

# Lattice QCD and New Physics

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## - *Outline*

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① Flavour Physics and New Physics:

➔ High Precision calculations: Lattice QCD

② Opportunity of new physics and Lattice QCD

➔ Charged Current modes:  $K \rightarrow \mu \nu$ ,  $B \rightarrow \tau \nu$ ,  $B \rightarrow D \tau \nu$ :  **$H^+$ , right-handed current ...**

➔ Flavour Changing Neutral Current observables:  $\Delta M_s$ ,  $\epsilon_K$ ,  
 $B \rightarrow K^* \mu \mu$ :  **$H^0$ , gluino ...**

# Flavour Physics and New Physics

Next Hints for the Quark Flavour sector mostly driven by the modes

①

CP asymmetry  
in  $B_s \rightarrow \psi\phi$ .  
 $\beta_s^{SM} \approx 0$

LHCb

②

$B_s \rightarrow \mu\mu$

③

$K \rightarrow \pi \nu\nu$

④

$K_{\mu 2}/K_{e 2}$

NA62-JPARC

➔ For these modes, hadronic uncertainties are not a big issue!!:

➔ However, to discriminate among NP models we will need much larger set of flavour-changing processes =>

$B \rightarrow \tau\nu$ ,  $B \rightarrow D\tau\nu$ ,  $B \rightarrow \pi(\rho)l\nu$ ,  $\Delta M_s$ ,  $\varepsilon_K$ ,  $B \rightarrow K^*\mu\mu$

Super B

➔ **KEY POINT:** to keep under very good control hadronic uncertainties

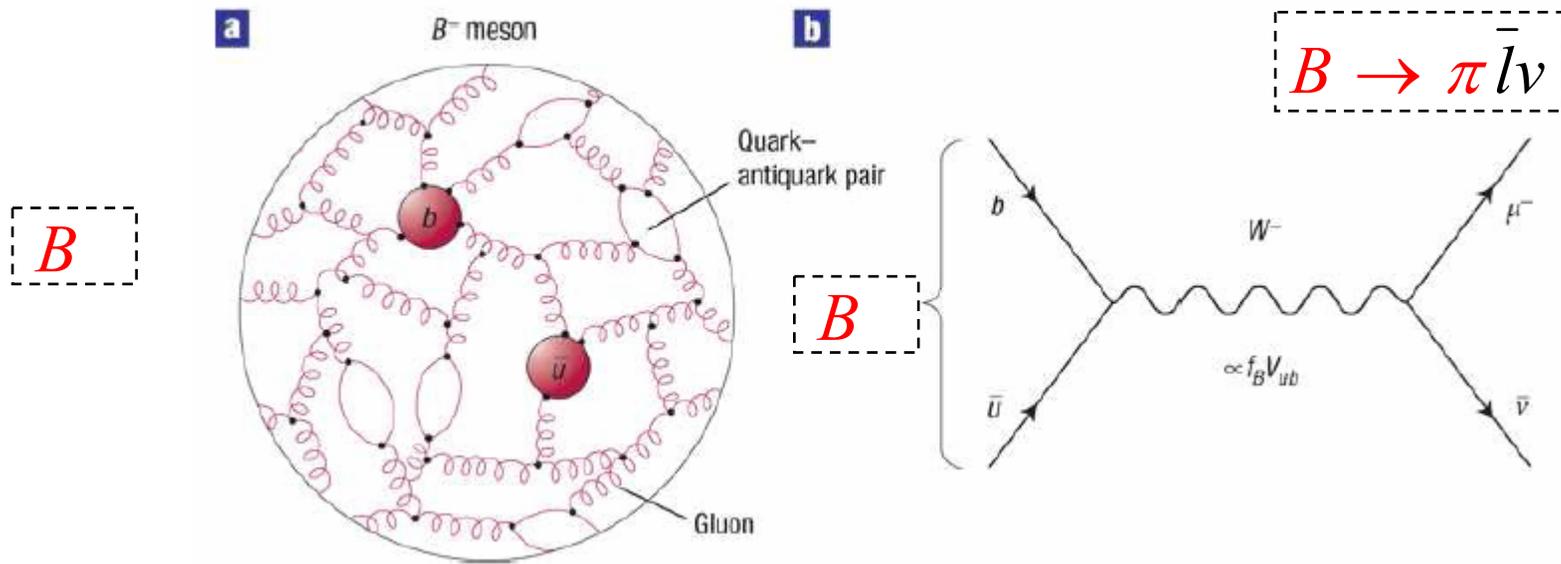


LATTICE QCD?

# Flavour Physics: New Physics vs QCD effects

☺ Changes of quark flavour inside a hadron are weak interaction processes:

$\rightarrow L_{\text{eff}}(M_W) \rightarrow \text{OPE}$

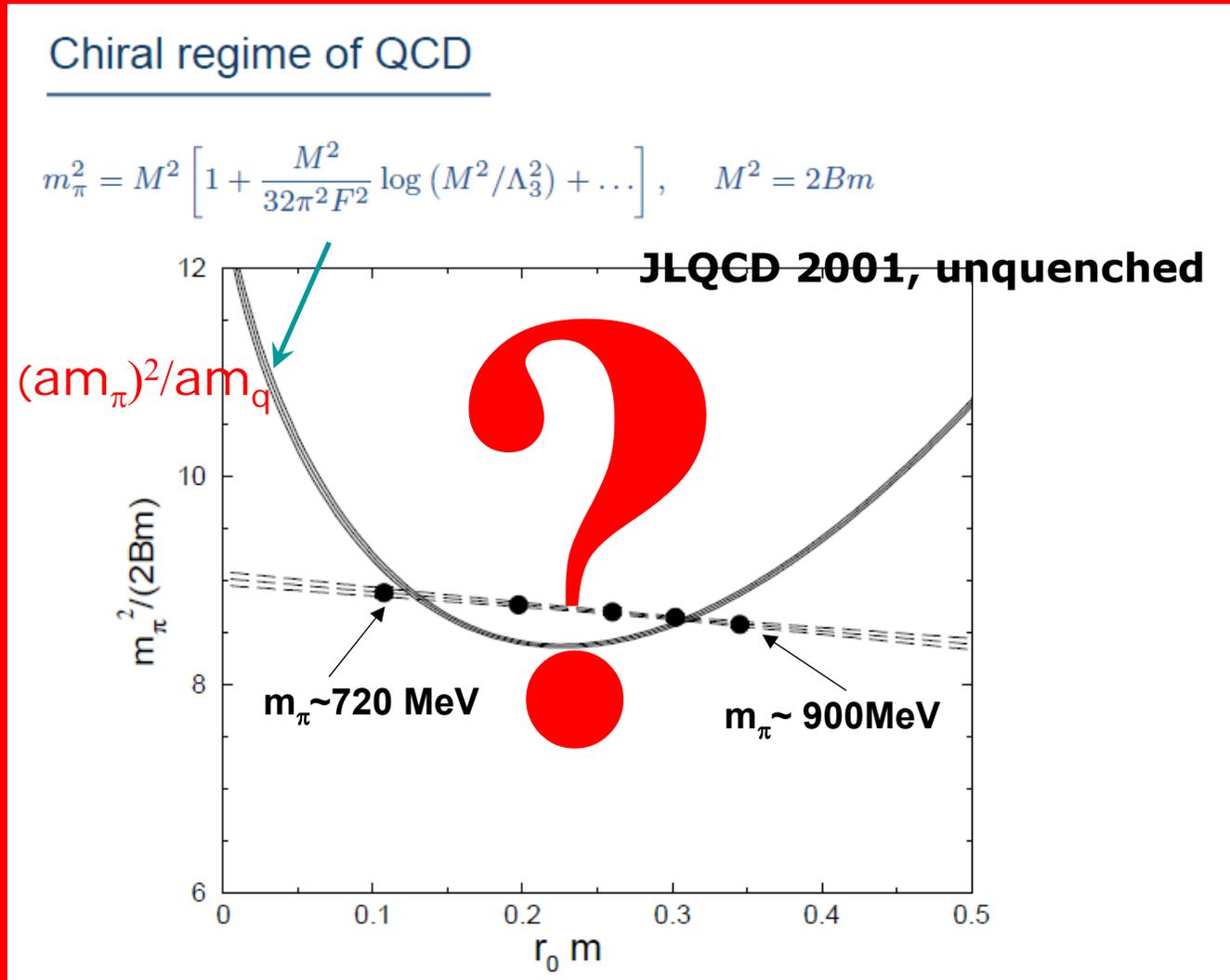


- ⇒ Due to confinement, QCD effects are significant and non-perturbative ☹
- ⇒ QCD effects absorbed into hadronic matrix elements

A task for Lattice QCD  $\rightarrow$  tremendous progress in the last years!

# Chiral Regime of QCD

A problem with (old) lattice simulations:  
Heavy pion mass!



