An attempt to clarify the neutrino abundance and mass impact on CMB and P(k)

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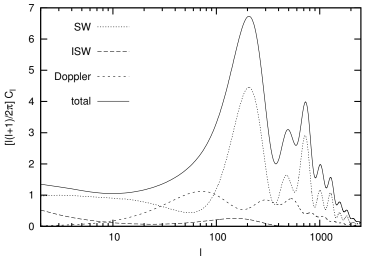
EPFL & CERN

Benasque, 16.08.2012

1. CMB
   - CMB parameter dependence
   - Impact of $N_{\rm eff}$
   - Impact of M_ν

2. Matter power spectrum
   - $P(k)$ parameter dependance
   - Impact of $N_{\rm eff}$
   - Impact of M_ν
   - Impact of mass splitting

In neutrinoless ΛCDM model, C_l^{TT} controlled by 8 effects/quantitites:



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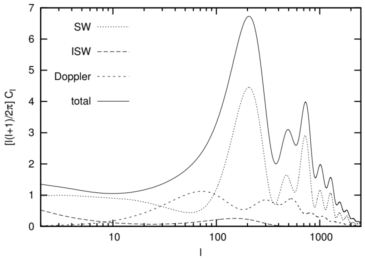

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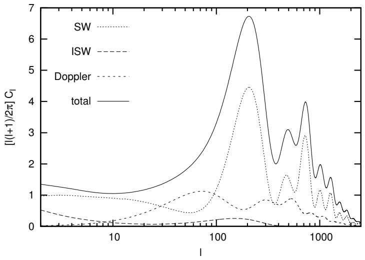

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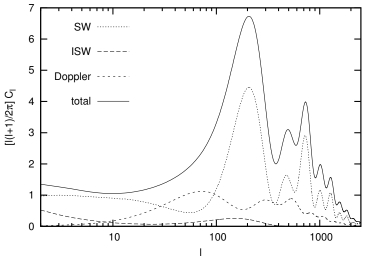

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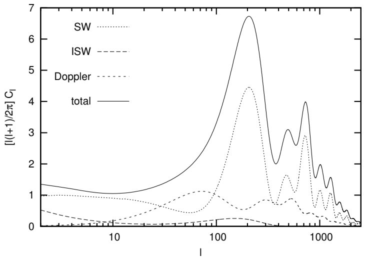

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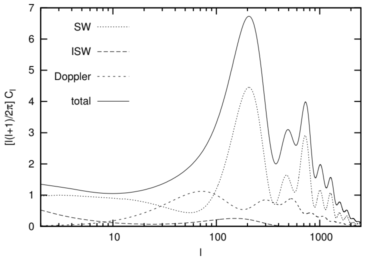

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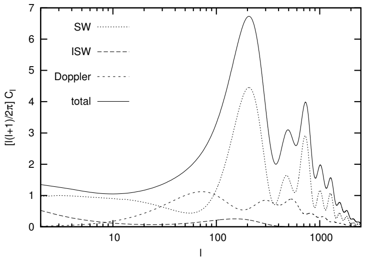

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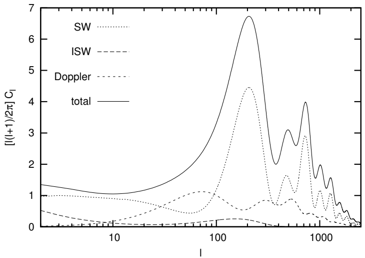

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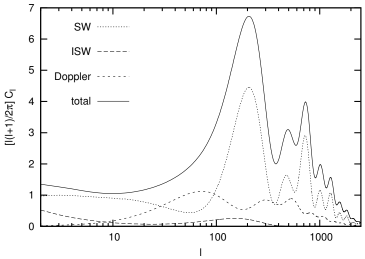

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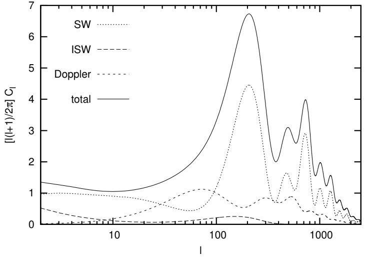
In terms of parameters $\{\omega_m, \omega_b, \Omega_\Lambda, A_s, n_s, \tau_{reio}\}$:

(with $h = \sqrt{\omega_m/(1-\Omega_\Lambda)}$ and $\omega_m = \omega_b + \omega_c$)



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This method allows to see if the parameter is really detectable, and to "isolate" the direct perturbation effect. Applicable to other physical ingredients...

