The Quest for Primordial Non-Gaussianity

Overview and some recent developments;
skewed toward observations

Dragan Huterer

University of Michigan

Why study non-Gaussianity (NG)?

1. NG presents a window to the very early universe. For example, NG can distinguish between physically distinct models of inflation.

2. Conveniently, NG can be constrained/measured using CMB anisotropy maps and LSS. In particular, there is a rich set of observable quantities that are sensitive to primordial NG.
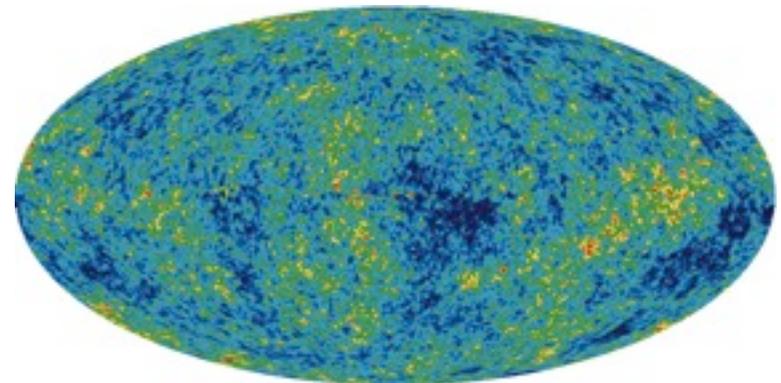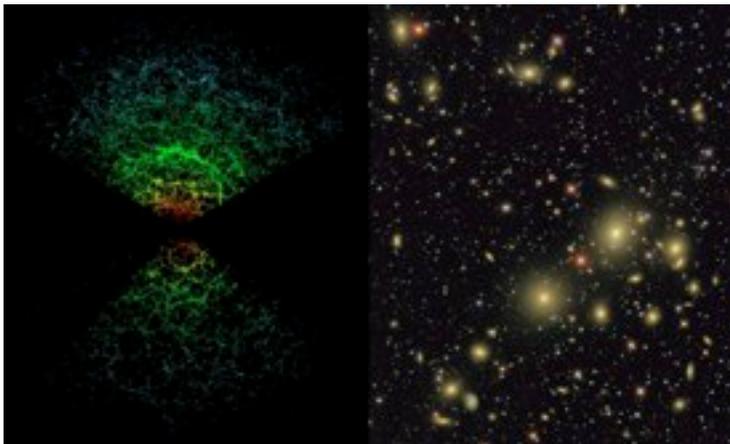
Non-Gaussianity papers
in the past 10 years

# of articles with
"Non-Gaussian"
in the title
on the ADS data base

Large-Scale Structure

CMB

Inflation / Theory

non-primordial NG

Planck

WMAP7
(1.5 σ)

?

Dalal et al.

Yadav & Wandelt (2.8σ ?)

WMAP5
(1.7 σ)

WMAP3
(0.7 σ)

WMAP1
(0.8 σ)

COBE
(≪1σ)

Produced by Emiliano Sefusatti
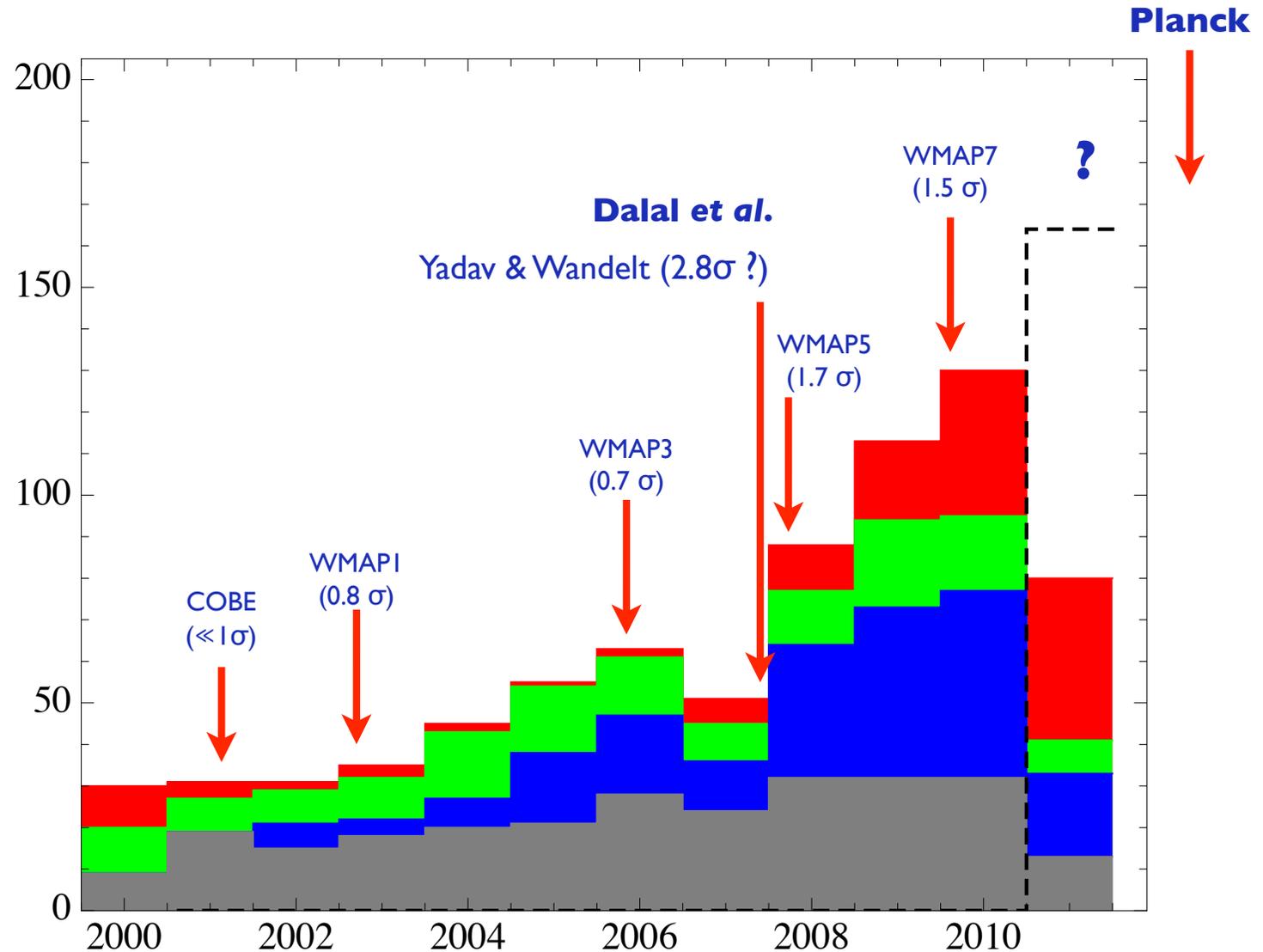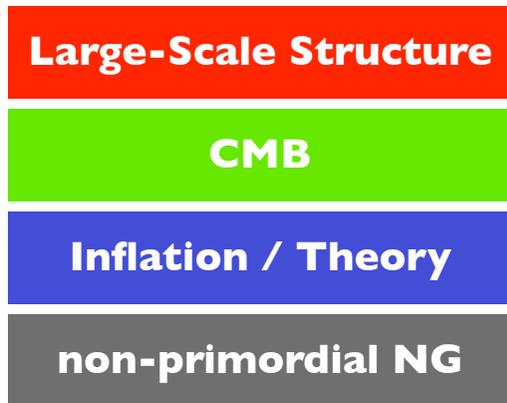
Initial conditions in the universe

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta, \phi) \qquad \ell \simeq \frac{180°}{\theta}$$
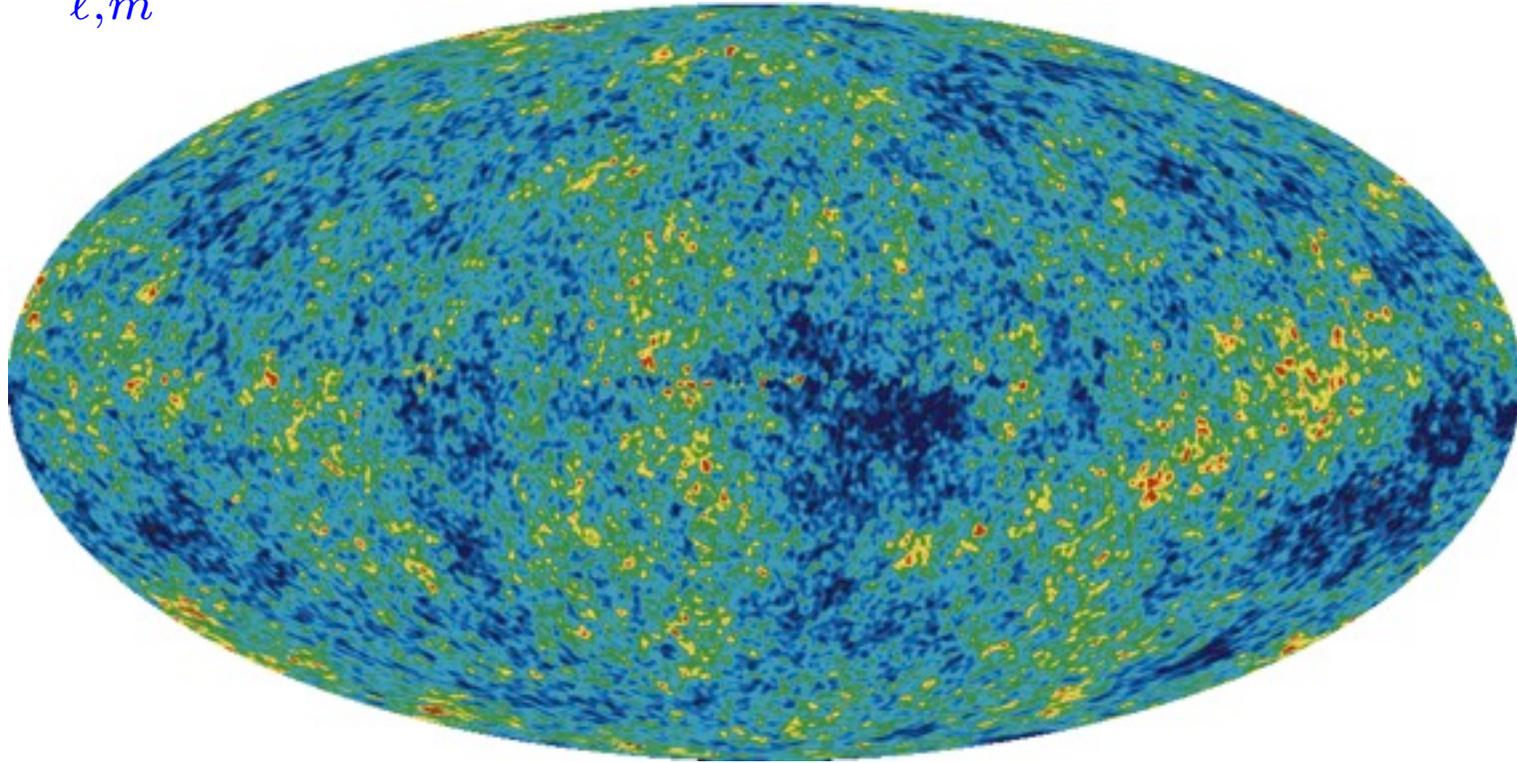
Generic inflationary predictions: Statistical Isotropy:

$$\langle a_{\ell m}\, a_{\ell' m'} \rangle \equiv C_{\ell \ell' m m'} = C_\ell \delta_{\ell \ell'} \delta_{m m'}$$

- Nearly scale-invariant spectrum of density perturbations
- Background of gravity waves   Gaussianity:
- (Very nearly) gaussian initial conditions: $\langle a_{\ell m}\, a_{\ell' m'}\, a_{\ell'' m''} \rangle = 0$

Standard Inflation, with...

1. a single scalar field

2. the canonical kinetic term

3. always slow rolls

4. in Bunch-Davies vacuum

5. in Einstein gravity

produces **unobservable** NG

Therefore, measurement of nonzero NG would point to a **violation** of one of the assumptions above

e.g. Maldacena 2003, X. Chen, Adv. Astronomy, 2010; Komatsu et al, arXiv:0902.4759

NG from 3-point correlation function
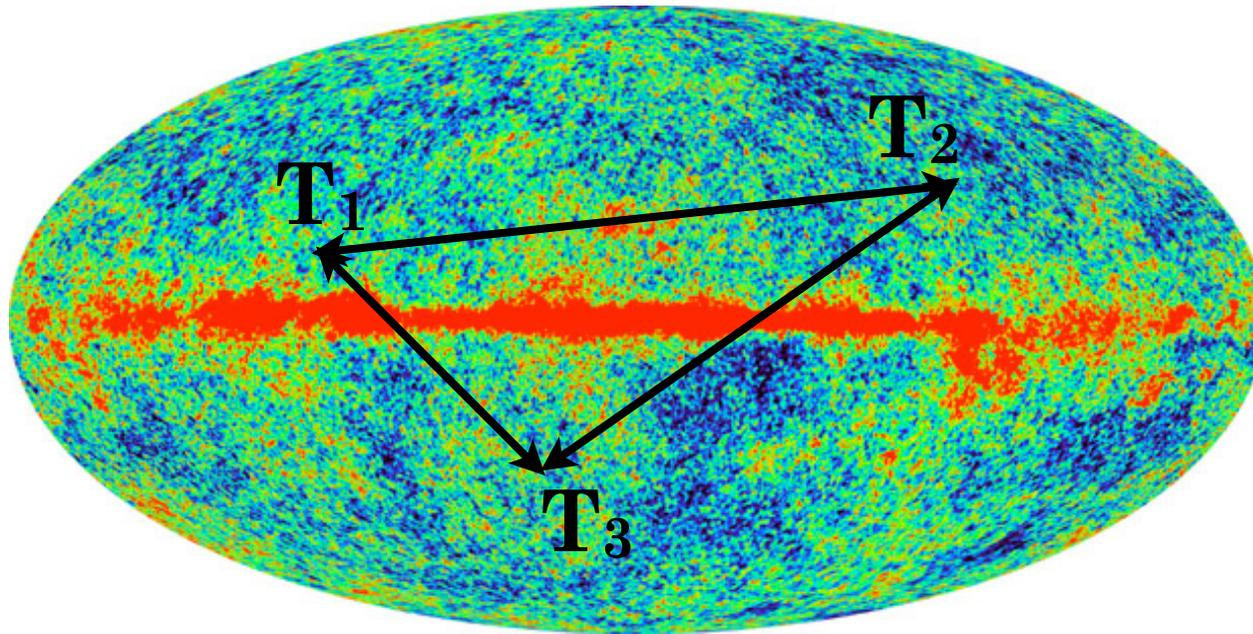
Commonly used "local" model of NG

$$\Phi = \Phi_G + f_{\mathrm{NL}} \left(\Phi_G^2 - \langle \Phi_G^2 \rangle \right)$$