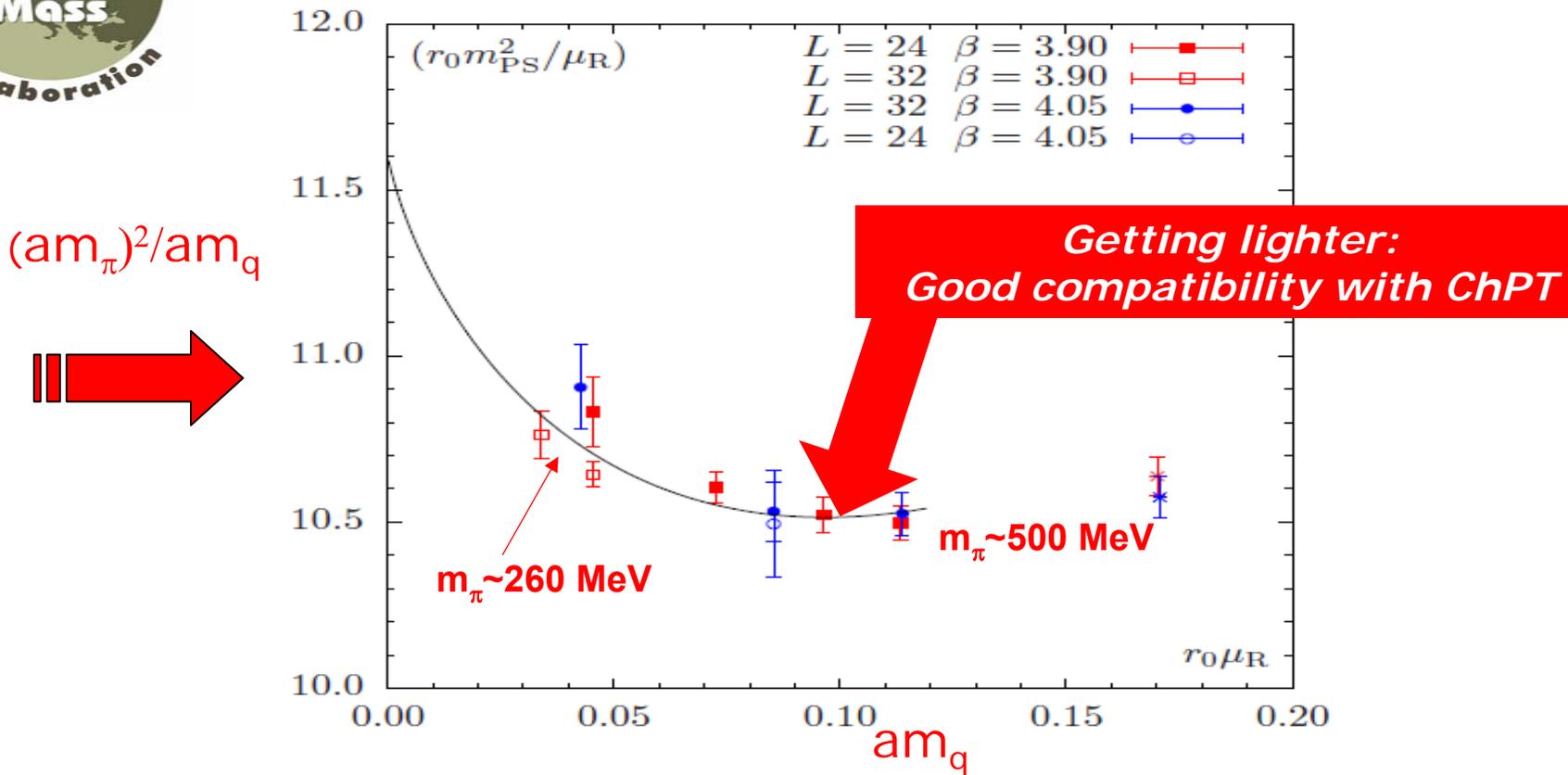
**Incompatibility with ChPT?**

# Chiral Regime of QCD

No longer problems with (new) lattice simulations  
 Good compatibility with ChPT



$$m_\pi^2 = M^2 \left[ 1 + \frac{M^2}{32\pi^2 F^2} \log(M^2/\Lambda_3^2) + \dots \right], \quad M^2 = 2Bm$$



ETMC arXiv:0803.0224

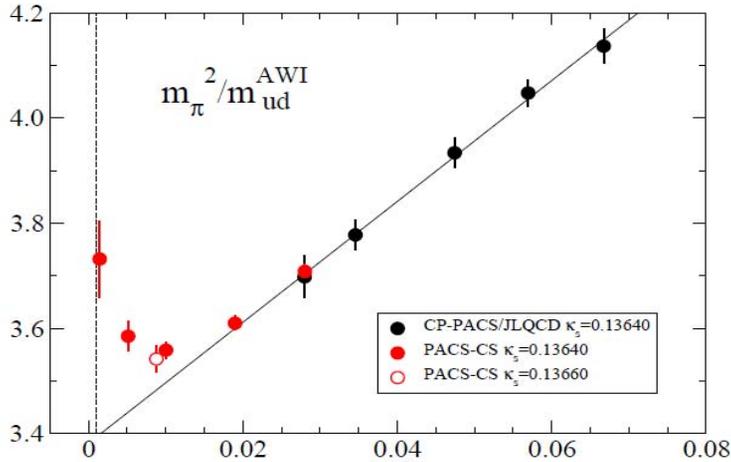
2006-08, unquenched, Wilson-like fermions ☺

# Similar plots by other collaborations (using different approaches)

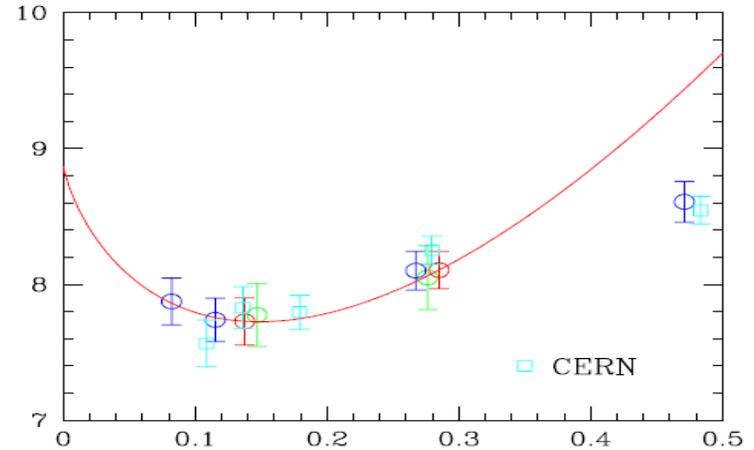
Chiral regime of QCD

$$(am_\pi)^2/am_q$$

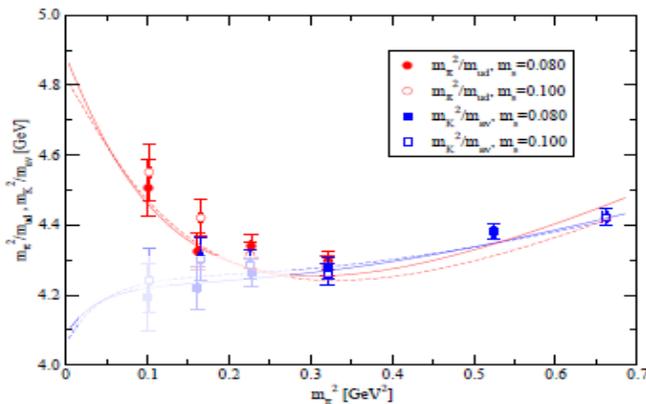
Hints of chiral Logs



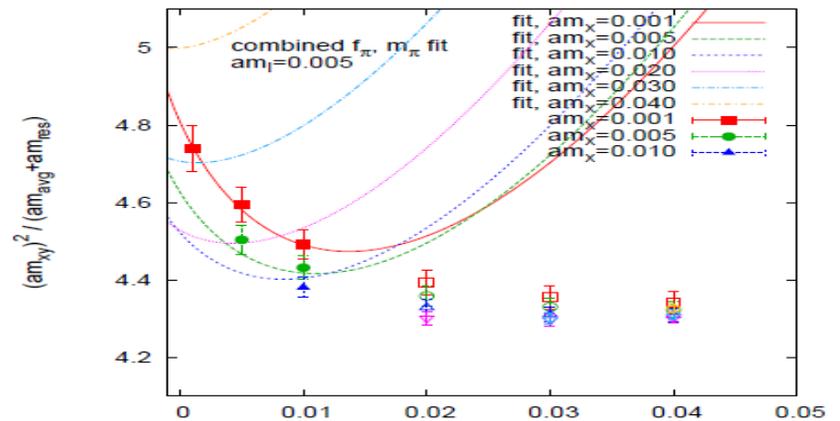
**PACS-CS'08,  $N_F=2+1$ , Clover:**  
 $a=0.09$  fm,  $M_\pi=156$  MeV,  $LM_\pi \sim 2.3!!$



**QCDSF'08,  $N_F=2$ , Clover:**  
 $a=0.07$  fm,  $M_\pi=313$  MeV,  $LM_\pi \sim 4$



**JLQCD/TWQCD'08,  $N_F=2+1$ , Overlap:**  
 $a=0.11$  fm,  $M_\pi=300$  MeV,  $LM_\pi \sim 2.6!!$



**RBC/UKQCD'06,  $N_F=2+1$ , DWF**  
 $a=0.114$  fm,  $M_\pi=310$ ,  $LM_\pi \sim 3-4$

# What about phenomenology?

★ Up to now, only a few observables available from first principle QCD action (2006 - on)

⇒ **Wilson-like & Domain-Wall fermions**

■ some observables are still in the “quenched approximation” (1974-2002);

● most unquenched estimates are from staggered fermions (**MILC: 2002 – on**)

☺ **First realistic QCD study => MILC opened up unquenched era!!**

☺ *but problematic for a SuperB era!*

⇒ *staggered ansatz:  $\det[D_{NF=1}] \cong \det[D_{stag}]^{1/4}$  to eliminate spurious “tastes”:*

⇒ *non-local theory! Shamir, Bernard, Golterman, Sharpe, Creutz '04-'08*

⇒ **QCD when lattice spacing goes 0?**

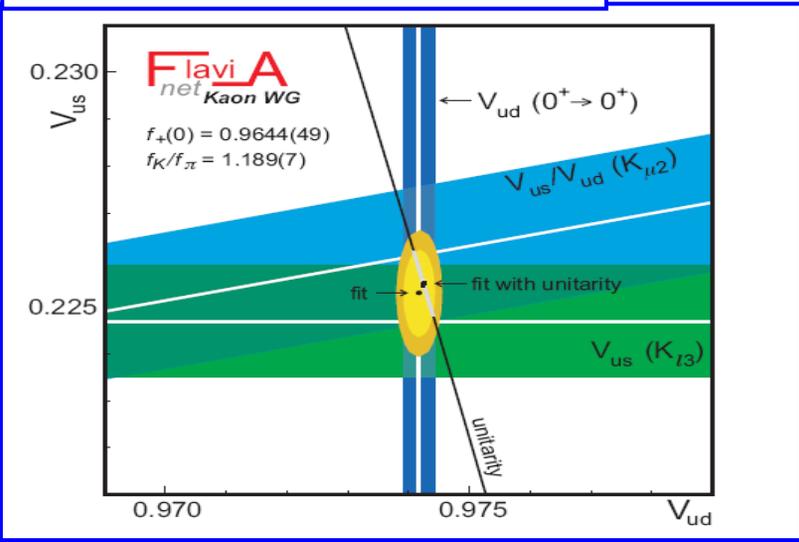
⇒ **at finite  $a$ , significant lattice artefacts: to subtract by complicated fit**

☺ *Lattice study from first principle action still need some time but progress already visible*

