Local density of states in quantum Hall systems with a smooth disordered potential landscape

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SUMMARY:

- The quantum Hall effect
- Vortex Green's function theory
- Applications: Local spectroscopy
- Conclusion/Perspectives

ACKNOWLEDGMENTS

Theory part

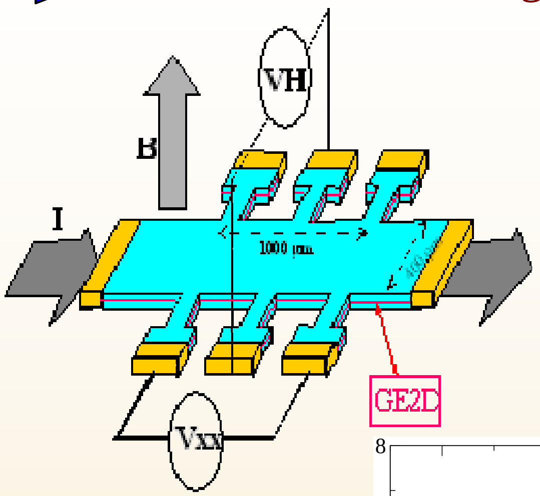
- Serge Florens @ Néel institute – Grenoble/France
- Rudolf Römer @ University of Warwick – Coventry/UK
- Mikhail Raikh @ University of Utah – Salt Lake City/USA
- Jascha Ulrich (Diplomarbeit) @ Aachen University – Aachen/Germany

Experimental part

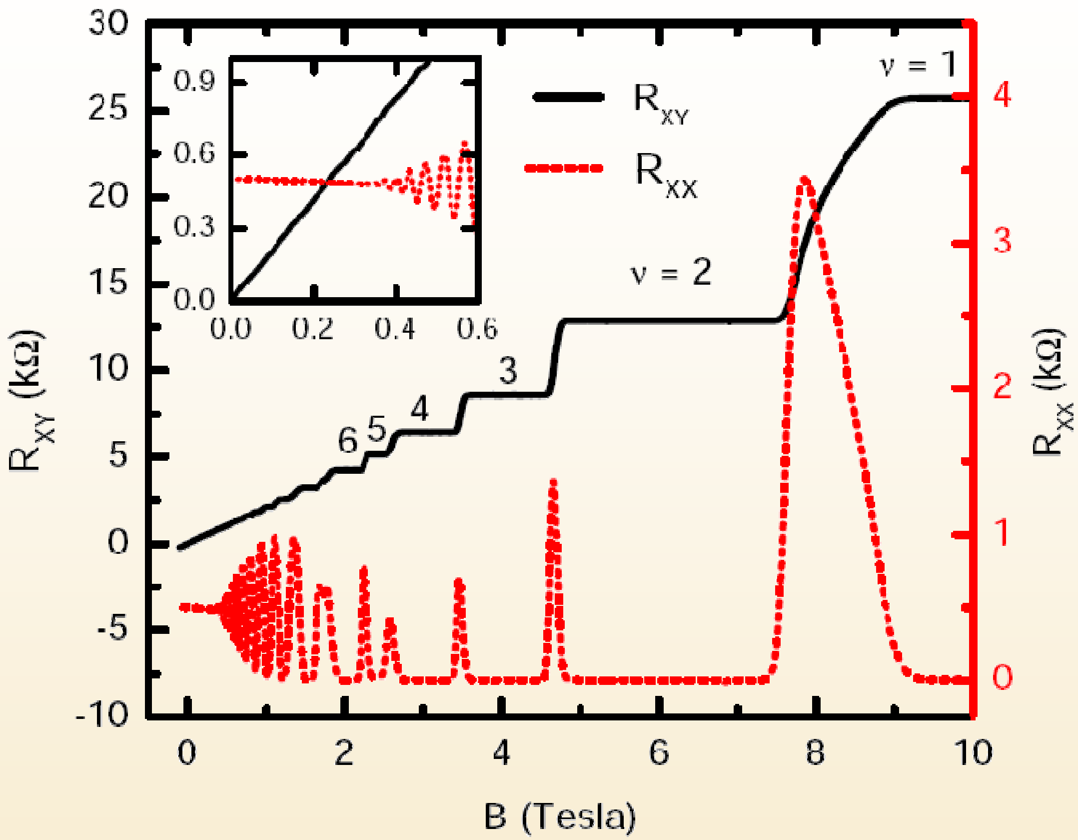
- Katsushi Hashimoto @ Tohoku University – Sendai/Japan
- Markus Morgenstern @ Aachen University – Aachen/Germany

WHY STUDY 2D ELECTRON GASES UNDER MAGNETIC FIELDS NOW?

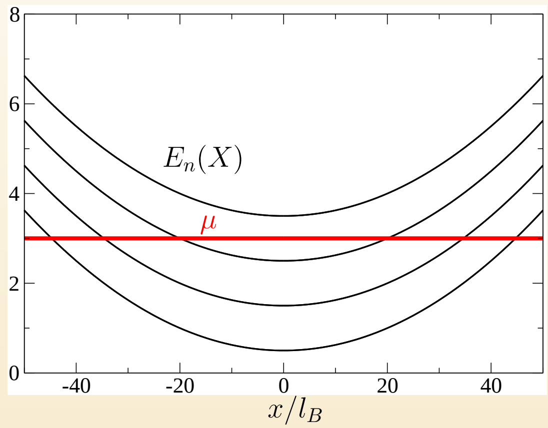
- IQHE :Von Klitzing *et al.* (1980)



T < 1 Kelvin
B > 1 Tesla



Edge states explanation:



- **Experiments since 2000 (far from being exhaustive):**
  - New effects: microwave induced zero-resistance states
  - New probes: local sensing techniques in the IQHE regime
  - New systems: graphene, topological insulators, 2DEG surface states

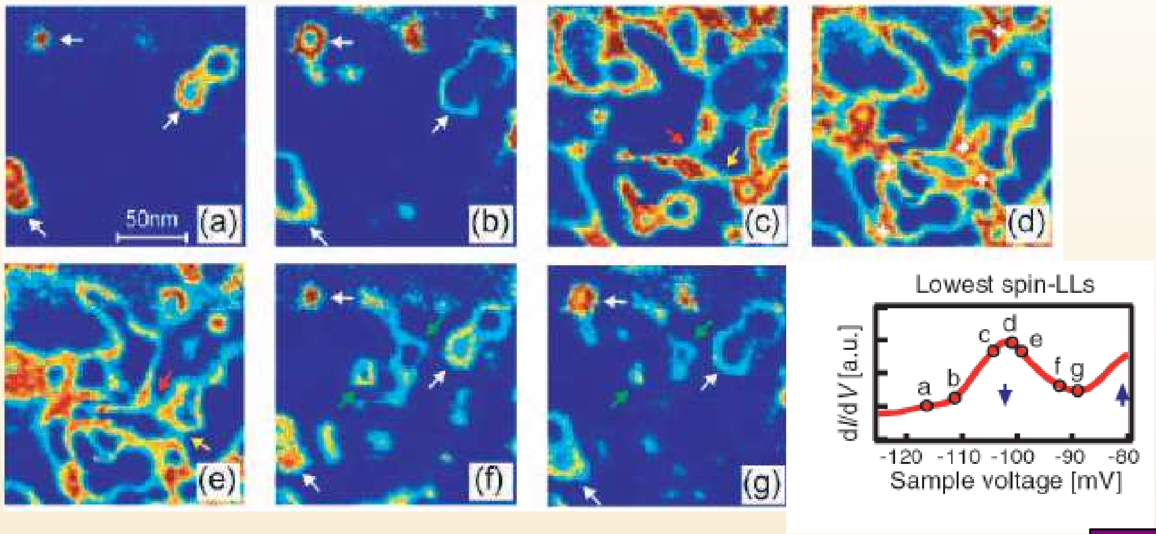
WHY STUDY 2D ELECTRON GASES UNDER MAGNETIC FIELDS NOW?

- STM Experiment: Local DOS in the IQHE regime (InSb surface states) B=12 T

Quantum Hall Transition in Real Space: From Localized to Extended States

K. Hashimoto,[1,2,3,*] C. Sohrmann,[4] J. Wiebe,[1] T. Inaoka,[5] F. Meier,[1,†] Y. Hirayama,[2,3] R. A. Römer,[4] R. Wiesendanger,[1] and M. Morgenstern[6,7]

PRL **101**, 256802 (2008)



(LOWEST LANDAU LEVEL)

- Percolation features
- Broad structures close to saddle points of the potential landscape

Disorder plays an important role!

- Theory:
  - Many fundamental aspects (e.g. for the IQHE) well understood
  - But: how do we calculate stuff? (quantitative **microscopic** theory to develop!)