Salopek & Bond 1990; Verde et al 2000; Komatsu & Spergel 2001; Maldacena 2003

Then the 3-point function is related to f_{NL} via (in k-space)

$$B(k_1, k_2, k_3) \sim f_{\mathrm{NL}} \left[P(k_1)P(k_2) + \mathrm{perm.} \right]$$

$f_{NL}= 0$

$f_{NL}= -5000$

$f_{NL}= -500$

$f_{NL}= +5000$

$f_{NL}= +500$

Using publicly available NG maps by Elsner & Wandelt
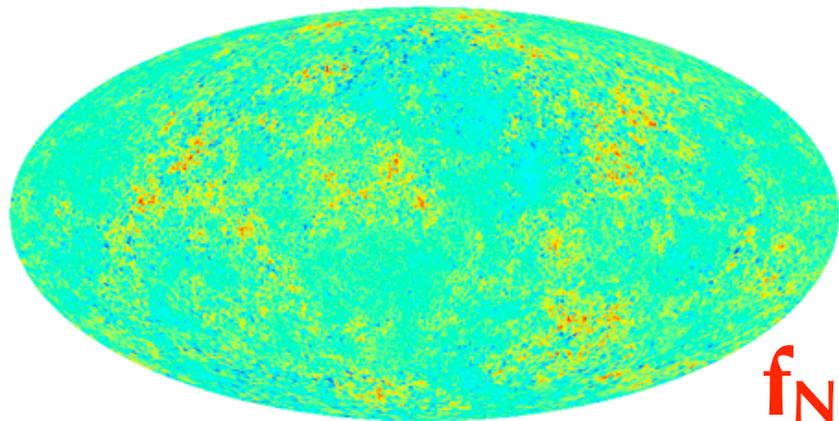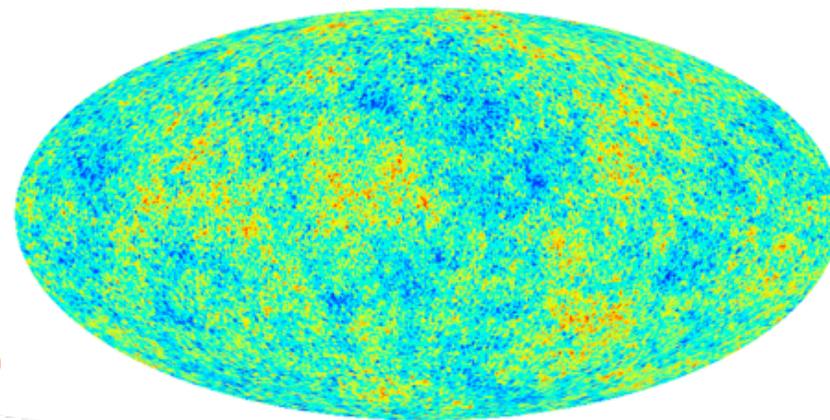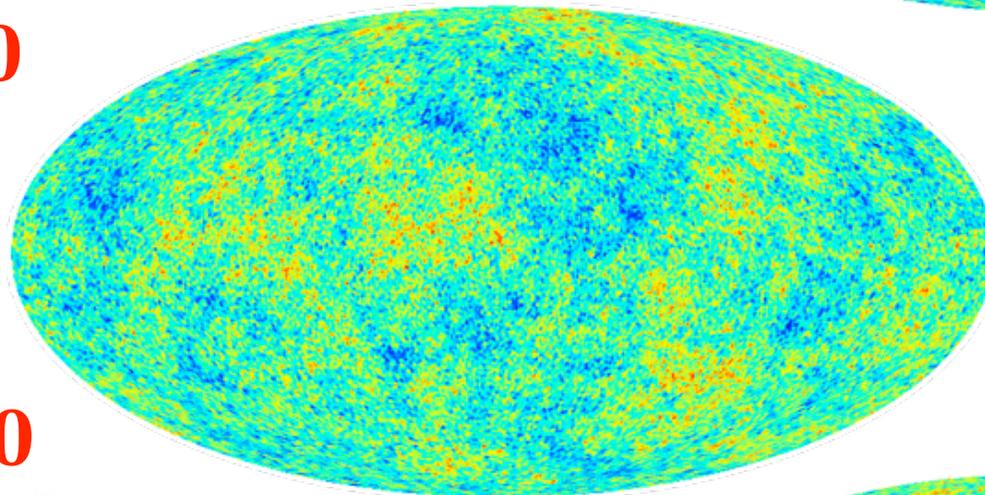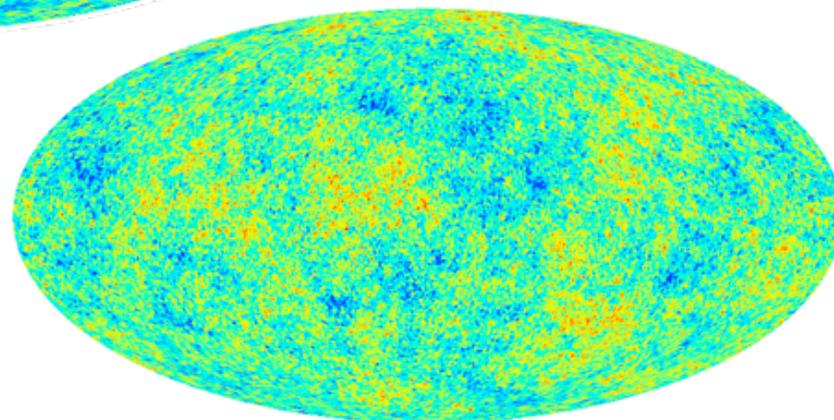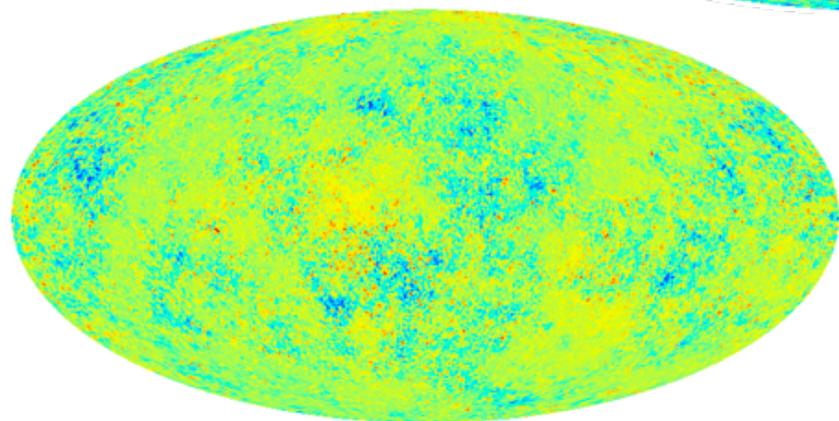
3-pt correlation function of CMB anisotropy ⇒ direct window into inflation

e.g. Luo & Schramm 1993

"local"
(eg. from sharp features in V(φ))

k_1 k_2

k_3
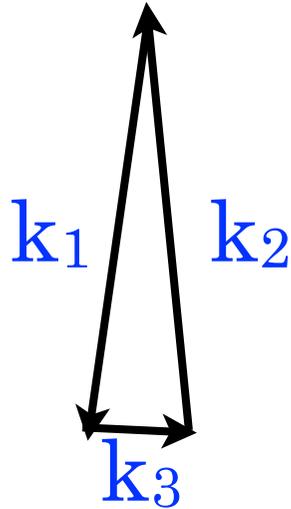
"equilateral"
(eg. higher-derivative action)

k_1 k_2

k_3

Local
x_2
$F(x_2, x_3)$
x_3

Higher Deriv.
x_2
$F(x_2, x_3)$
x_3
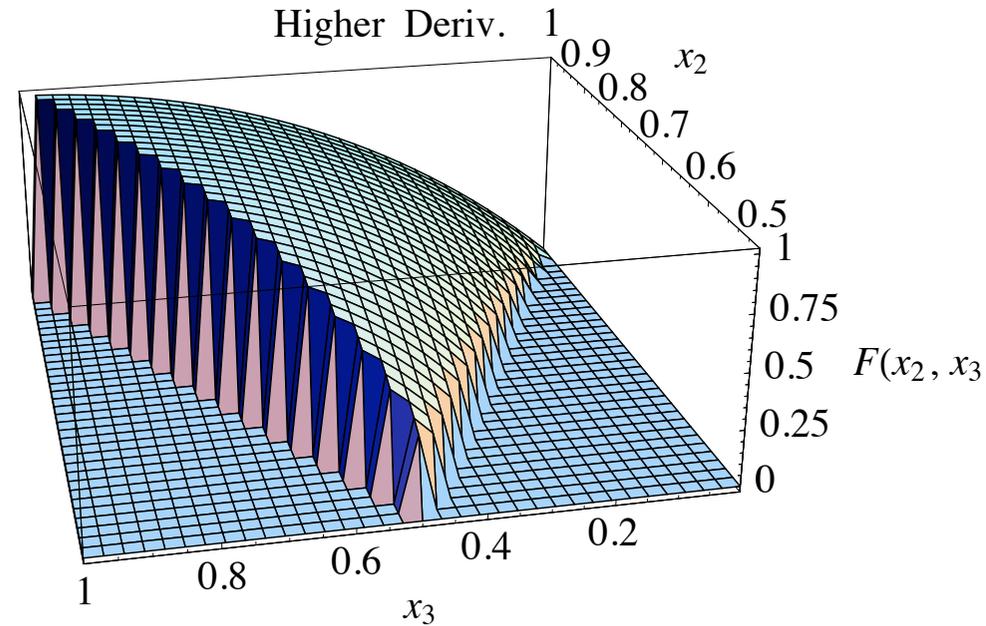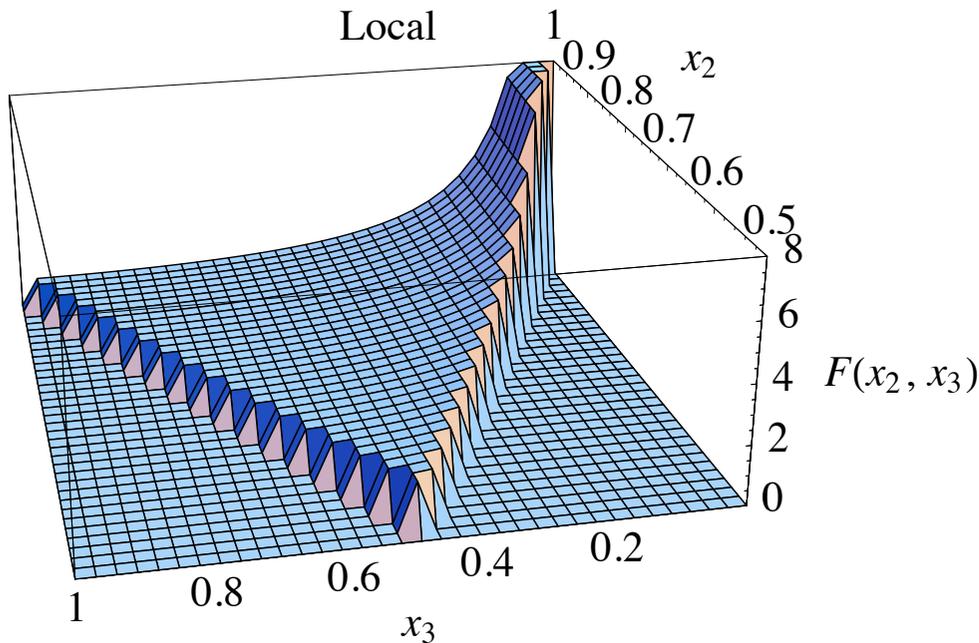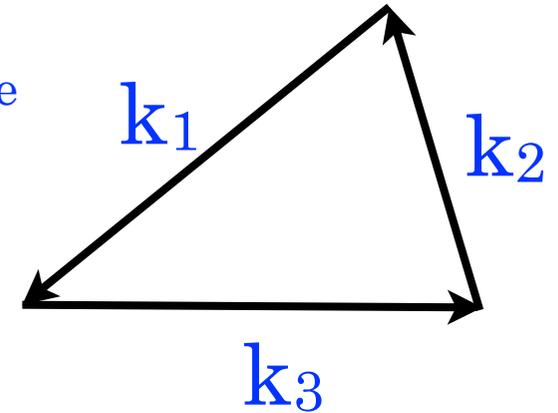
Babich, Creminelli & Zaldarriaga 2004

Brief history of NG measurements: 1990's

Early 1990s; COBE: Gaussian CMB sky (Kogut et al 1996)

1998; COBE: claim of NG at l=16 equilateral bispectrum
(Ferreira, Magueijo & Gorski 1998)

but explained by a known systematic effect!
(Banday, Zaroubi & Gorski 1999)

(and anyway isn't unexpected given all
bispectrum configurations you can measure;
Komatsu 2002)

Number of Gaussian MCs

Bispectrum value
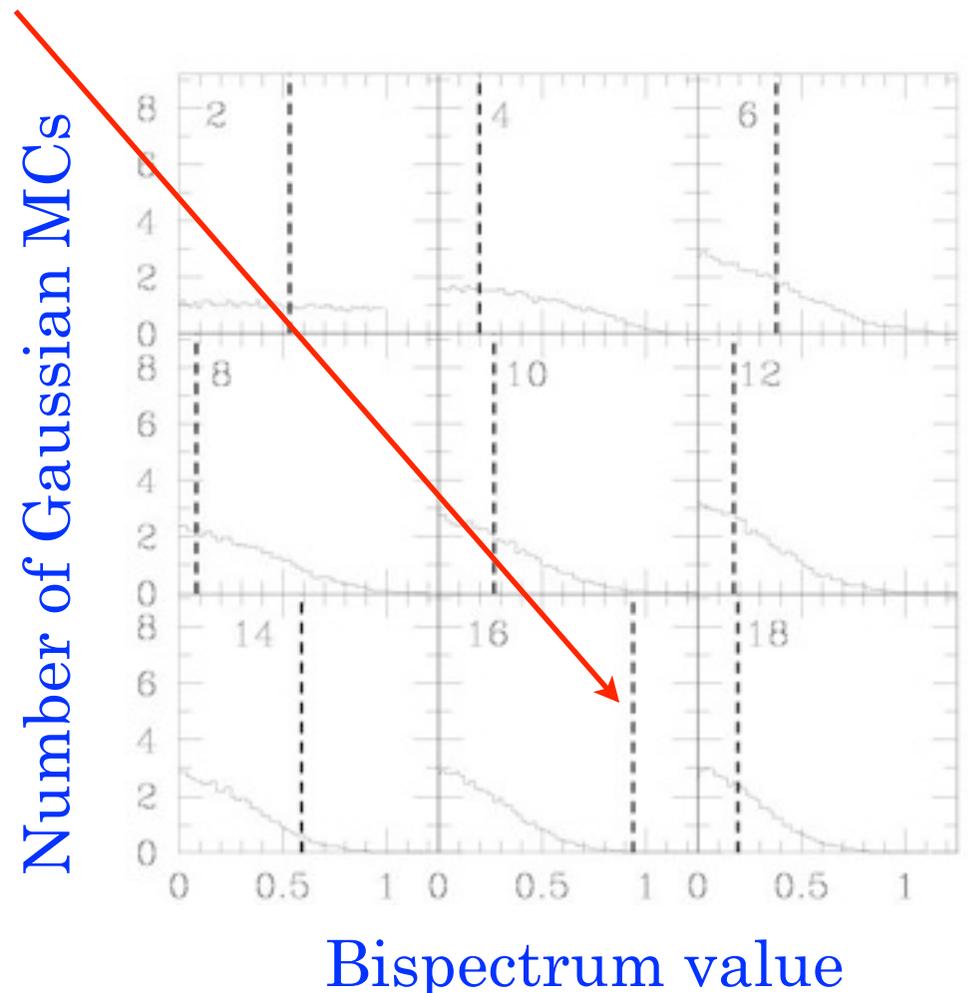
Brief history of NG measurements: 2000's

