Onsager relations in a two-dimensional electron gas with spin-orbit coupling

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Outline

Spin-orbit in a 2DEG

- What for?
- Spin-Hall effect(s)
- Role of disorder

G. Vignale, J. Supercond. Nov. Magn. (2010)

T. Jungwirth, J. Wunderlich and K. Olejník, Nat. Mat. (2012)

Charge vs. spin

- Spin currents
- Onsager relations

L. Y. Wang, A. G. Mal'shukov, C. S. Chu, PRB (2012)

C. Gorini, R. Raimondi, P. Schwab, arXiv (2012)

Spin-orbit in a 2DEG

Electronics (charge) $\longrightarrow$ Spintronics (charge&spin)

Why the spin?

- Long lifetime, low power consumption
- Multi-functional (semiconductor) devices
- Quantum computing

Nat. Mat. Insight (2012)

The main question&some possible answers

How to manipulate spins in solid state systems?

- Magnetic fields/materials
- Optical methods
- Spin-orbit coupling

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Nat. Mat. Insight (2012)

The main question&some possible answers

Spin-orbit coupling $\Rightarrow$

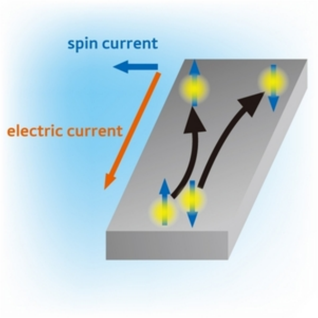
1. Electrical handle on spin
2. Spin not conserved

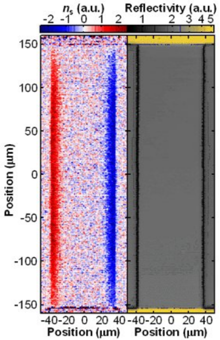
Spin-orbit in a 2DEG

The spin-Hall effect

An electric field **E** generates a transverse (pure) spin current

M. I. Dyakonov and V. I. Perel, Phys. Lett. A (1971)





Physicsworld (2006)

Y. K. Kato et al., Science (2004)

Spin-orbit in a 2DEG

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Y. K. Kato et al., Science (2004)

J. Wunderlich et al., PRL (2005)

S. O. Valenzuela and M. Tinkham, Nature (2006)

E. Saitoh et al., Appl. Phys. Lett. (2006)

T. Seki et al., Nat. Mat. (2008)

K. Ando et al., J. Appl. Phys. (2009)

E. S. Garlid et al., PRL (2010)

K. Olejník et al., PRL (2012)

...

Spin-orbit in a 2DEG

The spin-Hall effect

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M. I. Dyakonov and V. I. Perel, Phys. Lett. A (1971)

Intrinsic Spin Hall effect in the Rashba model

$$J_y^{s_z} = \sigma_{sH} E_x$$

Sinova et al., PRL (2004)

- No disorder $\Rightarrow \sigma_{sH} = \frac{e}{8\pi}$ “universal” result (interesting)
- *Any* disorder$\Rightarrow \sigma_{sH} = 0$ “universal” result (boring)

Disruptive role of impurities

Spin-orbit in a 2DEG

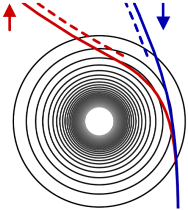
Intrinsic origin

- Crystal structure (broken bulk inversion symmetry)
- Device structure (broken structure inversion symmetry)

Extrinsic origin

- Mott skew-scattering
- Side-jump

Constructive role of impurities



Adapted from Engel et al., PRL (2005)

Intrinsic vs. extrinsic

Nontrivial dependence on physical parameters

Spin-orbit in a 2DEG

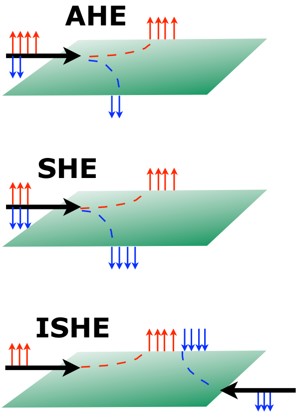
- a spin-up current generates a current in the transverse direction

$$\delta j_{y\uparrow} = 2\gamma_\uparrow j_{x\uparrow}$$

- a spin-down current generates a current in the transverse opposite direction

$$\delta j_{y\downarrow} = -2\gamma_\downarrow j_{x\downarrow}.$$

$\gamma_\uparrow, \gamma_\downarrow \leftrightarrow$ Intrinsic and/or extrinsic



Spin-orbit in a 2DEG

The main points

- Spin non-conservation
- Fundamental role of disorder
- Nontrivial interplay intrinsic-extrinsic

Two (related) questions

- Definition of the spin current?
- Reciprocity spin-Hall $\leftrightarrow$ inverse spin-Hall?