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We vary $N_{\rm eff}$ with fixed z_{eq}, z_Λ. One can easily show that:

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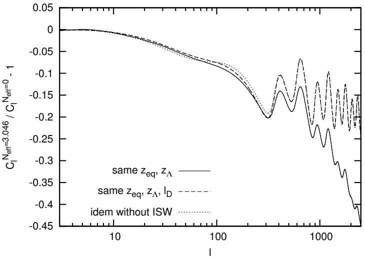
(C4) cannot be compensated in minimal ΛCDM (.. but it can be kept fixed if Y_{He} is decreased Bashinsky & Seljak 2004)

Example: $N_{\rm eff}$ increased from 0 to 3.046 with constant Y_{He} (solid) or (unrealistically) small Y_{He} (dashed)

JL, Mangano, Miele, Pastor, in press



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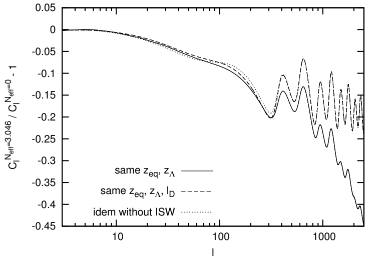

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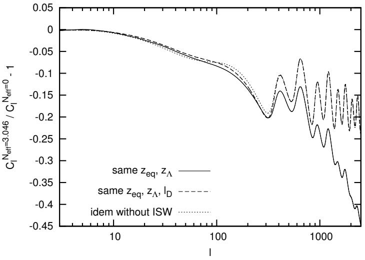

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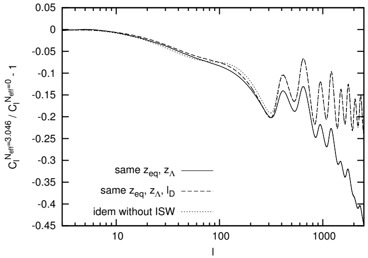

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- peak scale shifted because neutrinos propagate at $c > c_s$: "effective sound speed" enhanced (neutrino drag effect):

$$\Delta l \sim -3 \Delta N_{\rm eff}$$

Bashinsky & Seljak 2004

Conclusions:

- $N_{\rm eff}$ clearly detectable with CMB due to background and perturbation effects
- true for minimal ΛCDM and beyond (perturbation effects)
- accurate data at high-l helps
- BBN prior on Y_{He} helps
- H_0 prior helps (if h fixed, cannot keep z_{eq}, z_Λ fixed)

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Even without lensing information, WMAP gives $M_\nu < 1.3$ eV (95% C.L.) and Planck expected to give $M_\nu < 0.4$ eV (95% C.L.)

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- **(C2)** Ratio of odd-to-even peaks: ω_b/ω_γ — FIXED
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- **(C5)** Global amplitude: A_s — FIXED
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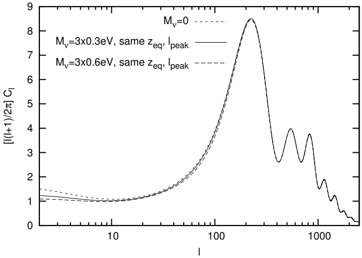
In the parameter set $\{M_\nu, \omega_c, \omega_b, \Omega_\Lambda, A_s, n_s, \tau_{reio}\}$, still have possibility to vary Ω_Λ in order to fix either

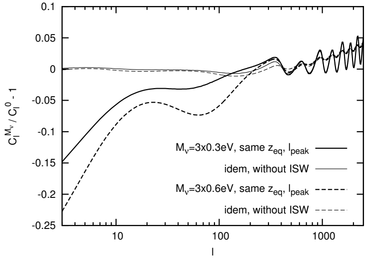
- $d_A(\eta_{LS})$ and **(C1)**+**(C4)**,
- or z_Λ and **(C7)**.