Pre-WMAP CMB: all is gaussian (e.g. MAXIMA; Wu et al 2001)

WMAP pre-2008: all is gaussian
(Komatsu et al. 2003; Creminelli, Senatore, Zaldarriaga & Tegmark 2007)

$-36 < f_{NL} < 100$ (95% CL)

Dec 2007, claim of NG in WMAP
(Yadav & Wandelt arXiv:0712.1148)

$27 < f_{NL} < 147$ (95% CL)
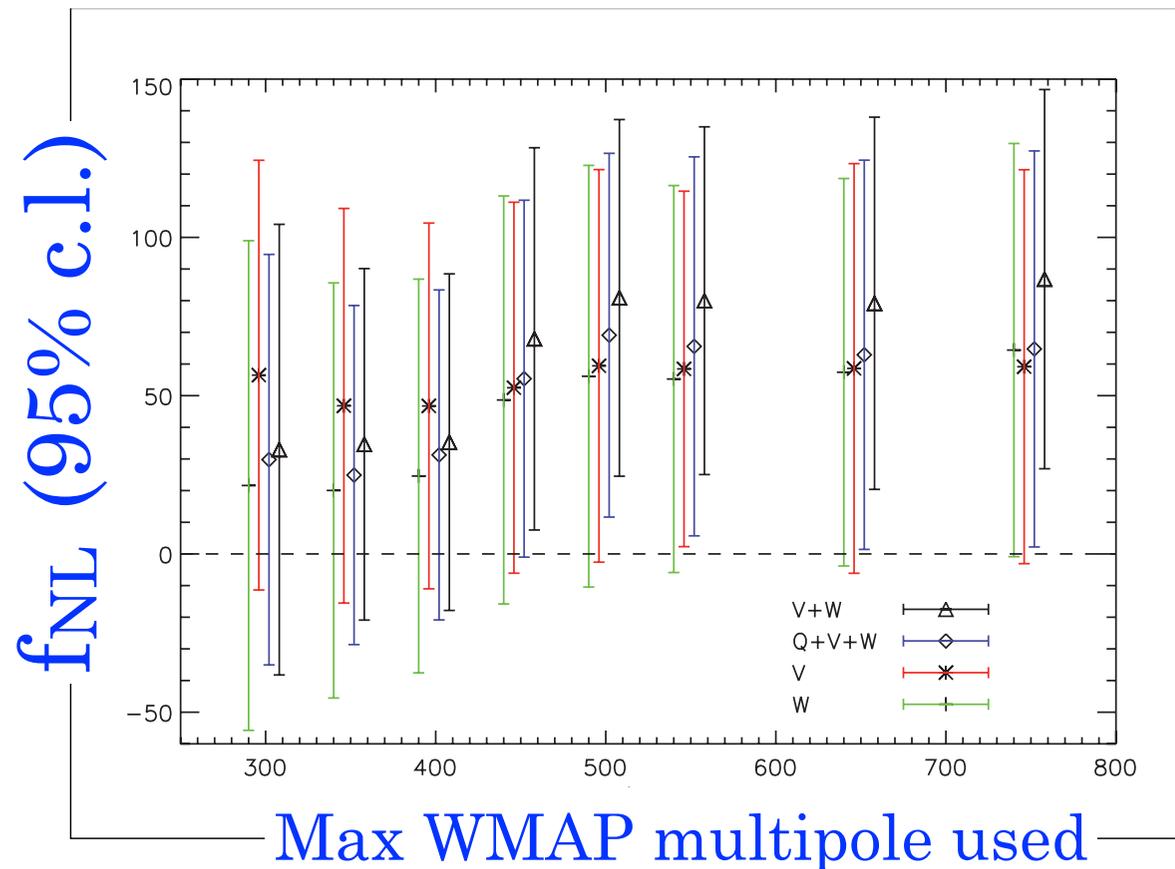
Current constraints from WMAP

Band	Foreground[b]	f_{NL}^{local}	f_{NL}^{equil}	f_{NL}^{orthog}	b_{src}
V+W	Raw	59 ± 21	33 ± 140	-199 ± 104	N/A
V+W	Clean	42 ± 21	29 ± 140	-198 ± 104	N/A
V+W	Marg.[c]	32 ± 21	26 ± 140	-202 ± 104	-0.08 ± 0.12
V	Marg.	43 ± 24	64 ± 150	-98 ± 115	0.32 ± 0.23
W	Marg.	39 ± 24	36 ± 154	-257 ± 117	-0.13 ± 0.19

Komatsu et al. 2010

Future: much better constraints expected, $\sigma(f_{NL}) < O(10)$ with Planck

Galaxy cluster counts' sensitivity to NG

$P(\delta\phi/\phi)$

$\delta\phi/\phi$
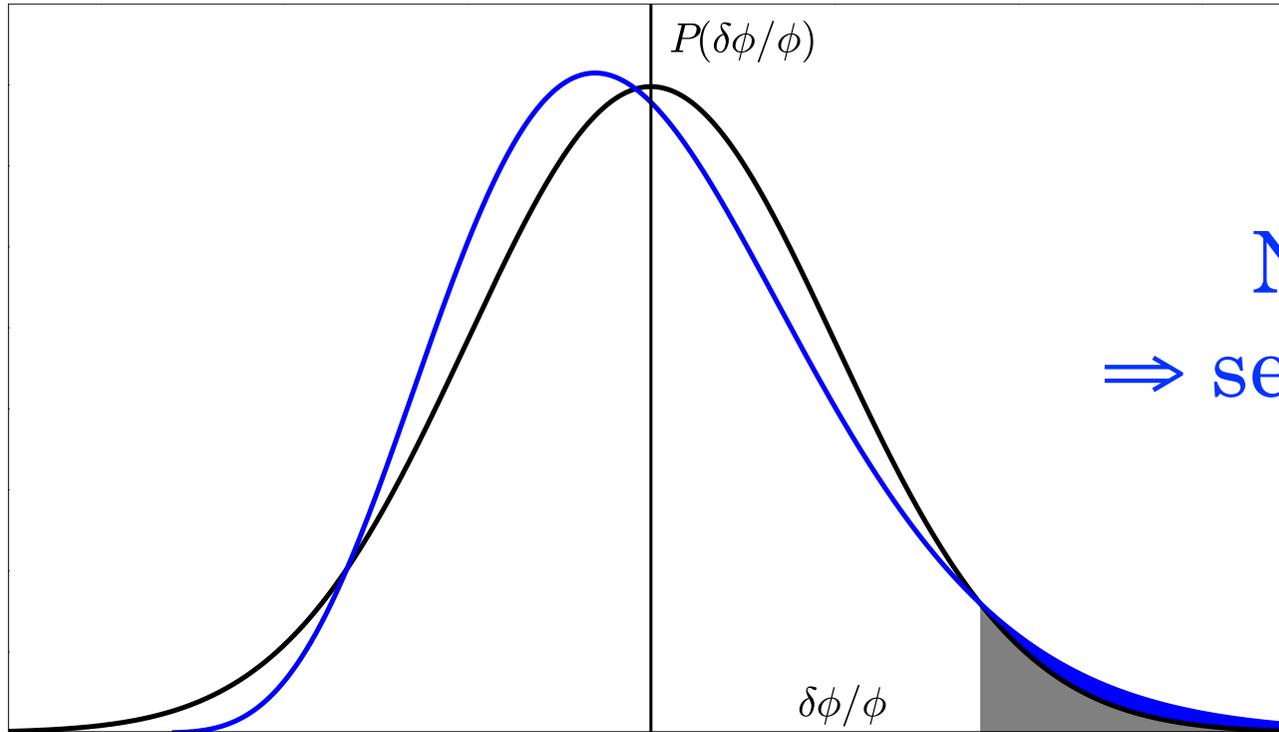
NG initial PDF
$\Rightarrow$ sensitivity to counts
"on the tail"

(amount of NG shown is >100× bigger than allowed by data!)

Lots of effort in the community to calibrate
the non-Gaussian mass function -
dn/dlnM(M, z) - of DM halos
(analytic extensions of Press-Schechter + simulations)

DM halo gets more massive with fNL>0 (and v.v.)

$f_{NL}=+5000$
$M=1.2 \ 10^{16} \ M_\odot$

$f_{NL}=+500$
$M=5.9 \ 10^{15} \ M_\odot$

Mapping between
M_G and $M \equiv M_{NG}$:

$f_{NL} = 500$

$M \ (h^{-1} \ M_{sun})$

$M_G \ (h^{-1} \ M_{sun})$
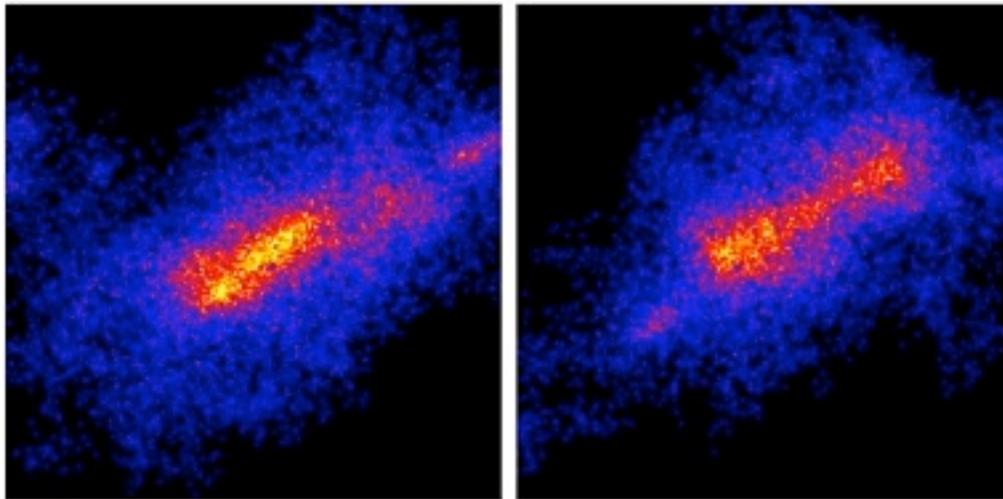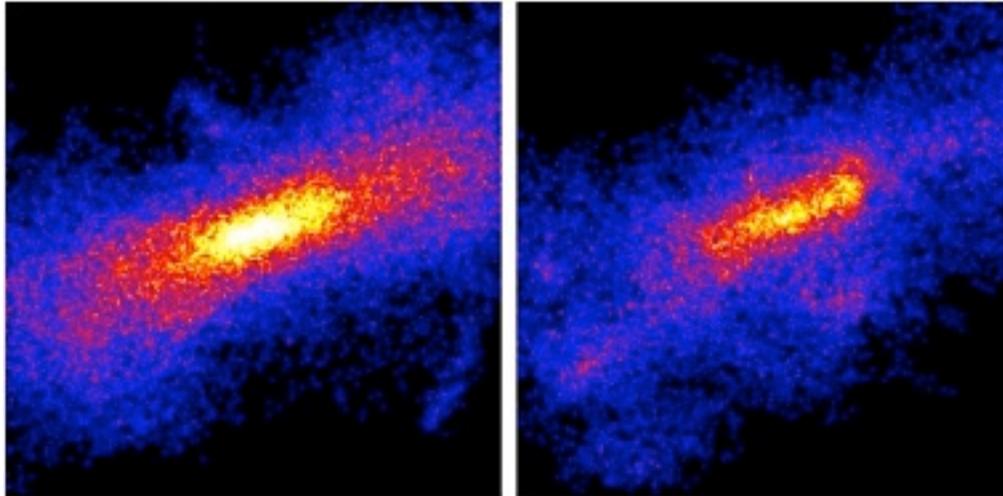
$f_{NL}=0$
$M=5.1 \ 10^{15} \ M_\odot$

