Halo abundances and shear in void models.

David Alonso - UAM/IFT

Based on the work: astro-ph:1204.3532 In collaboration with: J. García-Bellido, A. Knebe, T. Haugbølle

August 17th 2012



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Void models



 $\rho(\mathbf{r},\mathbf{t}_0)$



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Cosmological models.

Friedmann-Robertson-Walker (FRW)

$$ds^{2} = -dt^{2} + a^{2}(t) \left[\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\Omega^{2} \right], \quad a = \frac{1}{1 + z}$$
$$H^{2} \equiv \left(\frac{\dot{a}}{a}\right)^{2} = H^{2}_{0} \left[\Omega_{M} a^{-3} + \Omega_{k} a^{-2} + \Omega_{\Lambda} a^{-3(1+w)}\right]$$

Lemaître-Tolman-Bondi (LTB)

$$ds^{2} = -dt^{2} + \frac{(A'(t,r))^{2}}{1-k(r)}dr^{2} + A^{2}(t,r)d\Omega^{2}, \quad H_{T} \equiv \frac{\dot{A}}{A}, \quad H_{L} \equiv \frac{\dot{A}'}{A'}$$
$$H_{T}^{2}(t,r) = H_{0}^{2}(r) \left[\Omega_{M}(r) \left(\frac{A_{0}(r)}{A(t,r)}\right)^{3} + (1 - \Omega_{M}(r)) \left(\frac{A_{0}(r)}{A(t,r)}\right)^{2}\right]$$

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Shear



- Comoving geodesics have non-vanishing shear.
- Expansion + deformation

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• Crucial for perturbations



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Perturbation theory.

Perturbation theory in LTB models is complicated¹.

- No background homogeneity \Rightarrow No *SO*(3) splitting.
- Scalar and tensor modes coupled through the background shear.

$$\ddot{\Psi}$$
 + 4 $H_T \dot{\Psi}$ - $\frac{2 k(r)}{A^2} \Psi \propto \sigma^{\mu\nu} \tau^{\text{pert}}_{\mu\nu}$ + (...)

 Background shear can be characterised by a normalized shear-to-expansion ratio:

$$\epsilon \equiv \sqrt{\frac{2}{3} \frac{\sigma_{\mu\nu} \sigma^{\mu\nu}}{\theta^2}} = \frac{H_T - H_L}{2 H_T + H_L}$$

¹arXiv:0903.5040,1206.1602 and Sean's talk!



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LTB void models

N-body simulations of LTB models.

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Perturbation theory





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Perturbation theory.

But...

• 0-shear approximation: FRW perturbation theory at each *r*.

 $\delta_0(t,r) \propto D(\Omega_M(r), A(t,r)/A_0(r))$

Small shear effects could be encoded in a first-order correction

$$\delta_{\alpha}(t,r) \propto D(\Omega_{M}(r), A(t,r)/A_{0}(r)) \left[1 + \frac{\alpha}{\alpha} \epsilon(t,r)\right]$$

Where, in principle

$$\alpha = \alpha(t, r, \Omega, \ldots)$$

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Background evolution Perturbation theory The mass function

Mass function

- n(M) dM: number density of halos of mass $M \in (M, M + dM)$.
- Press & Schechter theory: abundance of non-linear structures is determined by spherical collapse + linear perturbation theory + gaussian δ .
- Bottomline: the mass function is a **universal** function of the variance of the linear density contrast: *σ*(*M*, *z*).

$$n(M,z) = \frac{\rho_M}{M}g(\sigma) \left| \frac{d \ln \sigma}{dM} \right|,$$

• Different parametrisations of the mass function can be found in the literature.



