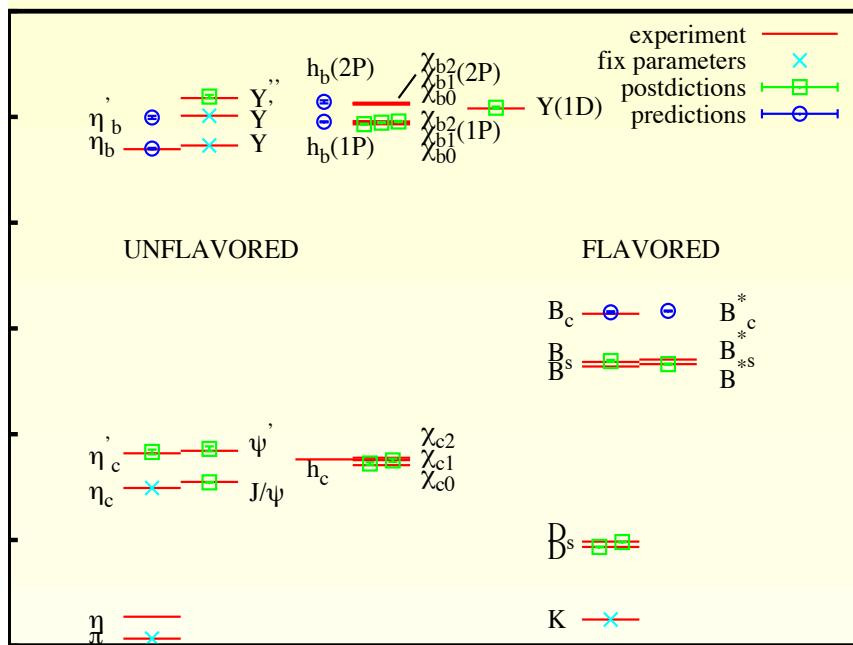
$c_F$ ,  $c_D$  and  $c_S$  are short distance matching coefficients which  
depend on  $m_Q$  and  $\mu$  (factorization scale)



## NRQCD(Cont.)

### ● Spectroscopy → Lattice NRQCD

The gold-plated meson spectrum from lattice QCD - HPQCD collaboration 2009



Gregory et al. [HPQCD] PRL.104:022001(2010)





## NRQCD(Cont.)

Problems:





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# pNRQCD

$\Lambda_{QCD} \lesssim m_Q v^2$  : weak coupling regime

$$\begin{aligned}\mathcal{L}_{\text{pNRQCD}} = & \int d^3\mathbf{r} \operatorname{Tr} \left\{ \mathbf{S}^\dagger (i\partial_0 - h_s(\mathbf{r}, \mathbf{p}, \mathbf{P}_R, \mathbf{S}_1, \mathbf{S}_2, \mu)) \mathbf{S} + \right. \\ & \left. + \mathbf{O}^\dagger (iD_0 - h_o(\mathbf{r}, \mathbf{p}, \mathbf{P}_R, \mathbf{S}_1, \mathbf{S}_2, \mu)) \mathbf{O} \right\} \\ & + V_A(r, \mu) \operatorname{Tr} \left\{ \mathbf{O}^\dagger \mathbf{r} \cdot g \mathbf{E} \mathbf{S} + \mathbf{S}^\dagger \mathbf{r} \cdot g \mathbf{E} \mathbf{O} \right\} + \\ & + \frac{V_B(r, \mu)}{2} \operatorname{Tr} \left\{ \mathbf{O}^\dagger \mathbf{r} \cdot g \mathbf{E} \mathbf{O} + \mathbf{O}^\dagger \mathbf{O} \mathbf{r} \cdot g \mathbf{E} \right\}\end{aligned}$$

$h_s, h_o$  = quantum mechanical Hamiltonians with scale dependent potentials calculable in perturbation theory in  
 $\alpha_s(m_Q v)$



## pNRQCD (Cont.)

$\Lambda_{QCD} \lesssim mv$  : strong coupling regime

$$L_{\text{pNRQCD}} = \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 S^\dagger (i\partial_0 - h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2)) S,$$

$$h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) = \frac{\mathbf{p}_1^2}{2m_Q} + \frac{\mathbf{p}_2^2}{2m_Q} + V_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2),$$

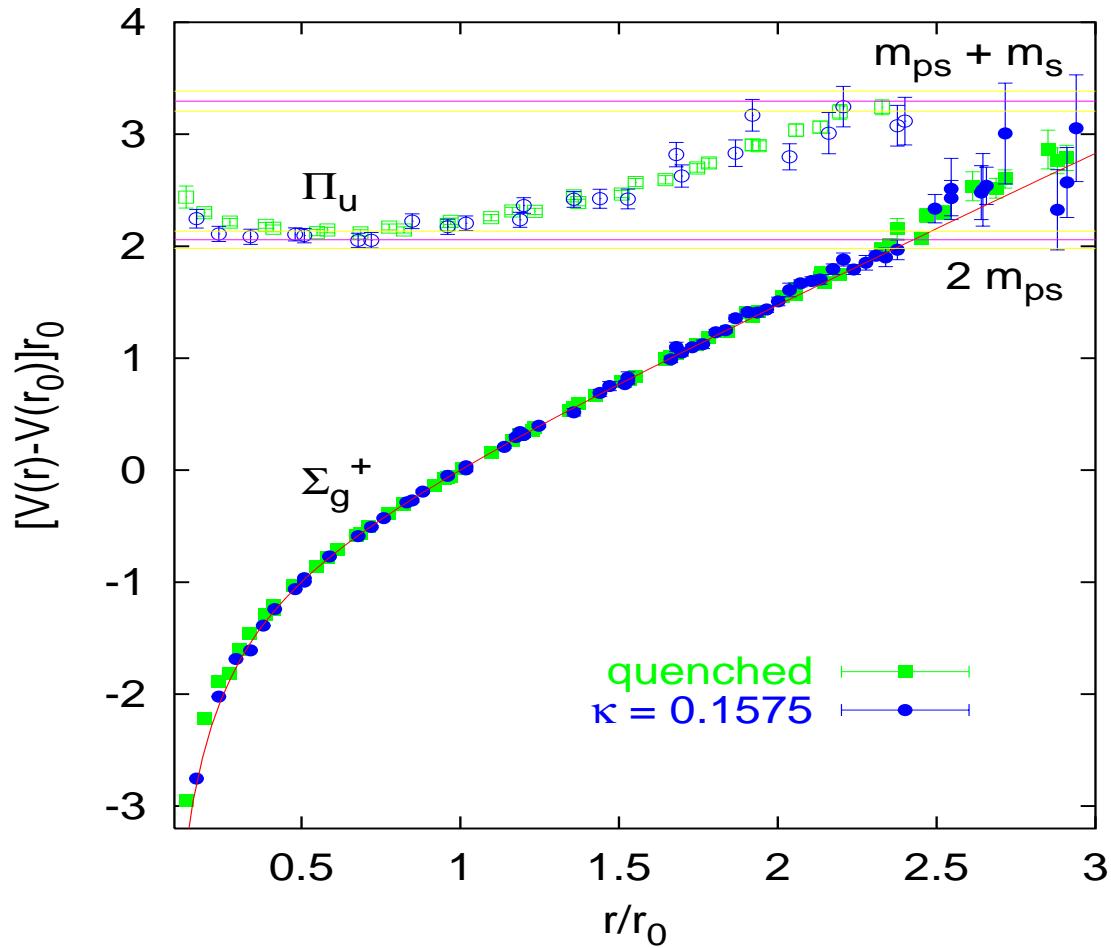
$$V_s = V_s^{(0)} + \frac{V_s^{(1)}}{m_Q} + \frac{V_s^{(2)}}{m_Q^2} + \dots,$$