$f_{NL}=-500$
$M=4.3 \ 10^{15} \ M_\odot$
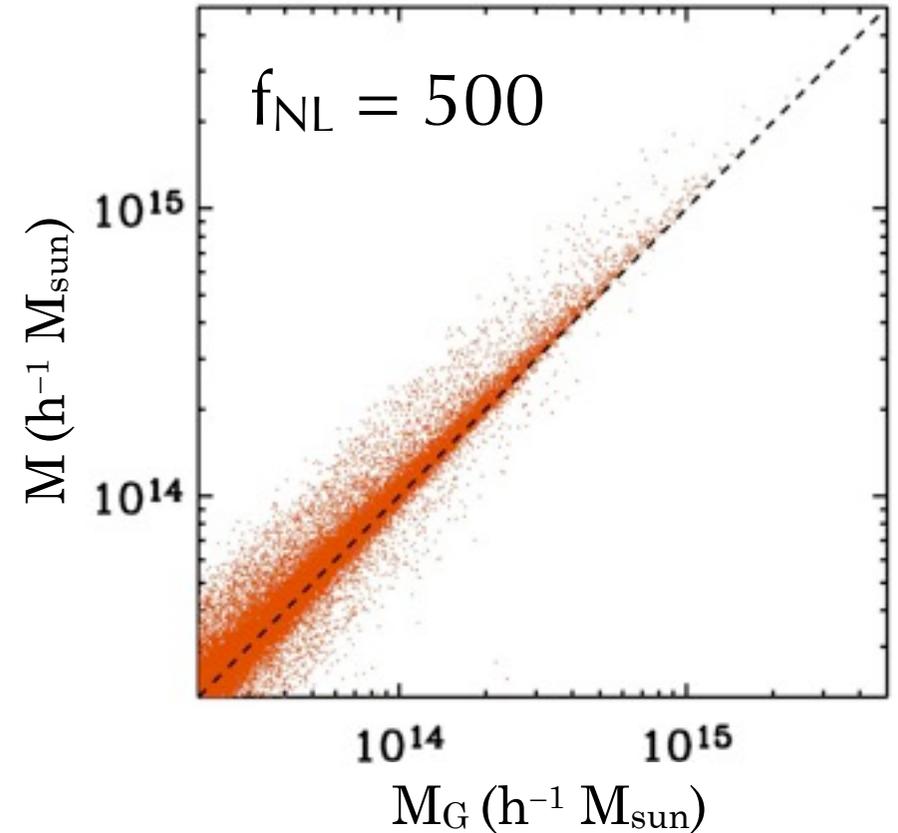
$\Rightarrow$ NG mass function:

$$\frac{dN}{dM} = \int \frac{dP(M|M_G)}{dM} \frac{dN}{dM_G} dM_G$$

Dalal, Doré, Huterer & Shirokov 2008
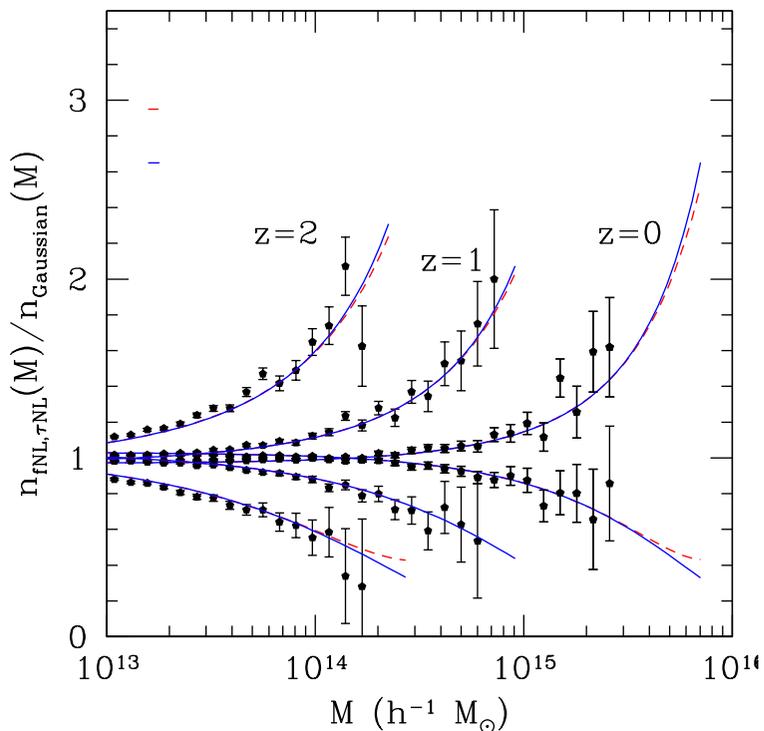
NG/Gaussian mass function ratios:
for fixed M, more sensitivity
at higher redshift

Smith & LoVerde 2011; Pillepich, Porciani and Hahn 2009;
many others going back to 1990s

Unfortunately, cluster counts are **weakly** sensitive to NG

e.g. Sefusatti et al. 2007 forecasted the depressing $\sigma(f_{NL})=145$ from SDSS
e.g. $\sigma(f_{NL})=450$ measured from SPT (Williamson et al 2010)

Nevertheless, it is true that a (large) amount of (local model) NG can boost the number of 'pink elephant' clusters

High-z, high-M - "pink elephant" - clusters of galaxies

- SPT-CL J0546-5045: z=1.067, M≈(8.0±1.0)·10^{14} M$_{sun}$
- XMMU J2235.3-2557: z=1.39, M≈(8.5±1.7)·10^{14} M$_{sun}$
- SPT-CL J2106-8544: z=1.132, M≈(1.3±0.2)·10^{15} M$_{sun}$

Some authors have claimed the existence of these clusters is in conflict with LCDM, but can be explained with (huge; f$_{NL}$~500) **non-Gaussianity**

Hoyle, Jimenez & Verde (2011);
Cayon, Gordon & Silk (2011);
Holz & Perlmutter 2011
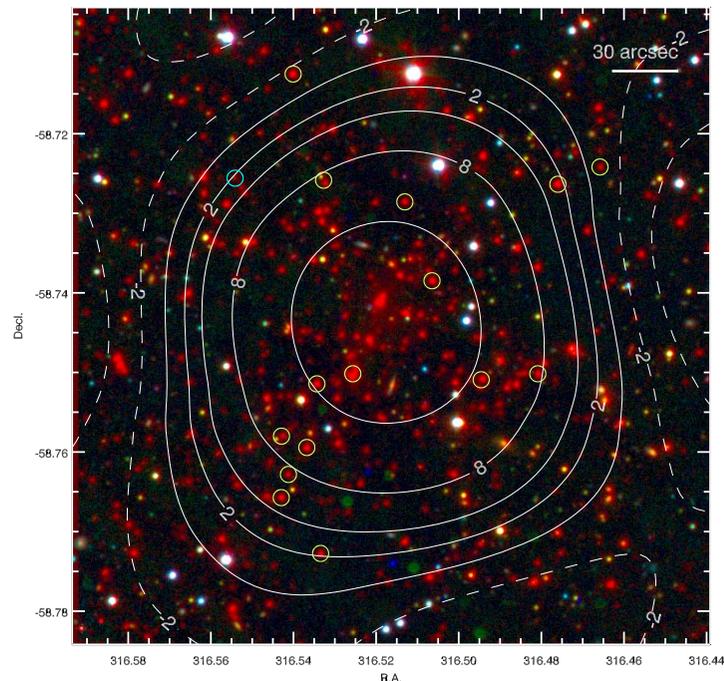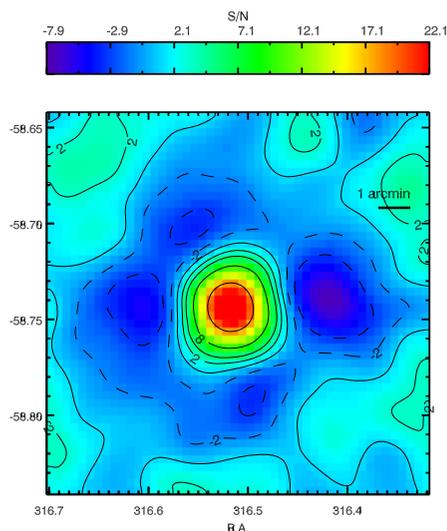
Are the pink elephants in conflict with LCDM?!
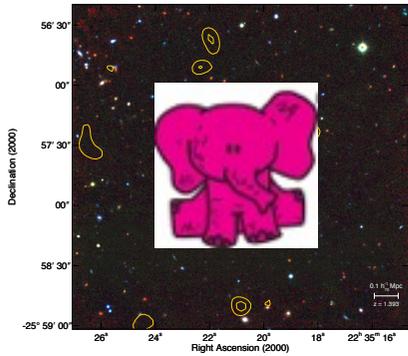
4 things to account for:

1. **Sample variance** - the Poisson noise in counting rare objects in a finite volume

2. **Parameter variance** - uncertainty due to fact that current data allow cosmological parameters to take a range of values

3. **Eddington bias** - mass measurement error will preferentially 'scatter' the cluster into higher mass

4. **Survey sky coverage** - needs to be fairly assessed

N.B. If a cluster rules out LCDM, it will rule out quintessence too!

No conflict - for now.

M_{200} (10^{15} M_{sol} h_{70}^{-1}) vs Redshift

Full sky, $p = 95\%$
2500 deg^2, $p = 95\%$
2500 deg^2, $p = 32\%$

J0658
J2248
J0102
J0615
J2106
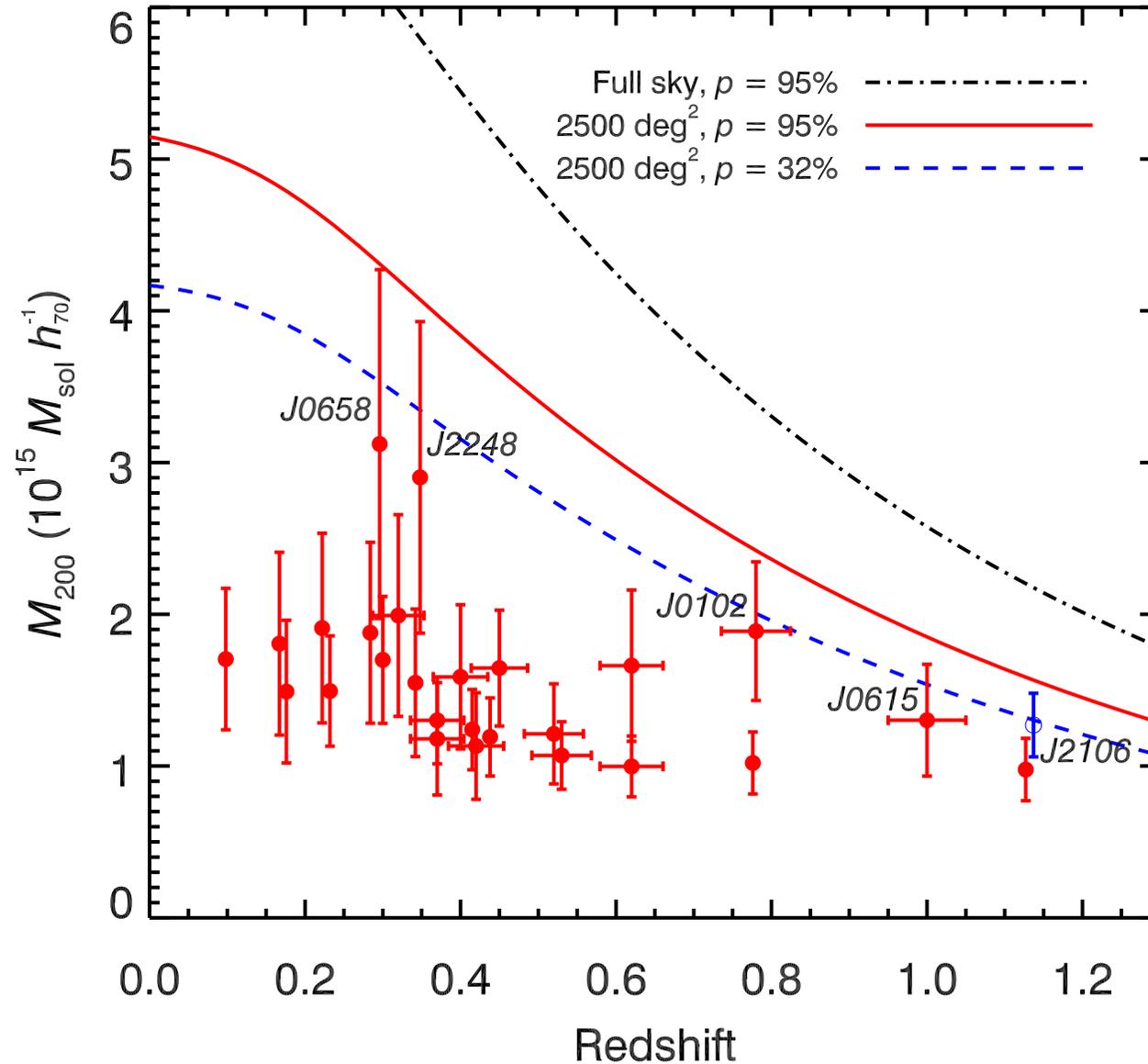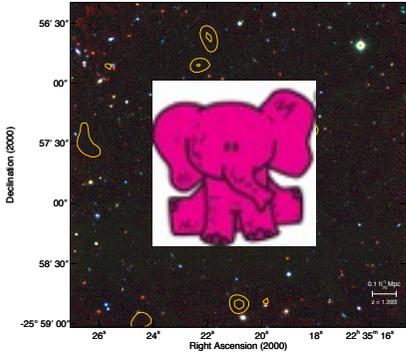
Foley et al 2011 arXiv:1101.1286 (SPT team);
Mortonson, Hu & Huterer 2011

Effects of primordial NG
on the bias of virialized objects

Simulations with non-Gaussianity (f_{NL})

f_{NL}= -5000

f_{NL}= -500

f_{NL}= 0

f_{NL}= +500

f_{NL}= +5000
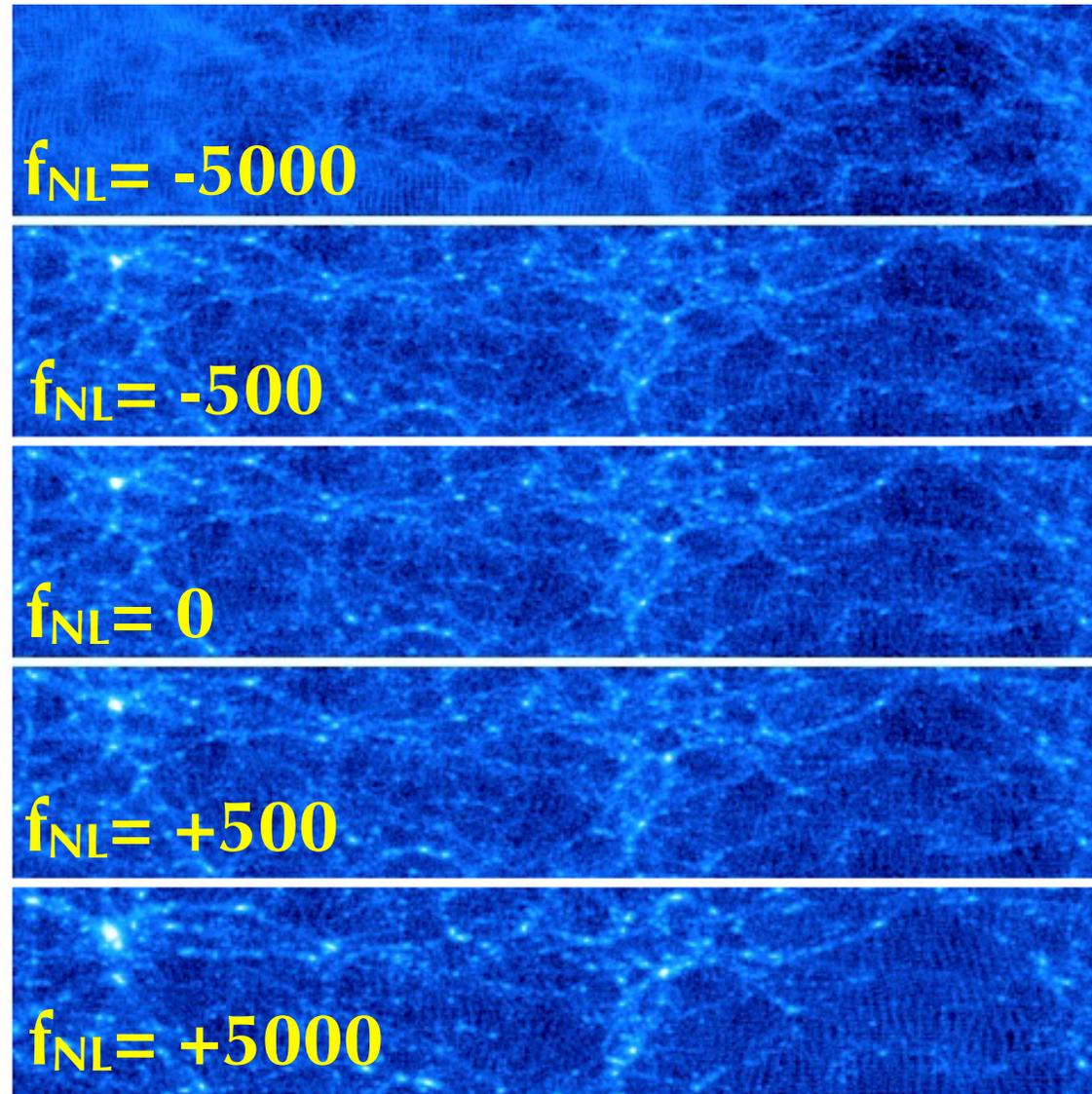
80 Mpc/h

375 Mpc/h

- Under-dense region evolution decrease with f_{NL}

- Over-dense region evolution increase with f_{NL}

- Same initial conditions, different f_{NL}
- Slice through a box in a simulation N_{part}=512^3, L=800 Mpc/h

