Anderson localization in disordered rods.

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Team of Waves in Elastic Systems

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Plan

1. Introduction: vibrations of uniform rods
2. Building symmetry:
   a) Locally periodic rods
3. Destroying the symmetry
   a) Topological defect
   b) Wannier-Stark Ladders
   c) Anderson localization
4. Conclusions

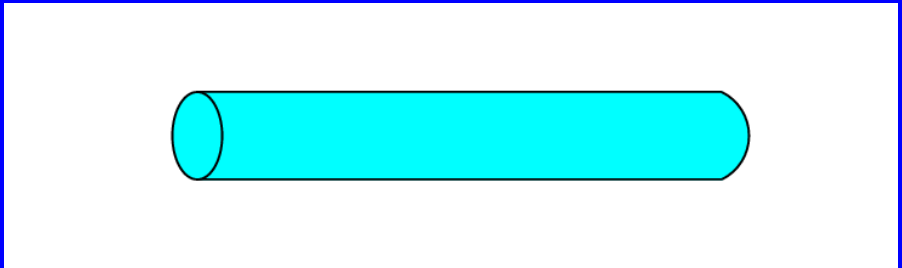
Waves in rods

$$\frac{\partial^2 u_z}{\partial z^2} - \frac{\rho}{E}\frac{\partial^2 u_z}{\partial t^2} = 0$$

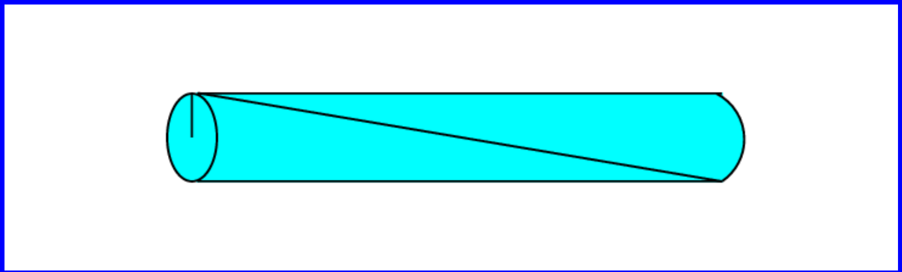
$$\frac{\partial^2 \theta}{\partial z^2} - \frac{\rho}{G}\frac{\partial^2 \theta}{\partial t^2} = 0$$

$$\frac{\partial^4 \xi}{\partial z^4} + \frac{\rho}{E R_g^2}\frac{\partial^2 \xi}{\partial t^2} = 0$$