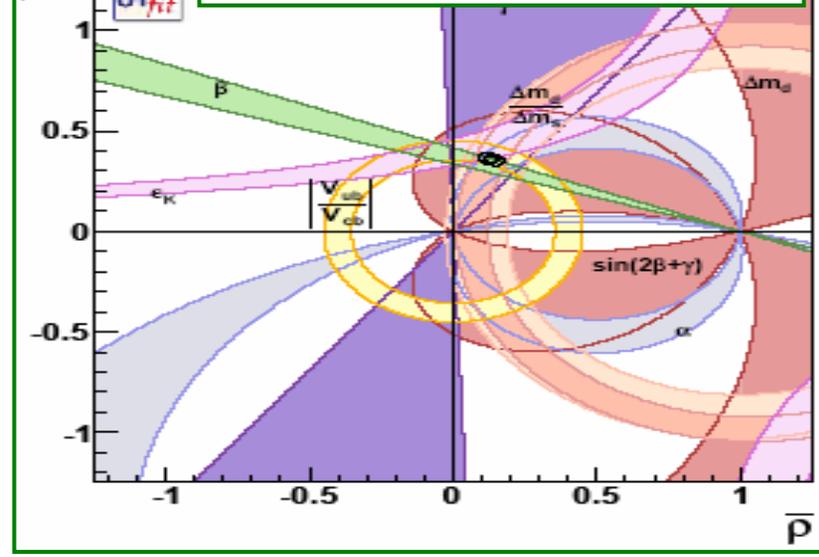


# Lattice QCD and Flavour Physics

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$



$$V_{cd}V_{cb}^* + V_{ud}V_{ub}^* + V_{td}V_{tb}^* = 0$$



**$K \rightarrow lv$ : exp. err 0.2%** ★★

lattice err. on  $f_K/f_\pi$ :  
**2-3% (2006) → 0.5% (2010)**

**$K \rightarrow \pi lv$ : exp. err 0.2%** ★

lattice err. on  $f_+(0)$ :  
**0.9% (2006) → 0.5% (2010)**

**$B \rightarrow D(D^*)lv$ : exp. err 1.5%** ●

lattice err. on  $F(1) \& G(1)$ :  
**5% (2006) → 2.5% (2010)** ■

**$\epsilon_K$ :  $K^0-\bar{K}^0$  mixing**  
 exp. err 0.5%

lattice err. on  $B_K$ :  
**11% (2006) → 4% (2010)** ★

**$\Delta M_d$ :  $B^0_d-B^0_d$  mixing**  
 exp. err 1%

lattice err. on  $f_{B_d}\sqrt{B_{B_d}}$ :  
**13% (2006) → 6% (2010)** ●

**$\Delta M_s$ :  $B^0_s-B^0_s$  mixing**  
 exp. err 0.7%

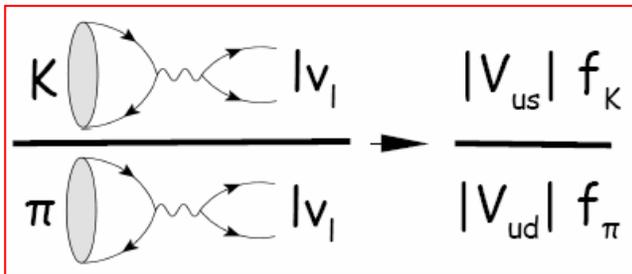
lattice err. on  $f_{B_s}\sqrt{B_{B_s}}$ :  
**13% (2006) → 6% (2010)** ●

**$B \rightarrow \pi(\rho)lv$ :**  
 exp. err 10%

lattice err. on  $f_+(q^2)$ :  
**11% (2007)!** ●

★ Simulation from first-principle action. ■ Quenched simulation. ● Staggered fermions

$f_K/f_\pi$ : an example of Modern lattice measure



Hadronic uncertainties from

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 u | K \rangle = p^\mu f_K$$

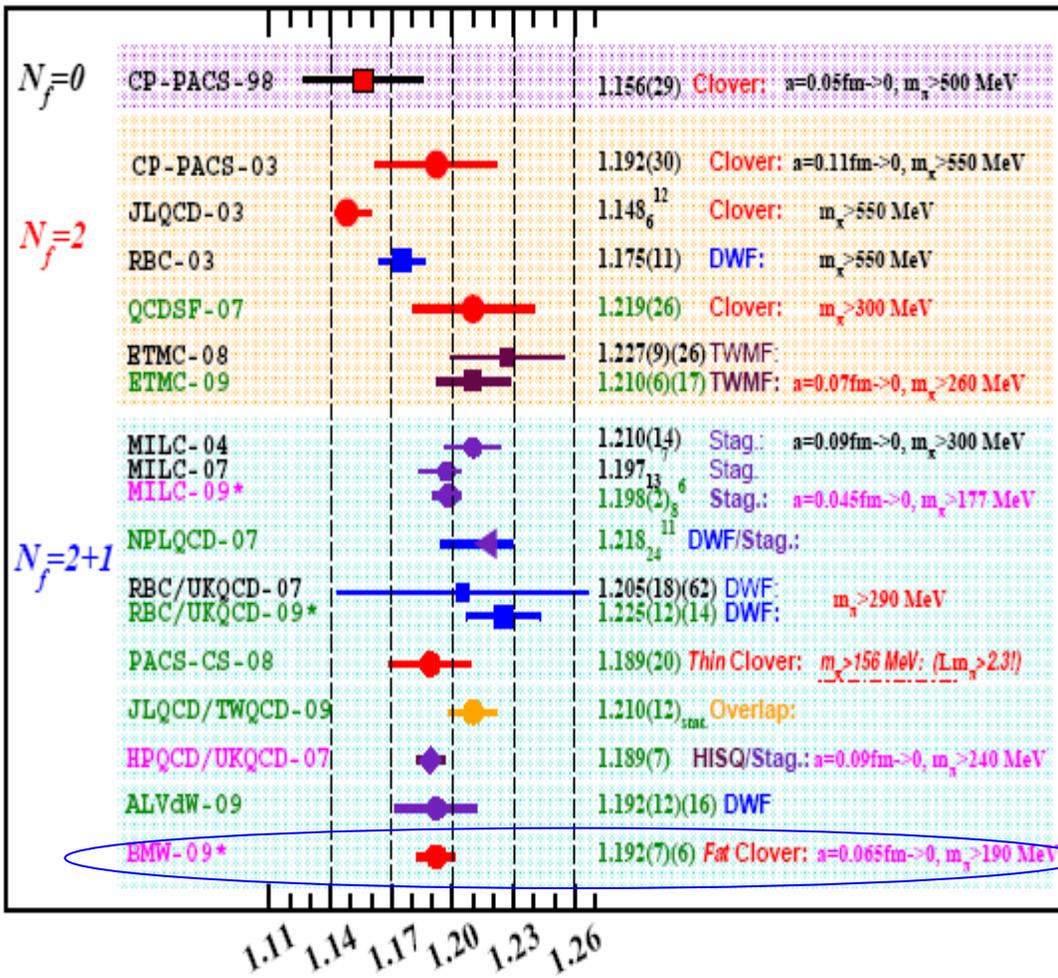
$$\langle 0 | \bar{d} \gamma^\mu \gamma_5 u | \pi \rangle = p^\mu f_\pi$$

# $f_K/f_\pi$ : lattice enters high precision era

❖ No Competition from non-Lattice approaches:

$$f_K/f_\pi = 1 + (m_K^2 - m_\pi^2) \times \text{unknown-coef.}$$

$f_K/f_\pi$



## Lattice systematic under control!

- ❑ Independent determinations (Wilson-like, Domain Wall, Stag.);
- ❑ Many lattice spacings (Continuum Limit reached);
- ❑ Light  $m_\pi \sim 190\text{ MeV}$ ;
- ❑ Finite size effects under control.

$$f_K/f_\pi = 1.193 \pm 0.006 \text{ [0.5%];}$$

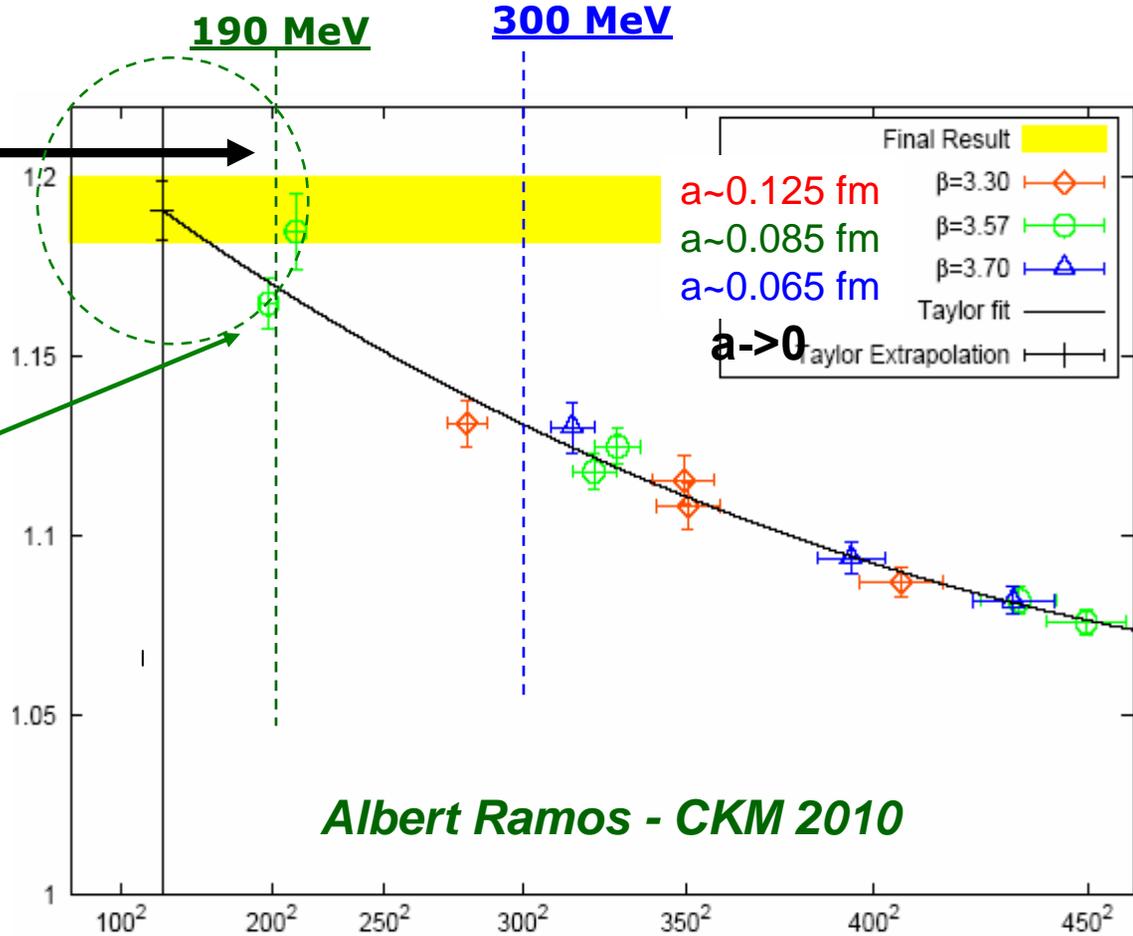
$$f_K/f_\pi - 1 = 0.193 \pm 0.006 \text{ [3%]}$$

good prospects to improve  $f_K/f_\pi - 1$  to 1% accuracy?

# $f_K/f_\pi$ - Evaluations

- getting very close to the physical point; **190 MeV**

The final result extrapolated at the physical pion is only 2% above the lightest point simulated!



