Non-Gaussianity & Inflation

Detection of NG would prove wrong at least one of these assumptions:

- single scalar field inflation
- slow roll
- canonical kinetic term
- Bunch-Davies vacuum

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Measurements so far leave room for some local non-Gaussianity

WMAP7	$-10 < f_{NL} < 74 \quad \text{at} \quad 95\% \text{ level}$	Komatsu et al., AJS (2011)
SDSS	$-29 < f_{NL} < 70 \quad \text{at} \quad 95\% \text{ level}$	Slosar et al., JCAP (2008)
PLANCK	$\Delta f_{NL} \simeq 5$	Tauber et al., A&A (2010)

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Are we ready for that?

Contaminants

Any non-linearity naturally produces non-Gaussian features

- Galactic foregrounds
- SZ effect
- Cosmic reionization
- Detector-induced noise
- Lensing – ISW correlation
- etc.

e.g. Komatsu, CQG, 2010
Liguori et al., AiA 2010

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We shall focus on particularly tricky one

Non-linear dynamics at recombination

Second-order perturbation theory

At first order the initial conditions are linearly propagated:

$$a_{lm} = \Phi(\mathbf{k})\,\mathcal{T}_l(k)\,Y_{lm}(\mathbf{k})$$

Non-Gaussianity is in the potential

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Non-Gaussianity is in the potential

... non vanishing bispectrum linked to primordial non-Gaussianity .

$$\langle\,\Phi(k_1)\,\Phi(k_2)\,\Phi(k_2)\,\rangle = 0 \quad \Rightarrow \quad \langle\,a_{l_1 m_1}\,a_{l_2 m_2}\,a_{l_3 m_3}\,\rangle = 0$$

Second-order perturbation theory

At second order:

$$a_{lm}^{(2)}(\mathbf{k}) \;=\; \mathcal{K}\left[\,\mathcal{T}_{lm}(\mathbf{k},\,\mathbf{k_1},\,\mathbf{k_2})\;\Phi^{(1)}(\mathbf{k_1})\,\Phi^{(1)}(\mathbf{k_2})\right]$$

Quadratic in the primordial fluctuations

Second-order transfer function

Second-order perturbation theory

At second order:

Quadratic in the primordial fluctuations

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Second-order transfer function

... Gaussian initial conditions are propagated nonlinearly into the observed CMB anisotropies .

$$\langle\,\Phi(k_1)\,\Phi(k_2)\,\Phi(k_2)\,\rangle = 0 \quad \not\Rightarrow \quad \langle\,a_{l_1m_1}\,a_{l_2m_2}\,a_{l_3m_3}\,\rangle = 0$$

Second-order perturbation theory

It is crucial to predict the shape and amplitude of second-order effects in order to subtract them from the data

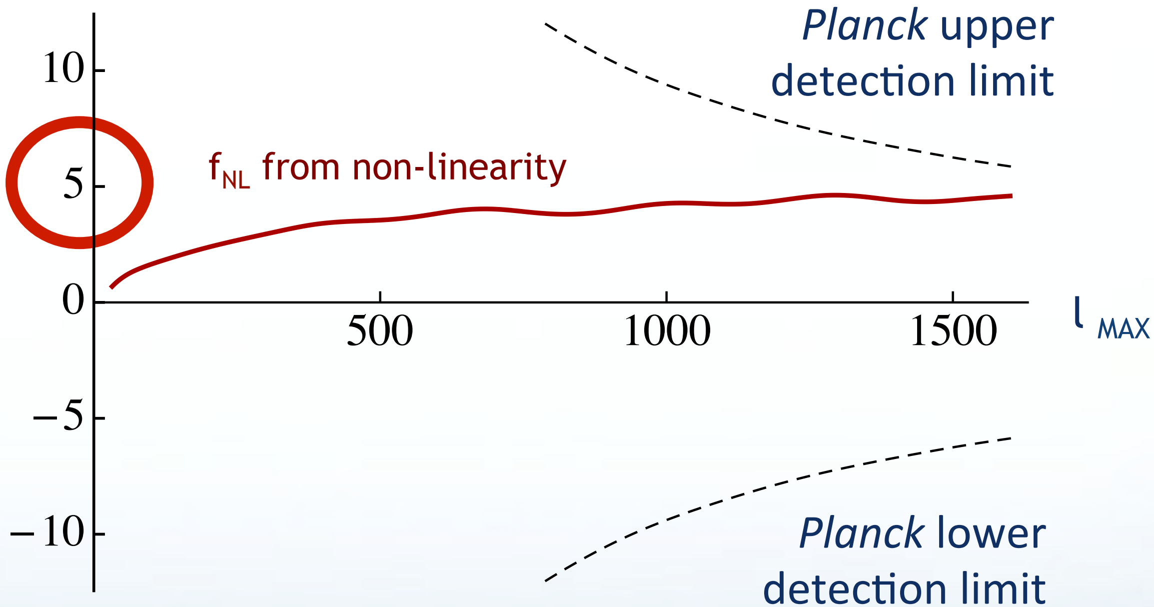
Previous attempts

Existing approximations either neglect part of the source terms or focus on a particular configuration