All  $V_s$ s can be, and most of them have been, calculated on the lattice





## pNRQCD(Cont.)



G.S. Bali at al. (TXL Collaboration), Phys. Rev.  
D62,(2000):054503

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- Charmonium:  $v^2 \sim 0.3$

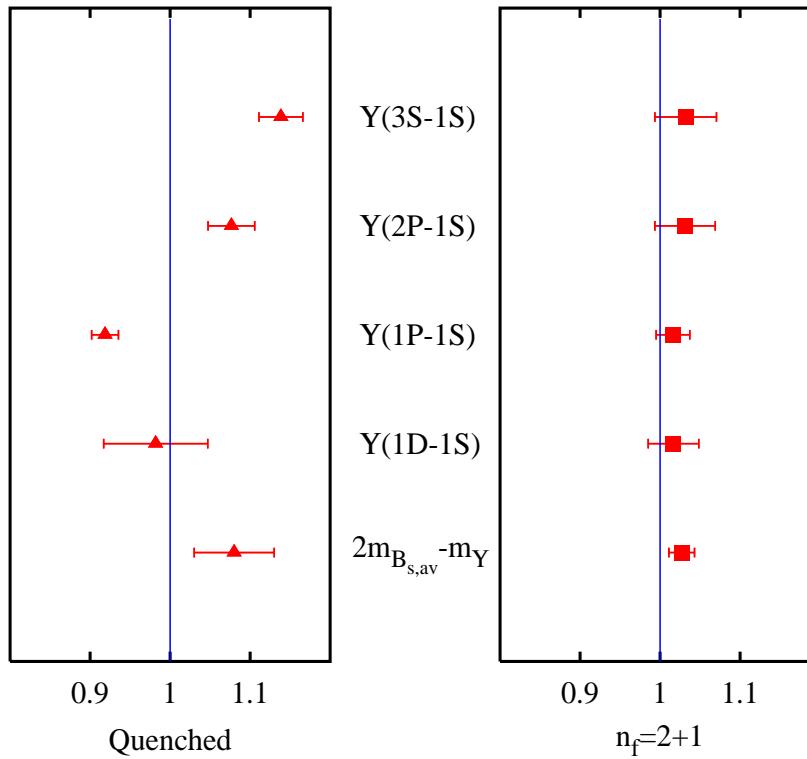
$$N_{QCD}^4 \sim 10N_{NRQCD}^4 \sim 100N_{pNRQCD}^4$$

- Bottomonium:  $v^2 \sim 0.1$

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# Lattice NRQCD



A. Gray, I. Allison, C.T.H. Davies, Emel Dalgic, G.P. Lepage, J. Shigemitsu, M. Wingate, Phys.Rev.D72:094507,2005.



# Spectrum at weak coupling

- LO:  $m_Q \alpha_s^2$ , trivial
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- NNLO:  $m_Q \alpha_s^4$ , (Pineda, Yndurain, 1997)
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- NNNLO:  $m_Q \alpha_s^5$ , almost complete, (Kniehl, Penin, Smirnov, Steinhauser, 2002; Beneke, Kiyo, Schuller, 2005 )





## The static potential

$$\begin{aligned}
 V_s(r, \mu) = & -\frac{C_F \alpha_s(1/r)}{r} \left\{ 1 + \frac{\alpha_s(1/r)}{4\pi} [\tilde{a}_1] + \left( \frac{\alpha_s(1/r)}{4\pi} \right)^2 [\tilde{a}_2] \right. \\
 & + \left( \frac{\alpha_s(1/r)}{4\pi} \right)^3 [a_3^L \log \mu r + \tilde{a}_3] \\
 & + \left. \left( \frac{\alpha_s(1/r)}{4\pi} \right)^4 [a_4^{L2} \log^2 \mu r + a_4^L \log \mu r + \tilde{a}_4] + \dots \right\},
 \end{aligned}$$

$\tilde{a}_1$ , W. Fischler, Nucl. Phys. B **129** (1977) 157; A. Billoire, Phys. Lett. B **92** (1980) 343

$\tilde{a}_2$ , M. Peter, Phys. Rev. Lett. **78** (1997) 602; Nucl. Phys. B **501** (1997) 471; Y. Schröder, Phys. Lett. B **447** (1999) 321

$a_3^L$ , N. Brambilla, A. Pineda, JS, A. Vairo, Phys. Rev. D60, 091502 (1999)

$\tilde{a}_3$ , ???

$a_4^{2L}$ , A. Pineda and JS, Phys. Lett. B **495** (2000) 323

$a_4^L$ , N. Brambilla, X. Garcia i Tormo, JS, A. Vairo, Phys. Lett. B **647**, 185 (2007)