Charge vs. spin

2D effective Hamiltonian

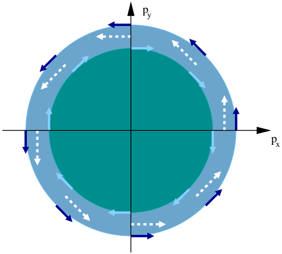
$$H = \underbrace{p^2/2m + H_{\rm intr}}_{\text{band/device structure}} + \underbrace{V_{\rm imp} + H_{\rm extr}}_{\text{impurities}}$$

$$H_{\rm intr} = -\mathbf{b}(\mathbf{p}) \cdot \boldsymbol{\sigma}, \quad H_{\rm extr} = -\frac{\lambda^2}{4\hbar}\left[\mathbf{p} \times \nabla V_{\rm imp}(\mathbf{r})\right] \cdot \boldsymbol{\sigma}$$

$$\epsilon_\pm(\mathbf{p}) = \frac{p^2}{2m} \pm |\mathbf{b}(\mathbf{p})|$$

$$|\mathbf{b}| \sim \text{meV} \sim 10^{-1}\epsilon_F$$

$$(\lambda/\lambda_c)^2 \sim 10^6$$



Charge vs. spin

2D effective Hamiltonian

$$H = \underbrace{p^2/2m + H_{\text{intr}}}_{\text{band/device structure}} + \underbrace{V_{\text{imp}} + H_{\text{extr}}}_{\text{impurities}}$$

$$H_{\text{intr}} = -\mathbf{b}(\mathbf{p}) \cdot \boldsymbol{\sigma}, \quad H_{\text{extr}} = -\frac{\lambda^2}{4\hbar}\left[\mathbf{p} \times \nabla V_{\text{imp}}(\mathbf{r})\right] \cdot \boldsymbol{\sigma}$$

Spin current

$$J_i^a \equiv \frac{1}{2}\{s^a, v_i\} \quad \Rightarrow \quad \partial_t s^a + \nabla \cdot \mathbf{J}^a = T^a$$

$$v_i = \nabla_{p_i} H, \quad a\,(\text{spin}), i\,(\text{space}) = x, y, z$$

Spin non-conservation $\Rightarrow$ "spin-torque" T^a

Charge vs. spin

Spin-orbit ↔ non-Abelian gauge fields

$$H = \frac{p^2}{2m} + \alpha\,(p_y\sigma^x - p_x\sigma^y) = \frac{(p+\eta\boldsymbol{\mathcal{A}}^a\sigma^a/2)^2}{2m} + \mathrm{const.}$$

$$\eta\mathcal{A}_y^x = -\eta\mathcal{A}_x^y = 2m\alpha, \quad \mathrm{else}\ \mathcal{A}_i^a = 0$$

Advantages

- Unambiguous definition of $\mathbf{J}_s$:“colour” current
- Spatially/time-modulated spin-orbit or Zeeman fields
- Interplay intrinsic-extrinsic

But...

Limited to linear-in-momentum spin-orbit (?)

N. J. Fröhlich and U. M. Studer, Rev. Mod. Phys. (1993), I. V. Tokatly, PRL (2008)

C. Gorini et al., PRB (2010)

Charge vs. spin

Charge

$$\Phi, \mathbf{A} \Rightarrow \mathbf{E}, \mathbf{B}, \mathbf{J}_c$$

Spin

$$\Psi, \mathcal{A} \Rightarrow \mathcal{E},\ \mathcal{B}, \mathbf{J}_s$$

$$\tilde{\partial}_t \mathbf{J}_s + \tilde{\nabla} \cdot \mathbf{J}_s = 0, \quad \mathbf{J}_s = \mathbf{J}_{s,\mathrm{intr}} + \mathbf{J}_{s,\mathrm{extr}}$$

$$\tilde{\partial}_t = \partial_t \underbrace{-\mathrm{i}\eta[\Psi, \ldots]}_{\text{Zeeman}} \qquad \tilde{\nabla} = \nabla \underbrace{+\mathrm{i}\eta[\mathcal{A}, \ldots]}_{\text{spin-orbit}}$$

Hall effects

- $\mathbf{J}_c = -\frac{e\tau}{m} \mathbf{J}_c \times \mathbf{B}$ Hall
- $\mathbf{J}_s = -\frac{\eta\tau}{4m} \mathbf{J}_c \times \mathcal{B}$ Spin-Hall
- $\mathbf{J}_c = -\frac{\eta\tau}{m} \mathbf{J}_s \times \mathcal{B}$ Inverse spin-Hall

Gorini et al., PRB (2010), Raimondi et al., Ann. Phys. (Berlin) (2012)

Charge vs. spin

Onsager relations

- Original formulation: fluxes and forces

  L. Onsager, Phys. Rev. (1931)
- Microscopic approach: Kubo formula

  N. Nagaosa et al., Rev. Mod. Phys. (2010)

$$\delta H = \mathbf{J}_\beta \cdot \mathbf{A}_\beta \quad \Rightarrow \quad \delta \mathbf{J}_\alpha = \underbrace{< \mathbf{J}_\alpha \mathbf{J}_\beta >}_{K_{\alpha\beta}} \mathbf{A}_\beta$$

Time-reversal: $J_\alpha \to \epsilon_\alpha J_\alpha, \;\; \epsilon_\alpha = \pm 1$

$$K_{\alpha\beta} = \epsilon_\alpha \epsilon_\beta K_{\beta\alpha}$$

$\alpha, \beta =$charge, spin, heat...