Approximation	Reference	f_{NL} contamination
Perturbed recombination	Khatri et al. 2009	- 1
Perturbed recombination	Senatore et al. 2009	- 3.5
Only quadratic terms	Nitta et al. 2009	+ 1
Squeezed limit	Bartolo et al. 2011	negligible for $L_{LONG} < 100$
Squeezed limit	Creminelli et al. 2011	
Squeezed limit	Lewis 2012	

First non-approximated result

... found by Pitrou et al. 2010, who wrote the Mathematica code CMBQuick



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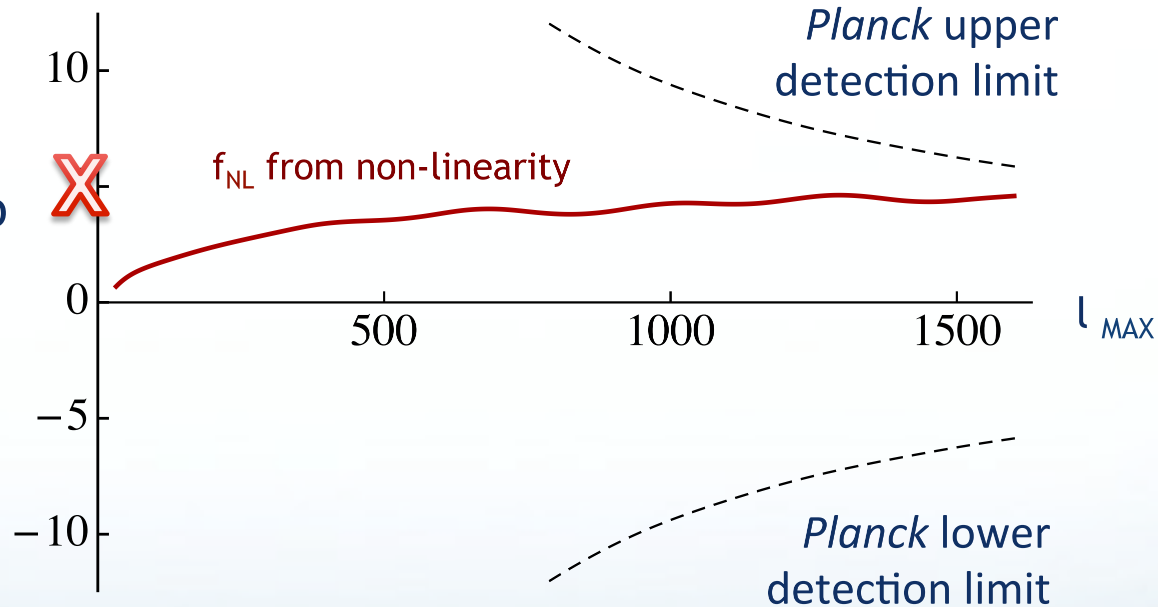
Recently corrected to 3



Why is it so complicated?

Recipe for a 2^{nd}-order Boltzmann code

✓ Derive Boltzmann equation at 2^{nd} order

Deriving Boltzmann equation

Beneke & Fidler, 2010
Pitrou et al. 2009
Senatore, Tassev & Zaldarriaga, 2009
Bartolo, Matarrese & Riotto, 2006-7

Theoretical complications (often forgotten at first order):

- Tetrad formalism for Boltzmann equation

$$G^{\mu}{}_{\nu} = K\, e_{\underline{A}}{}^{\mu}\, e^{\underline{B}}{}_{\nu}\, T^{\underline{A}}{}_{\underline{B}}$$