- BMW result:  $N_F=2+1$ , Clover,  $a=0.06 \rightarrow 0$ ,  $m_\pi \geq 190$  MeV,  $L m_\pi \sim 4$**   
 $f_K/f_\pi = 1.192(7) (6) \Rightarrow \sigma_{\text{rel}} \sim 0.75\%$  ( $\delta_{SU(3)} \sim 5\%$ )

Constrain on scalar densities:  $SM + (\bar{s}_R u_L)(\bar{\ell} \nu_L)$

$$\frac{B(K \rightarrow \mu\nu)}{B(\pi \rightarrow \mu\nu)} \times \frac{B(n \rightarrow p\nu)}{B(K \rightarrow \pi\nu)} = \left( \frac{f_K}{f_\pi} \frac{1}{f_+(0)} \right)^2 \times \left( 1 + g^H \frac{m_K^2}{m_{H^+}^2} \right)^2$$

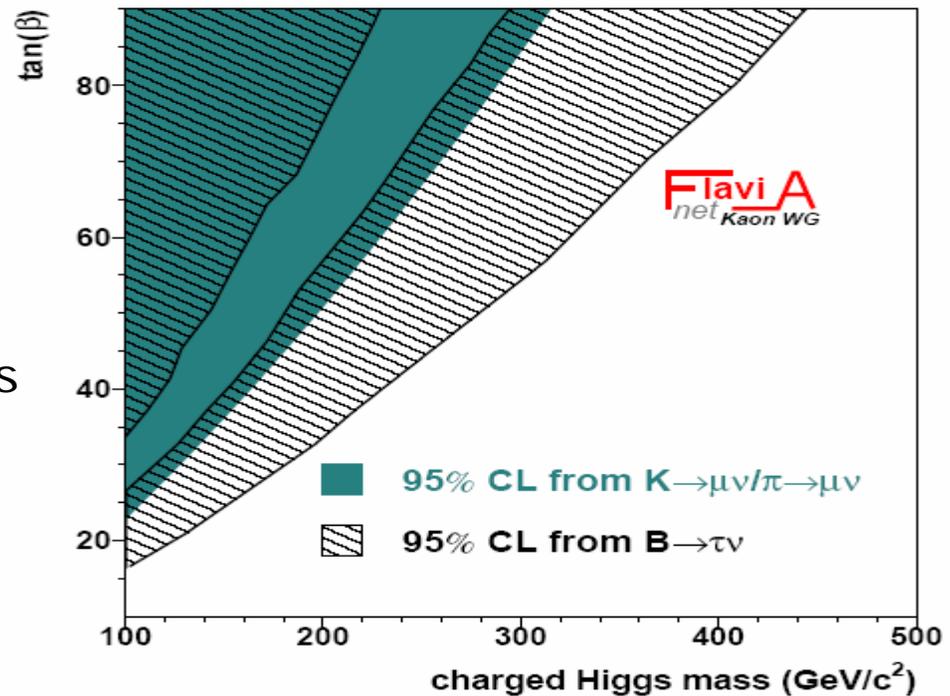
$$Br(B \rightarrow \tau\nu) \propto |V_{ub}|^2 f_B^2 m_B m_\tau^2 \times \left( 1 + g^H \frac{m_B^2}{m_{H^+}^2} \right)^2$$

Example MFV@large  $\tan\beta$

$$g^H = -\tan\beta^2 / (1 + \varepsilon_0 \tan\beta)$$

**Kaon:** NP highly suppressed => but favourable exp. and th. scenario

**B-physics:** large NP effects => but exp. and th. accuracy under progress



Present picture very reassuring and compatible between B and K physics.

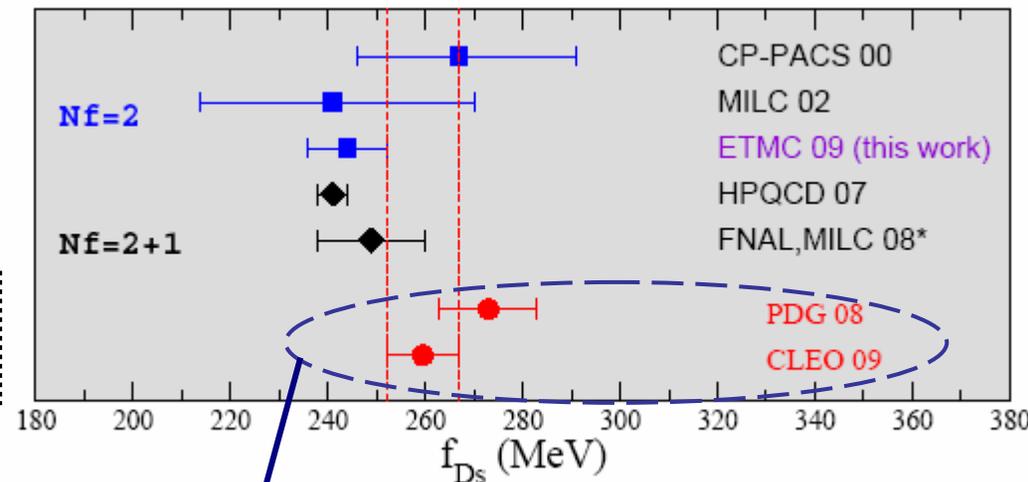
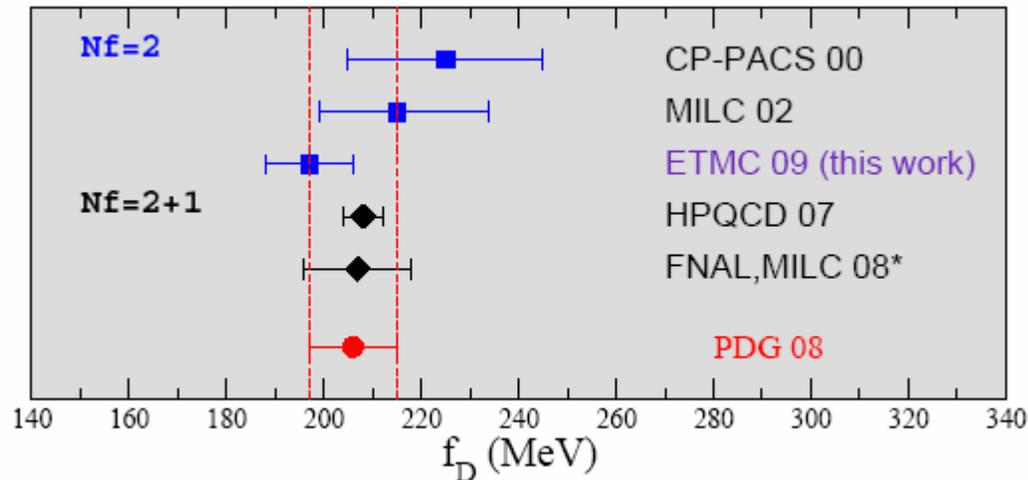
*2008  $f_{D_s}$  puzzle: 3-4 $\sigma$  deviation with experiments?*

# 2008 $f_{D_S}$ puzzle: 3-4 $\sigma$ deviation with experiments?

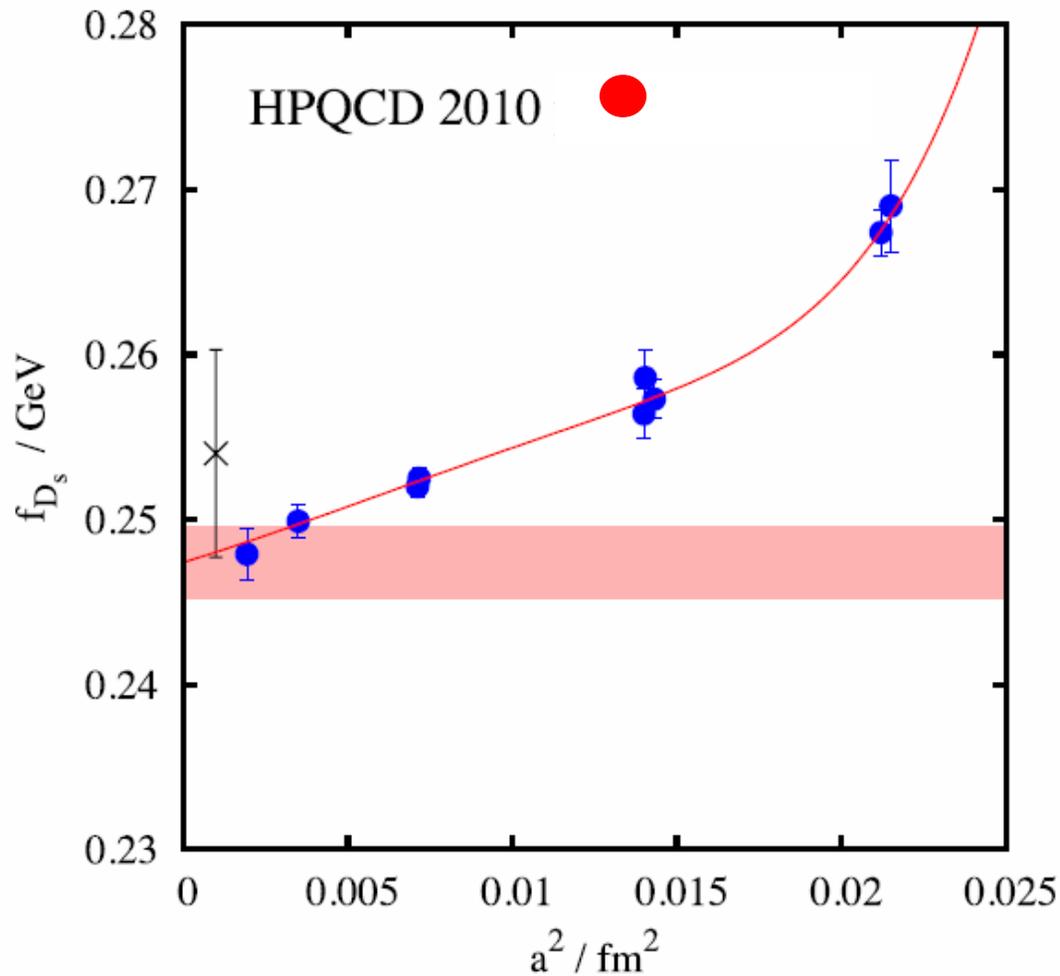
$f_D$  and  $f_{D_S}$  recently measured at CLEO, BaBar and Belle with good accuracy! (assuming CKM unitarity)

Key cross check of Lattice QCD before reach B-physics.