First option better motivated (cosmic variance).

We vary M_ν with fixed ω_b, ω_c, $d_A(\eta_{LS})$:

JL, Mangano, Miele, Pastor, in press





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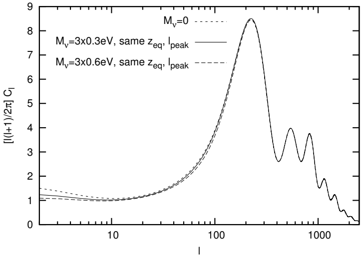
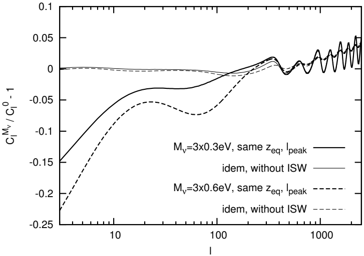




Only modified late ISW effect **(C7)** plus direct perturbation effects of extra d.o.f. Later mainly consists in extra early ISW ($20 < l < 200$) due to metric variations when neutrinos become non-relativistic after decoupling. Amplitude:

$$\Delta C_l / C_l \sim [m_\nu / 10 \text{ eV}].$$

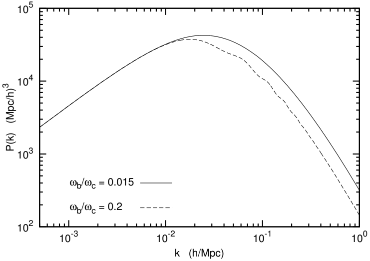
Also effects at $l > 200$ due to the fact that neutrino not fully relativistic prior to recombination.

Conclusions:

- M_ν difficult to measure: background effect (= LISW) masked by cosmic variance, and perturbation effect very small.
- extra priors (H_0, BAO...) help: not possible to keep $z_{eq} + d_A(\eta_{LS})$ fixed... then, significant background effect... that could still be compensated in more general cosmology (spatial curvature)
- detecting mass splitting in CMB is hopeless
- lensing extraction helps a lot

Matter power spectrum

In neutrinoless ΛCDM model, linear $P(k)$ (in $[\mathrm{Mpc}/h]^3$ vs $[h/\mathrm{Mpc}]$) controlled by 5 effects/quantitites:



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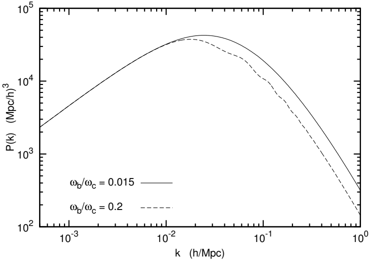

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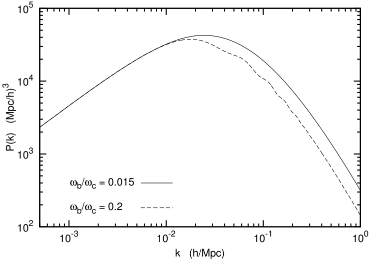

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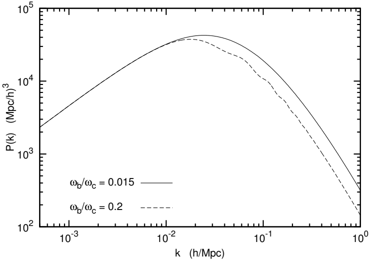

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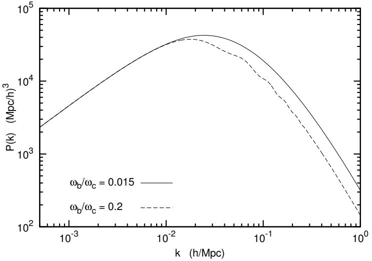