Dalal, Doré, Huterer & Shirokov 2008

Does galaxy/halo bias depend on NG?

$$\text{bias} \equiv \frac{\text{clustering of galaxies}}{\text{clustering of dark matter}} = \frac{\left(\frac{\delta\rho}{\rho}\right)_{\text{halos}}}{\left(\frac{\delta\rho}{\rho}\right)_{\text{DM}}}$$

cosmologists measure

usually nuisance parameter(s)

theory predicts

$D \leq 4$

× $P(b) = 1$

● $P(b) = 10^{0.3(1-\text{csc}b)}$

$300\, r^{-1.8}$

$\xi_{gg} = 20 r^{-1.8}$
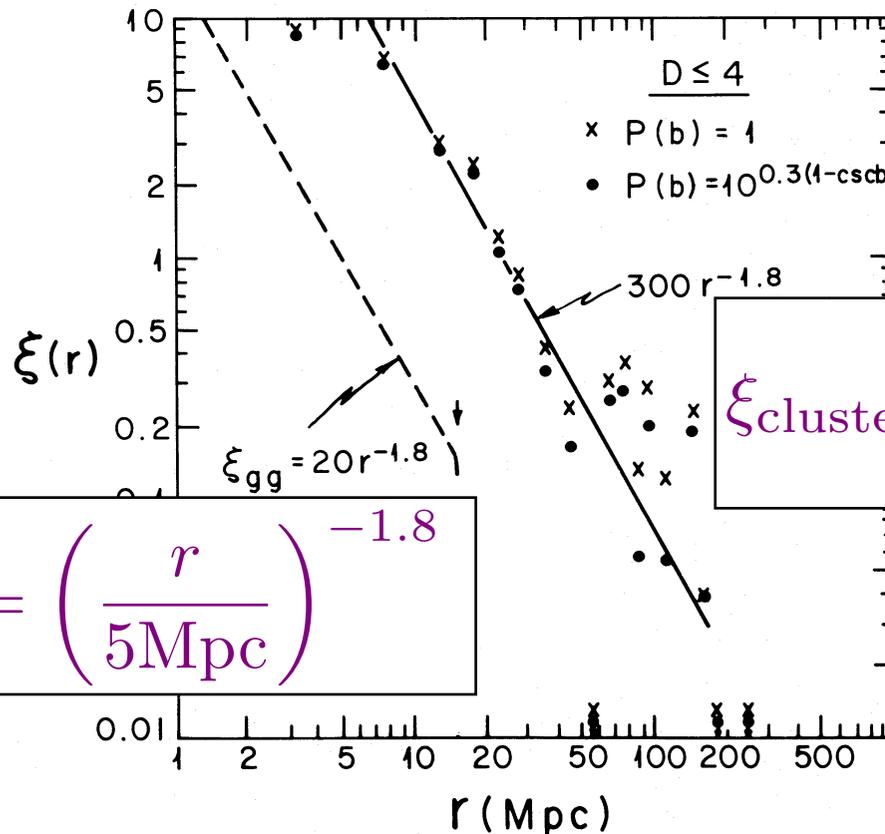
$$\xi_{\text{clusters}}(r) = \left(\frac{r}{25\text{Mpc}}\right)^{-1.8}$$

$$\xi_{\text{galaxies}}(r) = \left(\frac{r}{5\text{Mpc}}\right)^{-1.8}$$

Bahcall & Soneira 1983

Bias of dark matter halos

$$P_h(k, z) = \boxed{b^2(k, z)} \, P_{\mathrm{DM}}(k, z)$$

Enhanced "Peaks"

δ_c

Large Scale "Background"

x

figure credit: Wayne Hu
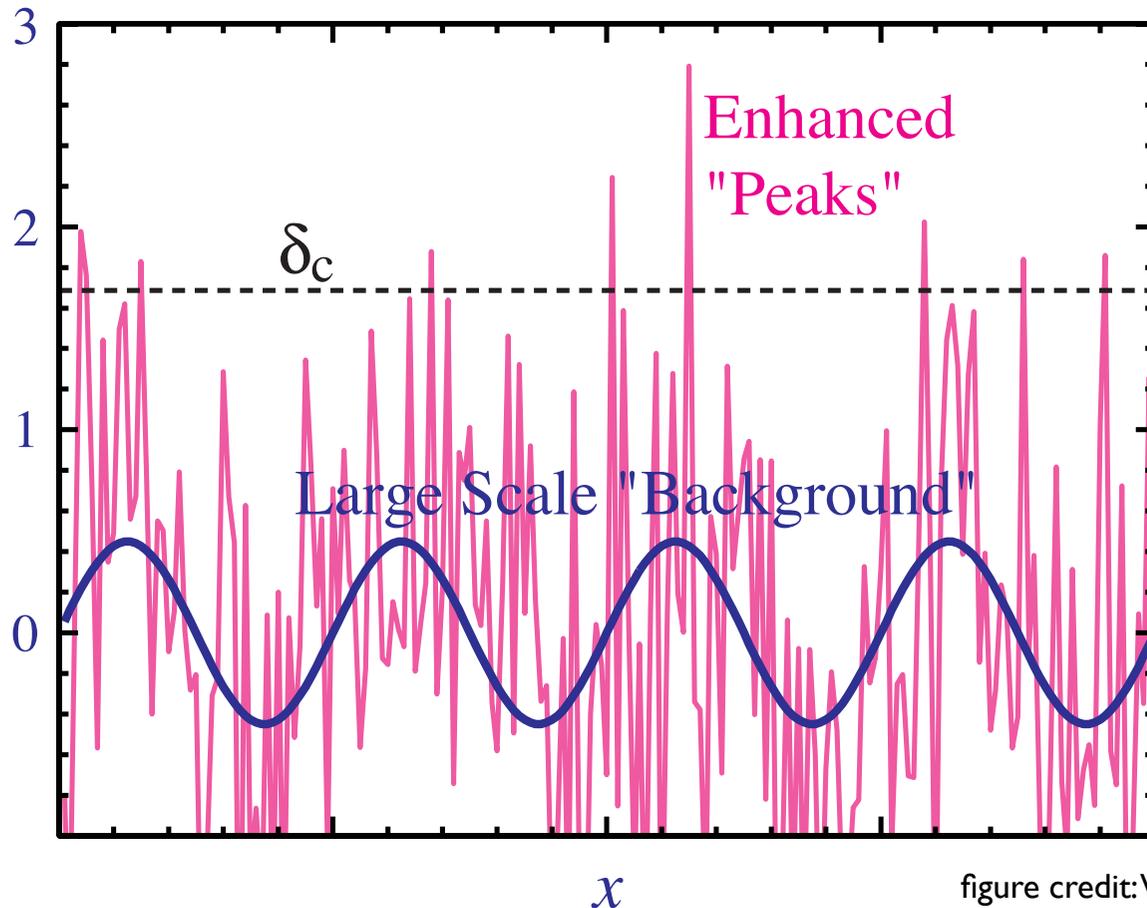
Simulations and theory both say: large-scale bias is scale-independent
(theorem if halo abundance is function of local density
and if the short and long modes are uncorrelated)

Scale dependence of NG halo bias
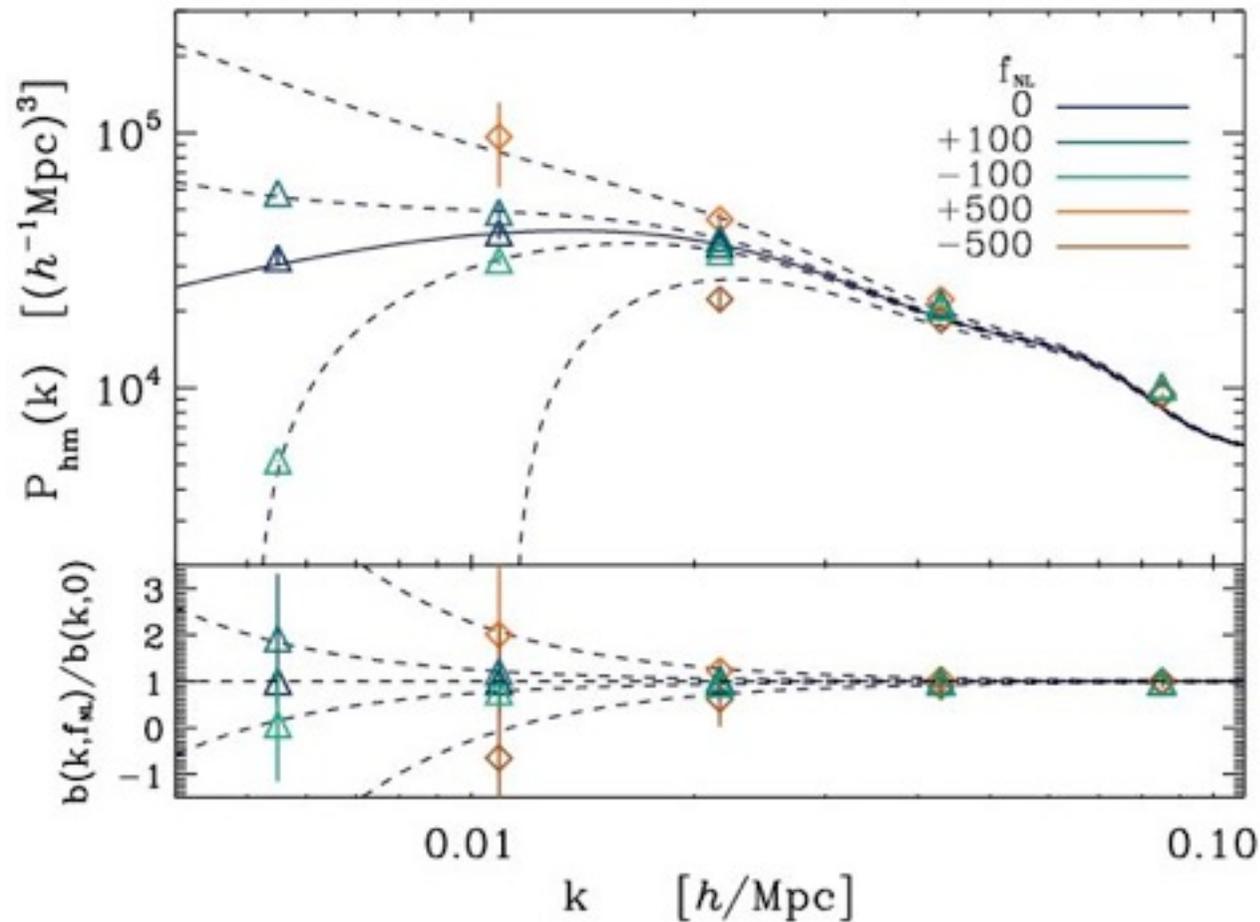
$$b(k) = b_{\mathrm{G}} + f_{\mathrm{NL}} \frac{\text{const}}{k^2}$$

Verified using a variety of theoretical
derivations and numerical simulations.

Dalal, Doré, Huterer & Shirokov 2008

$$\Delta b(k) = f_{\mathrm{NL}}(b_G - 1)\,\delta_c\,\frac{3\,\Omega_M H_0^2}{T(k)D(a)k^2}$$

Implications:

▸ Unique $1/k^2$ scaling of bias; no free parameters

▸ Distinct from effect of all other cosmo parameters

▸ Straightforwardly measured (clustering of any type of halo autocorrelation, cross-correlation with CMB,...

▸ Derived theoretically several different ways

▸ Extensively tested with numerical simulations; good agreement found

Dalal et al.; Matarrese & Verde; Slosar et al; Afshordi & Tolley; Desjacques et al; Giannantonio & Porciani; Grossi et al; McDonald; ....