Overall consistency of lattice results;



$f_{D_S}$  "puzzle" weakened by CLEO-c 2009 accurate measurement



● Staggered fermions

"Puzzle" seems to disappear:

No conclusive evidence for New Physics in the charm quark sector yet, but the  $D_{(s)}$  leptonic decays will continue to help constraining SM extensions

$\Delta F=2$  observables:  
 $\varepsilon_K (B_K)$ ,  $\Delta M_d$  and  $\Delta M_s$  ( $fB$  and  $Bb$ )

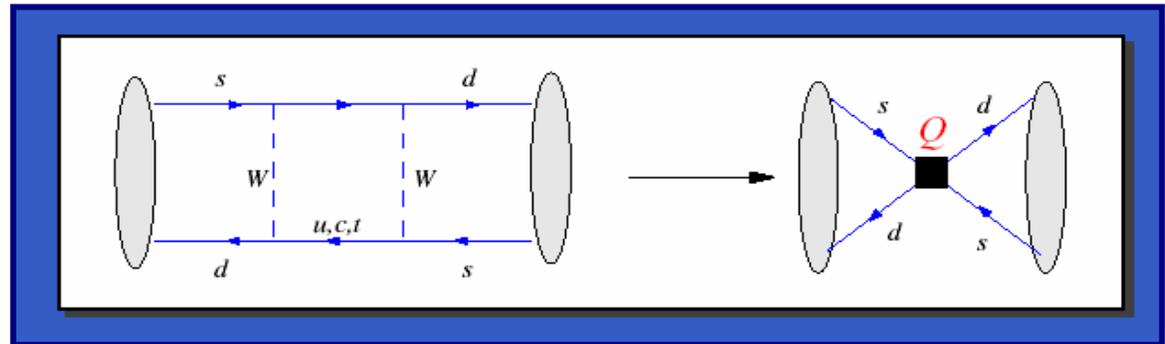
- Important role for the SM Unitarity Triangle analysis [UTfit and CKMfitter] :
- ❖ **strong constraints for non-minimal flavour models:**
  - ⇒ **MSSM** -> generic down squark insertion [Gabbiani et al. 1994, ...]
  - ⇒ **Little Higgs** [A. Buras et al. 2004]

*Lattice role and New Physics  
Opportunities!*

# CP-Violation in $K - \bar{K}$ Mixing: $\varepsilon_K$ and $B_K$

$$K_L \sim \overset{\text{CP}=-1}{(K^0 - \bar{K}^0)} + \varepsilon_K \overset{\text{CP}=+1}{(K^0 + \bar{K}^0)} \quad \varepsilon_K \rightarrow \text{indirect CP-violation}$$

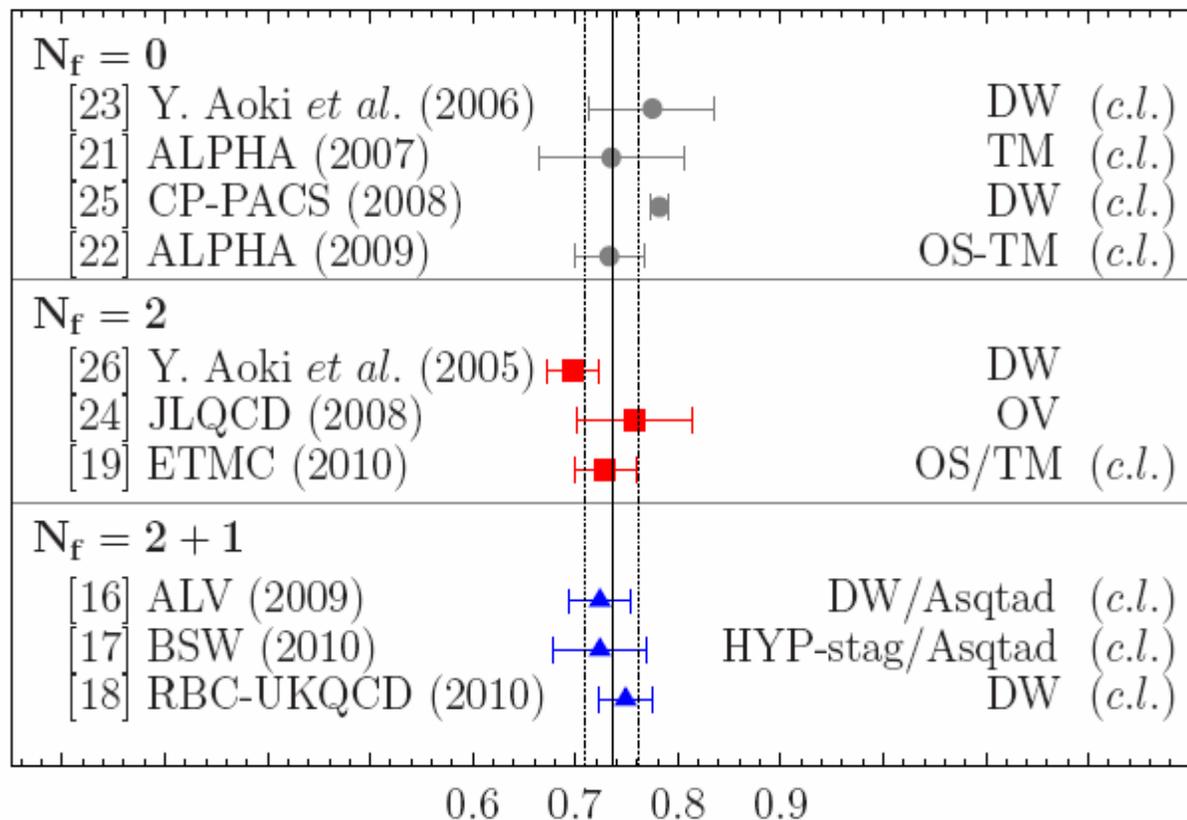
**The Effective  
 $\Delta S=2$   
Hamiltonian**



$$\varepsilon_K \sim \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C(\mu) \cdot \langle \bar{K}^0 | \overbrace{\bar{s} \gamma_\mu (1 - \gamma_5) d \bar{s} \gamma_\mu (1 - \gamma_5) d}^{Q(\mu)} | K^0 \rangle$$

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

# Dramatic progress in $B_K$ evaluation from recent simulations



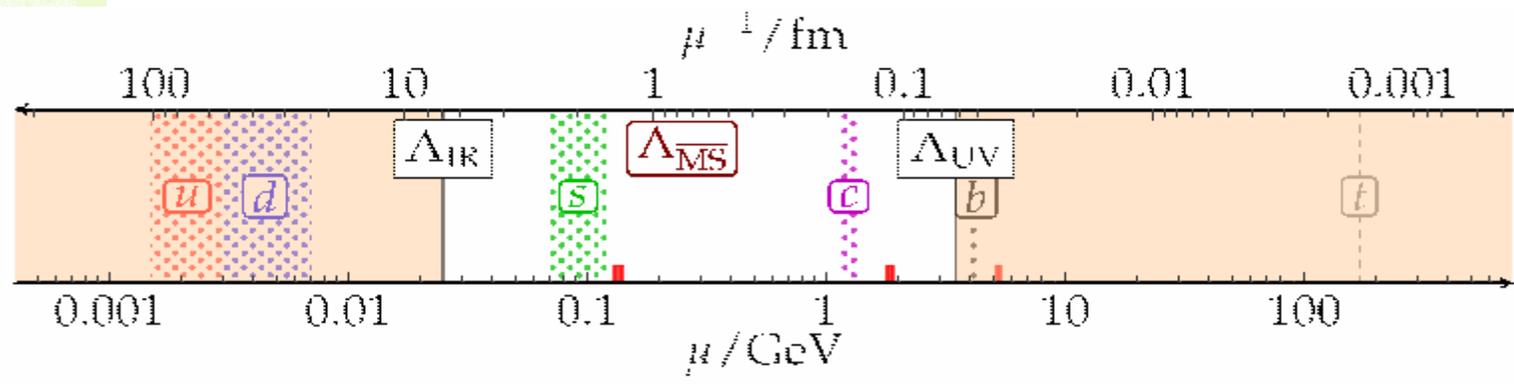
**Average by FLAG WG** (1011.4408 [hep-lat]):  $B_K = \mathbf{0.724(30)}$  (4%)

➤ *current value lower than old  $N_f=0$ :  $B_K \sim 0.83(6)$*

**key observation (2000):** to exploit  $\chi$ -improved action ->  
among others Carlos Pena (UAM), Pilar Hernandez (IFIC)

➤ increase the tension in the UTA

# Challenge of B-physics: the multi scale-problem of QCD



hierarchy of disparate physical scales to be covered:

$$\Lambda_{IR} = L^{-1} \ll m_\pi, \dots, m_D, m_B \ll a^{-1} = \Lambda_{UV}$$

$$\left\{ O(e^{-Lm_\pi}) \Rightarrow L \gtrsim \frac{4}{m_\pi} \sim 6 \text{ fm} \right\} \rightsquigarrow L/a \gtrsim 120 \rightsquigarrow \left\{ am_D \lesssim \frac{1}{2} \Rightarrow a \approx 0.05 \text{ fm} \right\}$$

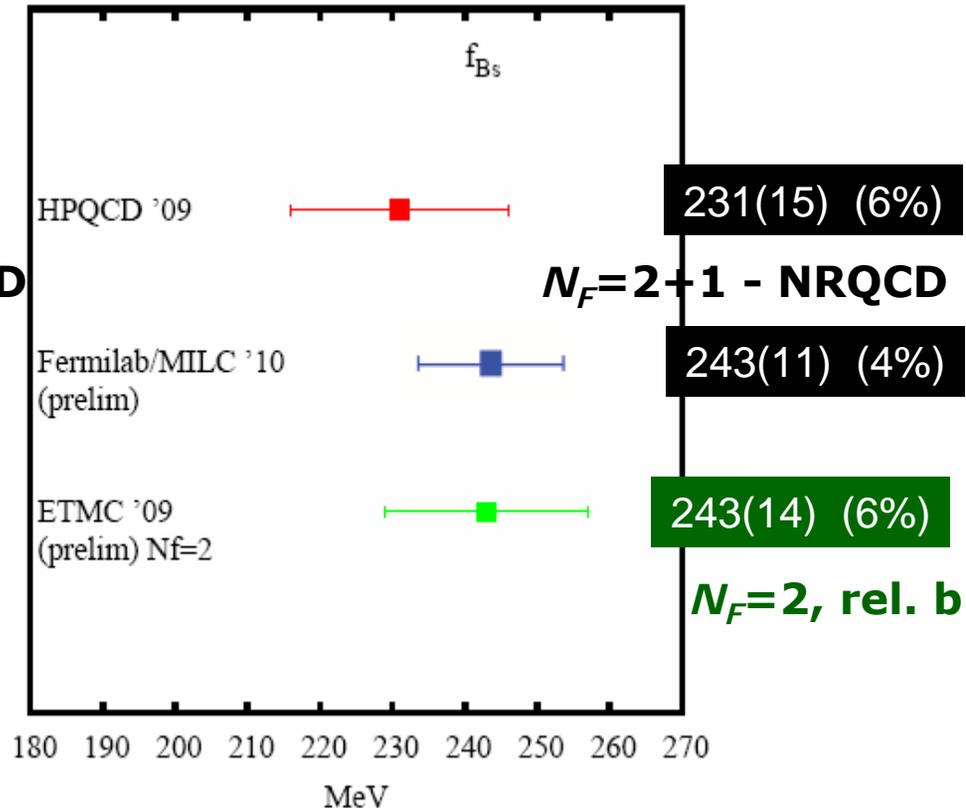
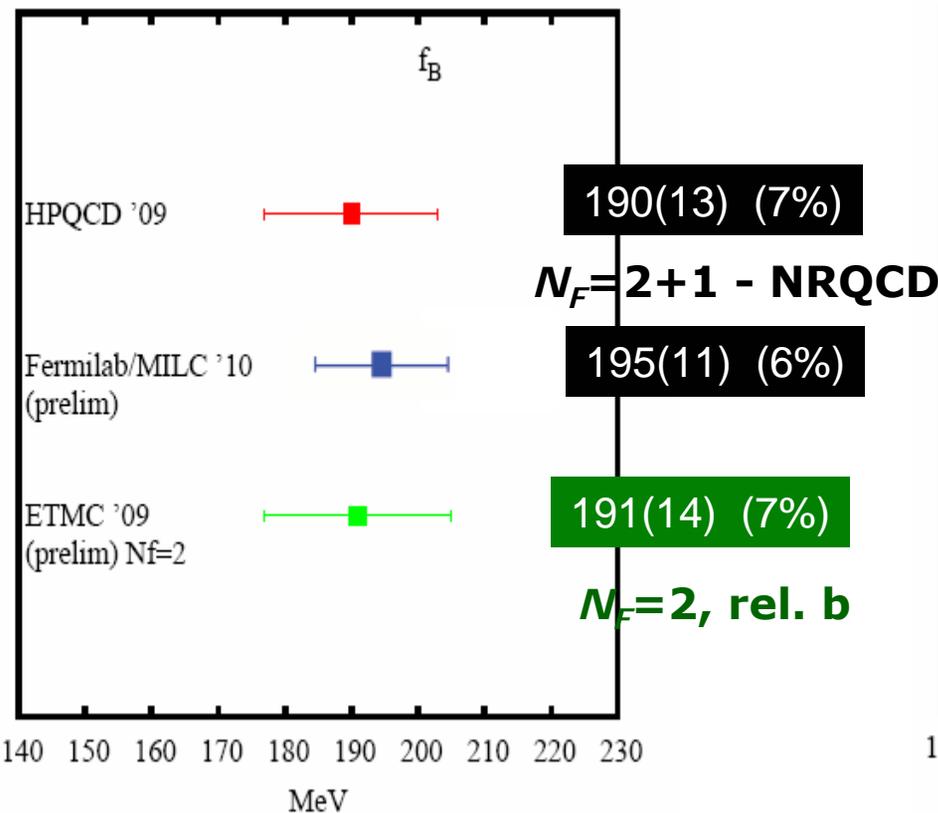
Currently, **b** quarks cannot be directly simulate at their physical mass due to large discretization errors ( $a m_b \ll 1$ )