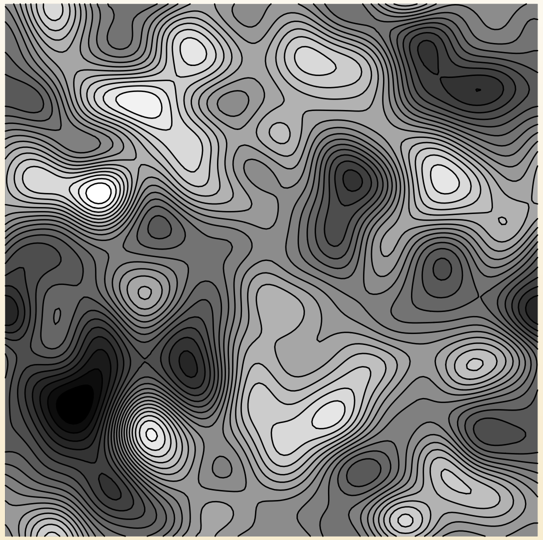
This talk

Goal:

Find an (approximate) analytical solution to the problem

$$H = H_0 + V(\mathbf{r})$$

arbitrary potential energy



$$H_0 = \frac{1}{2m^*}\left(-i\hbar\nabla_{\mathbf{r}} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2$$

$$\nabla \times \mathbf{A}(\mathbf{r}) = B\hat{\mathbf{z}}$$

Theoretical difficulties:

- Disorder averaging is questioned (*at microscopic scale*)

  question of origin of irreversibility and dissipation (crucial for transport)

- We are in a nonperturbative regime at high magnetic fields

  (kinetic energy frozen + degeneracy of Landau levels)

- Smooth disorder (finite correlation length)

  Complexity of diagrammatrics at high magnetic fields (unsolved problem)

  Raikh & Shabhazyan, PRB (1993)

- We are at the border between classical and quantum mechanics

  The wave function as a basic dynamical object is questioned

⇨ Need to develop a new approach/method to tackle the problem

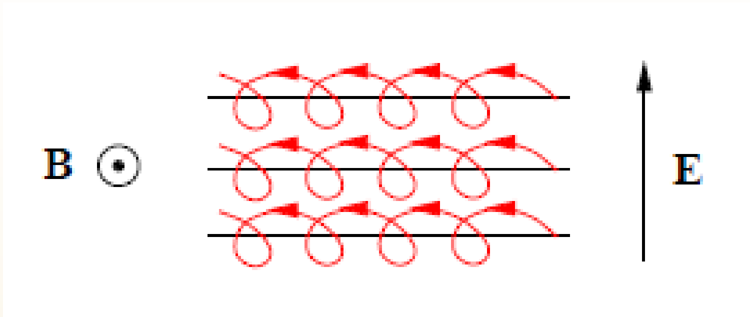
Standard theoretical approaches (I):

Semiclassical limit

CLASSICAL MOTION IN HIGH PERPENDICULAR MAGNETIC FIELD

Two degrees of freedom with very different timescales

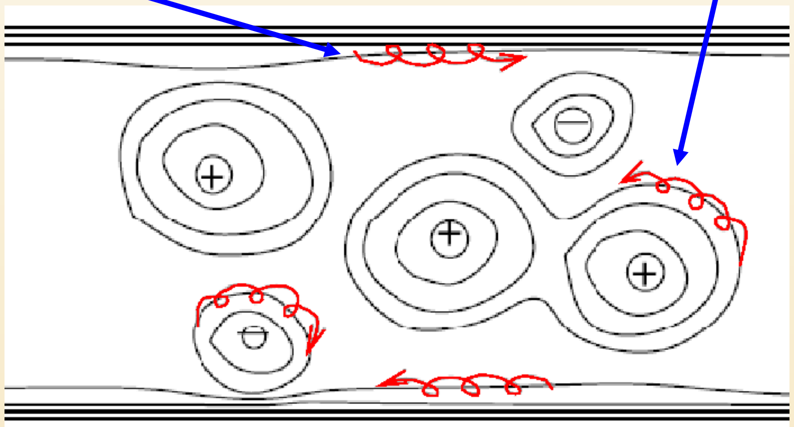
- fast cyclotron motion: $\dot{\theta} = \omega_c = |e|B/(m^* c)$
- slow drift: $\mathbf{v}_d = \frac{1}{B}\mathbf{E} \times \hat{\mathbf{z}}$

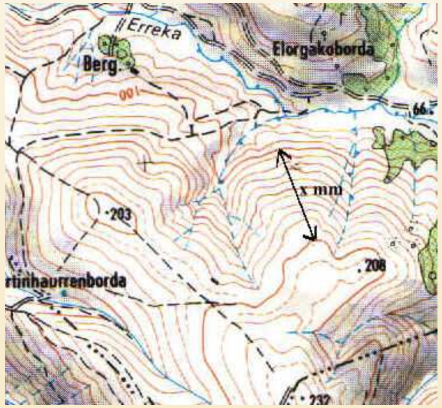


▶ *Decoupling in the limit* $B \to \infty$

Edges: delocalized skipping orbits

Disordered bulk: localization on closed equipotential lines





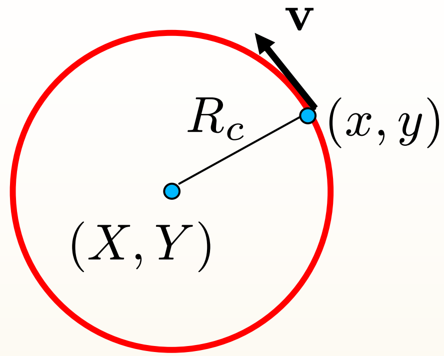
Remark: motion regular and integrable in the limit $B \to \infty$!

Averaging over disordered potential configurations questionable here!

SEMICLASSICAL MOTION : THE GUIDING CENTER PICTURE

change in variables:

$$\begin{cases} x = X + \zeta = X + v_y/\omega_c \\ y = Y + \eta = Y - v_x/\omega_c \end{cases} \qquad (x, p_x), (y, p_y) \to (X, Y), (\zeta, \eta)$$

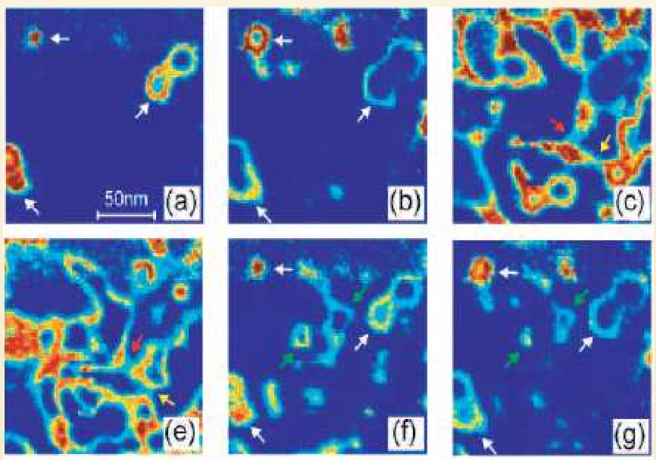


$$H = \frac{1}{2} m^* \mathbf{v}^2 + V(X + \zeta, Y + \eta)$$

then quantization $\begin{cases} [\hat{v}_x, \hat{v}_y] = -i\hbar\omega_c/m^* \\ [\hat{X}, \hat{Y}] = i l_B^2 \end{cases}$

Semiclassical high field picture (V smooth): $\boxed{l_B^2 = \hbar c/(|e|B) \to 0}$ $\quad [\hat{X}, \hat{Y}] \to 0$

(*X* and *Y* treated as classical variables)



PRL **101**, 256802 (2008)

Hashimoto *et al.*, (2008)

Effective energy: $E_{n,\mathbf{R}} = \hbar\omega_c(n + 1/2) + V(\mathbf{R})$

Limitations:

- *No quantization of energies (e.g. in quantum dot)*
- *No transverse spread + no tunneling effects (e.g. in QPC)*
- *Problems to formulate a consistent transport theory*
- *Captures only the high temperature regime*

LDoS in the IQHE regime follows potential landscape

but quantum percolation features

MOTIVATION FOR A HIGH MAGNETIC FIELD EXPANSION

At large magnetic field:

- Magnetic length $l_B = 8 \text{ nm at } 10 \text{ T}$
- Correlation length of the disordered potential in heterostructures: $\xi \geq 100 \text{ nm}$

$$\boxed{\text{The random potential is smooth on the scale } l_B}$$

The idea of using l_B/ξ *as a small parameter is not new. The real challenge is to go beyond the strict limit* $l_B/\xi = 0$ *!*

Some attempts:

- *Effective Hamiltonian theory*
  - Haldane & Yang, PRL (1997)
  - Apenko & Lozovik, J. of Phys. C (1984)

  - limited to energy
  - includes only virtual transitions = no Landau-level mixing taken into account

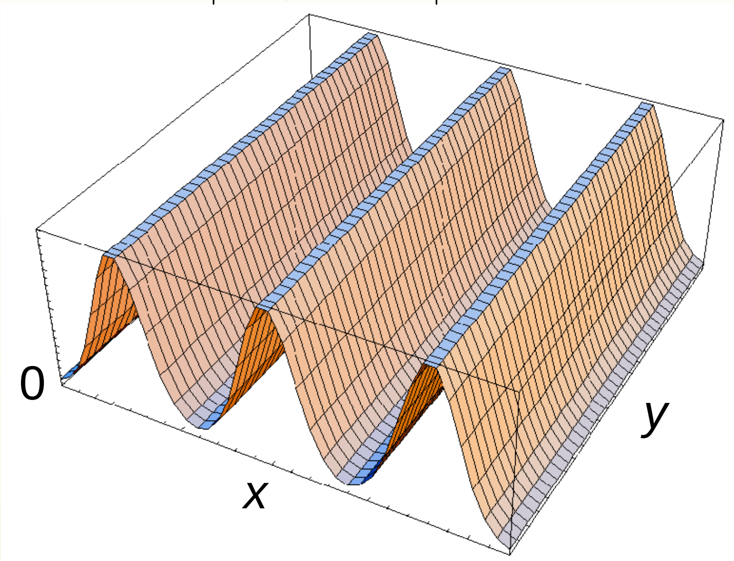
Standard theoretical approaches (II):

Wave functions

TRANSLATION INVARIANT LANDAU STATES

$$H_0 = \frac{1}{2m^*}\left(-i\hbar\nabla_{\mathbf{r}} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2 \quad \text{with} \quad \nabla \times \mathbf{A}(\mathbf{r}) = B\hat{\mathbf{z}}$$