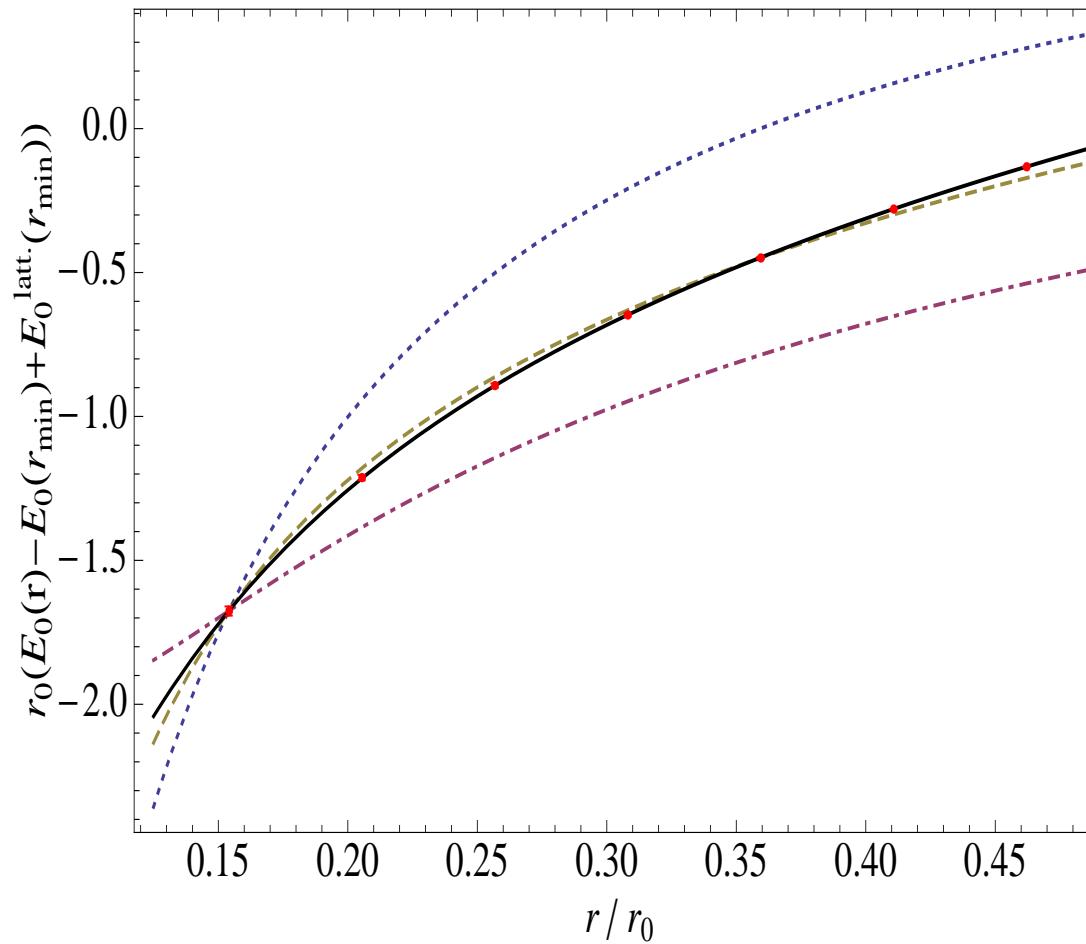
# The 3-loop potential

$$\tilde{a}_3 = \tilde{a}_3^{(0)} + \tilde{a}_3^{(1)} n_f + \tilde{a}_3^{(2)} n_f^2 + \tilde{a}_3^{(3)} n_f^3$$

- $\tilde{a}_3^{(i)}$ ,  $i = 1, 2, 3$ : Smirnov, Smirnov, Steinhauser, 2008
- $\tilde{a}_3^{(0)}$ : Padé estimate  $\sim 313 \times 4^3$ , Chishtie, Elias, 2001
- $\tilde{a}_3^{(0)}$ : Renormalon estimate  $\sim 292 \times 4^3$ , Pineda, 2001
- $\tilde{a}_3^{(0)}$ : NNNLL calculation + lattice data =  
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- $\tilde{a}_3^{(0)}$ : NNNLO  $\sim 209.885(1) \times 4^3$ , Anzai, Kiyo, Sumino, arXiv:0911.4335; Smirnov, Smirnov, Steinhauser, arXiv:0911.4742





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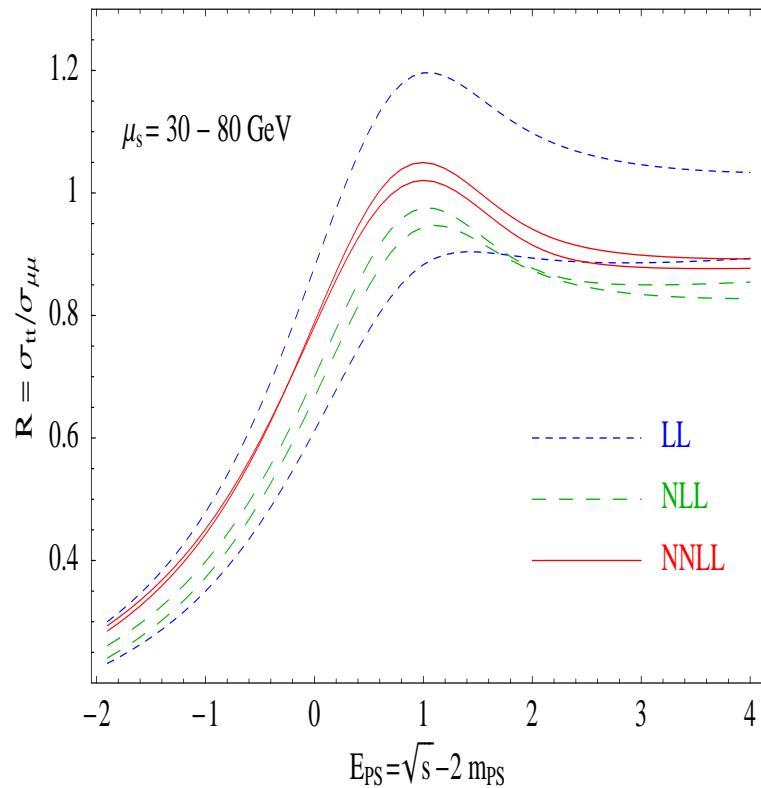
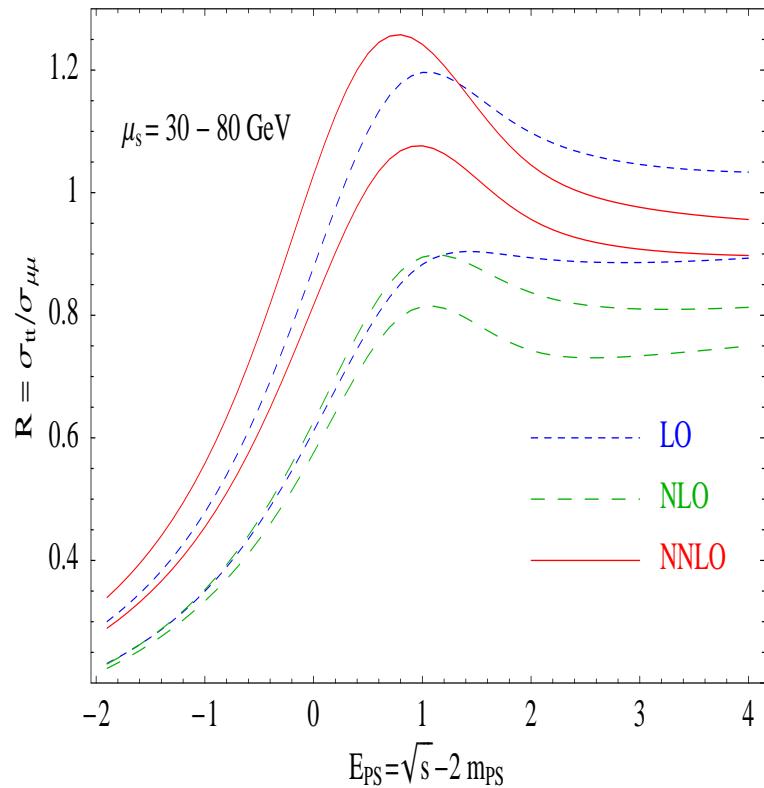