- ❑ effective theory: like NRQCD action
- ❑ simulate heavy quark in the charm region and extrapolate to the B:

# $f_B - f_{B_s}$ : unquenched results

- ☺ Overall picture satisfactory
- ☺ Continuum Limit

- ☹ A few independent determinations (Wilson-like, Stag.);
- ☹ Still coarse lattice spacing at  $N_F=2+1$



**Average (Laiho, Lunghi, Van de Water '10)**

$$f_B = 192.8(9.9) \text{ MeV (5\%)}$$

$$f_{B_s} = 238.8(9.5) \text{ MeV (4\%)}$$

## Comments:

### - Effective theory approach: discretized NRQCD action

#### ➤ *Quite Sophisticated procedure!*

⇒ *larger set of  $1/m_Q$  corrections on the lattice w.r.t the continuum*

➤ *Improved through  $O[1/m_Q^2, a^2]$  and leading terms  $O[1/m_Q^3]$*

➤  *$O[\alpha_s^n/(am_Q)]$  divergences to be subtracted to get the continuum limit*

➤ *On the other hand, large experience by HPQCD*

*(Eduardo Follana, Elvira Gamiz (Granada)).*

- *Practical approach (a la San Tommaso) welcome*

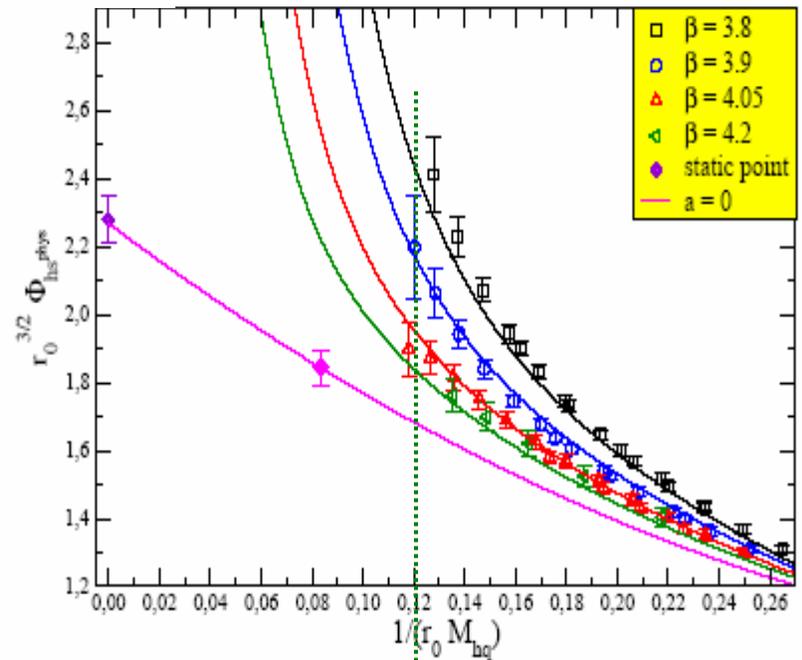
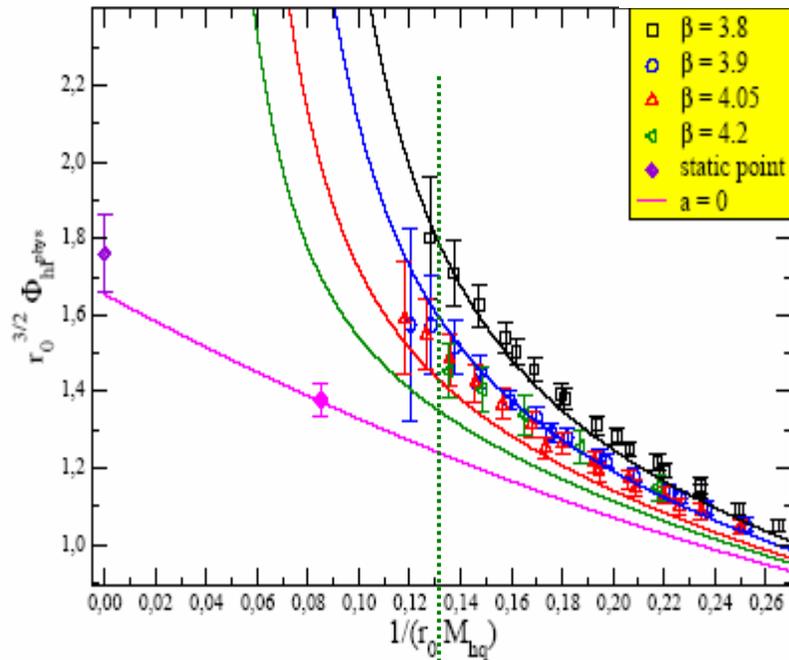


- Practical approach (a la San Tommaso): **ETMC**

$f_B$

$$\Phi = f_B \sqrt{M_B}$$

$f_{Bs}$



safe windows  $am_Q < 1$

- HQET-guided extrapolation
- Combined fit with the static point,  $m_b = \infty$
- Larger safe windows with smaller  $a \sim 6$  GeV

# $\Delta M_d$ and $\Delta M_s$

$$\Delta M_s : \langle B | \bar{b}_L \gamma^\mu s_L \bar{b}_L \gamma^\mu s_L | \bar{B} \rangle \propto \hat{B}_{Bs} f_{Bs}^2.$$

$$\Delta M_d : \langle B | \bar{b}_L \gamma^\mu d_L \bar{b}_L \gamma^\mu d_L | \bar{B} \rangle \propto \hat{B}_{Bd} f_{Bd}^2$$

☹ **Only one  $NF=2+1$  calculation available from HPQCD – Staggered fermions**

**(Eduardo Follana, Elvira Gamiz)**

☹ **Continuum Limit – still coarse lattice spacing** ☹

$$\hat{B}_{Bd} = 1.26(11) \rightarrow (9\%) \quad \hat{B}_{Bs} = 1.33(6) \rightarrow (5\%)$$

❖ **reassuring scenario but we need further calculation from several groups!**

**Average (Laiho, Lunghi, Van de Water '10)**

$$f_{Bs} \sqrt{B_{Bs}} = 275(13) \rightarrow 5\%!!? \quad \xi = 1.243(28) \rightarrow 2\%$$

$$\xi \equiv \frac{f_{Bs} \sqrt{B_{Bs}}}{f_{Bd} \sqrt{B_{Bd}}}$$

$\Delta F=1$   $b \rightarrow u, b \rightarrow s$  transitions:

$$B \rightarrow \pi (\rho) l \nu,$$

$$B \rightarrow K^* \mu^+ \mu^-,$$

$$B \rightarrow K^* (\phi) \gamma$$

☹ *very preliminary unquenched activities*

-> *pratically as bad as quenched era*

☹ *further complication -> form factors =  $f(M_B, M_{\pi}, q^2)$*

$$A(B \rightarrow K^* \mu\mu) = \langle K^* | \text{Heff} | B \rangle = \begin{cases} C_7 \times \bar{b} \sigma^{\mu\nu} s F_{\mu\nu} \\ C_9 \times \bar{b} \gamma_L^\mu s \bar{l} \gamma^\mu l + C_{10} \times \bar{b} \gamma_L^\mu s \bar{l} \gamma^\mu \gamma_5 l \\ C_S \times \bar{b}_L s_R \bar{l} l \end{cases}$$