$\left|\Psi_{n,k}(x,y)\right|^2$



Landau states: take $\mathbf{A}(\mathbf{r}) = xB\hat{\mathbf{y}}$

$$E_{n,k} = \hbar\omega_c(n + \tfrac{1}{2})$$

$$\Psi_{n,k}(x,y) = e^{iky}\exp\left[-\frac{(x-kl_B^2)^2}{2l_B^2}\right] H_n\left(\frac{x-kl_B^2}{l_B}\right)$$

Landau (1930)

- Translationally invariant along *y*
- Localized on the scale l_B along *x*

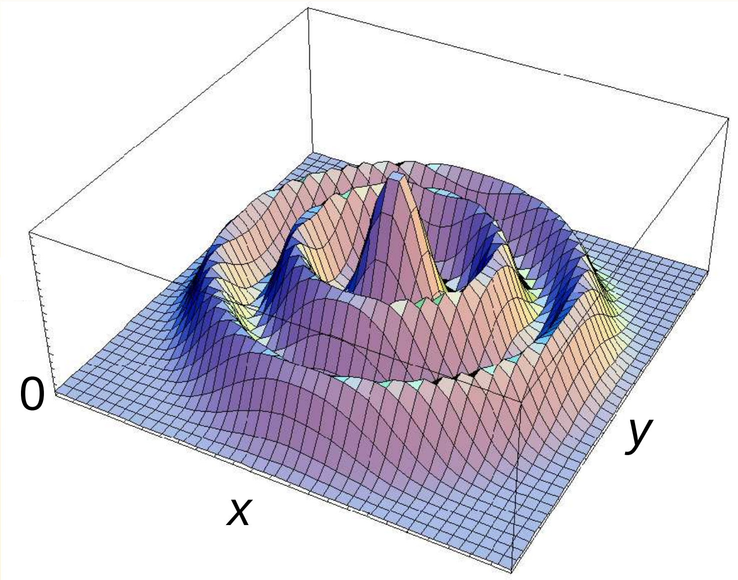
Remarks:

- **Huge degeneracy of Landau levels**
- Magnetic field enters in wave functions only via $l_B = \sqrt{\hbar c/|e|B}$
- Landau states problematic for quantum/classical correspondence

CIRCULARLY INVARIANT STATES

- Other possible eigenstates of H_0

$\left|\Psi_{l,m}(r,\theta)\right|^2$



Circular states: take $\mathbf{A}(\mathbf{r}) = \mathbf{B} \times \mathbf{r}/2$

$$E_{l,m} = \hbar\omega_c\left(l + \frac{|m|+m+1}{2}\right)$$

$$\Psi_{l,m}(r,\theta) = r^{|m|} \exp\left[\frac{-r^2}{4l_B^2}\right] L_l^{|m|}\left(\frac{r^2}{2l_B^2}\right) e^{im\theta}$$

- Rotationally invariant around the origin
- Localized on a scale l_B along r

Remark:

- States still problematic for quantum/classical correspondence at $l_B \to 0$

DIGRESSION ON THE LANDAU LEVEL INDEX

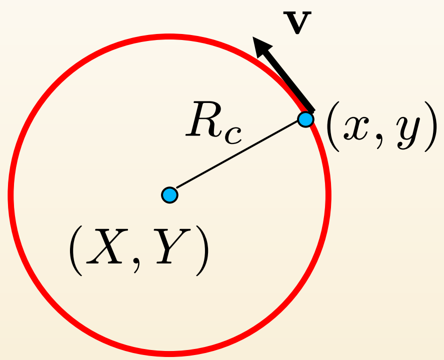
What is the physical meaning of the Landau level index *n*?

- Translation-invariant states: *n* comes from $|\Psi|^2 \to 0$ (degree of Hermite polynomial)

- Rotation-invariant states: $n = l + \frac{|m|+m}{2}$ angular momentum (single-valuedness)

  degree of Laguerre polynomial

- A semi-classical answer:



$$
\begin{aligned}
E &= \frac{1}{2} m^* v^2 = \frac{1}{2} m^* (R_c \omega_c)^2 \\
&= \hbar\omega_c \frac{R_c^2 m^* \omega_c}{2\hbar} = \hbar\omega_c \frac{\Phi_c}{\Phi_0}
\end{aligned}
$$

$\Phi_c = \pi R_c^2 B$

$\Phi_0 = hc/|e|$

$\omega_c = |e|B/(m^* c)$

To compare with $E = \hbar\omega_c \left(n + \frac{1}{2}\right)$ $\Longrightarrow$ $\boxed{\Phi_c = n\,\Phi_0}$

Cyclotron Flux is quantized

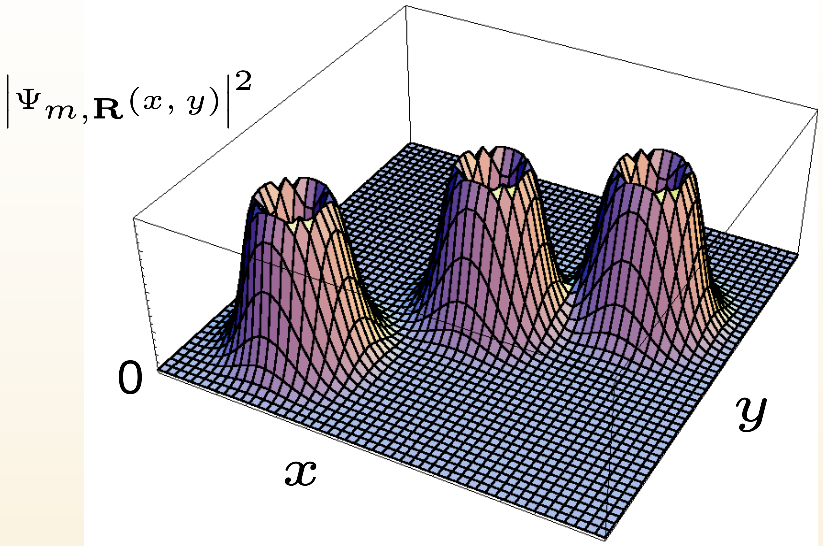
- Can we build the right basis of eigenstates where *n* is only related to the accumulated phase?

YES : the vortex states basis

Champel & Florens, PRB (2007)

VORTEX (SEMI-COHERENT) EIGENSTATES

- Other possible eigenstates of H_0



Vortex states: $\Psi_{m,\mathbf{R}}(\mathbf{r}) = \langle \mathbf{r} | m, \mathbf{R} \rangle$

$$\Psi_{m,\mathbf{R}}(\mathbf{r}) = |\mathbf{r}-\mathbf{R}|^m \, e^{im \arg(\mathbf{r}-\mathbf{R})} \times \exp\left[-\frac{(\mathbf{r}-\mathbf{R})^2 - 2i\hat{\mathbf{z}}\cdot(\mathbf{r}\times\mathbf{R})}{4l_B^2}\right]$$

$$E_{m,\mathbf{R}} = \hbar\omega_c(m+\tfrac{1}{2})$$

Overcomplete semicoherent states basis:

$$\begin{cases} \langle m_1, \mathbf{R_1} | m_2, \mathbf{R_2} \rangle = \delta_{m_1,m_2} \overbrace{\exp\left[-\frac{(\mathbf{R}_1-\mathbf{R}_2)^2 - 2i\hat{\mathbf{z}}\cdot(\mathbf{R}_1\times\mathbf{R}_2)}{4l_B^2}\right]}^{\langle \mathbf{R}_1 | \mathbf{R}_2 \rangle} \\ \displaystyle\sum_{m=0}^{+\infty} \int \frac{d^2\mathbf{R}}{2\pi l_B^2} |m,\mathbf{R}\rangle\langle m,\mathbf{R}| = 1 \end{cases}$$

Remarks:

- **States OK for quantum/classical correspondence**
- **States with no preferred symmetry: can adapt to arbitrary V(r)**

Semi-coherent vortex states Green's function formalism

Champel, Florens & Canet, PRB (2008)
Champel & Florens, PRB (2009)
Champel & Florens, PRB (2010)

THEORY: VORTEX GREEN'S FUNCTIONS

Our approach: electron dynamics projected in the vortex representation

▶ Vortex states: $\Psi_{m,\mathbf{R}}(\mathbf{r}) = \langle \mathbf{r} | m, \mathbf{R} \rangle$

$$\boxed{\sum_{m=0}^{+\infty} \int \frac{d^2\mathbf{R}}{2\pi l_B^2} |m, \mathbf{R}\rangle\langle m, \mathbf{R}| = 1}$$

Exact result:

$$G(\mathbf{r}, \mathbf{r}', \omega) = \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_{m_1} \sum_{m_2} K_{m_1;m_2}(\mathbf{R}, \mathbf{r}, \mathbf{r}') \, \tilde{g}_{m_1;m_2}(\mathbf{R}, \omega)$$

to determine

known $\quad = e^{-\frac{l_B^2}{4}\Delta_{\mathbf{R}}} \left[\Psi^*_{m_2,\mathbf{R}}(\mathbf{r}') \Psi_{m_1,\mathbf{R}}(\mathbf{r}) \right]$ (cyclotron motion)