## pNRQCD: weak coupling regime (Cont.)

- Hyperfine splitting at NNLL accuracy (Kniehl, Penin, Pineda, Smirnov, and Steinhauser (03))
  - $m_{J/\psi} - m_{\eta_c(1S)} \sim 112 \pm 40 (\delta\alpha_s) \text{ MeV}$   
[Exp:  $116.6 \pm 1.0$ , MeV]  
[MILC/Fermilab:  $116 \pm 7.4$ , MeV]
  - $m_{\Upsilon(1S)} - m_{\eta_b(1S)} = 39 \pm 11 (\text{th})^{+9}_{-8} (\delta\alpha_s) \text{ MeV}$   
[Exp:  $69.6 \pm 2.9$  MeV]  
[HPQCD:  $61 \pm 14$ , MeV]  
[MILC/Fermilab:  $54 \pm 12.4$ , MeV]  
[RBC/UKQCD:  $60.3 \pm 7.7$ , MeV]
  - $m_{B_c^*} - m_{B_c} \sim 40 \pm 17^{+15}_{-12}$   
[HPQCD:  $53 \pm 7 \pm 2 \pm 6$ , MeV]



# $t\bar{t}$ at ILC



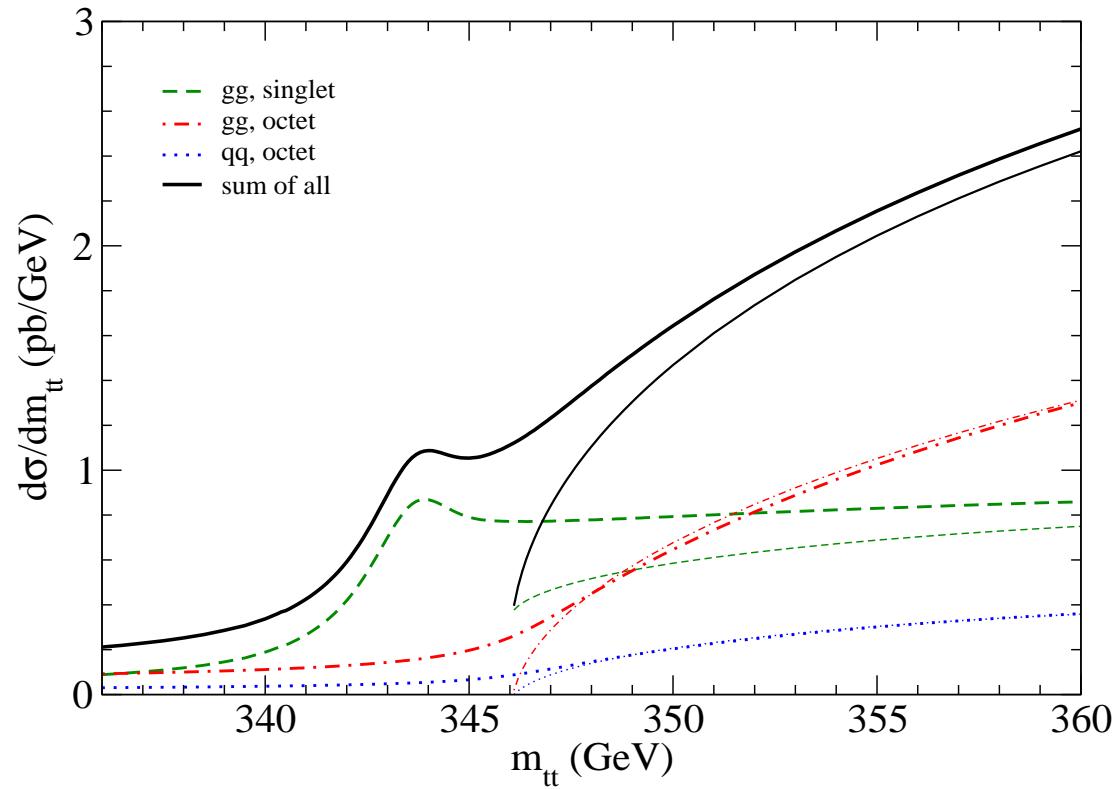
(Pineda, Signer, 2006)



# $t\bar{t}$ at LHC ?



# $t\bar{t}$ at LHC ?



(Hagiwara, Sumino, Yokoya, 2008)



# pNRQCD: strong coupling regime

$\Lambda_{QCD} \lesssim mv$  : strong coupling regime

$$L_{\text{pNRQCD}} = \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 S^\dagger (i\partial_0 - h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2)) S,$$

$$h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) = \frac{\mathbf{p}_1^2}{2m_Q} + \frac{\mathbf{p}_2^2}{2m_Q} + V_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2),$$

$$V_s = V_s^{(0)} + \frac{V_s^{(1)}}{m_Q} + \frac{V_s^{(2)}}{m_Q^2} + \dots,$$

All  $V_s$ s can be, and most of them have been, calculated on the lattice

# Lattice pNRQCD





# Lattice pNRQCD

Example: the  $1/m_Q$  potential (N. Brambilla, A. Pineda, JS, A. Vairo, Phys. Rev. D63:014023,2001)

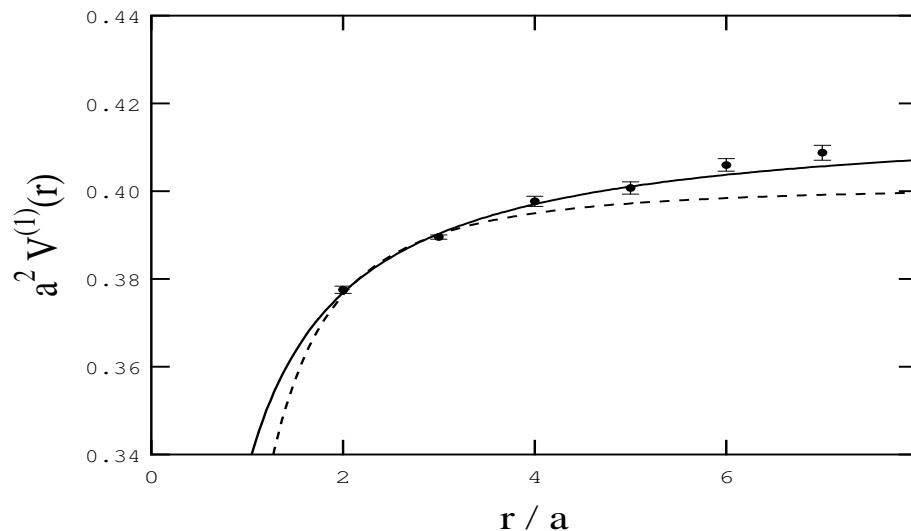
$$V^{(1)}(r) = -\frac{1}{2} \int_0^\infty dt t \langle\langle g \mathbf{E}_1(t) \cdot g \mathbf{E}_1(0) \rangle\rangle_c,$$



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Dashed :  $V^{(1)}(r) = \frac{a}{r^2} + c$  , Solid :  $V^{(1)}(r) = \frac{a}{r} + c$

# Lattice pNRQCD

Constraints on  $V^{(1)}(r)$ :





# Lattice pNRQCD

Constraints on  $V^{(1)}(r)$ :

- $r \ll 1/\Lambda_{QCD}$ :  $V^{(1)}(r) \sim \alpha_s^2(1/r)/r^2$ , from perturbation theory