➤ Matrix elements  $\Leftrightarrow$  7 form factors

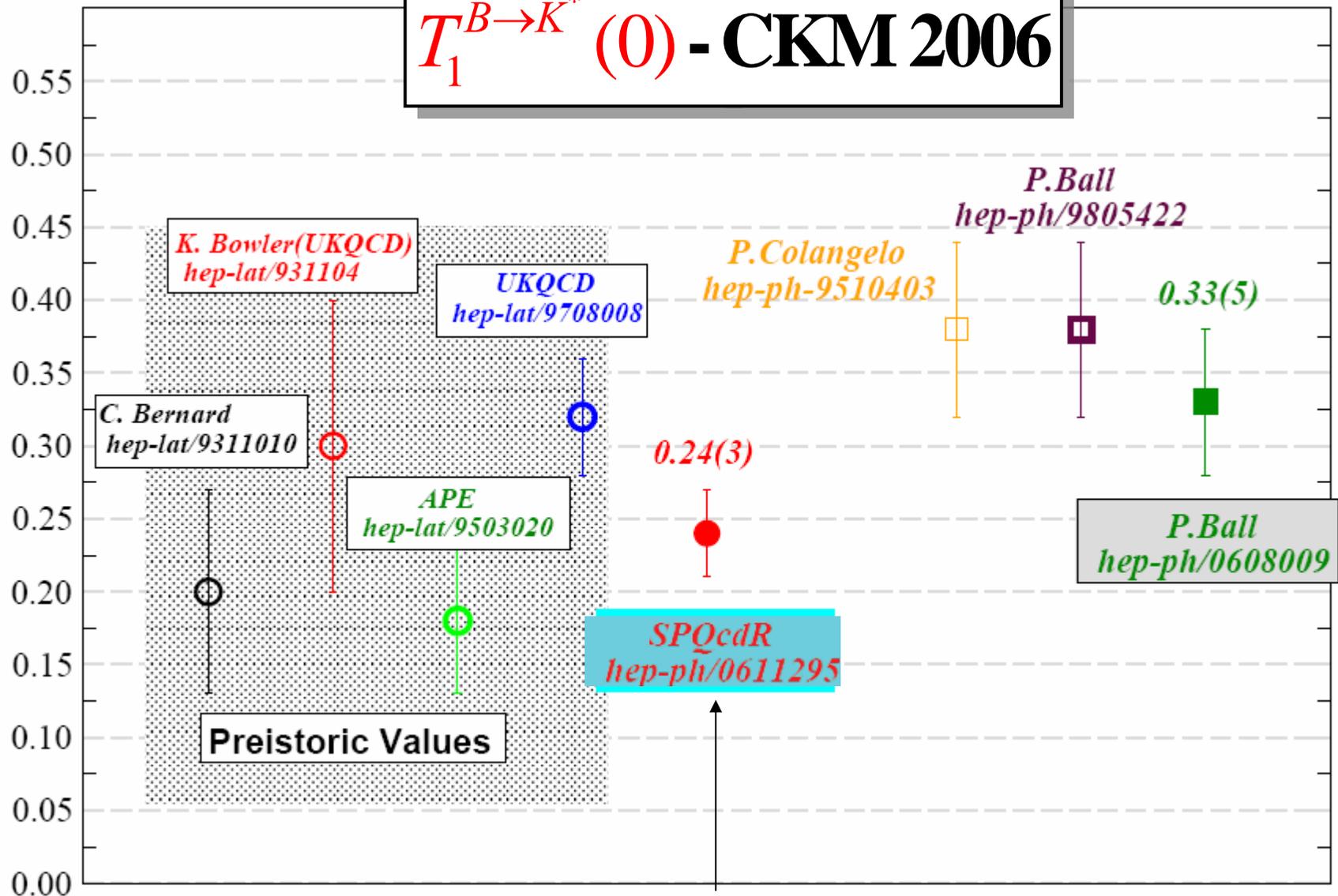
Lattice QCD

$$\begin{aligned} \langle K^*(p_{K^*}) | \bar{s} \gamma_\mu P_{L,R} b | B(p) \rangle &= i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu*} p^\alpha q^\beta \frac{V(q^2)}{m_B + m_{K^*}} \mp \\ &\mp \frac{1}{2} \left\{ \epsilon_\mu^* (m_B + m_{K^*}) A_1(q^2) - (\epsilon^* \cdot q) (2p - q)_\mu \frac{A_2(q^2)}{m_B + m_{K^*}} - \right. \\ &\quad \left. - \frac{2m_{K^*}}{q^2} (\epsilon^* \cdot q) [A_3(q^2) - A_0(q^2)] q_\mu \right\}, \end{aligned}$$

$$\begin{aligned} \langle K^*(p_{K^*}) | \bar{s} i \sigma_{\mu\nu} q^\nu P_{R,L} b | B(p) \rangle &= -i \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu*} p^\alpha q^\beta T_1(q^2) \pm \\ &\pm \frac{1}{2} \left\{ [\epsilon_\mu^* (m_B^2 - m_{K^*}^2) - (\epsilon^* \cdot q) (2p - q)_\mu] T_2(q^2) + \right. \\ &\quad \left. + (\epsilon^* \cdot q) \left[ q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (2p - q)_\mu \right] T_3(q^2) \right\}. \end{aligned}$$

☹ Up to now, only  $T_1(0)$  on the Lattice but quenched (Becirevic, Lubicz, F.M. '06)

# $T_1^{B \rightarrow K^*}(0)$ - CKM 2006



— LATTICE — — SUM RULES —

$A_{FB}(B_d \rightarrow K^* l^+ l^-)$ , already plays a special role:

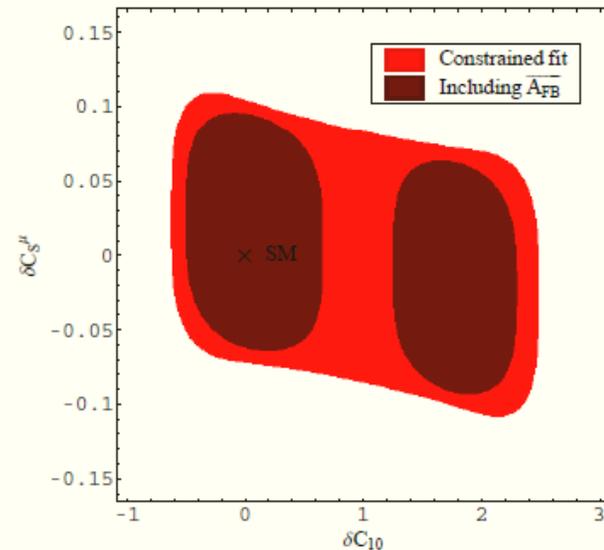
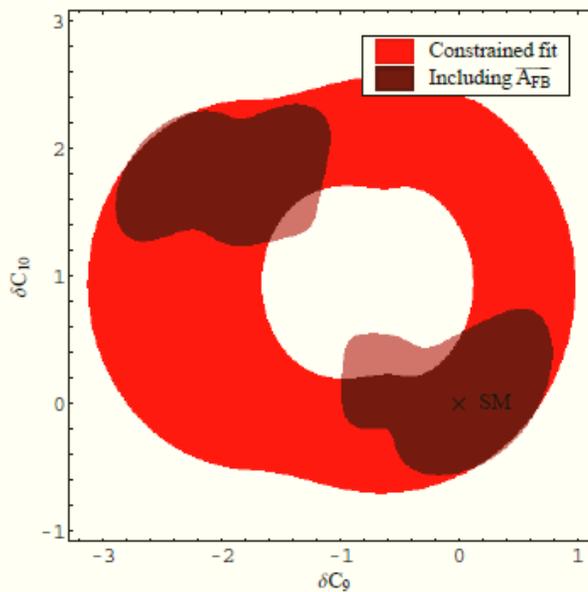
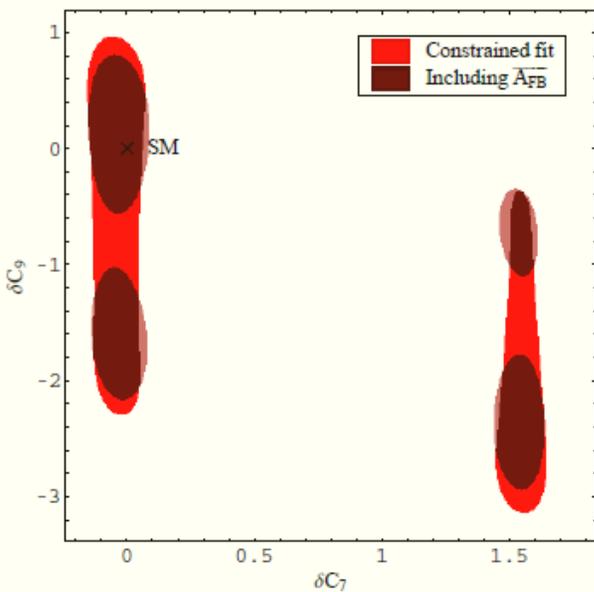
❖ very large experimental error;

➔ **Necessary to improve lattice form factors!**

$A_{FB}(B_d \rightarrow K^* l^+ l^-)$ : 2 bins

exp (large): Babar+Belle '09

$[q^2 < 6.25 \text{ GeV}^2]$	$0.24^{+0.19}_{-0.24}$	$-0.01 \pm 0.02$
$[q^2 > 10.24 \text{ GeV}^2]$	$0.76^{+0.53}_{-0.34}$	$0.20 \pm 0.08$



**Model-independent Analysis in MFV!**

**FCNC constraints from  $Br(B_d \rightarrow X_s \gamma)$ ,  $Br(B_d \rightarrow X_s l^+ l^-)$ ,  $Br(B_d \rightarrow \mu^+ \mu^-)$ ,  $\Delta M_s$**

Hurth, Isidori, Kamenik, F.M '08

$$D \rightarrow \pi \ell \nu_\ell$$

Lattice QCD for D, NF=2 unquenched

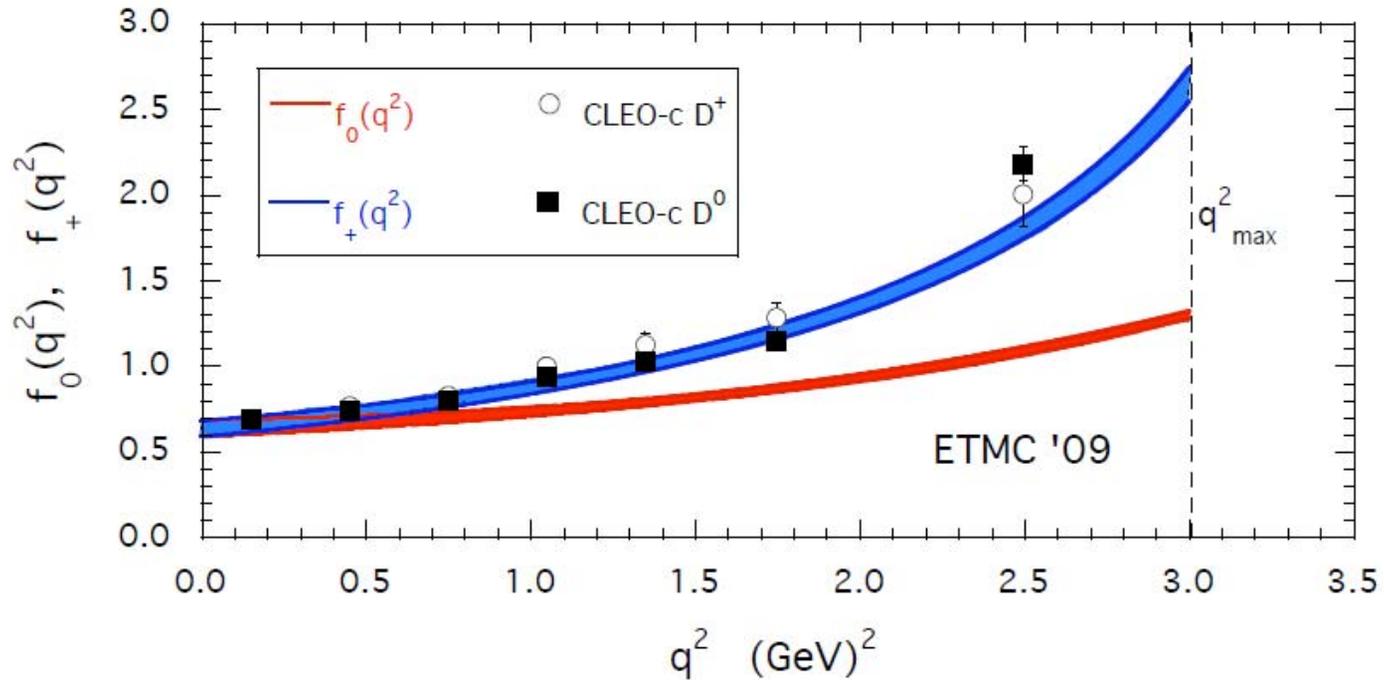


Figure 3: Vector [ $f_+(q^2)$ ] and scalar [ $f_0(q^2)$ ] form factors for the  $D \rightarrow \pi \ell \nu_\ell$  decay versus the squared

ETMC et al., '09

# *Lattice Flavour Physics in Spain*

☺ 3 international collaborations: **ETMC**, **HPQCD**, **CLS**

☹ **1 Mare Nostrum!**



# Conclusions:

LATTICE QCD -> touchable progress in recent years:

➡ reliable unquenched simulations with pions close to the physical point =>  $m_\pi=156$  MeV (PACS-CS),  $m_\pi=190$  MeV (BMW)

➡  $f_K/f_\pi$  paradigm of present lattice progress!