▶ « Vortex Dyson's » equation:

$$\boxed{(\omega - E_{m_1} + i0^+) \tilde{g}_{m_1;m_2}(\mathbf{R}, \omega) = \delta_{m_1,m_2} + \sum_{m_3} \tilde{v}_{m_1;m_3}(\mathbf{R}) \star \tilde{g}_{m_3;m_2}(\mathbf{R}, \omega)}$$

with the **star-product** $\star = \exp\left[i \frac{l_B^2}{2} \left(\overleftarrow{\partial}_X \overrightarrow{\partial}_Y - \overleftarrow{\partial}_Y \overrightarrow{\partial}_X \right) \right]$

connection to the deformation (Weyl) quantization theory

THEORY: VORTEX GREEN'S FUNCTIONS

High magnetic field regime: $(\omega_c \to \infty$ while keeping l_B finite$)$ *LL mixing negligible*

Effective potential

$$\boxed{(\omega - E_m + i0^+)\tilde{g}_m(\mathbf{R}) = 1 + \tilde{v}_m(\mathbf{R}) \star \tilde{g}_m(\mathbf{R})}$$

$$\star = \exp\left[i\frac{l_B^2}{2}\left(\overleftarrow{\partial}_X \overrightarrow{\partial}_Y - \overleftarrow{\partial}_Y \overrightarrow{\partial}_X\right)\right]$$

Trivial for 1D potentials: $\tilde{g}_m(\mathbf{R}) = [\omega - E_m - \tilde{v}_m(\mathbf{R}) + i0^+]^{-1}$

Trugman, PRB (1983)
Raikh & Shahbazayan, PRB (1995)

Some non trivial questions: *- How to get quantized energies for a closed system?*

- How to get tunneling effects in QPC?

$$V(\mathbf{R}) = V(\mathbf{R}_0) + [\mathbf{R} - \mathbf{R}_0] \cdot \nabla V(\mathbf{R}_0) + \frac{1}{2}\left[(\mathbf{R} - \mathbf{R}_0) \cdot \nabla\right]^2 V(\mathbf{R}_0)$$

everything is encoded in quadratic (curvature) terms!

Dyson's equation up to second-order derivatives of V:

$$1 = \left[\omega - E_m - V(\mathbf{R}) - \frac{2m+1}{4} l_B^2 \Delta_{\mathbf{R}} V + i0^+\right] \tilde{g}_m(\mathbf{R})$$
$$+ \frac{l_B^4}{8}\left[\partial_Y^2 V \partial_X^2 + \partial_X^2 V \partial_Y^2 - 2\partial_X \partial_Y V \partial_X \partial_Y\right] \tilde{g}_m(\mathbf{R})$$

This ugly equation can be exactly solved!

Champel & Florens, PRB (2009)

EXACT SOLUTION FOR ANY QUADRATIC POTENTIAL

Solution (m=0):

$$\boxed{\tilde{g}_m(\mathbf{R}) = -i\int_0^{+\infty} dt\, \frac{e^{i\frac{\eta(\mathbf{R})}{\gamma}\left[t-\tan(\sqrt{\gamma}t)/\sqrt{\gamma}\right]}}{\cos(\sqrt{\gamma}t)}\, e^{it\left[\omega - V(\mathbf{R}) - l_B^2 \Delta V(\mathbf{R})/4 + i0^+\right]}}$$

where

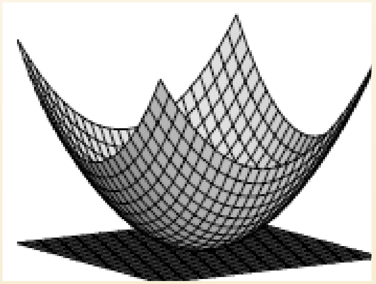
$$\gamma = \frac{l_B^4}{4}\left[\partial_{XX}V\partial_{YY}V - (\partial_{XY}V)^2\right]$$

Related to the Gaussian curvature of V

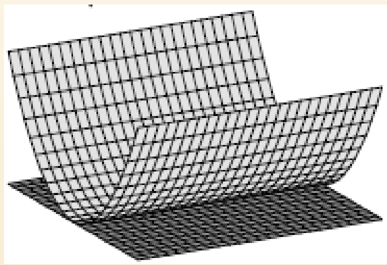
$$\eta(\mathbf{R}) = \frac{l_B^4}{8}\left[\partial_{XX}V(\partial_Y V)^2 + \partial_{YY}V(\partial_X V)^2 - 2\partial_{XY}V\partial_X V\partial_Y V\right]$$

Solution embraces all possible cases of quadratic potentials

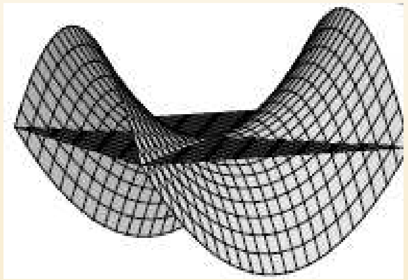
Stability of vortex quantum numbers



$\gamma > 0$



$\gamma = 0$



$\gamma < 0$

Solution periodical in time

$\Rightarrow$ energy quantization

Solution with lifetime

$\Rightarrow$ tunneling and dissipation

Open and closed quantum mechanics unified!

Champel & Florens, PRB (2009)

Applications of vortex formalism (I):

Local density of states

Champel & Florens, PRB Rapid Com (2009)
Champel & Florens, PRB (2009)
Champel & Florens, PRB (2010)
Hashimoto, Champel, Florens, *et al.*, PRL (2012)

LOCAL DENSITY OF STATES

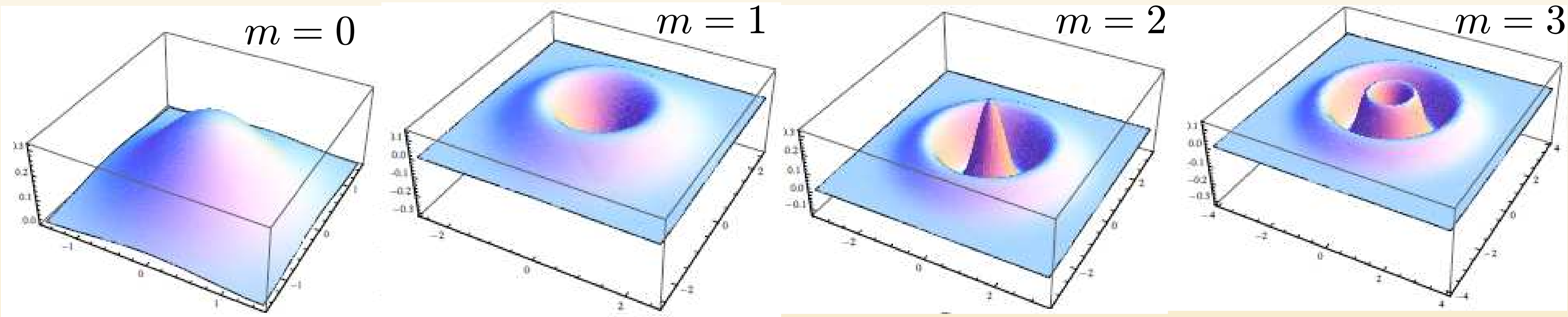
- Vortex view of LDoS at high field: $\tilde{g}_{m,m} = \tilde{g}_m \, \delta_{m,m}$

$$\rho(\mathbf{r}, E) = -\frac{1}{\pi} \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_{m=0}^{+\infty} F_m(\mathbf{R} - \mathbf{r}) \, \mathrm{Im} \, \tilde{g}_m(\mathbf{R}, E)$$

cyclotron orbit

guiding center drifting

with structure factor: $F_m(\mathbf{R}) = \dfrac{(-1)^m}{\pi l_B^2} L_m \left(\dfrac{2\mathbf{R}^2}{l_B^2} \right) e^{-\mathbf{R}^2/l_B^2}$ can be negative (Wigner's distribution)



- Lowest order result for vortex Green's function (local 1D drift):

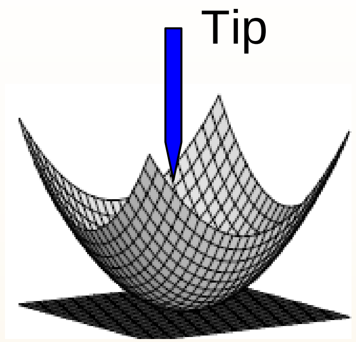
$$-\frac{1}{\pi} \mathrm{Im} \, \tilde{g}_m(\mathbf{R}, E) = \delta[E - E_m - \tilde{v}_m(\mathbf{R})]$$

for smooth potential and lowest LL

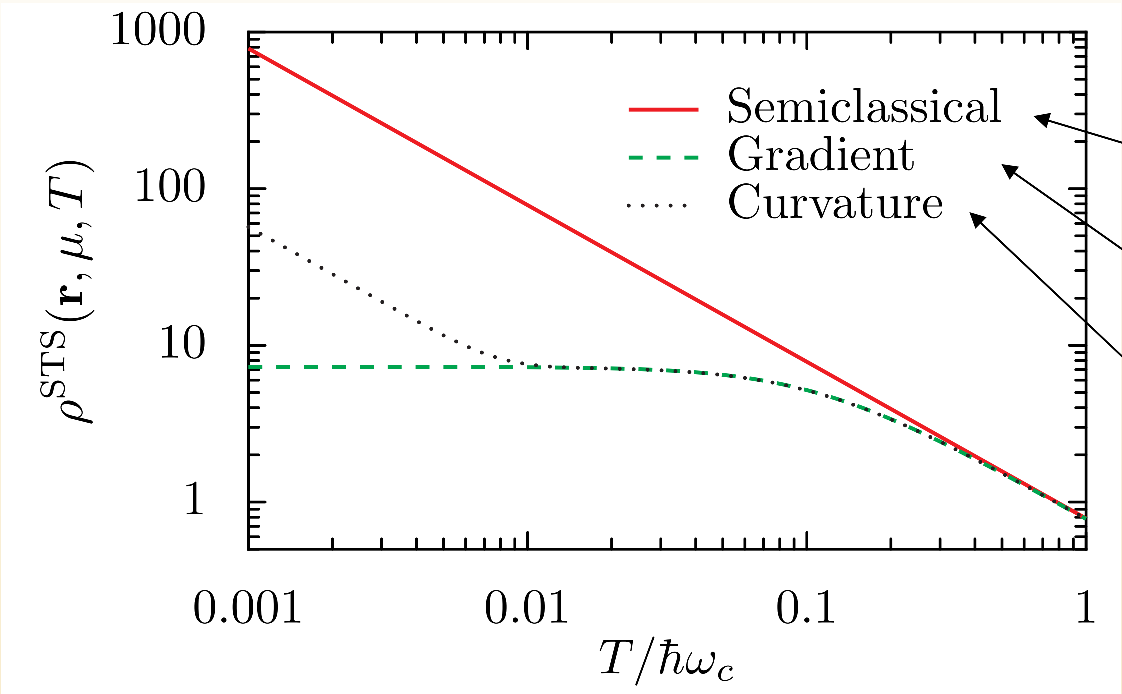
$$= \int d^2\mathbf{r} F_m(\mathbf{R} - \mathbf{r}) V(\mathbf{r}) \approx V(\mathbf{R})$$