Constraints from **current** data: SDSS
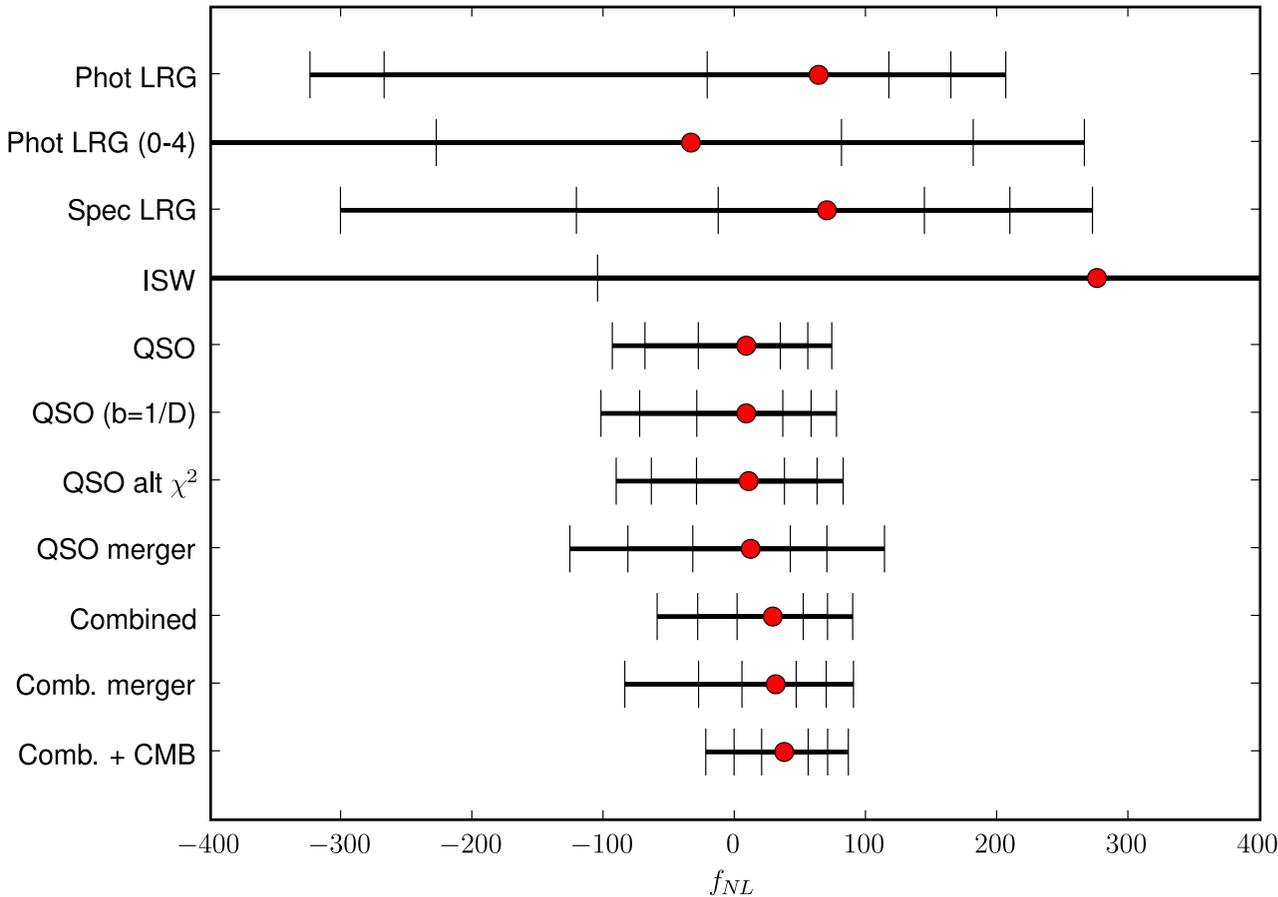
f_{NL} = 8 +/- 30 (68%, QSO)

f_{NL} = 23 +/- 23 (68%, all)

Slosar et al. 2008
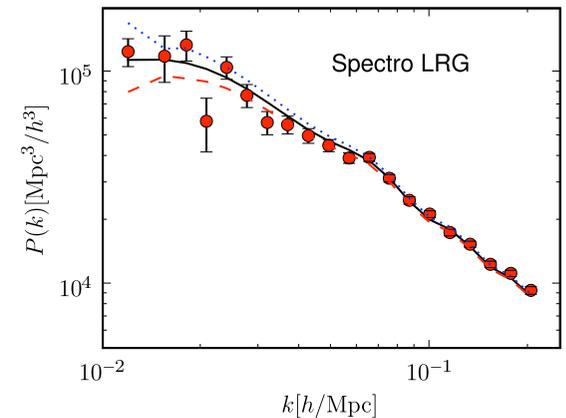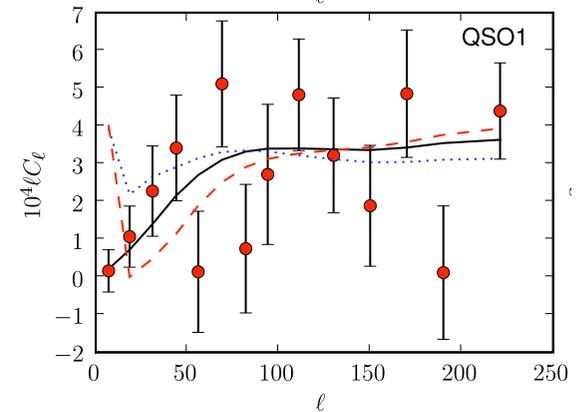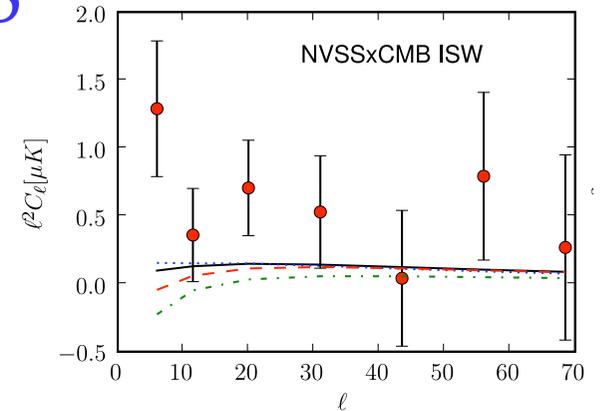
Future data forecasts for LSS: $\sigma(f_{NL}) \approx O(\text{few})$
(at least?) as good as, and highly complementary, to Planck CMB

Nongaussianity form clustering of galaxy clusters

NG can survive marginalization over numerous systematic effects
e.g:
- relation of mass of cluster and its observable quantity (T, flux, etc)
- redshift evolution of bias

Dark Energy Survey cluster forecasts

Nuisance parameters		Counts			Covariance			Counts + Covariance		
				Marginalized errors—Full Covariance						
Halo bias	M_{obs}	$\sigma(\Omega_{DE})$	$\sigma(w)$	$\sigma(f_{NL})$	$\sigma(\Omega_{DE})$	$\sigma(w)$	$\sigma(f_{NL})$	$\sigma(\Omega_{DE})$	$\sigma(w)$	$\sigma(f_{NL})$
Marginalized	Marginalized	∞	∞	∞	∞	∞	∞	0.069	0.23	**6.0**
Known	Marginalized	0.097	0.33	$\mathbf{2.1 \times 10^3}$	0.13	0.43	**12**	0.065	0.22	**5.4**
Marginalized	Known	∞	∞	∞	0.099	0.34	**7.0**	0.0036	0.014	**3.8**
Known	Known	0.0051	0.023	**94**	0.042	0.13	**5.1**	0.0036	0.014	**1.8**

Counts mainly probe DE parameters
Covariance mainly probes f_{NL}

Cunha, Huterer & Doré 2010

P(k), and the bispectrum, are also affected by NG on smaller scales

Power spectrum

Bispectrum

$Ratio\ P(f_{NL} = 100)\ /\ P(f_{NL} = 0)$

$Ratio\ B(f_{NL} = 100)\ /\ B(f_{NL} = 0)$

Total

mass function

halo profile
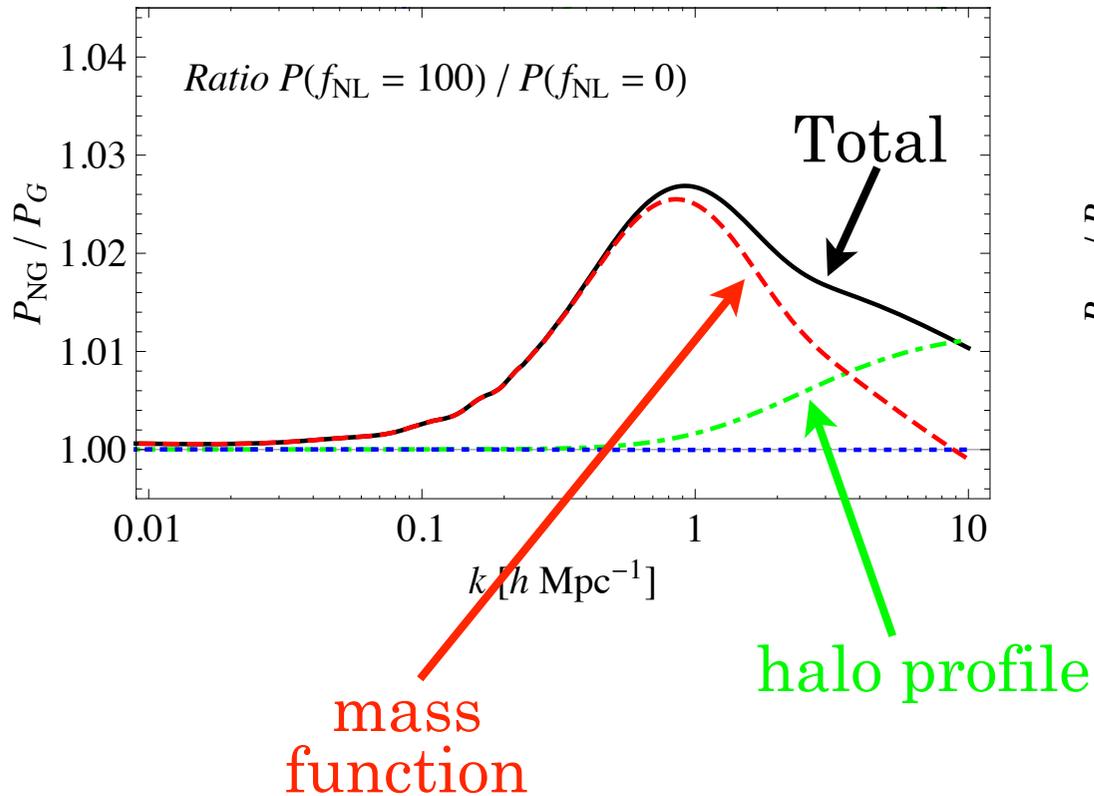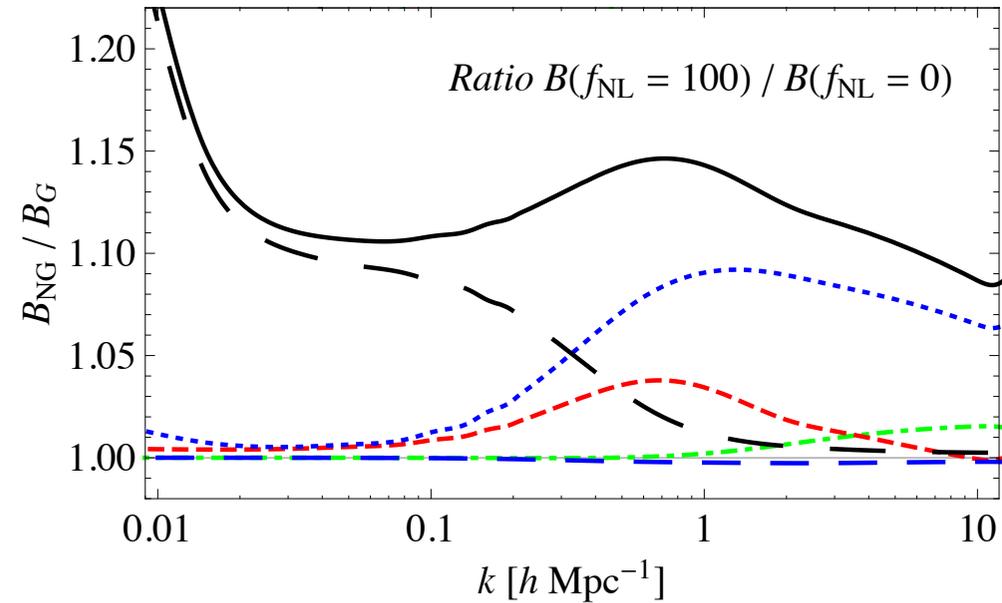
More general NG models: beyond f_{NL}

Scale-dependent nongaussianity?
Generalized local ansatz

- Motivated by multi-field inflationary models
- In general, even if you are considering standard single-field inflation, interactions may lead to scale-dependence of f_{NL}

(Usual) local model...

$$\Phi(x) = \phi_G(x) + f_{\mathrm{NL}} \left[\phi_G^2(x) - \langle \phi_G^2 \rangle \right]$$

...we generalize to a scale dependent (non-local) model

$$\Phi(x) = \phi_G(x) + f_{\mathrm{NL}}(x) * \left[\phi_G^2(x) - \langle \phi_G^2 \rangle \right]$$

$$\Phi(k) = \phi_G(k) + f_{\mathrm{NL}}(k) \int \frac{d^3 k'}{(2\pi)^3} \phi_G(k') \phi_G(k - k')$$

A complete basis for $f_{NL}(k)$: piecewise-constant bins
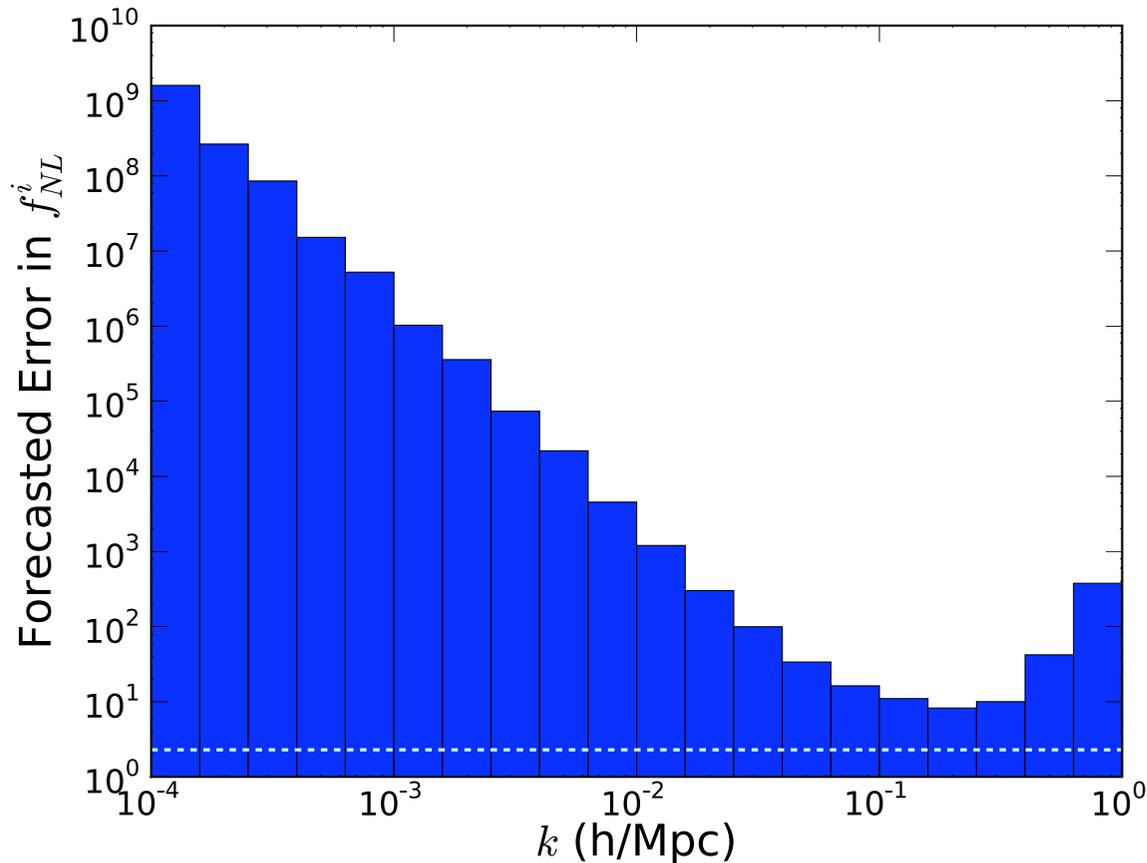
Measurement forecasts
from
DES-type survey