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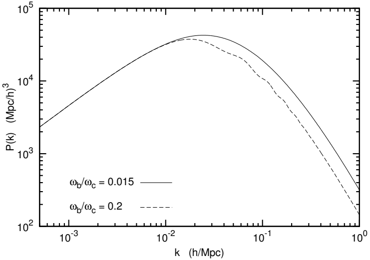

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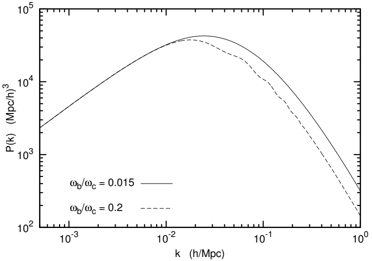
In terms of parameters $\{\omega_m, \omega_b, \Omega_\Lambda, A_s, n_s, \tau_{reio}\}$:

(with $h = \sqrt{\omega_m/(1-\Omega_\Lambda)}$ and $\omega_m = \omega_b + \omega_c$)



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- **(P4)** Overall amplitude — Ω_Λ, A_s
- **(P5)** Global tilt — n_s

Impact of N_{eff}

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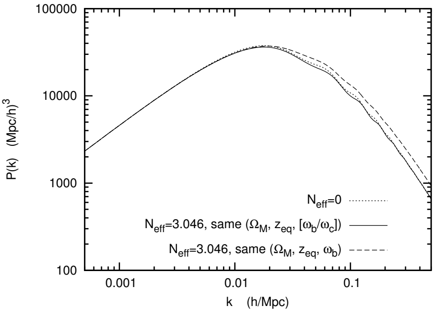
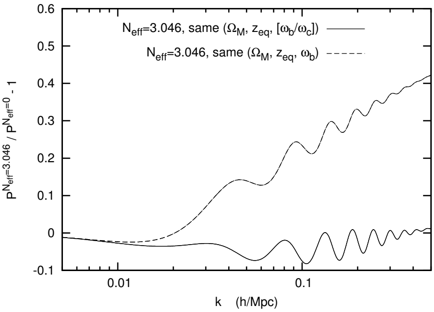
In both cases, background effect (**(P2)** or **(P3)**) adds up with direct perturbation effect: we expect the amplitude/phase shift observed in CMB to show up in BAOs.

We increase $N_{\rm eff}$ with fixed z_{eq}, Ω_Λ, and either fixed ω_b/ω_c or ω_b:

JL, Mangano, Miele, Pastor, in press





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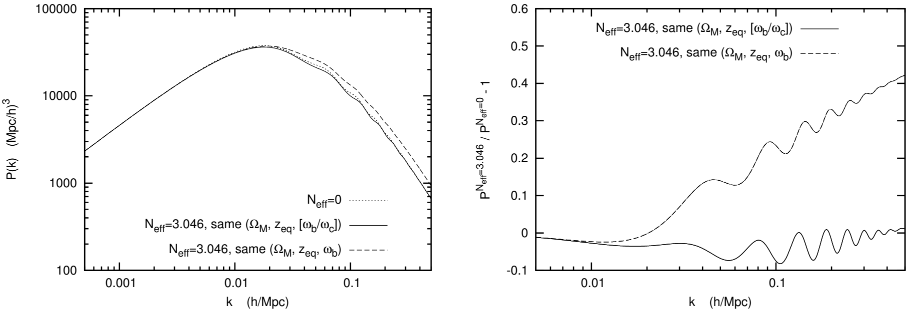

Background effect = either change of slope for $k \geq k_{eq}$ or in BAO phase.

Always an additional BAO phase shift from perturbation effect (neutrino drag).