HIERARCHY OF LOCAL ENERGY SCALES IN VORTEX REPRESENTATION

- LDoS for the lowest Landau level at the center (r = 0) within different approximation schemes



<u>Example:</u> $$V(\mathbf{r}) = \frac{U_0}{2} r^2$$



$$\rho(\mathbf{r}, \mu, T) = -\frac{1}{2\pi l_B^2} n_F'[V(\mathbf{r})]$$

$\gamma = \eta = 0$ (lowest order, curvature neglected)

exact expression

Existence of a hierarchy of local energy scales ⇒

controlled theory for a smooth arbitrary potential !

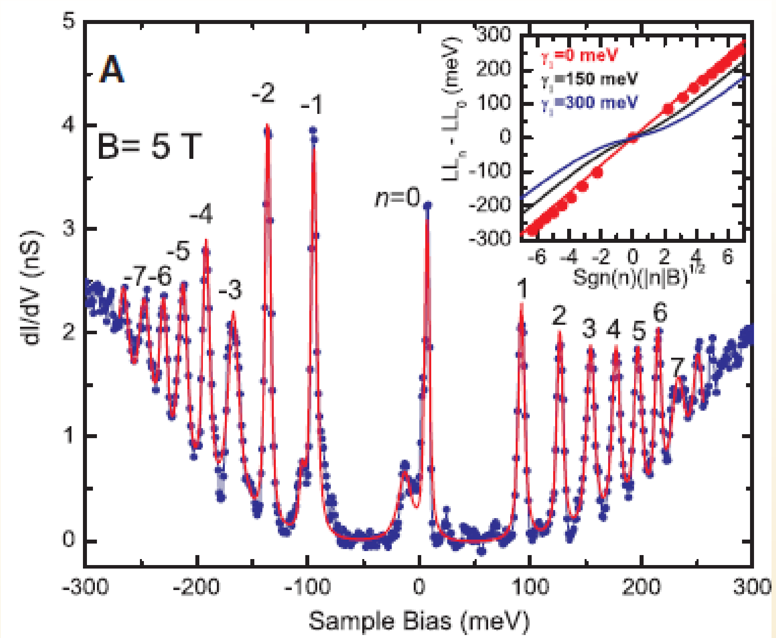
Champel & Florens, PRB (2009)
Champel & Florens, PRB (2010)

APPLICATION: LDOS IN GRAPHENE

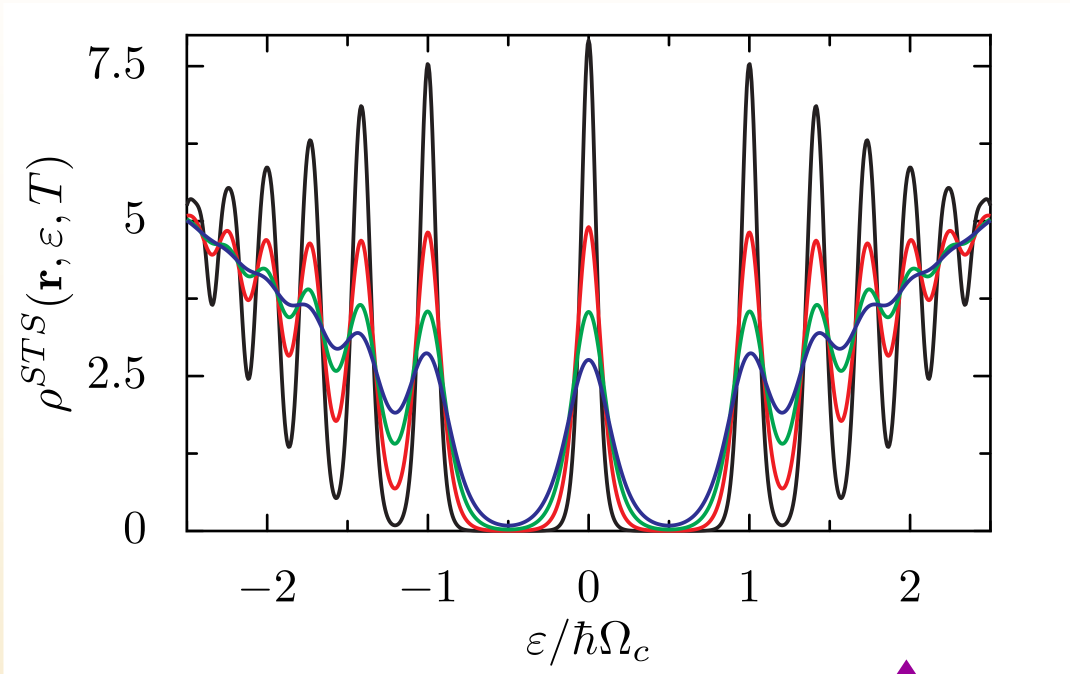
graphene

Experiment (at fixed Temp)

Theory: LDoS for different temperatures (with lowest order vortex Green's function)



Miller *et al.*, Science (2009)



Champel & Florens, PRB (2010)

at fixed tip position

Experimental features captured by theory:

- the width of the LDoS peaks at fixed tip position grows roughly as $\sqrt{m}$
- the heights of the LDoS peaks decrease with m

Origin: wave function broadening

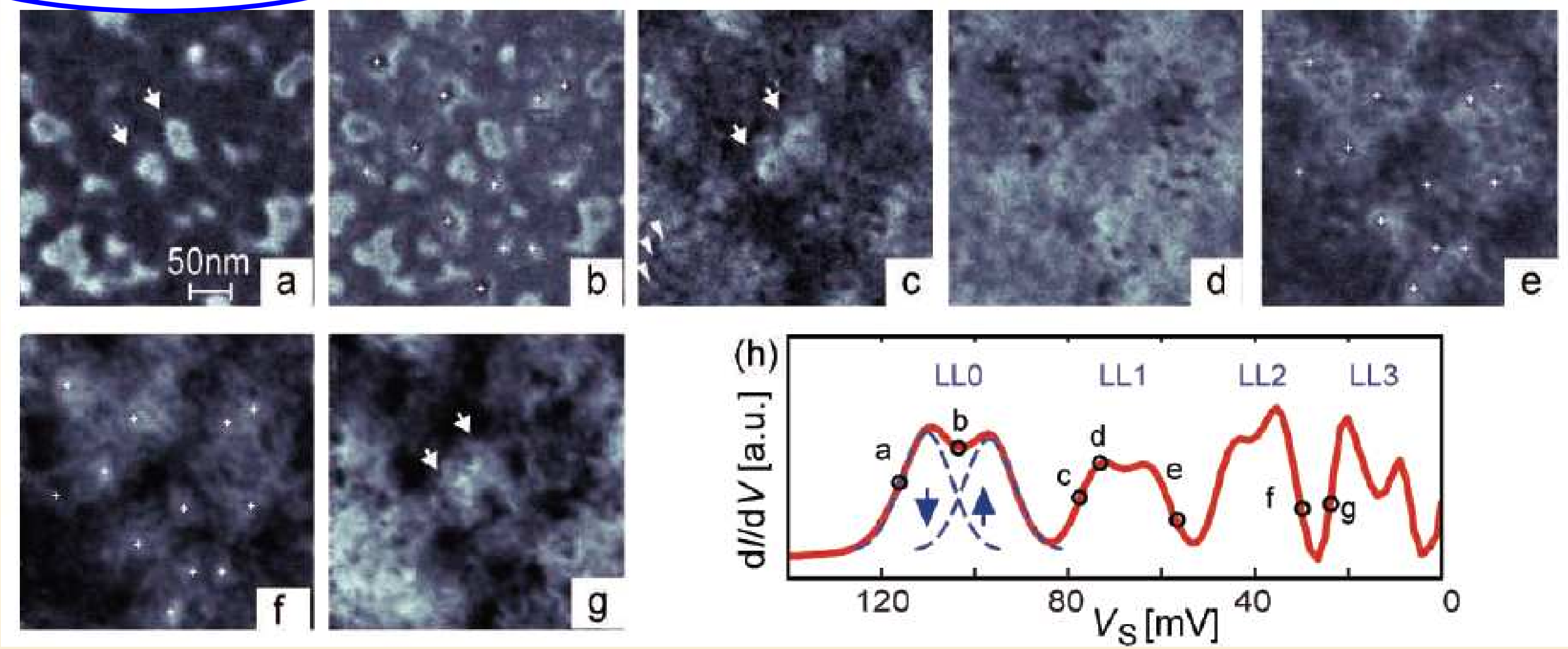
What about spatial dependence of LDOS?