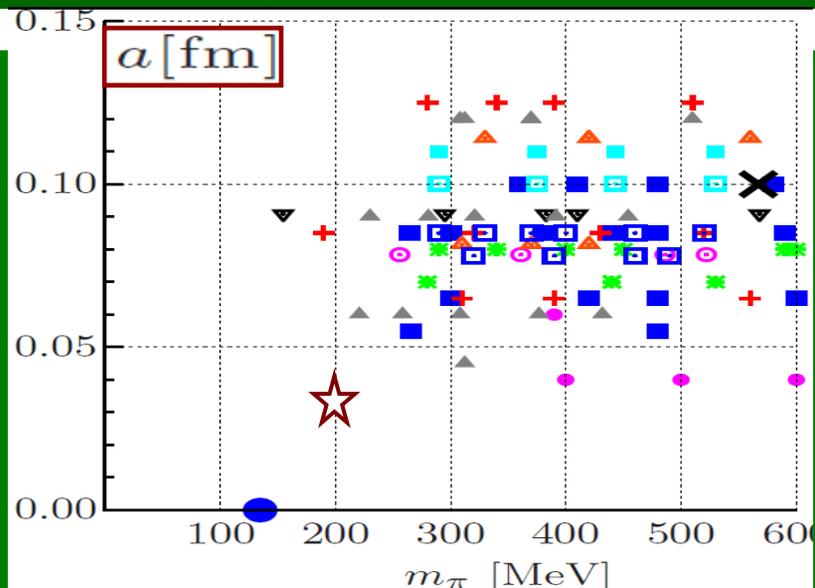
➡ many other calculations should be come soon!

In particular, for B-physics observables

Still a long work to assess 1%-precision needed at Super-B

- 1 discretization errors:  $a*m_B \ll 1$   
=>  $a \sim 0.033$  fm (6 GeV): ( $a \geq 0.07$  fm)
- 2 finite volume effects:  $L*m_\pi \gg 1$   
=>  $L \geq 4.5$  fm ( $L \leq 3$  fm)
- 3 chiral regime:  $200 \leq m_\pi \leq 300$  MeV

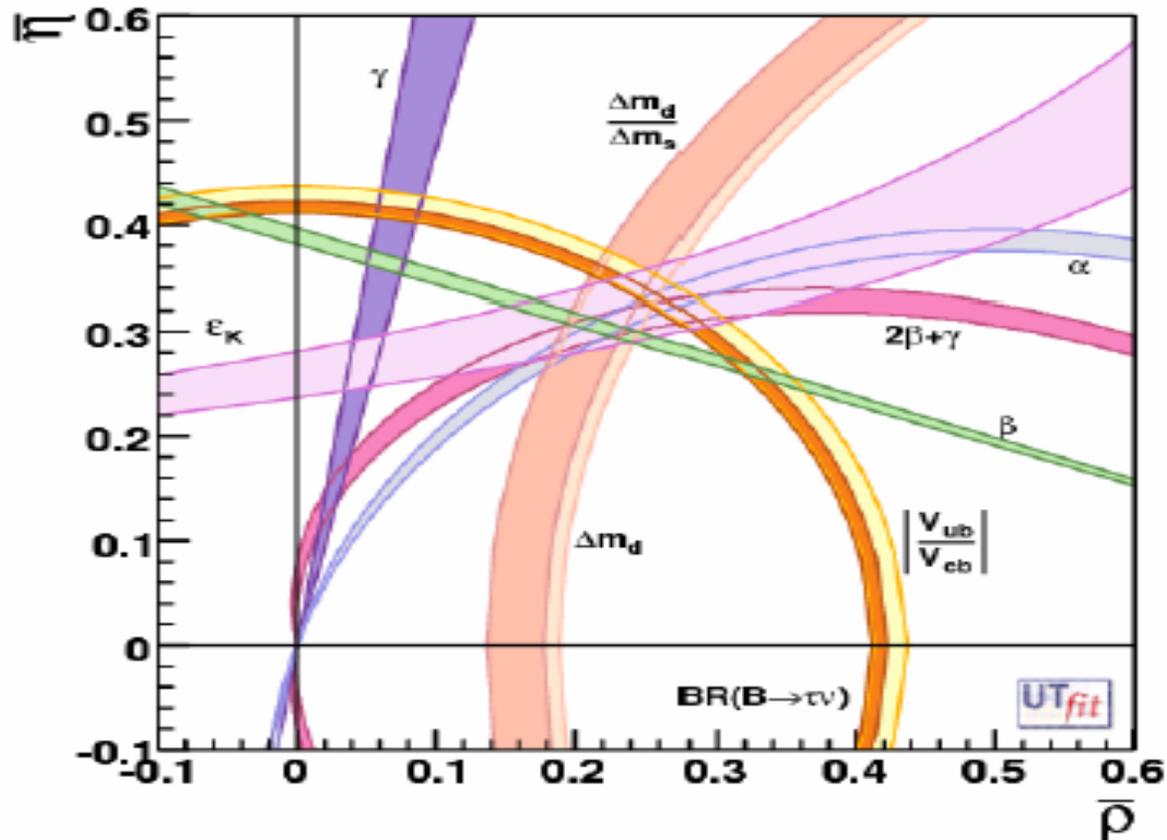
V. Lubicz Super-B report.



courtesy of G. Herdoiza

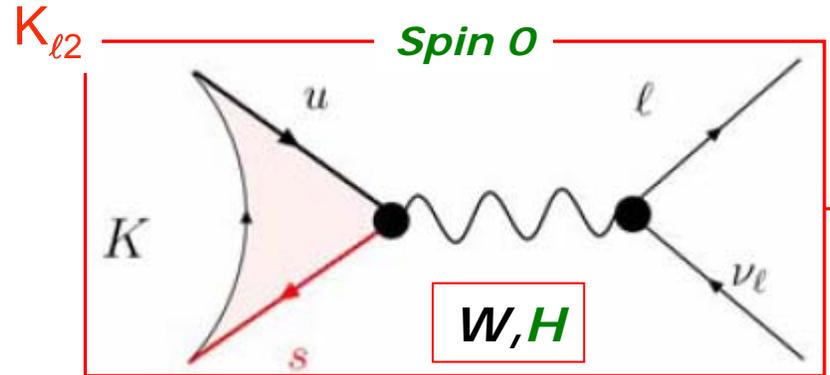
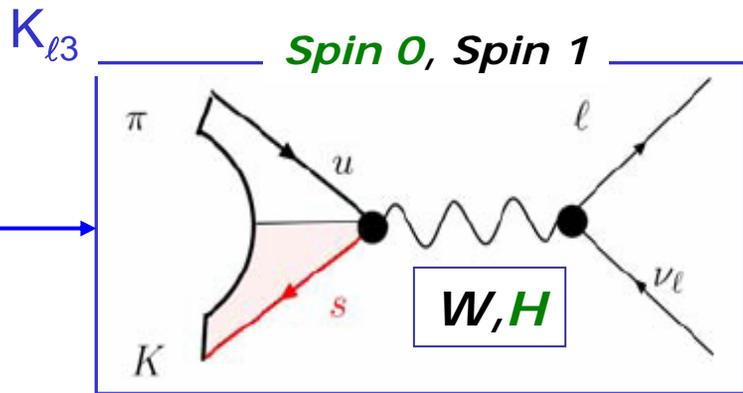
# Super B and Lattice QCD Alliance

➤ *Constraining  $V_{us}$ ,  $V_{ub}$ ,  $\rho$ ,  $\eta$  as precisely as possible from as many independent modes as possible in order to search for New Physics!*



2HDM, Susy ...: Charged Higgs =>

$$SM + (\bar{S}_R u_L) (\bar{\ell} \nu_L)$$



$K_{I3}$ : Higgs effects =>  $f_0(q^2) \rightarrow f_0(q^2) (1 + g^H q^2 / m^2_H)$

- to test  $g^H$  => calculate slope and curvature of  $f_0(q^2)$ !

$|V_{us}|^{KI3}$  is NP free as soon as we use  $f_0(q^2)$  from data

$K_{I2}$ : Higgs effects =>  $|V_{us}|^{KI2} \rightarrow |V_{us}|^{KI2} (1 + g^H m^2_K / m^2_H)$

- to test  $g^H$  =>  $f_K$  from theory:  $f_K / f_\pi$  better determined

constrain on scalar densities

Hadronic uncertainties 0.5%

$$\frac{B(K \rightarrow \mu\nu)}{B(\pi \rightarrow \mu\nu)} \times \frac{B(n \rightarrow pl\nu)}{B(K \rightarrow \pi l\nu)} = \left( \frac{f_K}{f_\pi} \frac{1}{f_+(0)} \right)^2 \times \left( 1 + g^H \frac{m_K^2}{m_{H^+}^2} \right)^2 \quad (\text{known f.s.})$$

similarly search on  $H^+$  from

$B^\pm \rightarrow \tau^\pm \nu$

$$Br(B \rightarrow \tau\nu) \propto |V_{ub}|^2 f_B^2 m_B m_\tau^2 \times \left( 1 + g^H \frac{m_B^2}{m_{H^+}^2} \right)^2$$

- hadronic uncertainty  $\rightarrow f_B$ :  
~5% accuracy from Lattice  
(Lunghi et al. 2009)

Best for indirect  $H^+$  searches  
but only feasible at ● SuperB

$B^\pm \rightarrow D \tau^\pm \nu$

$$\frac{d\Gamma(B \rightarrow D\tau\nu)}{dq^2} \propto |V_{cb}|^2 \rho_V(q^2) \times \left( 1 - \frac{m_\tau^2}{m_B^2} \left| 1 + g^H \frac{q^2}{m_{H^+}^2} \right|^2 \rho_S(q^2) \right)$$

- hadronic uncertainty  $\rightarrow \rho_V$  ●  
 $\rightarrow \rho_S$  ■

Only scalar component sensitivity  
to  $H^+$ : opportunity for Lhcb?

but different theoretical and experimental prospects

■ Quenched simulation.

● Staggered fermions