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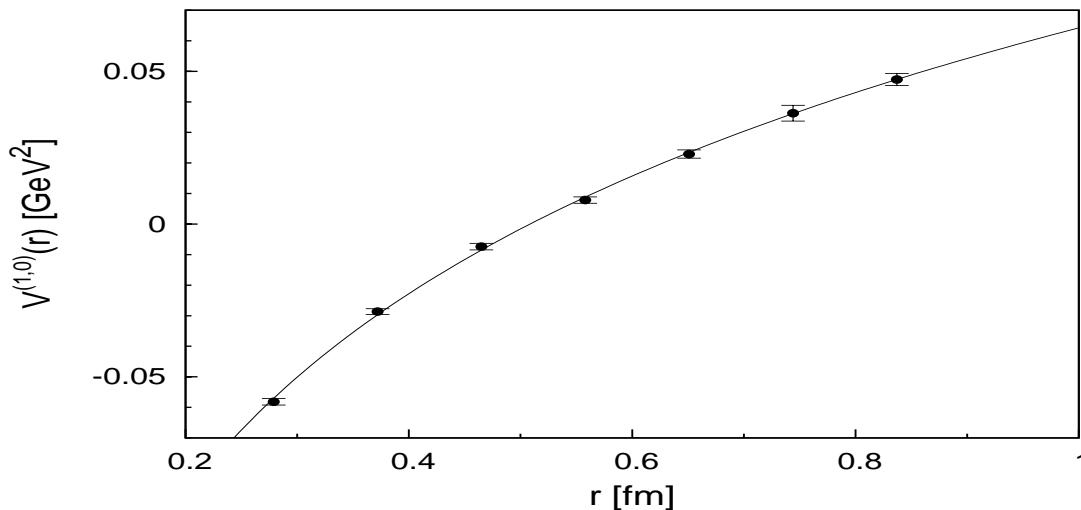
- $r \ll 1/\Lambda_{QCD}$ :  $V^{(1)}(r) \sim \alpha_s^2(1/r)/r^2$ , from perturbation theory
- $r \gg 1/\Lambda_{QCD}$ :  $V^{(1)}(r) \sim a \ln r + c$ , from the QCD effective string theory (Guillem Perez-Nadal, JS, Phys.Rev.D79:114002,2009)



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Z(3930), Y(3940), X(3940), X(4160), Y(4260), Y(4350),  
Y(4660)





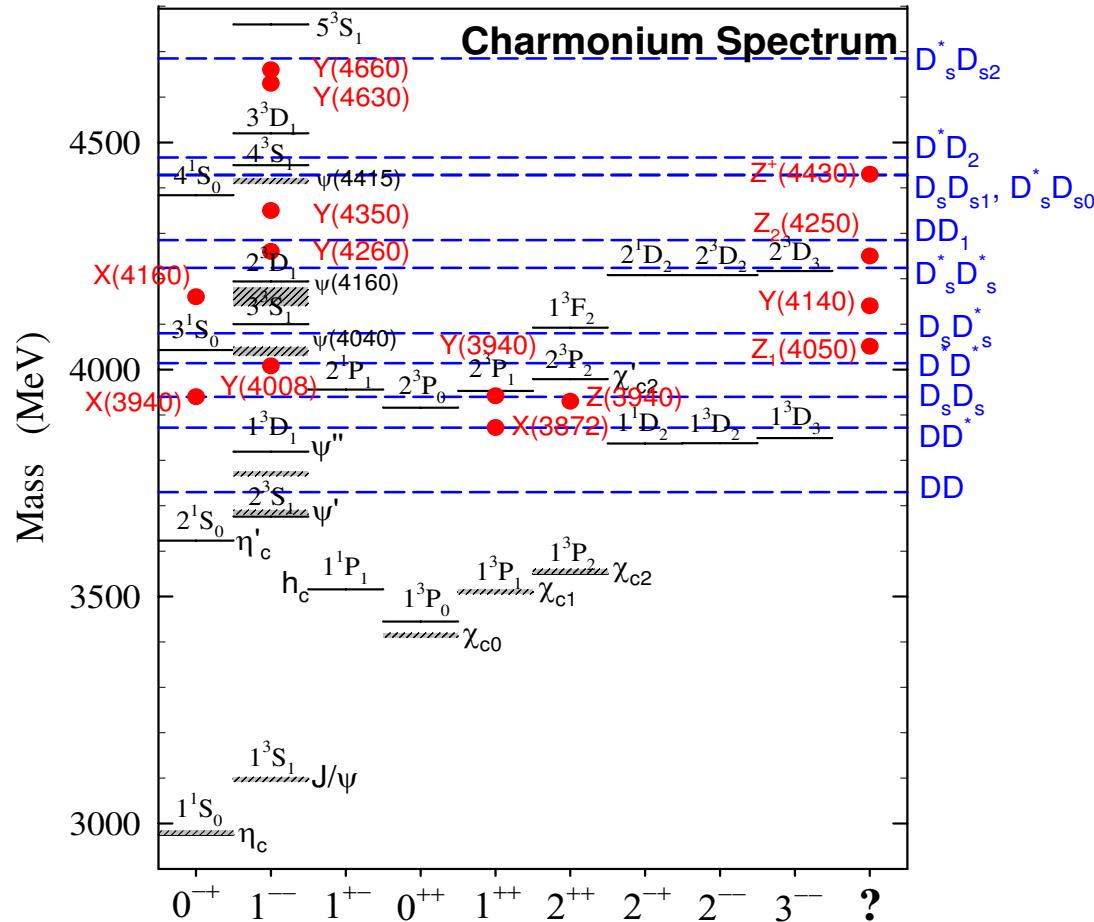
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Z(3930), Y(3940), X(3940), X(4160), Y(4260), Y(4350),  
Y(4660)
  - Do not fit potential model expectations. Theoretical  
possibilities: molecules, tetraquarks, hybrids,...