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- Fluid limit & multipoles

$$\frac{1}{\bar{p}} T_{(n)}{}^{\underline{0}}{}_{\underline{0}} = -\,{}_0\Delta_{00}^{(n)}$$

$$\frac{1}{\bar{p}} T_{(n)}{}^{\underline{i}}{}_{\underline{0}} = \frac{i}{3} \sum_m (-1)^m \xi^{i}_{-m}\; {}_1\Delta_{1m}^{(n)}$$

$$\frac{1}{\bar{p}} T_{(n)}{}^{\underline{i}}{}_{\underline{j}} = \frac{\delta_{ij}}{3}\, {}_2\Delta_{00}^{(n)} - \frac{1}{5} \sum_m (-1)^m \chi^{ij}_{2-m}\; {}_2\Delta_{2m}^{(n)}$$

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$$\frac{1}{\bar{\rho}} T_{(n)}{}^{\underline{0}}{}_{\underline{0}} = -\,{}_0\Delta_{00}^{(n)}$$

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- Horrible angular projections

$$\sum_{l_1=|l-1|}^{l+1} \sum_{m_1=-l_1}^{l_1} i^{l-l_1} \begin{pmatrix} l_1 & 1 & l \\ -s & 0 & -s \end{pmatrix} \begin{pmatrix} l_1 & 1 & l \\ m_1 & m_2 & m \end{pmatrix} f^{(2)}_{ab,l_1 m_1}(\boldsymbol{k}).$$

Recipe for a 2^{nd}-order Boltzmann code

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- ✓ Solve the Einstein-Boltzmann differential system at 2^{nd} order

Mode coupling

At second order, convolution integrals appear:

$$A(\mathbf{x})\,B(\mathbf{x}) \;\rightarrow\; \mathcal{K}\left[\,\tilde{A}(\mathbf{k}_1)\,\tilde{B}(\mathbf{k}_2)\,\right]$$

Typical equation:

Purely second-order terms

=

Convolution of quadratic source terms

Mode coupling

Example Anisotropic stresses in Newtonian gauge

$$\Psi(\mathbf{k}) - \Phi(\mathbf{k}) = 0$$

Mode coupling

Example Anisotropic stresses in Newtonian gauge

$$\Psi(\mathbf{k}) - \Phi(\mathbf{k}) = \mathcal{K}\Big[\quad -\frac{2}{k^4}\,\Psi(k_1)\Phi(k_2)\left(2\mu^2 k_1^2 k_2^2 + 3\mu k_1^3 k_2 + \mu k_1 k_2^3 + k_1^2 k_2^2 + k_1^4\right) + \text{symm}$$

$$+\frac{1}{k^4}\,\Phi(k_1)\Phi(k_2)\left(15\mu^2 k_1^2 k_2^2 + 14\mu k_1^3 k_2 + 14\mu k_1 k_2^3 + 5k_1^2 k_2^2 + 4k_1^4 + 4k_2^4\right)$$

$$+\frac{1}{k^4}\,\Psi(k_1)\Psi(k_2)\left(7\mu^2 k_1^2 k_2^2 + 6\mu k_1^3 k_2 + 6\mu k_1 k_2^3 + k_1^2 k_2^2 + 2k_1^4 + 2k_2^4\right) \quad \Big]$$

Solve the system for a (k1, k2, k) grid

Differential system

- Four relativistic hierarchies (I, E, B, N) with custom L_{MAX}
  + 2 non-relativistic ones (baryons and CDM)
- Full Boltzmann equation, including collision term & lensing
- Approx. 100 equations on a grid in (k1, k2, k)
- We use the first-order sources & integrator from CLASS

Blas, Lesgourgues & Tram, JCAP 2011

- Same results using transfer functions or Green functions

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Computation time: 40 minutes

Line-of-sight source functions

$$\dot{\Delta}_{lm} + i\,\mathbf{k}\cdot\mathbf{n}\,\Delta_{lm} + \dot{\kappa}\,\Delta_{lm} \;=\; S_{lm}$$