REAL SPACE LDOS DATA

InSb surface states

B=6 T

Hashimoto, Champel, Florens *et al.*, PRL (2012)



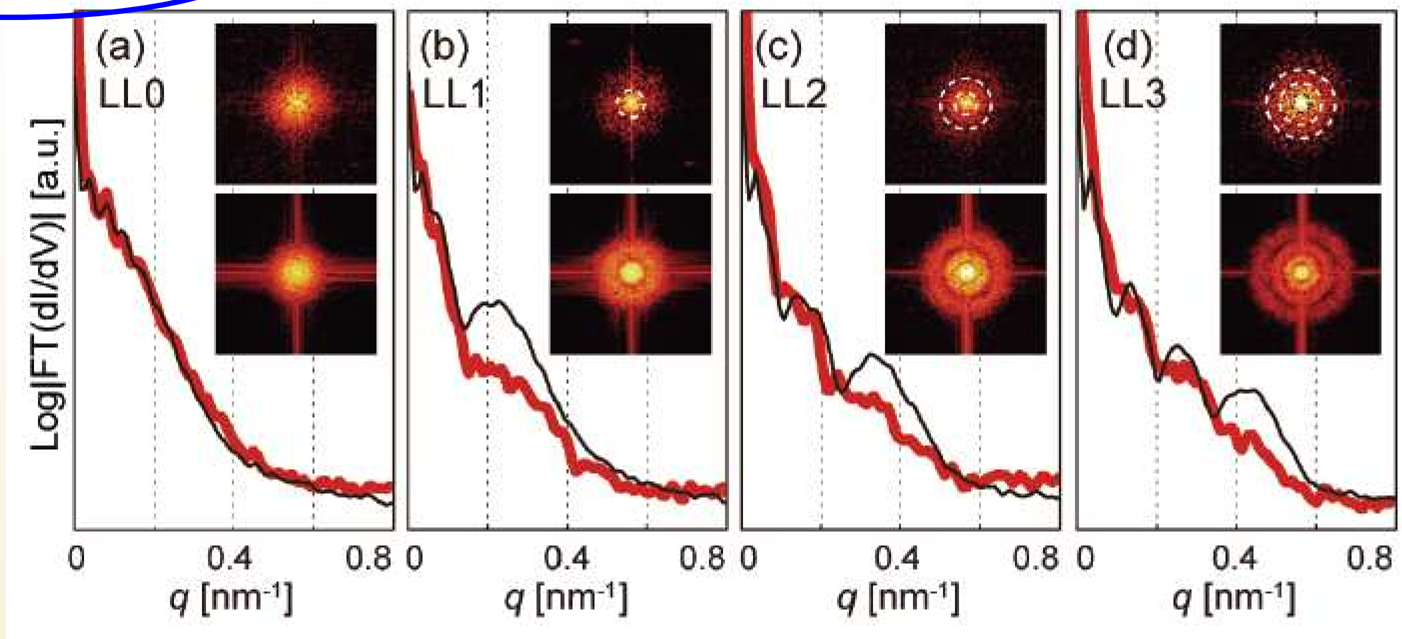
- 4 successive LLs are observed (spin resolved)
- The drift trajectories are blurred in the high LLs
  … but no obvious signature of the nodal structure associated to cyclotron motion

MOMENTUM-SPACE LDOS DATA

InSb surface states

B=6 T

Hashimoto, Champel, Florens *et al.*, PRL (2012)



- Structures appear at scale $1/l_B \approx 0.1\,\mathrm{nm}^{-1}$
- LLn shows n kinks in the momentum-dependence
- Good comparison experiment/simulations

REVEALING THE NODAL STRUCTURE OF CYCLOTRON MOTION

LDOS (theory):

Guiding-center spectral density (disorder-dependent, length scale ~ 50 nm)

$$\rho(\mathbf{r},E) = \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_{m=0}^{+\infty} F_m(\mathbf{R}-\mathbf{r}) \;\; A_m(\mathbf{R};E)$$

for B=6 T

Structure factor for cyclotron motion
(length scale l_B ~ 10 nm)

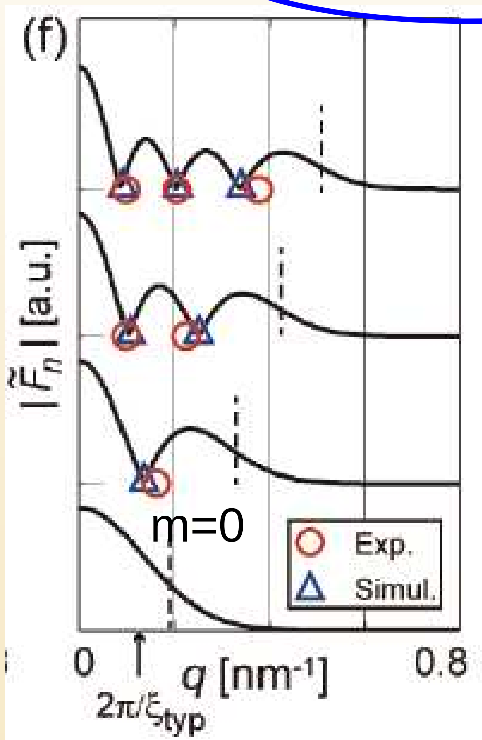
Deconvolution in Fourier space:

InSb surface states

$$\tilde{\rho}(\mathbf{q},E) = \sum_{n=0}^{+\infty} \tilde{F}_m(\mathbf{q}) \;\; \tilde{A}_m(\mathbf{q};E)$$

where $\quad \tilde{F}_n(\mathbf{q}) = L_m\left(\frac{l_B^2\mathbf{q}^2}{2}\right) e^{-l_B^2\mathbf{q}^2/4}$

Kinks of $\tilde{\rho}(\mathbf{q},E)$ follow the nodes of $\tilde{F}_m(\mathbf{q})$



m=3

m=2

m=1

- The nodal structure of LLs is robust to disorder
- Key property of quantum Hall states!

Hashimoto, Champel, Florens *et al.*, PRL (2012)

Other Applications of vortex formalism (II):

Averaged density of states and LDOS correlations

Champel & Florens, PRB (2010)
Champel, Florens & Raikh, PRB (2011)
Ulrich, Florens & Champel, in preparation (2012)

SAMPLE AVERAGED LDOS

Disorder correlator (Gaussian)

$$\langle V(\mathbf{R}_1)V(\mathbf{R}_2)\rangle = v^2\, e^{-\frac{|\mathbf{R}_1-\mathbf{R}_2|^2}{\xi^2}}$$

DOS with lowest order vortex result:

$$\langle \rho(\mathbf{r},\omega)\rangle = \frac{1}{2\pi l_B^2}\sum_{m=0}^{+\infty}\frac{e^{-\left(\frac{\omega-E_m}{\Gamma_m}\right)^2}}{\sqrt{\pi}\Gamma_m}$$