Conclusions:

- matter power spectrum = complementary probe of $N_{\rm eff}$.
- main signatures in slope (for fixed n_s) and in BAO phase.
- currently: $\sigma(N_{\rm eff}) \sim 0.7$, $1.0\sigma - 1.9\sigma$ excess.
  Planck with lensing extraction: $\sigma(N_{\rm eff}) \sim 0.3$,
  Planck + Euclid: $\sigma(N_{\rm eff}) \sim 0.1$.
  No prospects to test with precision standard value 3.046

Impact of M_ν

Famous neutrino free-streaming effect: on small scales, not only neutrinos do not cluster, but also the growth of CDM perturbations is modified (scale-dependent growth factor):

$$\delta_c \propto a^{1-\frac{3}{5}f_\nu}, \qquad f_\nu \equiv \omega_\nu/\omega_m.$$

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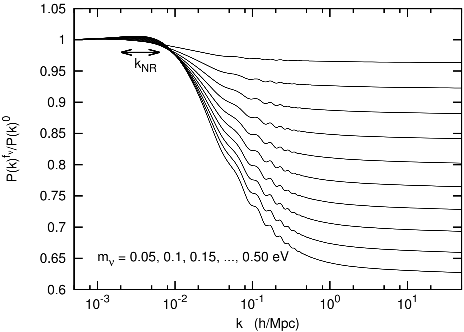
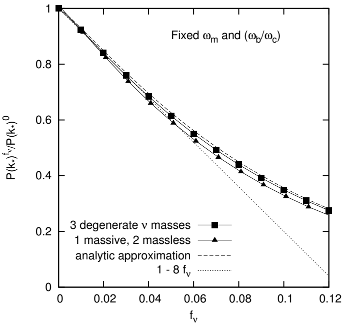
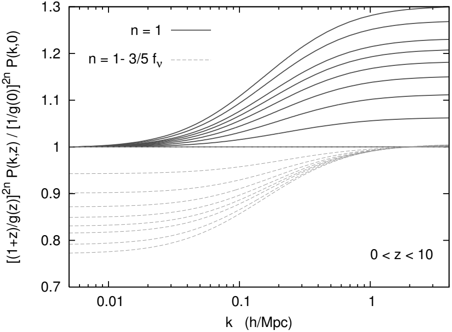
Best seen when most background effects are cancelled: below we fix ω_m, Ω_Λ ω_b/ω_c:

JL, Mangano, Miele, Pastor, in press



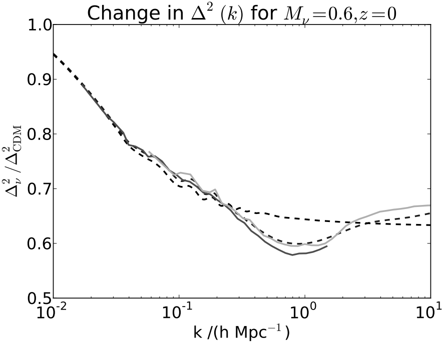


Effect is strongly redshift dependent:



with $f_\nu = 0.1$ ($M_\nu = 1$ eV)

Non-linear corrections well understood on mildly non-linear scales:



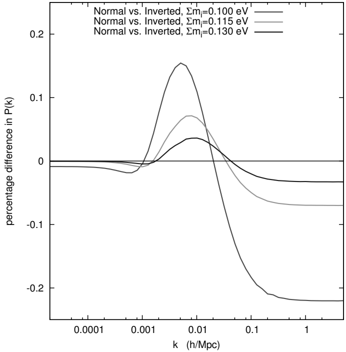
Bird, Haehnelt, Viel 2011; see also Brandbyge et al.2010

- $P(k)$ very sensitive to M_ν, $\Delta P/P \sim -8\,(10) f_\nu \sim [M_\nu/1\ \mathrm{eV}]$: at least 5% for normal hierarchy scenario, 10% for inverted hierarchy scenario.
- scale-dependent growth factor $g(z,k)$: data at different redshift helps; not degenerate with usual extensions of ΛCDM (curvature, running...)
- $\sigma(M_\nu) \sim 0.1$ eV for Planck with lensing extraction or with BOSS,
  0.06 with Planck+DES,
  0.03 with Planck + Euclid CS,
  0.015 with Planck + Euclid $P(k)$ ...

Impact of mass splitting

- times of non-relativistic transitions depend on individual masses
- hence 3 free-streaming scales depending on each mass
- total small-scale supression due to reduced cdm growth rate between non-relativistic transition and now: small dependendance on individual mass.

JL, Mangano, Miele, Pastor, in press



Unlikely to be ever detected even with CMBPol/Core + SKA