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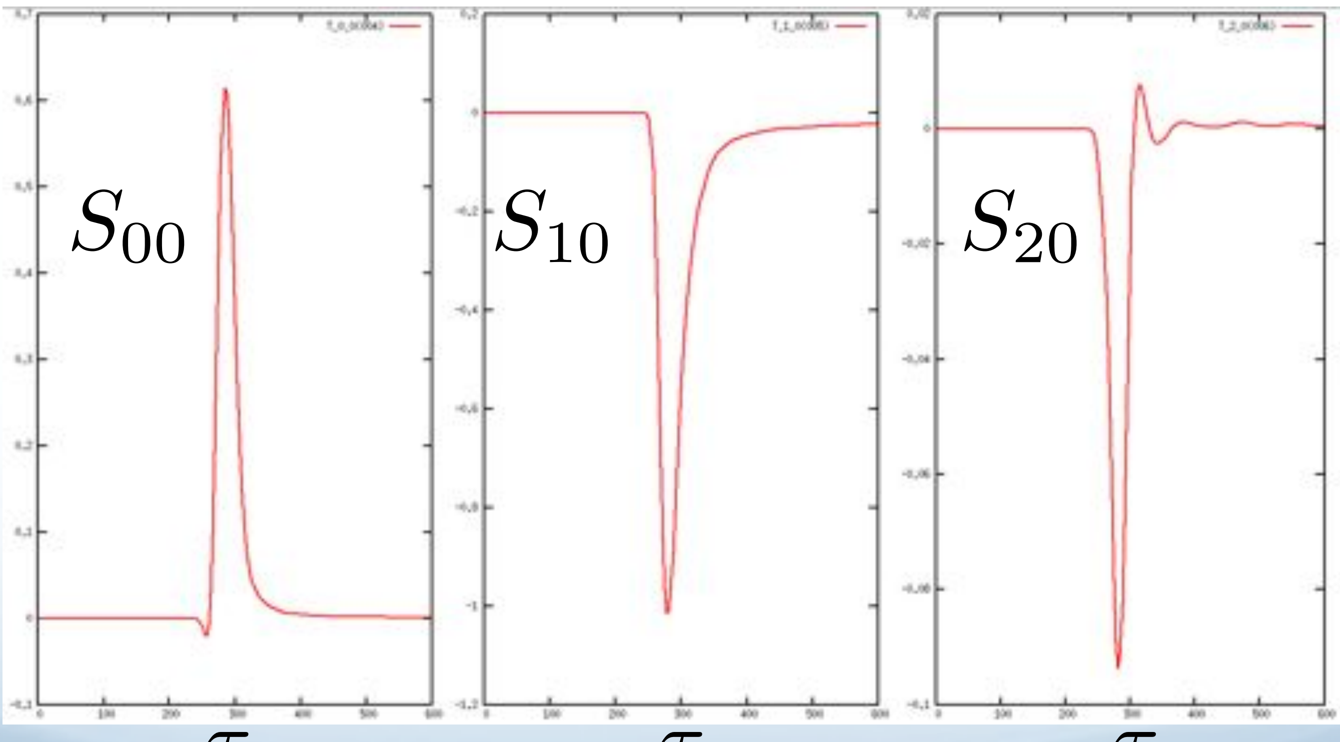

Recipe for a 2^{nd}-order Boltzmann code

- ✓ Derive Boltzmann equation at 2^{nd} order
- ✓ Solve the Einstein-Boltzmann differential system at 2^{nd} order
- ✓ Derive transfer functions using the line-of-sight formalism

Line-of-sight integral

At first order is a convolution over spherical Bessels:

Seljak & Zaldarriaga, 1996

$$\mathcal{T}_l(\mathbf{k}) \;=\; \int_0^{\tau_0} d\tau\, e^{-\kappa(\tau,\tau_0)}\, j_l(k(\tau-\tau_0))\;\; S(k)$$

First-order
line-of-sight sources
One scalar function!

Line-of-sight integral

Oscillatory integral over complicated projection functions

Beneke, Fidler & Klingmuller, 2011

Combination of spherical Bessels and 3j's

$$\mathcal{T}_{lm}(\mathbf{k},\,\mathbf{k}_1,\,\mathbf{k}_2) \;=\; \int_0^{\tau_0} d\tau\, e^{-\kappa(\tau,\tau_0)} \sum_{L=0}^{l_{\max}} J_{Llm}(k(\tau-\tau_0))\;\mathcal{S}_{Lm}(\mathbf{k},\,\mathbf{k}_1,\,\mathbf{k}_2)$$