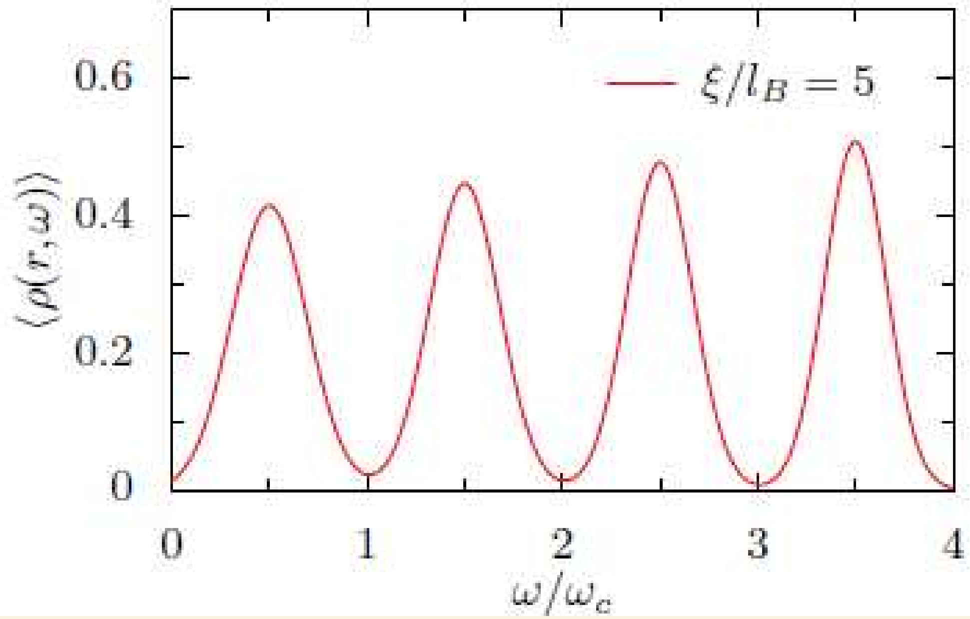
with

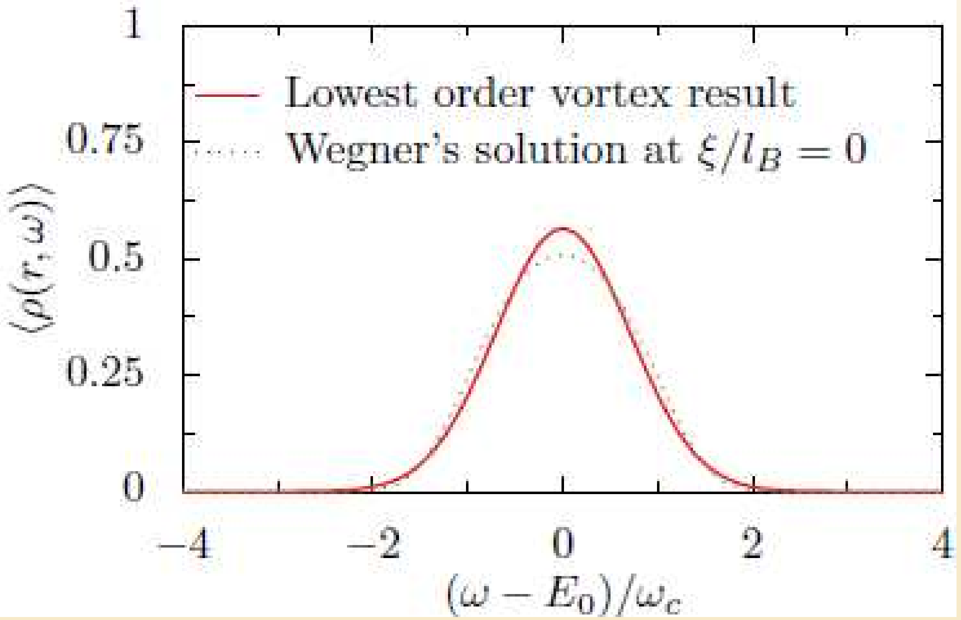
$$\Gamma_m^2 = \frac{\pi}{2}v^2\xi^2 l_B^4\int d^2\mathbf{q}\left[\tilde{F}_m\left(\frac{l_B^2\mathbf{q}}{2}\right)\right]^2 e^{-\frac{\xi^2 q^2}{4}}$$

- The lowest order Green's function is exact at $\xi \gg l_B$

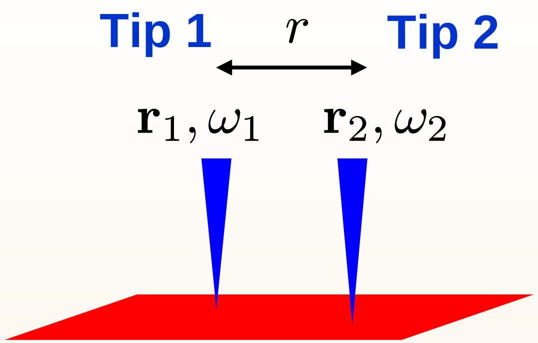
- Opposite limit $\xi \ll l_B$ analytically solved by Wegner

➡ most stringent test!





LDOS CORRELATIONS (I)

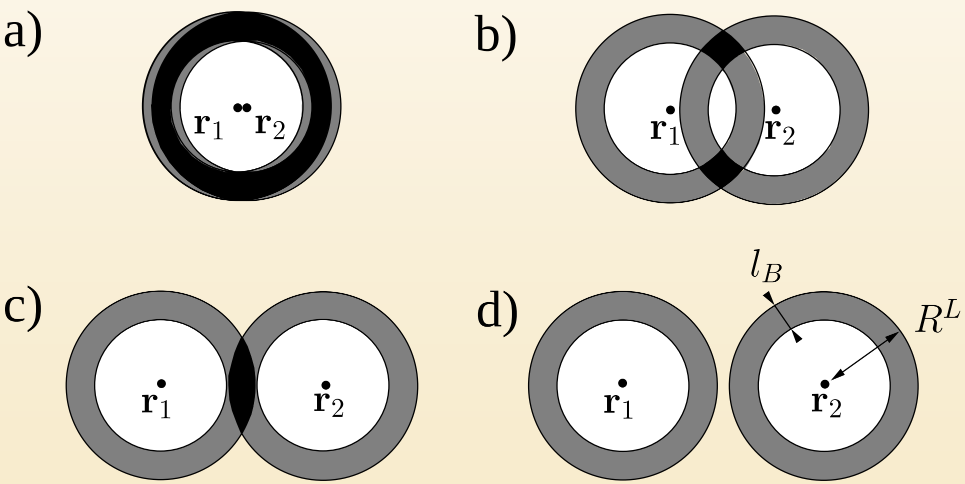


Two-point LDoS correlator:

Perform the following sample averaging of LDOS-LDOS signal

$$\chi(r, \omega_1, \omega_2) = \langle \rho(\mathbf{r}_1, \omega_1) \rho(\mathbf{r}_2, \omega_2) \rangle - \langle \rho(\mathbf{r}_1, \omega_1) \rangle \langle \rho(\mathbf{r}_2, \omega_2) \rangle$$

- Geometrical interpretation: overlap of quantum rings



Area for c) > Area for b)

$\Rightarrow$ χ peaks again for $r \approx 2R_c$

Robust way to reveal some nodes in real space?

LDOS CORRELATIONS (II)

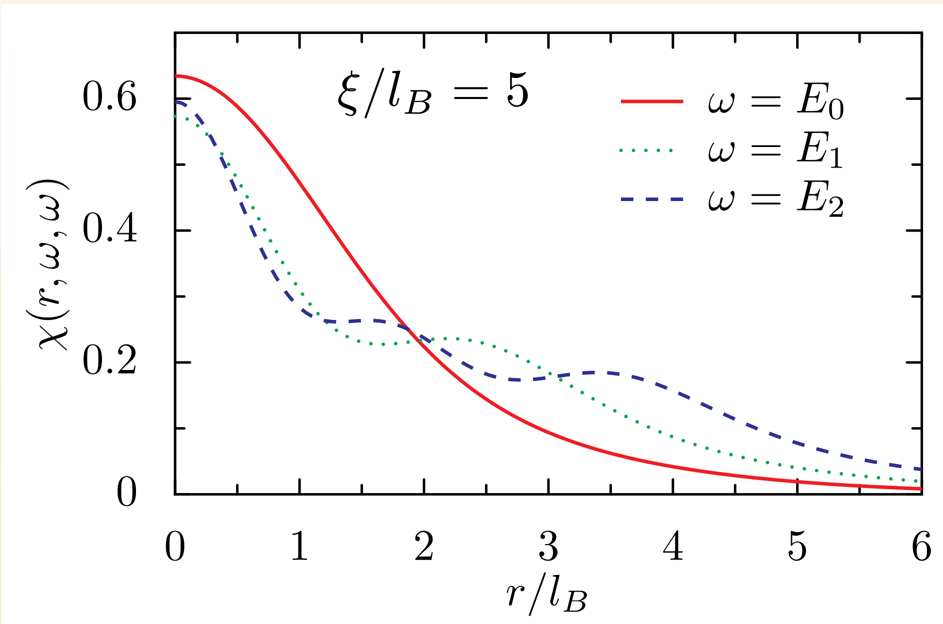
Procedure for computation

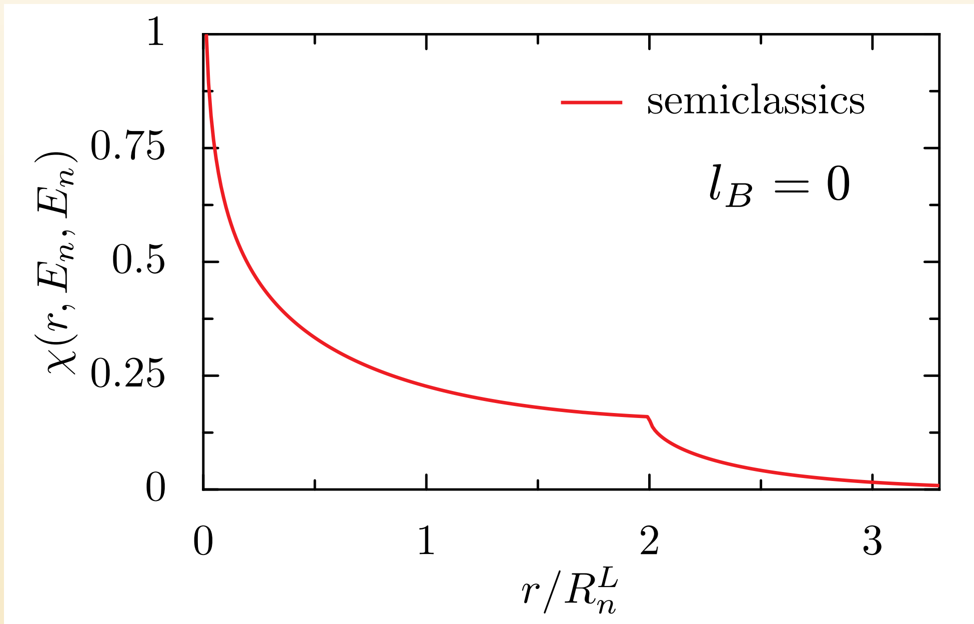
$$\langle \rho(\mathbf{r}_1,\omega_1)\rho(\mathbf{r}_2,\omega_2)\rangle = \int \frac{d^2\mathbf{R}_1}{2\pi l_B^2}\int \frac{d^2\mathbf{R}_2}{2\pi l_B^2}\sum_{m_1=0}^{+\infty}\sum_{m_2=0}^{+\infty} F_{m_1}(\mathbf{R}_1-\mathbf{r}_1)F_{m_2}(\mathbf{R}_2-\mathbf{r}_2)$$

$$\times \int \frac{dt_1}{2\pi}\int \frac{dt_2}{2\pi}\, e^{i(\omega_1-E_{m_1})t_1+i(\omega_2-E_{m_2})t_2}\left\langle e^{-i\left[\tilde{v}_{m_1}(\mathbf{R}_1)t_1+\tilde{v}_{m_2}(\mathbf{R}_2)t_2\right]}\right\rangle$$

this can be done analytically!