Charge vs. spin

$$\alpha = s\,(\text{spin}),\ \beta = c\,(\text{charge})$$

Time-reversal: $\mathbf{J}_s \to \mathbf{J}_s,\ \mathbf{J}_c \to -\mathbf{J}_c \ \Rightarrow\ K_{sc} = -K_{cs}$

Spin-Hall response

$$\delta \mathbf{J}_s = \underbrace{< \mathbf{J}_s \mathbf{J}_c >}_{K_{sc}} \mathbf{A}, \quad \delta \mathbf{J}_c = \underbrace{< \mathbf{J}_c \mathbf{J}_s >}_{-K_{sc}} \boldsymbol{\mathcal{A}}$$

"Spin gauge" dependent reciprocal relations

- Charge conservation: $\mathbf{J}_c \cdot \mathbf{A} = -e\Phi_c$
- Spin non-conservation: $\mathbf{J}_s \cdot \boldsymbol{\mathcal{A}} \neq -\eta\Psi \ \Rightarrow\ \sigma_{sH,A} \neq \sigma_{sH,\Phi}$

"Spin-gauge" $\leftrightarrow$ experimental setup

C. Gorini, R. Raimondi and P. Schwab, arXiv (2012)

Charge vs. spin

$$H = \frac{p^2}{2m} + H_{\mathrm{intr}} + V_{\mathrm{imp}} + H_{\mathrm{extr}}$$

Optical injection (homogeneous)

$$\delta H_1 = \sum_i \left[J_i \, e A_i + J_i^z \, \eta \mathcal{A}_i^z \right]$$

$$J_y = -\sigma_{sH} \, \eta \mathcal{E}_x^z \Leftrightarrow J_x^z = \sigma_{sH} \, e E_y$$

Scalar potentials (inhomogeneous)

$$\delta H_2 = -e\Phi - \eta\Psi$$

$$J_y = -\bar{\sigma}_{sH} \, \eta \mathcal{E}_x^z \Leftrightarrow \bar{J}_x^z = \bar{\sigma}_{sH} \, e E_y$$

J_x^z =standard (non-conserved), $\bar{J}_x^z$ =conserved

The End

To sum up

- Spin-orbit $\Rightarrow$ electrical handle on spin
- Fundamental role of impurities (intrinsic vs. extrinsic)
- Spin-charge Onsager: dependence on experimental setup

Something for the future

- Extension of non-Abelian formalism
- Spin-thermoelectric effects

G. Vignale, J. Supercond. Nov. Magn. (2010)

T. Jungwirth, J. Wunderlich and K. Olejník, Nat. Mat. (2012)

C. Gorini, R. Raimondi and P. Schwab, arXiv (2012)

This&That 1

$$
\begin{aligned}
\mathbf{J}^0 &= -D\Big(\nabla n + 2eN_0\mathbf{E}\Big) - D^a\Big([\tilde{\nabla} s]^a + \frac{\eta N_0}{2}\mathcal{E}^a\Big) + \\
&\quad -\frac{e\tau}{m}\mathbf{J}^0 \wedge \mathbf{B} - \frac{\eta\tau}{m}\mathbf{J}^a \wedge \mathcal{B}^a \\
\mathbf{J}^a &= -D^a\Big(\nabla n + 2eN_0\mathbf{E}\Big) - D\Big([\tilde{\nabla} s]^a + \frac{\eta N_0}{2}\mathcal{E}^a\Big) + \\
&\quad -\frac{e\tau}{m}\mathbf{J}^a \wedge \mathbf{B} - \frac{\eta\tau}{4m}\mathbf{J}^0 \wedge \mathcal{B}^a,
\end{aligned}
$$

$$
D = \text{charge diffusion constant}, \qquad D^a = \frac{D'}{n}s^a
$$

Rashba+disorder

$$\sigma_{sH}(\omega) = -\frac{\gamma\sigma}{e}\left[\frac{-i\omega + 1/\tau_s}{-i\omega + 1/\tau_{DP} + 1/\tau_s}\right]$$

$$\bar{\sigma}^{sH}(\omega) = -\frac{\gamma\sigma}{e}\left[\frac{-i\omega}{-i\omega + 2/\tau_{DP}}\right]\left[\frac{-i\omega - 1/\tau_{DP} + 1/\tau_s}{-i\omega + 1/\tau_{DP} + 1/\tau_s}\right]$$

$$\gamma = \gamma_{\text{intr}} + \gamma_{\text{extr}}, \quad 1/\tau_{DP} = (2m\alpha)^2 D, \quad 1/\tau_s = 1/\tau\,(\lambda p_F/4)^4$$

$$J_x^z = s^z\left(\frac{1}{i\hbar}[x, H]\right), \quad \bar{J}_x^z = \frac{1}{i\hbar}[s^z x, H]$$