Second-order line-of-sight sources

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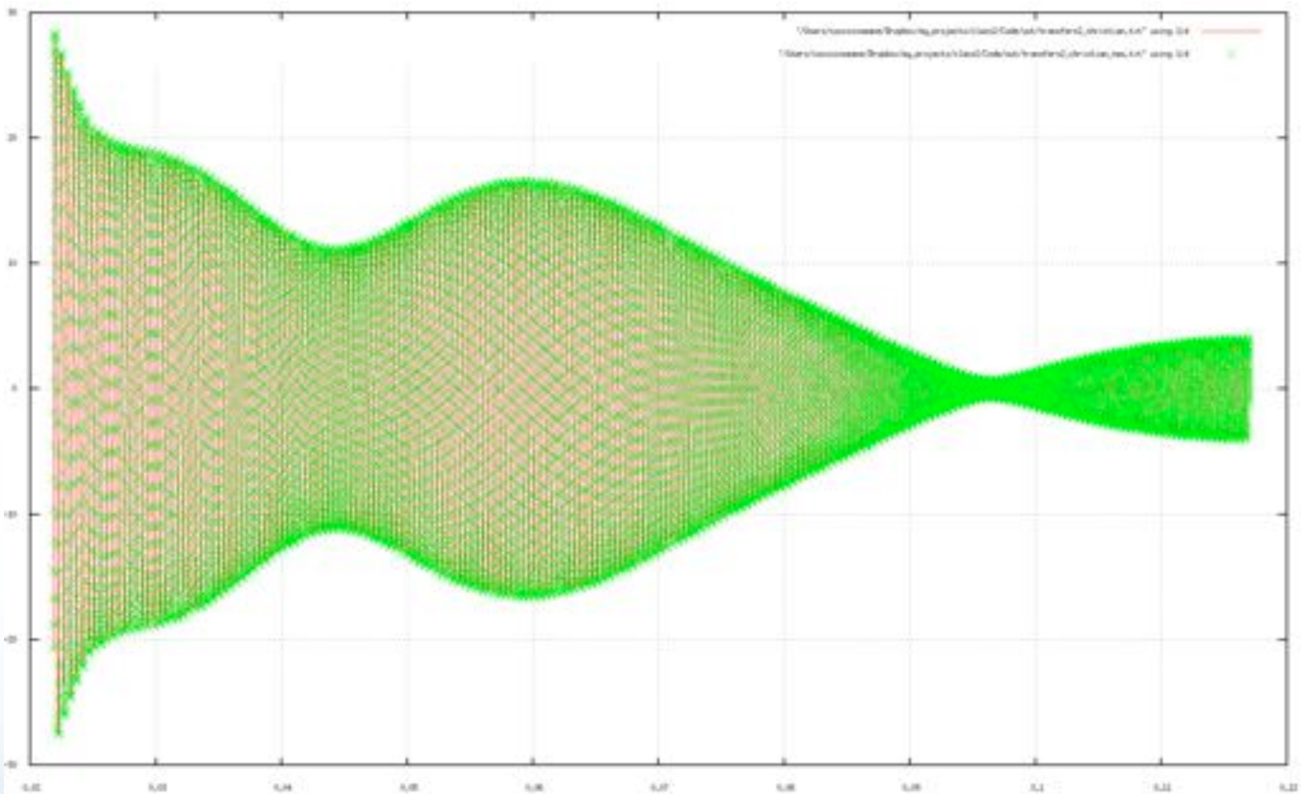
Second-order line-of-sight sources

Computation time: 15 minutes

Line-of-sight integral

$\mathcal{T}_{100}$



k [Mpc/h]

Recipe for a 2nd-order Boltzmann code

- ✓ Derive Boltzmann equation at 2nd order
- ✓ Solve the Einstein-Boltzmann differential system at 2nd order
- ✓ Derive transfer functions using the line-of-sight formalism
- ✓ Compute the temperature full-sky bispectrum

Angular power spectrum

Convolution of an oscillatory function with the smooth power spectrum

$$C_l \;\propto\; \int_0^\infty dk\, k^2\, P(k)\, |\, T_l(k)\, |^2$$

Full-sky bispectrum

Involved integral on 6 highly oscillatory functions

Fergusson & Shellard, 2007, 2009
Fregusson, Regan & Shellard 2011

$$
\begin{aligned}
b_{l_1 l_2 l_3} = \int dr\, r^2 \\
\int dk_1\, k_1^2\, T_{l_1}(k_1)\, P(k_1)\, j_{l_1}(r\, k_1) \\
\int dk_2\, k_2^2\, T_{l_2}(k_2)\, P(k_2)\, j_{l_2}(r\, k_2) \\
\int dk_3\, k_3^2\, T_{l_3}(k_1, k_2, k_3)\, j_{l_3}(r\, k_3)
\end{aligned}
$$