- Spatial dependence confirms previous expectations

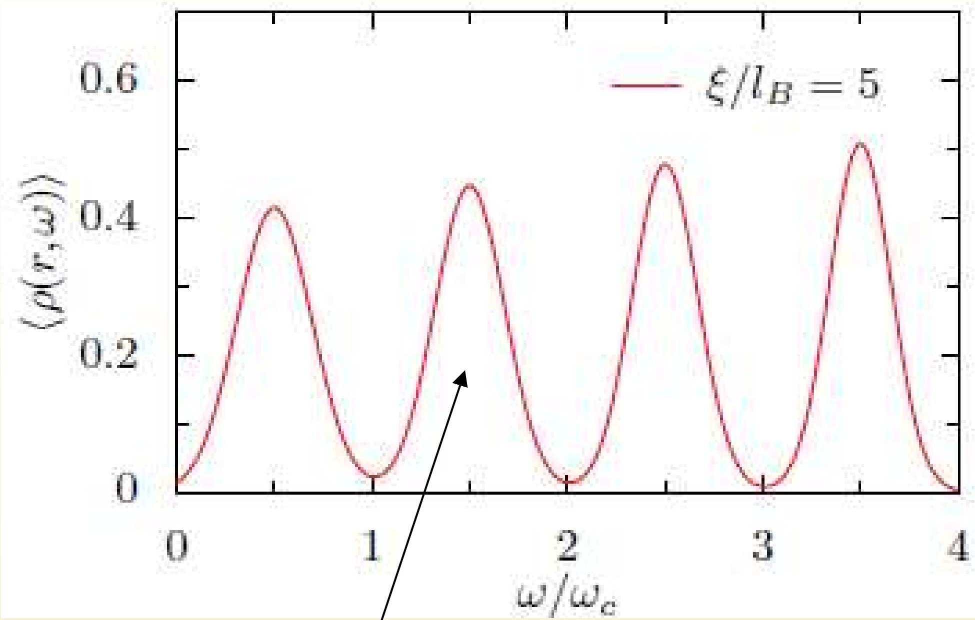


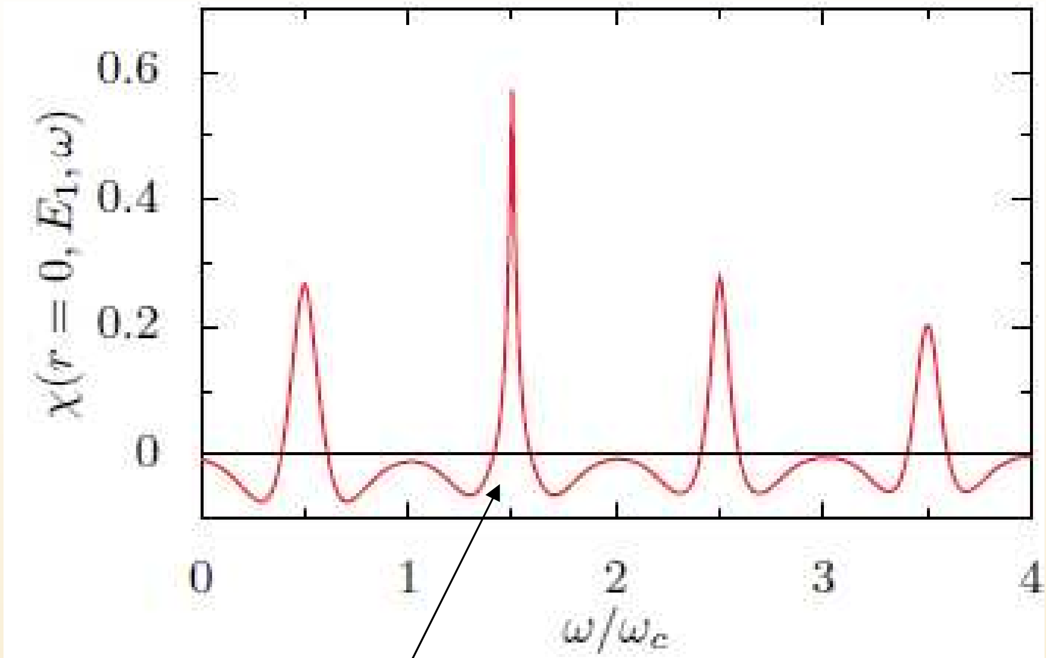


Champel, Florens, Raikh, PRB (2011)

LDOS CORRELATIONS (III)

Energy dependence: DOS versus LDOS correlations at equal position





has broad peaks with width $\sim \sqrt{\langle V^2 \rangle}$

has narrow resonances with width $\sim (l_B/\xi)\sqrt{\langle V^2 \rangle}$

Positive correlations if $\Delta\omega = n\hbar\omega_c$, negative otherwise

Champel, Florens, Raikh, PRB (2011)

▶ Prospects: compare analytic theory with experiments and numerics

PERSPECTIVES: PROBABILITY DISTRIBUTION FOR THE LDOS

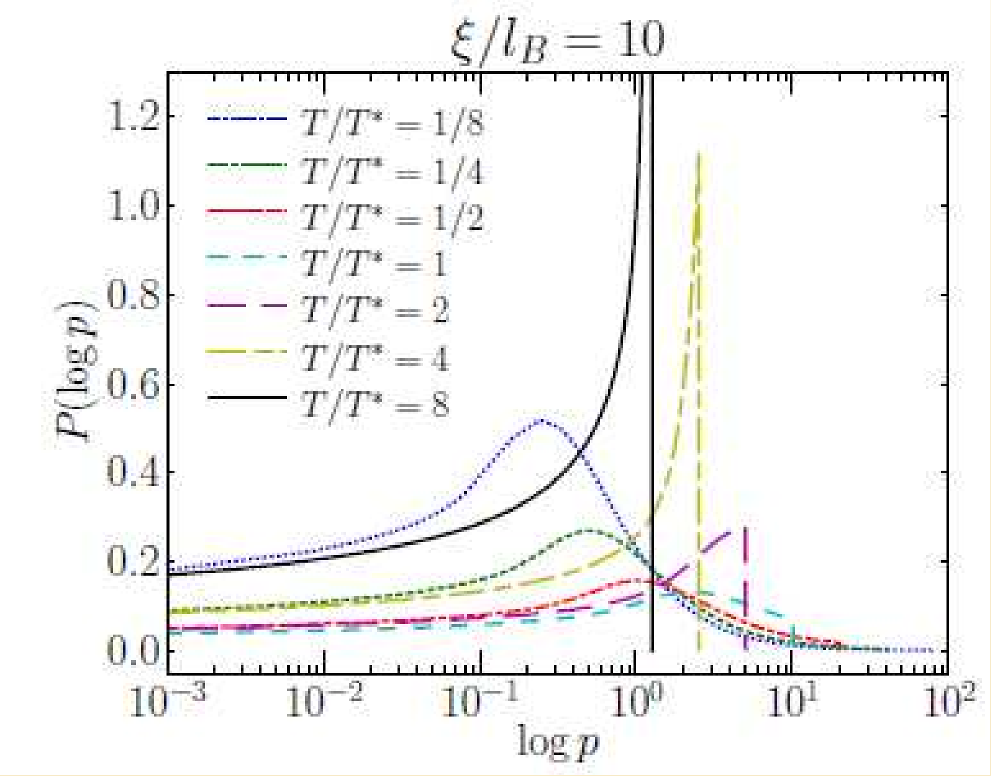
Probablity distribution for the LDOS:

$$P(\rho) = \int \frac{d\lambda}{2\pi} e^{i\lambda\rho} \sum_{n=0}^{\infty} \frac{(-i\lambda)^n}{n!} \langle [\rho(\omega, \mathbf{r} = \mathbf{0})]^n \rangle$$

On-going work with J. Ulrich and S. Florens

Higher moments of the LDOS correlations can be computed analytically at $\xi \gg l_B$

Preliminary results: Assumptions: LL0 + additional external broadening (temperature T)

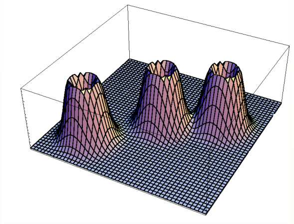


$$p = \rho / \langle \rho \rangle \qquad T^* = (l_B/\xi) v$$

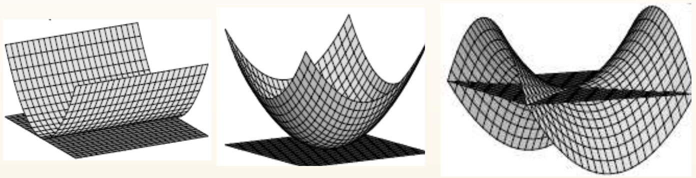
- Wide-stretched distribution at $T \ll T^*$
- Sharply peaked distribution at high *T* (LDOS gets more and more uniform accross the sample)

CONCLUSION

- The rigorous formulation of a quantum guiding center theory was established in terms of semi-coherent state Green's functions

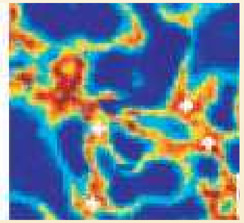


The overcompleteness of the vortex representation makes possible the unification of closed and open systems (bulk and edge states on the same footing)!



- Local equilibrium observables such as the LDOS can be calculated accurately from systematic gradient expansion using semi-coherent state Green's functions

- STM experiments show percolating states in 2DEG at high magnetic fields, and revealed the robust nodal structure of Landau levels



- Theory works well for capturing disordered averaged quantities (averaged DOS, LDOS correlations, probability distribution, …)