# "New" Charmonium Spectrum

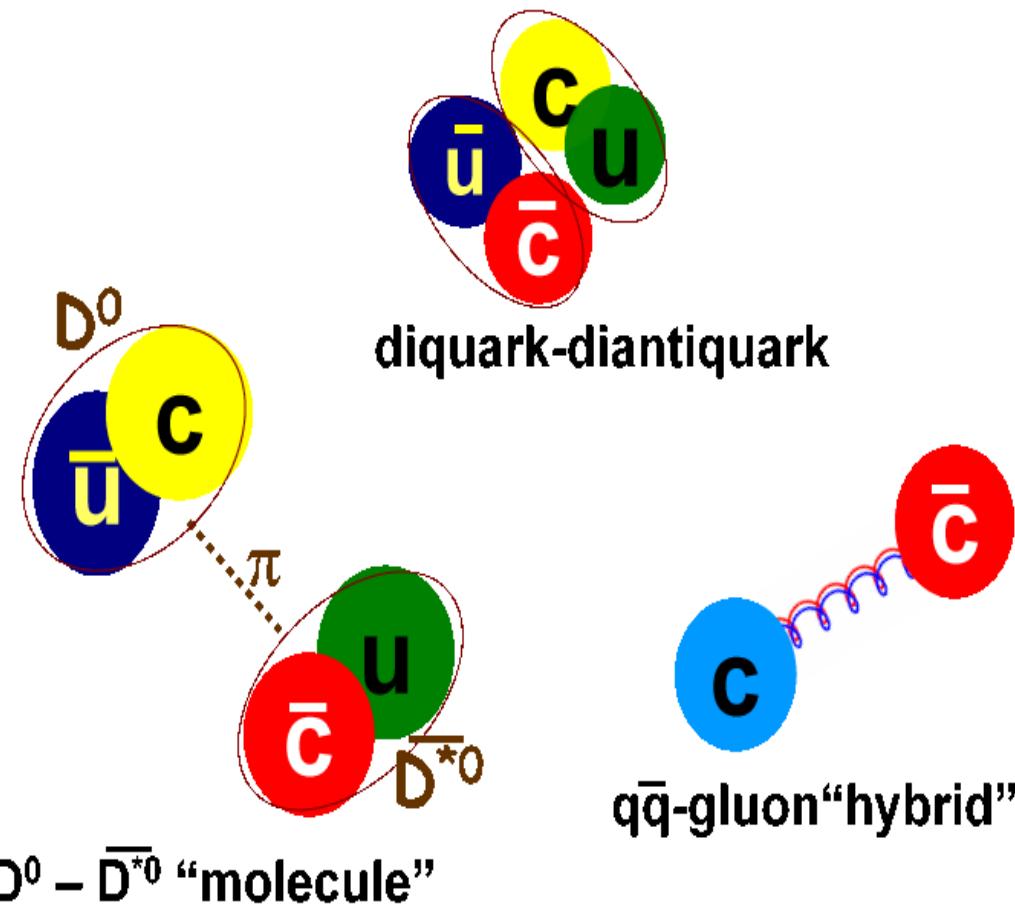


Godfrey, 2009





# Exotics

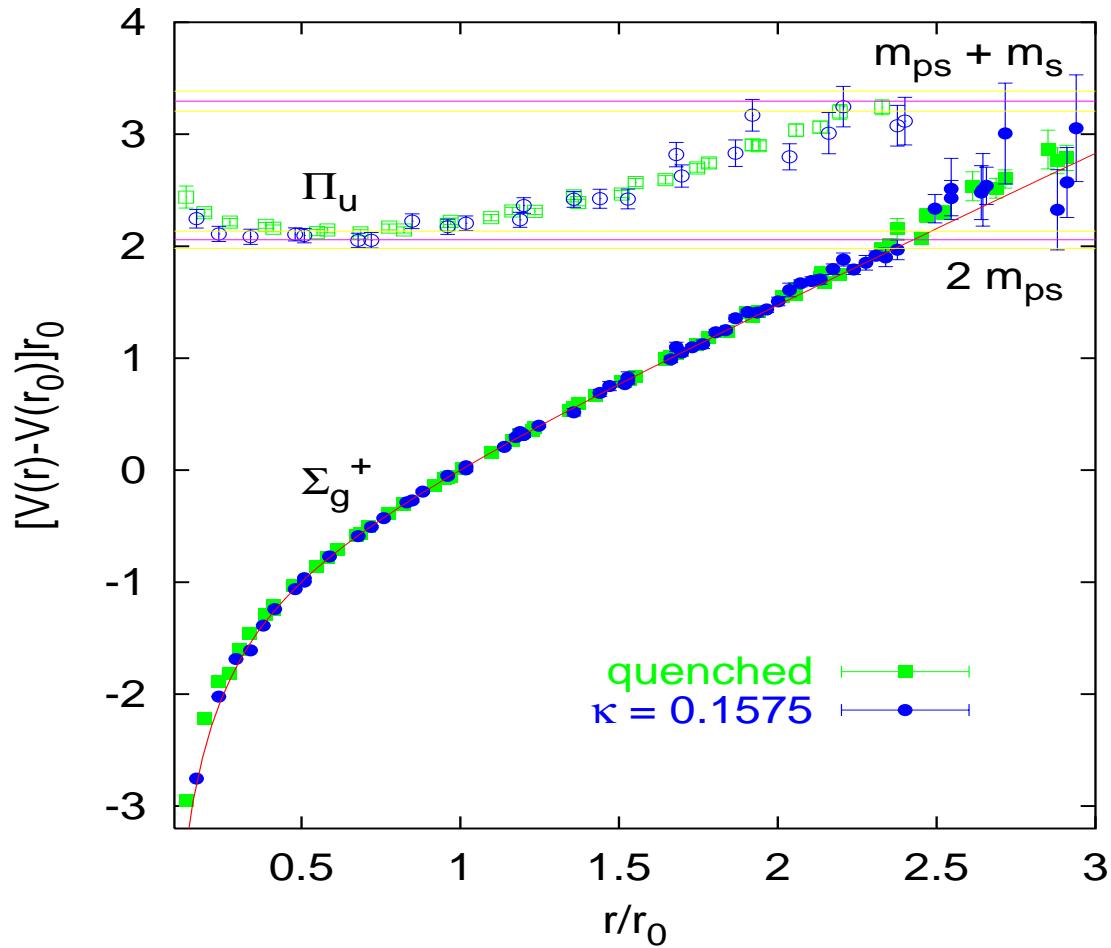


Godfrey, Olsen, 2008





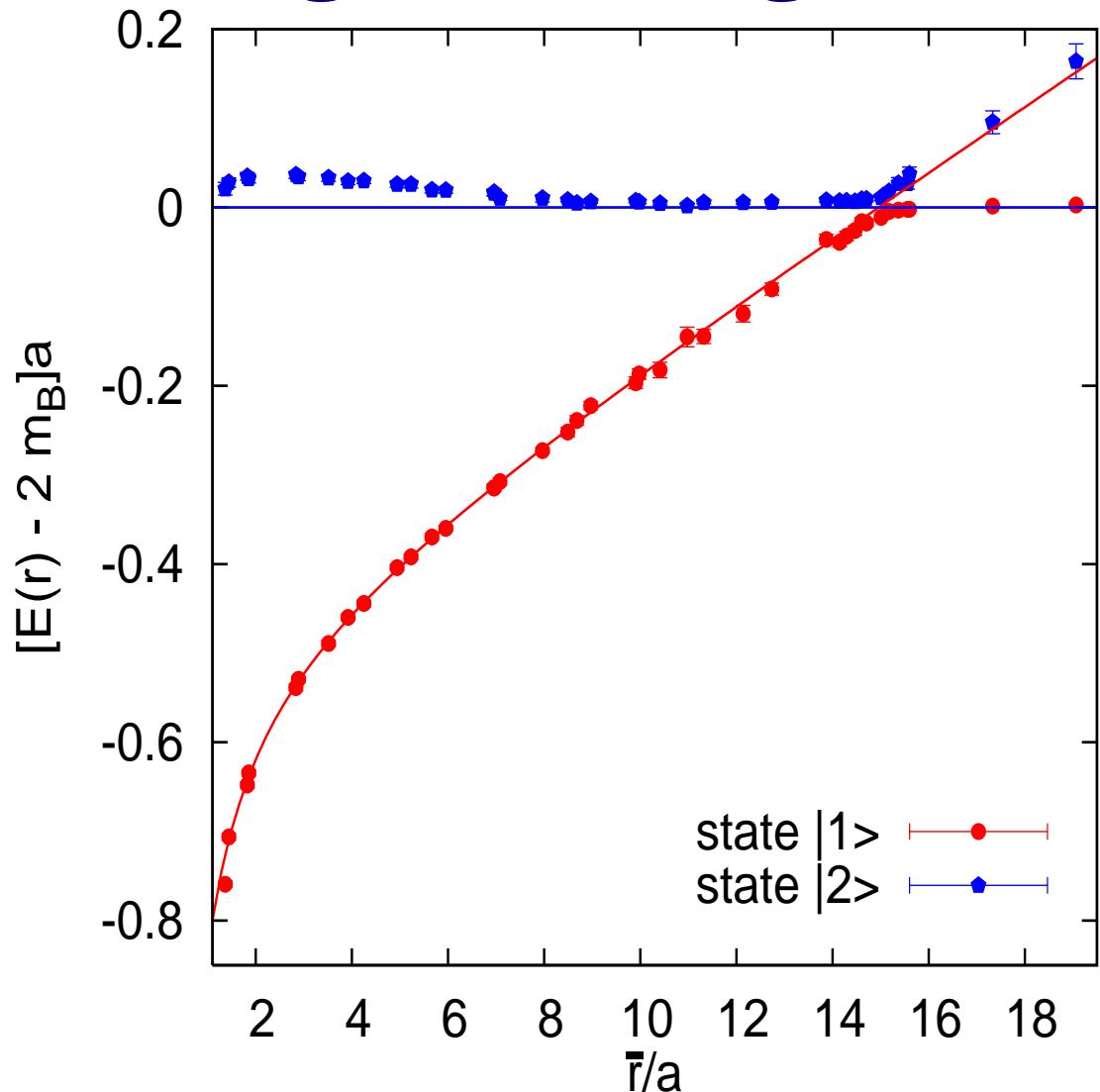
## pNRQCD(Cont.)



G.S. Bali at al. (TXL Collaboration), Phys. Rev.  
D62,(2000):054503



# String breaking



Bali, Neff, Düssel, Lippert, Schilling (2005)



# Unconventional charmonium states, (N. Brambilla et al., 1010.5827 )

State	$m$ (MeV)	$\Gamma$ (MeV)	$J^{PC}$	Process (mode)	Experiment (# $\sigma$ )	Year	Status
$X(3872)$	$3871.52 \pm 0.20$	$1.3 \pm 0.6$	$1^{++}/2^{-+}$	$B \rightarrow K(\pi^+\pi^- J/\psi)$ ( $<2.2$ ) $p\bar{p} \rightarrow (\pi^+\pi^- J/\psi) + \dots$ $B \rightarrow K(\omega J/\psi)$ $B \rightarrow K(D^{*0}\bar{D}^0)$ $B \rightarrow K(\gamma J/\psi)$ $B \rightarrow K(\gamma\psi(2S))$	<b>Belle</b> [85, 86] (12.8), <b>BABAR</b> [87] (8.6) <b>CDF</b> [88–90] (np), <b>DØ</b> [91] (5.2) <b>Belle</b> [92] (4.3), <b>BABAR</b> [93] (4.0) <b>Belle</b> [94, 95] (6.4), <b>BABAR</b> [96] (4.9) <b>Belle</b> [92] (4.0), <b>BABAR</b> [97, 98] (3.6) <b>BABAR</b> [98] (3.5), <b>Belle</b> [99] (0.4)	2003	OK
$X(3915)$	$3915.6 \pm 3.1$	$28 \pm 10$	$0/2^{?+}$	$B \rightarrow K(\omega J/\psi)$ $e^+e^- \rightarrow e^+e^-(\omega J/\psi)$	<b>Belle</b> [100] (8.1), <b>BABAR</b> [101] (19) <b>Belle</b> [102] (7.7)	2004	OK
$X(3940)$	$3942_{-8}^{+9}$	$37_{-17}^{+27}$	$?^{?+}$	$e^+e^- \rightarrow J/\psi(D\bar{D}^*)$ $e^+e^- \rightarrow J/\psi(\dots)$	<b>Belle</b> [103] (6.0) <b>Belle</b> [54] (5.0)	2007	NC!
$G(3900)$	$3943 \pm 21$	$52 \pm 11$	$1^{--}$	$e^+e^- \rightarrow \gamma(D\bar{D})$	<b>BABAR</b> [27] (np), <b>Belle</b> [21] (np)	2007	OK
$Y(4008)$	$4008_{-49}^{+121}$	$226 \pm 97$	$1^{--}$	$e^+e^- \rightarrow \gamma(\pi^+\pi^- J/\psi)$	<b>Belle</b> [104] (7.4)	2007	NC!