Second-order transfer function

Full-sky bispectrum

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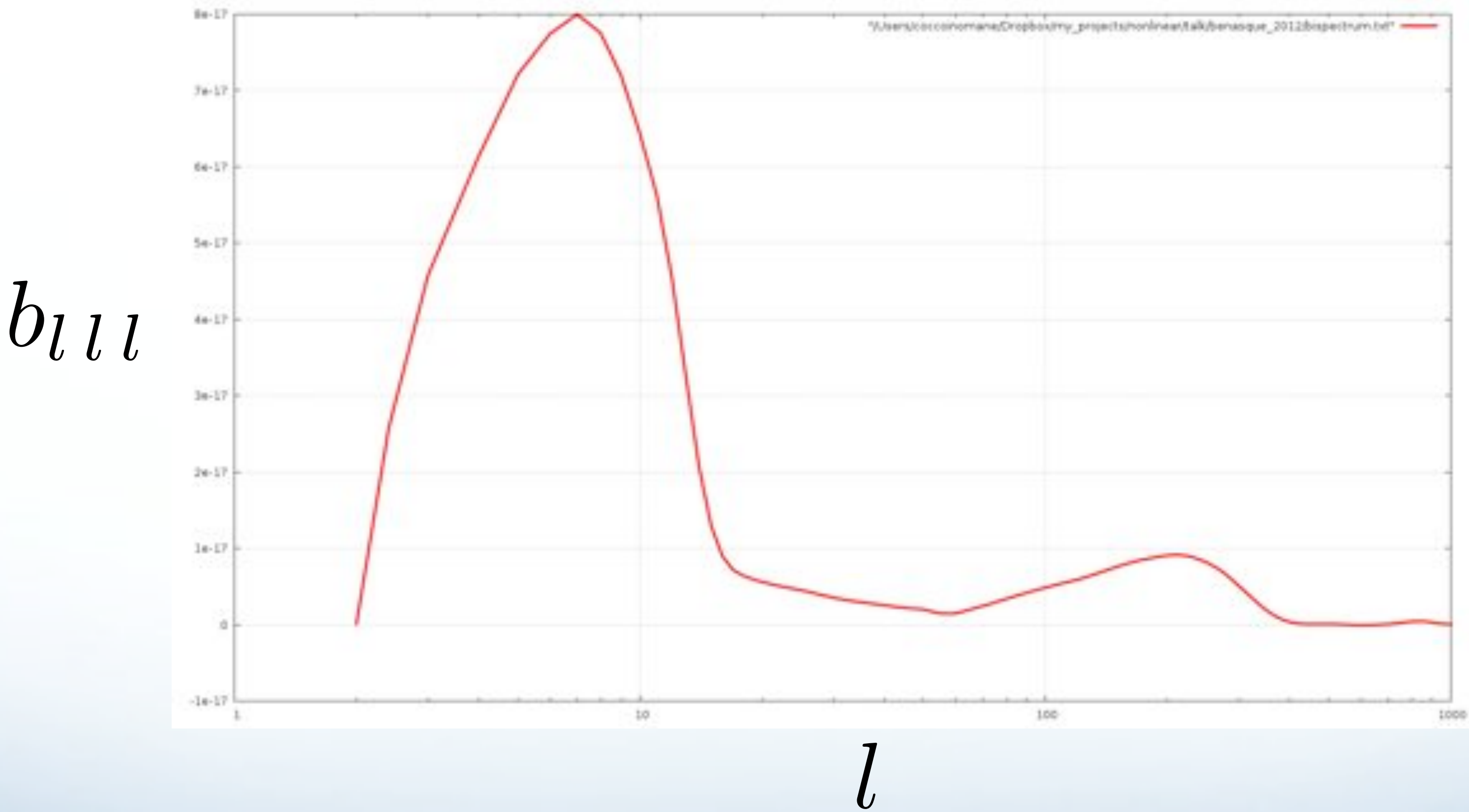
Second-order transfer function

Computation time: 25 minutes

Full-sky bispectrum



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- ✗ Compute f_{NL} contamination

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- ✗ Compute f_{NL} contamination

... wait until October

Conclusions

- Second-order corrections to the CMB bispectrum need to be computed
- The full second-order transfer function is needed to properly subtract the effect
- We have written a fast, parallel and flexible second-order Boltzmann code in Newtonian gauge
- From cosmology to bispectrum in less than 2 hours
- Expect the first science results on f_{NL} in October 2012

Current & future work

- Perform covergence tests on the bispectrum
- Check the bispectrum against squeezed analytical solutions
- Compute the bias on f_{NL}
- Include ISW in the line-of-sight sources
- Include lensing terms in the line-of-sight sources
- Implement a tight-coupling scheme
- Compute the bispectrum in a smarter way (e.g. make the integral separable using decomposition)
- Implement synchronous gauge

Thank you for your attention!