Given this basis, projecting forecasts onto any
parametrized $f_{NL}(k)$ model is now trivial

Warning, however: theoretical predictions are uncertain and
(always) have to be checked with simulations first

Becker, Huterer & Kadota, 2011, 2012

Future: using LSS to probe scale-dependent NG

▸ Scale-dep NG models are motivated by particle theory (single-field inflation with self-interaction; mixed curvaton-inflaton models)

▸ Effects on LSS are significant, but theory predictions are uncertain ⇒ ongoing theoretical and simulation work

▸ Understanding of astrophysics (of DM halos, etc) required in order to probe fundamental physics

Scale-dependent NG ansatz:

$$f_{\mathrm{NL}}(k) = f_{\mathrm{NL}}^{*} \left(\frac{k}{k_*} \right)^{n_{f_{\mathrm{NL}}}}$$

Halos of mass M probe NG on scale k~M$^{-1/3}$

$\sim 10^{15}\ h^{-1} M_{\mathrm{sun}}$

$\sim 10^{13.5}\ h^{-1} M_{\mathrm{sun}}$

DES

LSST

$\sigma[\ f_{\mathrm{NL}}(k)\ ]$

k (h Mpc^{-1})

Shandera, Dalal & Huterer, 2011
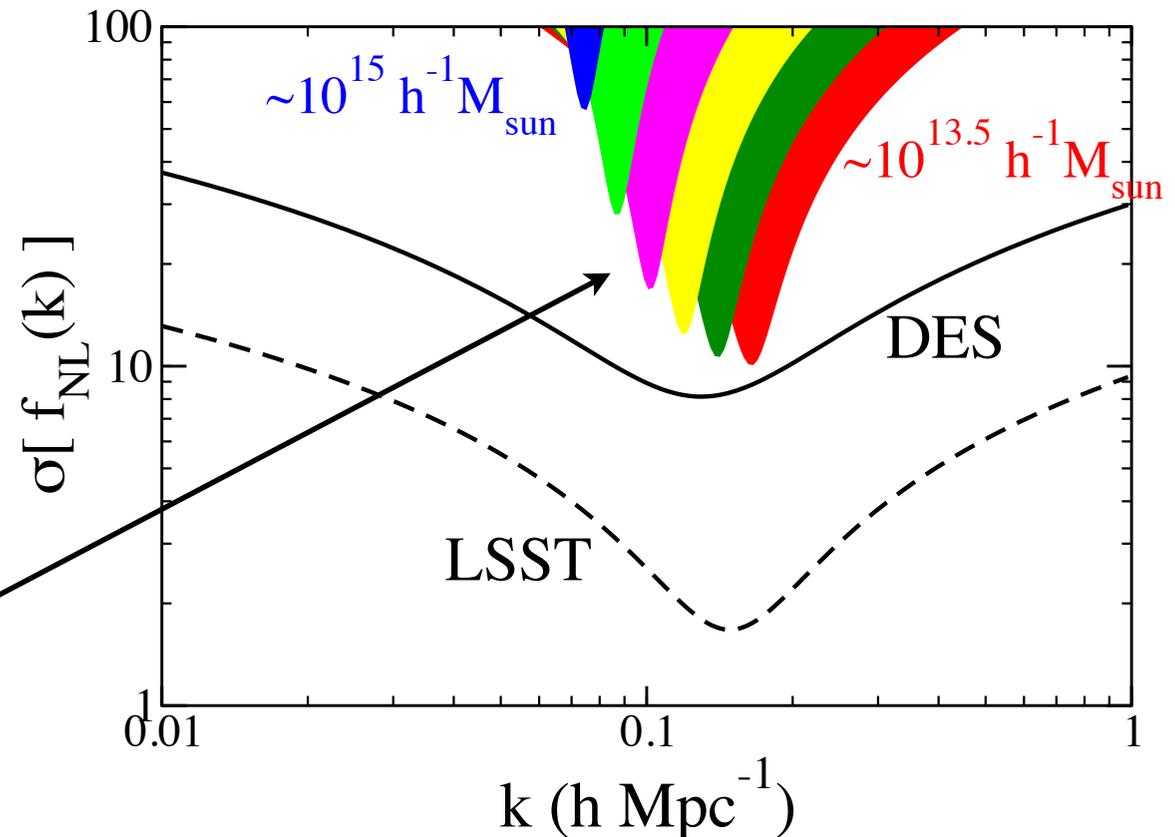
CMB, LSS, and CMB+LSS **forecasts**

$$f_{\mathrm{NL}}(k) = f_{\mathrm{NL}}^* \left(\frac{k}{k_*} \right)^{n_{f_{\mathrm{NL}}}}$$

DES + cosmological prior

Planck bispectrum

DES + Planck bispectrum

Fiducial $f_{\mathrm{NL}}(k) = 30$

LSS k_{max} $(z=0)$

$f_{\mathrm{NL}}(k)$

k (h/Mpc)

Becker, Huterer & Kadota, arXiv:1206.6165
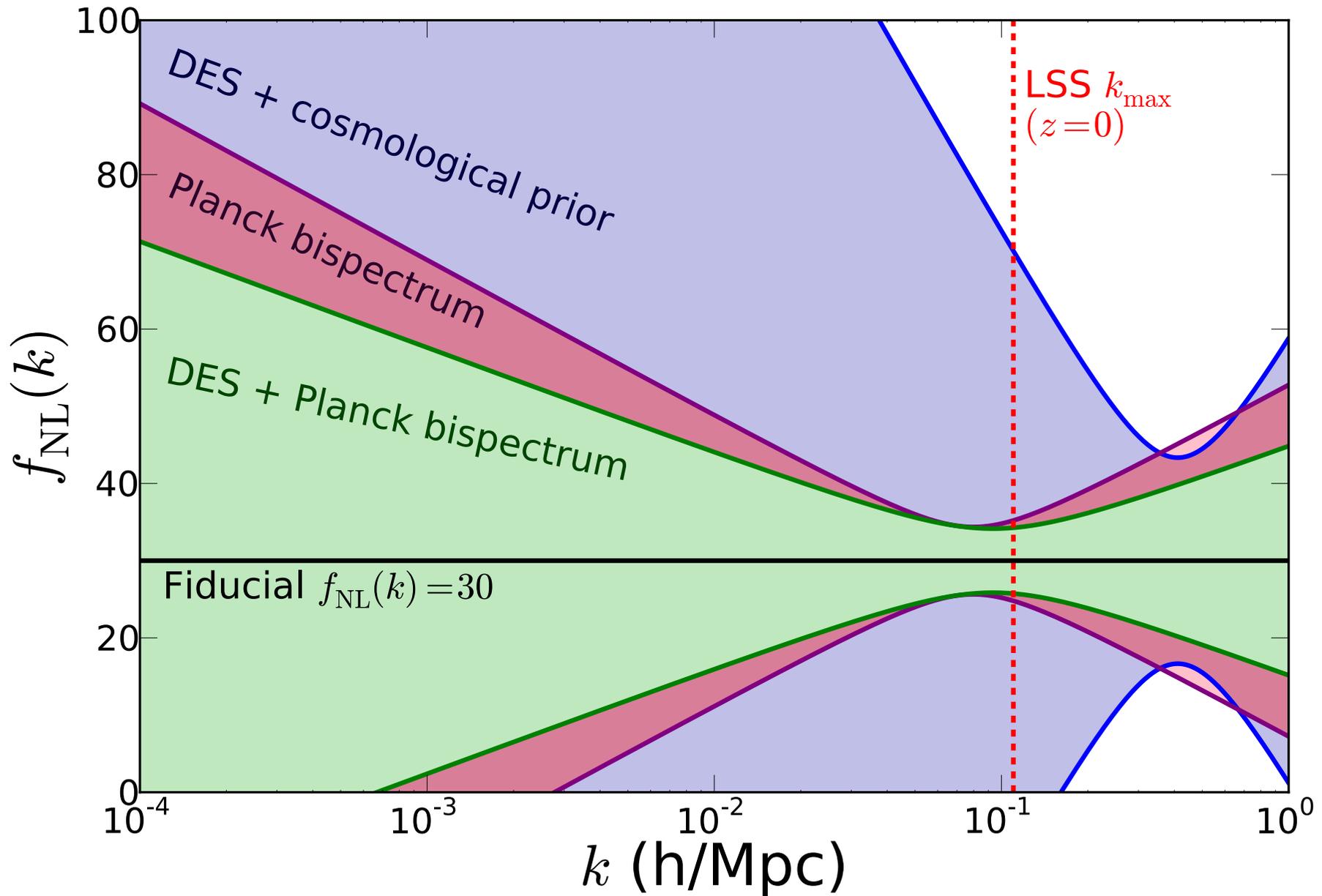
First constraints on the running of NG

WMAP7 data, modified KSW estimator
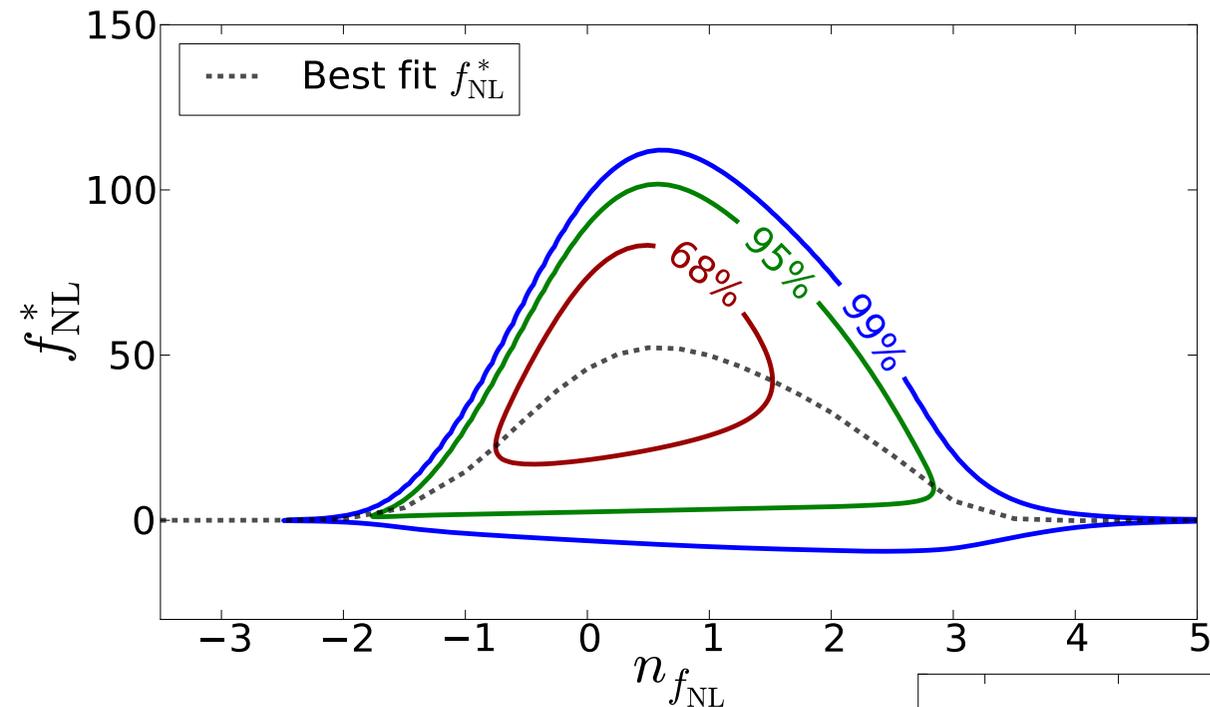
$$f_{\mathrm{NL}}(k) = f_{\mathrm{NL}}^* \left(\frac{k}{k_*} \right)^{n_{f_{\mathrm{NL}}}}$$

$$n_{f_{\mathrm{NL}}} = 0.3^{+1.9}_{-1.2}$$

at 95% CL

Becker & Huterer, arXiv:1207.5788
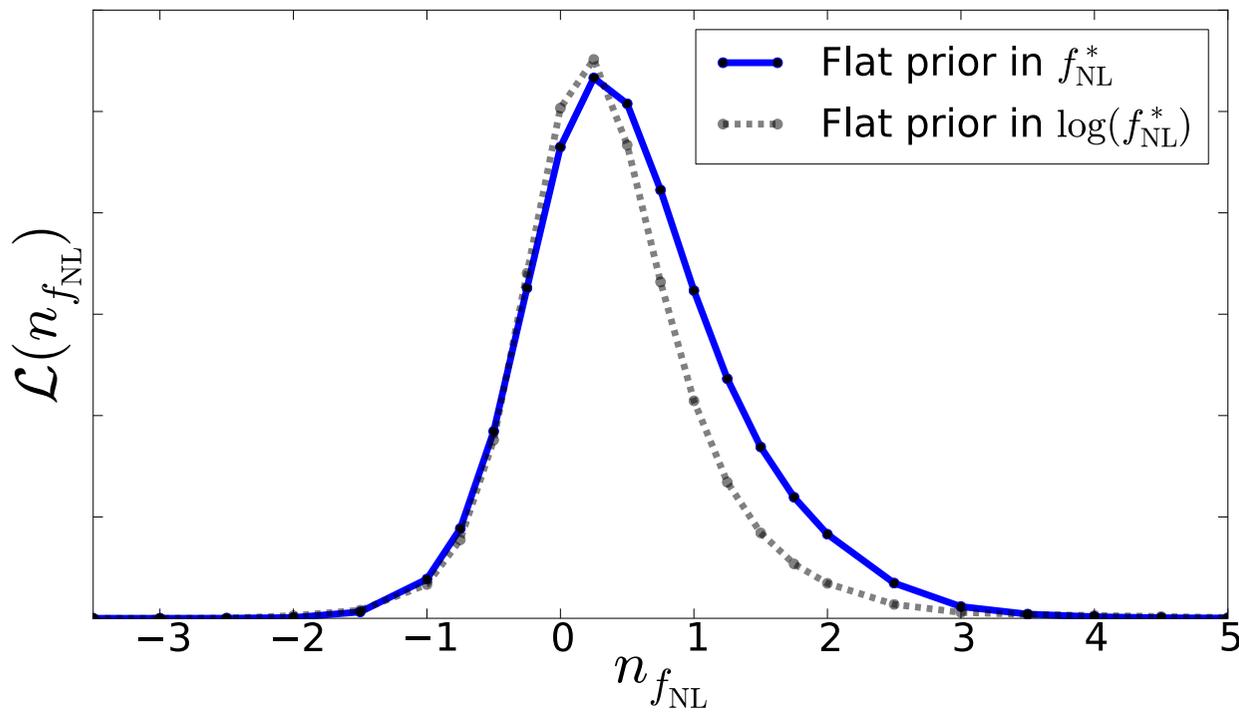
Forecasts for f$_{\mathrm{NL}}$(k)