$Z_1(4050)^+$	$4051_{-43}^{+24}$	$82_{-55}^{+51}$	?	$B \rightarrow K(\pi^+\chi_{c1}(1P))$	<b>Belle</b> [105] (5.0)	2008	NC!
$Y(4140)$	$4143.4 \pm 3.0$	$15_{-7}^{+11}$	$?^{?+}$	$B \rightarrow K(\phi J/\psi)$	<b>CDF</b> [106, 107] (5.0)	2009	NC!
$X(4160)$	$4156_{-25}^{+29}$	$139_{-65}^{+113}$	$?^{?+}$	$e^+e^- \rightarrow J/\psi(D\bar{D}^*)$	<b>Belle</b> [103] (5.5)	2007	NC!
$Z_2(4250)^+$	$4248_{-45}^{+185}$	$177_{-72}^{+321}$	?	$B \rightarrow K(\pi^+\chi_{c1}(1P))$	<b>Belle</b> [105] (5.0)	2008	NC!
$Y(4260)$	$4263 \pm 5$	$108 \pm 14$	$1^{--}$	$e^+e^- \rightarrow \gamma(\pi^+\pi^- J/\psi)$ $e^+e^- \rightarrow (\pi^+\pi^- J/\psi)$ $e^+e^- \rightarrow (\pi^0\pi^0 J/\psi)$	<b>BABAR</b> [108, 109] (8.0) <b>CLEO</b> [110] (5.4) <b>Belle</b> [104] (15) <b>CLEO</b> [111] (11) <b>CLEO</b> [111] (5.1)	2005	OK
$Y(4274)$	$4274.4_{-6.7}^{+8.4}$	$32_{-15}^{+22}$	$?^{?+}$	$B \rightarrow K(\phi J/\psi)$	<b>CDF</b> [107] (3.1)	2010	NC!
$X(4350)$	$4350.6_{-5.1}^{+4.6}$	$13.3_{-10.0}^{+18.4}$	$0.2^{++}$	$e^+e^- \rightarrow e^+e^-(\phi J/\psi)$	<b>Belle</b> [112] (3.2)	2009	NC!
$Y(4360)$	$4353 \pm 11$	$96 \pm 42$	$1^{--}$	$e^+e^- \rightarrow \gamma(\pi^+\pi^-\psi(2S))$	<b>BABAR</b> [113] (np), <b>Belle</b> [114] (8.0)	2007	OK
$Z(4430)^+$	$4443_{-18}^{+24}$	$107_{-71}^{+113}$	?	$B \rightarrow K(\pi^+\psi(2S))$	<b>Belle</b> [115, 116] (6.4)	2007	NC!
$X(4630)$	$4634_{-11}^{+9}$	$92_{-32}^{+41}$	$1^{--}$	$e^+e^- \rightarrow \gamma(\Lambda_c^+\Lambda_c^-)$	<b>Belle</b> [25] (8.2)	2007	NC!
$Y(4660)$	$4664 \pm 12$	$48 \pm 15$	$1^{--}$	$e^+e^- \rightarrow \gamma(\pi^+\pi^-\psi(2S))$	<b>Belle</b> [114] (5.8)	2007	NC!
$Y_b(10888)$	$10888.4 \pm 3.0$	$30.7_{-7.7}^{+8.9}$	$1^{--}$	$e^+e^- \rightarrow (\pi^+\pi^-\Upsilon(nS))$	<b>Belle</b> [37, 117] (3.2)	2010	NC!