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LTB void models Background evolution N-body simulations of LTB models. Perturbation theory Halo statistics The mass function

Mass function

Recipe for the mass function in LTB growing-mode voids:

- Calculate $\sigma(M)$ outside the void.
- Rescale it by a factor

$$g(t,r) = \frac{\delta(t,r)}{\delta(t,r=\infty)}$$

- Use your favourite mass function parametrisation with this rescaled variance.
- We will show results in terms of

$$n(>M,< r,z)\equiv rac{3}{4\pi\,r^3}\int_0^r r'^2 dr'\int_M^\infty dM'\,n(M',r',z).$$



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The simulation The halo catalog

Method.





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Alonso et al. arXiv:1010.3453 David Alonso - UAM-IFT,

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Method

In N-body simulations of FRW universes perturbations are generated by displacing the particles from their ideal comoving positions before the first timestep by using the Zel'Dovich approximation

$$\mathbf{q} = \mathbf{q}_i - i \sum_{\mathbf{k}} \frac{\delta_{\mathbf{k}}(t)}{k^2} \mathbf{k} e^{i\mathbf{k}\cdot\mathbf{q}}.$$

In our case we must also account for the displacement due to the background void. To do this:

- Calculate standard transfer function for perturbations in the flat EdS background and use existing software to calculate the initial gravitational potential $\phi_{\mathbf{k}}^{i}$.
- Add to this the potential due to the void from the exact LTB solution \(\phi_k^v\).
- Use existing software to calculate particle displacements with the total potential.



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Simulations performed.

Name	Z _{start}	Ω _{in}	$\Delta r/r_0$	#particles	Comments
\mathcal{H}	24	0.25	0.3	960 ³	High res sim
\mathcal{V}	49	0.25	0.3	512 ³	Void alone
<i>S</i> 24	24	0.25	0.3	512 ³	Void + matter
<i>S</i> 49	49	0.25	0.3	512 ³	Void + matter
S99	99	0.25	0.3	512 ³	Void + matter
<i>S</i> Ω125	49	0.125	0.3	512 ³	Void + matter
$S\Omega 063$	49	0.0625	0.3	512 ³	Void + matter
<i>S</i> Ω021	199	0.0208	0.3	512 ³	Void + matter
$S\Delta01$	49	0.125	0.1	512 ³	Void + matter
$S\Delta05$	49	0.125	0.5	512 ³	Void + matter
\mathcal{L}	49	0.25	0.3	768 ³	L=3600 Mpc h ⁻¹

In all cases $r_0 = 1100 \,\text{Mpc}$ and $H_0 = 64 \,\text{km s}^{-1}$, Mpc^{-1} . Mass resolution is $m_p = 4.3 \times 10^{12} \,M_{\odot} \,h^{-1}$ (\mathcal{H})





The simulation The halo catalog

Background evolution.

We checked that the simulations reproduce the background evolution correctly by comparing data and theoretical predictions for: density profile and expansion rates.



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The simulation The halo catalog

Background evolution.

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The halo catalog

The halos were extracted using AHF (Knollmann & Knebe, 2009, ApJS, 182, 608).

- Create adaptively smoothed density field from the particle content.
- 2 Locate possible halos as local overdensities.
- Extract gravitationally bound particles by iteratively solving Poisson's equation.
- Oetermine halo extent and mass through

$$M(r_{\rm vir}) = \frac{4\pi}{3} r_{\rm vir}^3 \, \rho_c \, \Delta$$

(Δ is a fudge factor. We use $\Delta = 200$).



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N-body simulations of LTB models. Halo statistics

Halo abundances





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Halo abundances

Conclusions.

- The theoretical description of the halo mass function can be extended to LTB universes by adding a parameter that accounts for the effect of the background shear on the growth of perturbations.
- The value of this parameter depends mildly on *t* and *r*, with α ~ 2. Its dependence on the void model is still under study.
- If α turned out to be cosmology-independent, halo abundances might be used to constrain the background cosmic shear.
- If applicable to $O(10 \,\mathrm{Mpc})$ -sized voids, this treatment could be used in the modelling of environmental effects.



Halo abundances

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THANKS FOR YOUR ATTENTION!



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