Projected errors $\sigma(f^*_{\mathrm{NL}})$ and $\sigma(n_{f_{\mathrm{NL}}})$, and the corresponding pivots

Variable	BigBOSS	BigBOSS+Planck C_ℓs	Planck bispec	**BigBOSS+all Planck**
$\sigma(f^*_{\mathrm{NL}})$	3.0	2.6	4.4	2.2
$\sigma(n_{f_{\mathrm{NL}}})$	0.12	0.11	0.29	0.078
$\mathrm{FoM}^{(\mathrm{NG})}$	2.7	3.4	0.78	5.8
k_{piv}	0.33	0.35	0.080	0.24
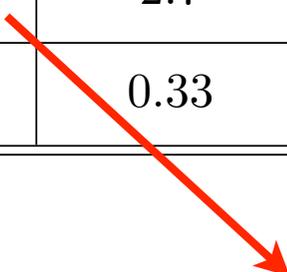
area in f$_{\mathrm{NL}}$*-n$_{\mathrm{fNL}}$ plane

NB: The LSS forecasts are very uncertain,
much more so than the CMB

Expansion of arbitrary bispectrum into modes

$$B(k_1, k_2, k_3) = \sum_{p,r,s} \alpha_{prs}\, q_p(k_1) q_r(k_2) q_s(k_3)$$
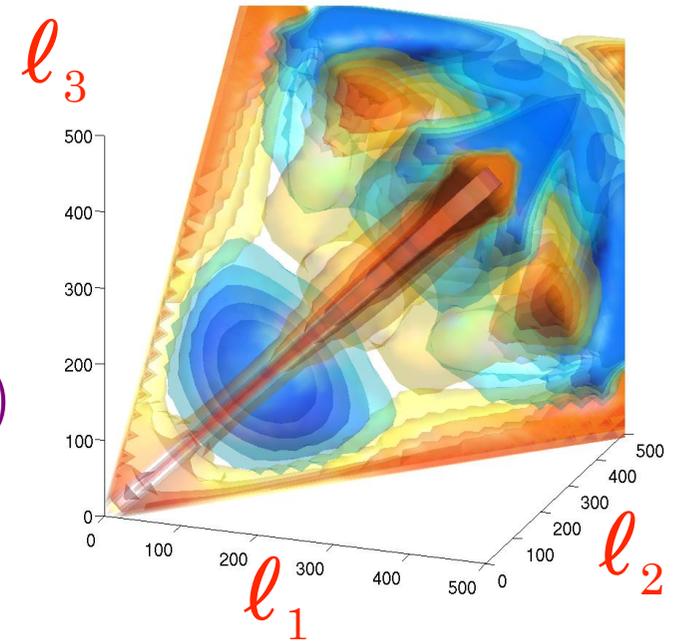
Amplitude from WMAP

Mode number

WMAP Modes
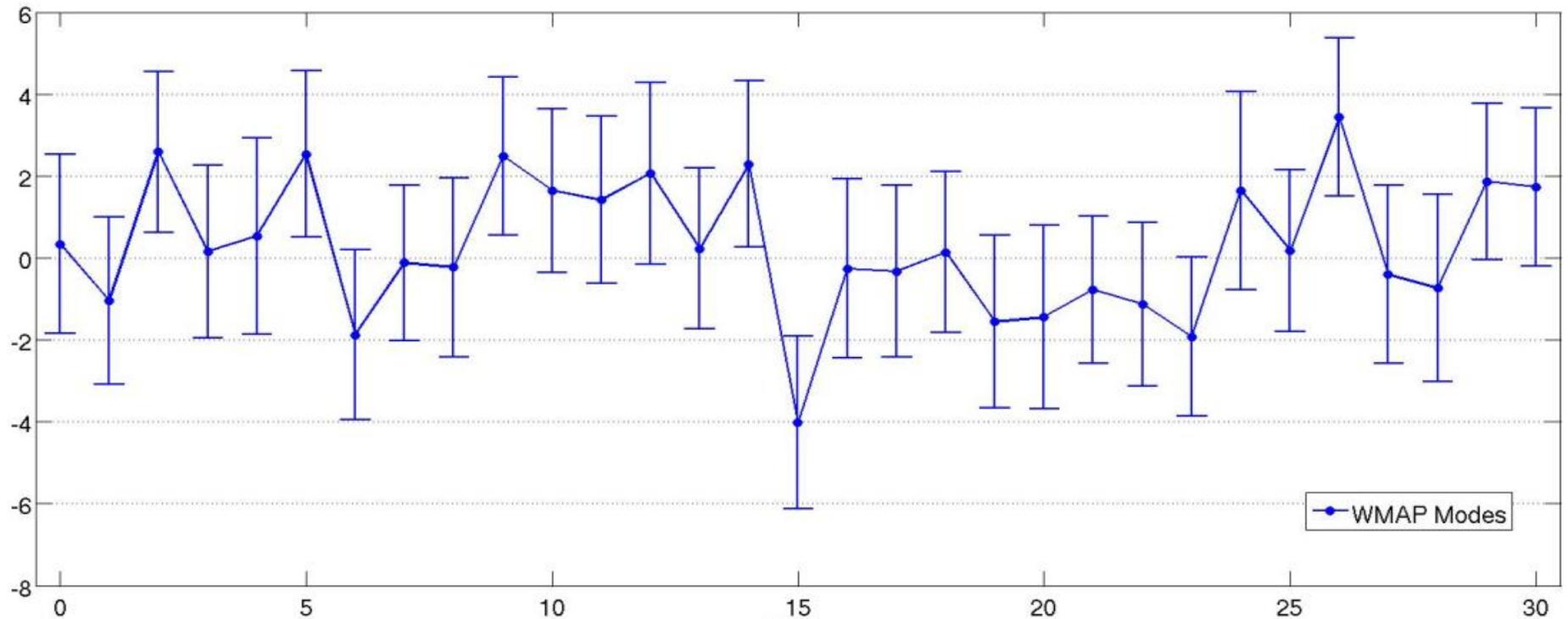
Dark Energy
Survey (2012)

Harvard-Cfa survey (1980s)

21cm mapp

First CfA Strip

26.5 ≤ δ < 32.5

m_g ≤ 15.5

Copyright 1998
Smithsonian
Astrophysical Observatory
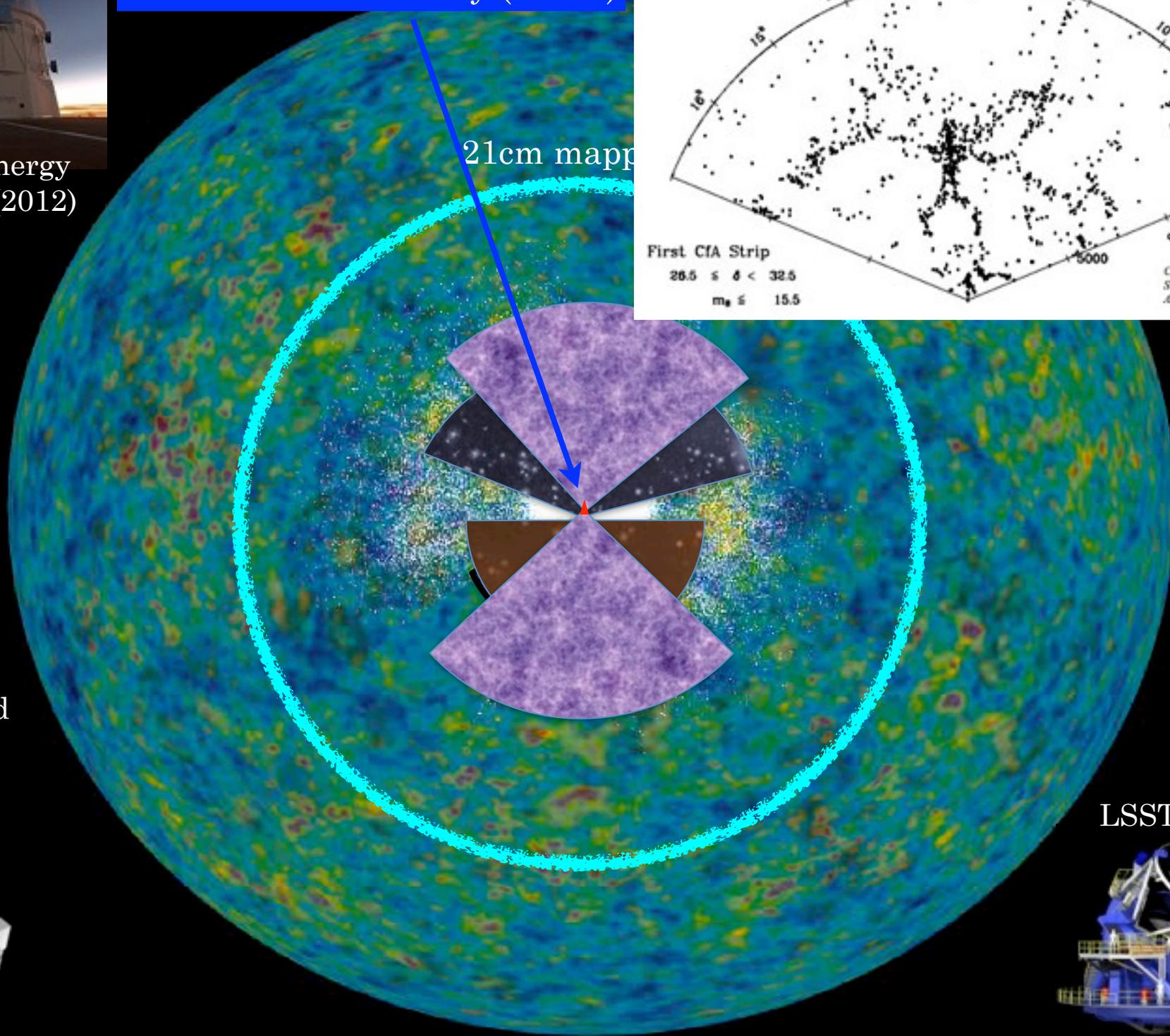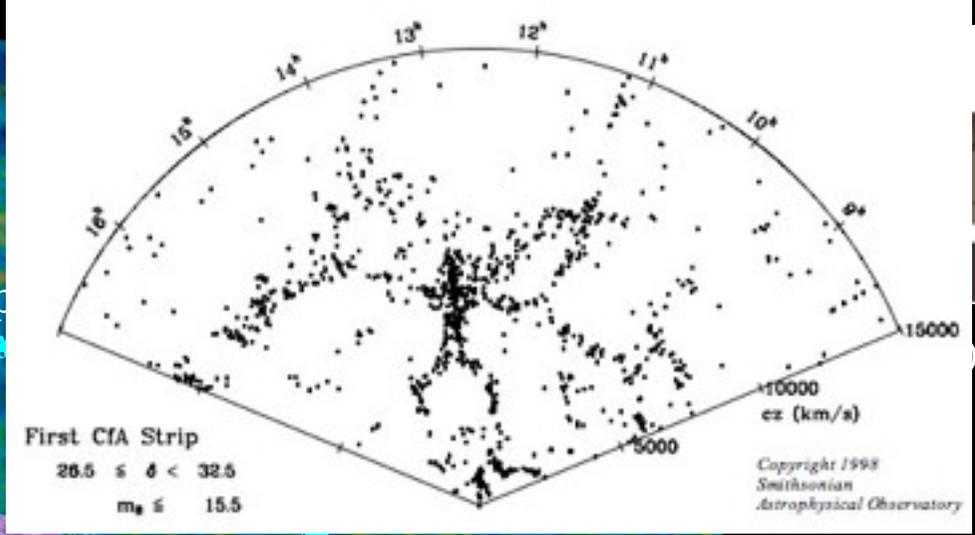
Euclid and
WFIRST
(~202X)
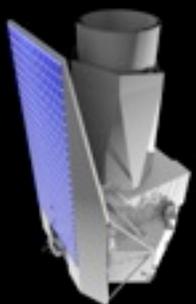
LSST (~2018)
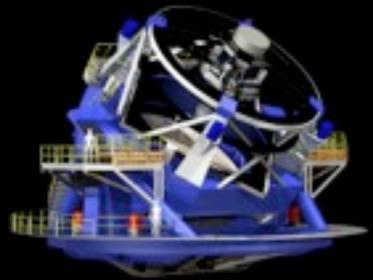
Advances in Astronomy special issue on

"Testing the Gaussianity and Statistical Isotropy of the Universe"

http://www.hindawi.com/journals/aa/2010/si.gsiu/

15 review articles (all also on arXiv)

Testing the Gaussianity and Statistical Isotropy of the Universe

Guest Editors: Dragan Huterer, Eiichiro Komatsu, and Sarah Shandera
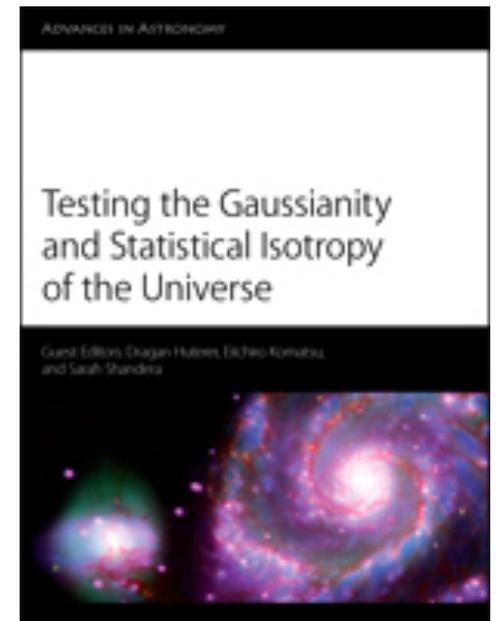
Challenges for NG program
... and approximate current status

- Motivate simple and more complicated NG models
  ✓ (single-field, multiple fields, self-interactions)

- Utilize a variety of observables in LSS and CMB to get at NG ✓

- Develop fast, near-optimal estimators to extract NG from the CMB ✓

- Develop theory to relate NG models to LSS observables ✓✗ (messy; still need to check with sims)

- Develop theory to use LSS info from quasi-linear scales ($k \gtrsim 0.1 \ h^{-1}$ Mpc) ✓✗

- Use LSS bispectrum to get at primordial NG ✗

- Control the systematic errors, esp large-scale LSS ✓✗