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$X(3872)$	$3871.52 \pm 0.20$	$1.3 \pm 0.6$	$1^{++}/2^{-+}$	$\pi^+\pi^- J/\psi$	$D^{*0}\bar{D}^0$ molecule (bound)	[121, 122]
				$D^{*0}\bar{D}^0$	$D^{*0}\bar{D}^0$ unbound	[383–385]
				$\gamma J/\psi, \gamma\psi(2S)$	if $1^{++}, \chi_{c2}(2P)$	[71]
				$\omega J/\psi$	if $2^{-+}, \eta_{c2}(1D)$	[81, 93, 126]
					charmonium + mesonic-molecule mixture	[381]
					QCDSR: $[cq]_3[\bar{c}\bar{q}]_3$ (S+A)	[381]
					QCDSR: $[c\bar{q}]_1[\bar{c}q]_1$ (P+V)	[381]
					QCDSR: $[c\bar{c}]_1(A) + [c\bar{q}]_1[\bar{c}q]_1$ (P+V)	[381]
					$D^{*+}D^{*-} + D^{*0}\bar{D}^{*0}$	[386]
					$\chi_{c2}(2P)$ ( <i>i.e.</i> , $2^3P_2 c\bar{c}$ )	[71]
					$1^3F_2 c\bar{c}$	
$X(3915)$	$3915.6 \pm 3.1$	$28 \pm 10$	$0, 2^{?+}$	$\omega J/\psi$	Coupled-channel effect	[34]
$Z(3930)$	$3927.2 \pm 2.6$	$24.1 \pm 6.1$	$2^{++}$	$D\bar{D}$		
$X(3940)$	$3942^{+9}_{-8}$	$37^{+27}_{-17}$	? $^{?+}$	$D\bar{D}^*$		
$G(3900)$	$3943 \pm 21$	$52 \pm 11$	$1^{--}$	$D\bar{D}$		
$Y(4008)$	$4008^{+121}_{-49}$	$226 \pm 97$	$1^{--}$	$\pi^+\pi^- J/\psi$	hadrocharmonium	[282, 355]
$Z_1(4050)^+$	$4051^{+24}_{-43}$	$82^{+51}_{-55}$	?	$\pi^+\chi_{c1}(1P)$	QCDSR: $[c\bar{q}]_1[\bar{c}q]_1$ (V+V)	[381]
$Y(4140)$	$4143.0 \pm 3.1$	$11.7^{+9.1}_{-6.2}$	? $^{?+}$	$\phi J/\psi$	QCDSR: $[c\bar{s}]_1[\bar{c}s]_1$ (V+V)	[381]
					$D_s^{*+}D_s^{*-}$	[386]
$X(4160)$	$4156^{+29}_{-25}$	$139^{+113}_{-65}$	? $^{?+}$	$D\bar{D}^*$	hadrocharmonium	[282, 355]
$Z_2(4250)^+$	$4248^{+185}_{-45}$	$177^{+321}_{-72}$	?	$\pi^+\chi_{c1}(1P)$	charmonium hybrid	[276–278]
$Y(4260)$	$4263 \pm 5$	$108 \pm 14$	$1^{--}$	$\pi^+\pi^- J/\psi$	$J/\psi f_0(980)$ bound state	[313]
				$\pi^0\pi^0 J/\psi$	$D_0\bar{D}^*$ molecular state	[375]
					$[cs][\bar{c}\bar{s}]$ tetraquark state	[345, 381]
					hadrocharmonium	[282, 355]
					QCDSR: $[c\bar{q}]_1[\bar{c}q]_1$ (S+V)	[381]
					QCDSR: $[c\bar{q}]_1[\bar{c}q]_1$ (P+A)	[381]
					(see $Y(4140)$ )	
$Y(4274)$	$4274.4^{+8.4}_{-6.7}$	$32^{+22}_{-15}$	? $^{?+}$	$B \rightarrow K(\phi J/\psi)$	hadrocharmonium	[282, 355]
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$0, 2^{++}$	$\phi J/\psi$	crypto-exotic hybrid	[71]
$Y(4360)$	$4353 \pm 11$	$96 \pm 42$	$1^{--}$	$\pi^+\pi^-\psi(2S)$	$Y_B(4360) = [cd][\bar{c}\bar{d}](1P)$ , baryonium	[321]
					QCDSR: $[cq]_3[\bar{c}q]_3$ (S+V)	[381]
					QCDSR: $[c\bar{s}]_1[\bar{c}s]_1$ (S+V)	[381]
					$D^{*+}\bar{D}_1^0$ molecular state	[378, 387]
					$[cu][\bar{c}\bar{d}]$ tetraquark state	[379]
					hadrocharmonium	[282, 355]
					QCDSR: $[cd]_1[cu]_1$ (V+A)	[381]
					QCDSR: $[cu]_3[\bar{c}d]_3$ (S+P)	[381]
					$Y_B(4660) = [cd][\bar{c}\bar{d}](2P)$ , baryonium	[321]
					$\psi(2S)f_0(980)$ molecule	[324]
					$\psi(2S)f_0(980)$ molecule	[306]
					$[cs][\bar{c}\bar{s}]$ tetraquark state	[375]
					hadrocharmonium	[282, 355]
					$Y_B(4660) = [cd][\bar{c}\bar{d}](2P)$ , baryonium	[321]
					QCDSR: $[cs]_3[\bar{c}s]_3$ (S+V)	[381]
					$\Upsilon(5S)$	[388]
					$b$ -flavored $Y(4260)$	[37, 117]
$Y_b(10888)$	$10888.4 \pm 3.0$	$30.7^{+8.9}_{-7.7}$	$1^{--}$	$\pi^+\pi^-\Upsilon(nS)$		

# Conclusions



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- "Known" physics is not that known
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- Will super-B uncover more?