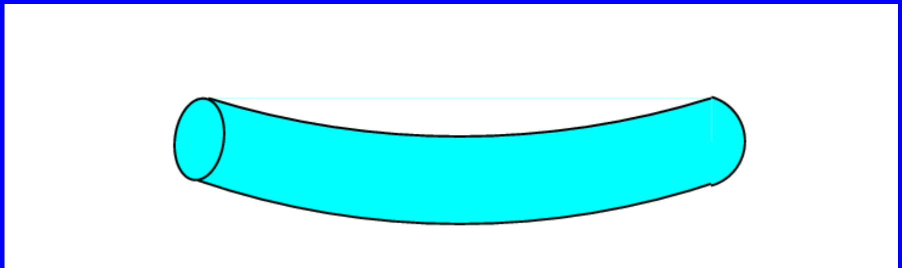
Compressional



Torsional



Bending



Timoshenko equation

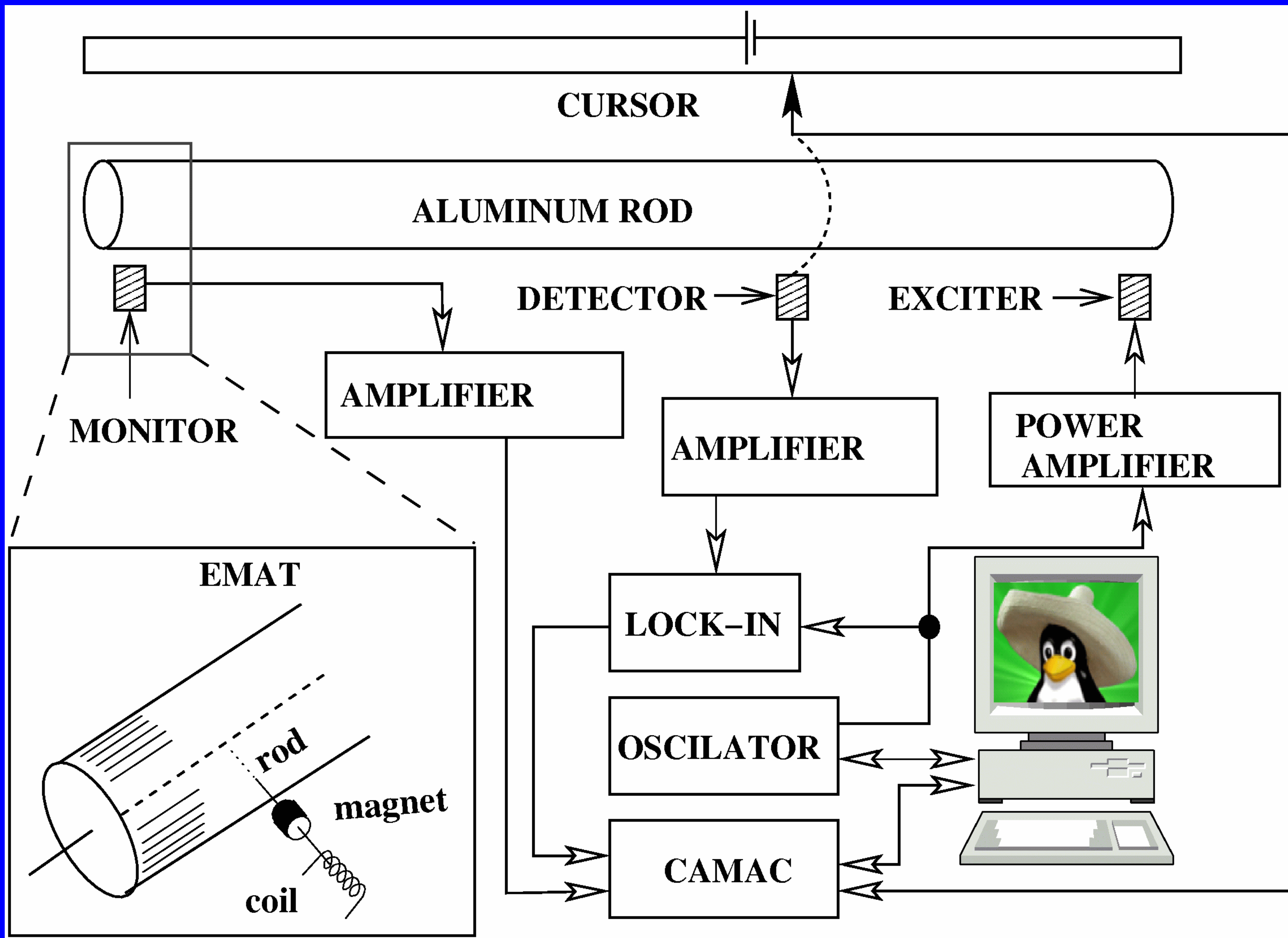
$$\frac{EI_y}{\rho S}\frac{\partial^4\xi}{\partial z^4} - \frac{I_y}{S}\left(1+\frac{E}{\kappa G}\right)\frac{\partial^4\xi}{\partial z^2\partial t^2} + \frac{\partial^2\xi}{\partial t^2} + \frac{\rho I_y}{\kappa G S}\frac{\partial^4\xi}{\partial t^4} = 0$$

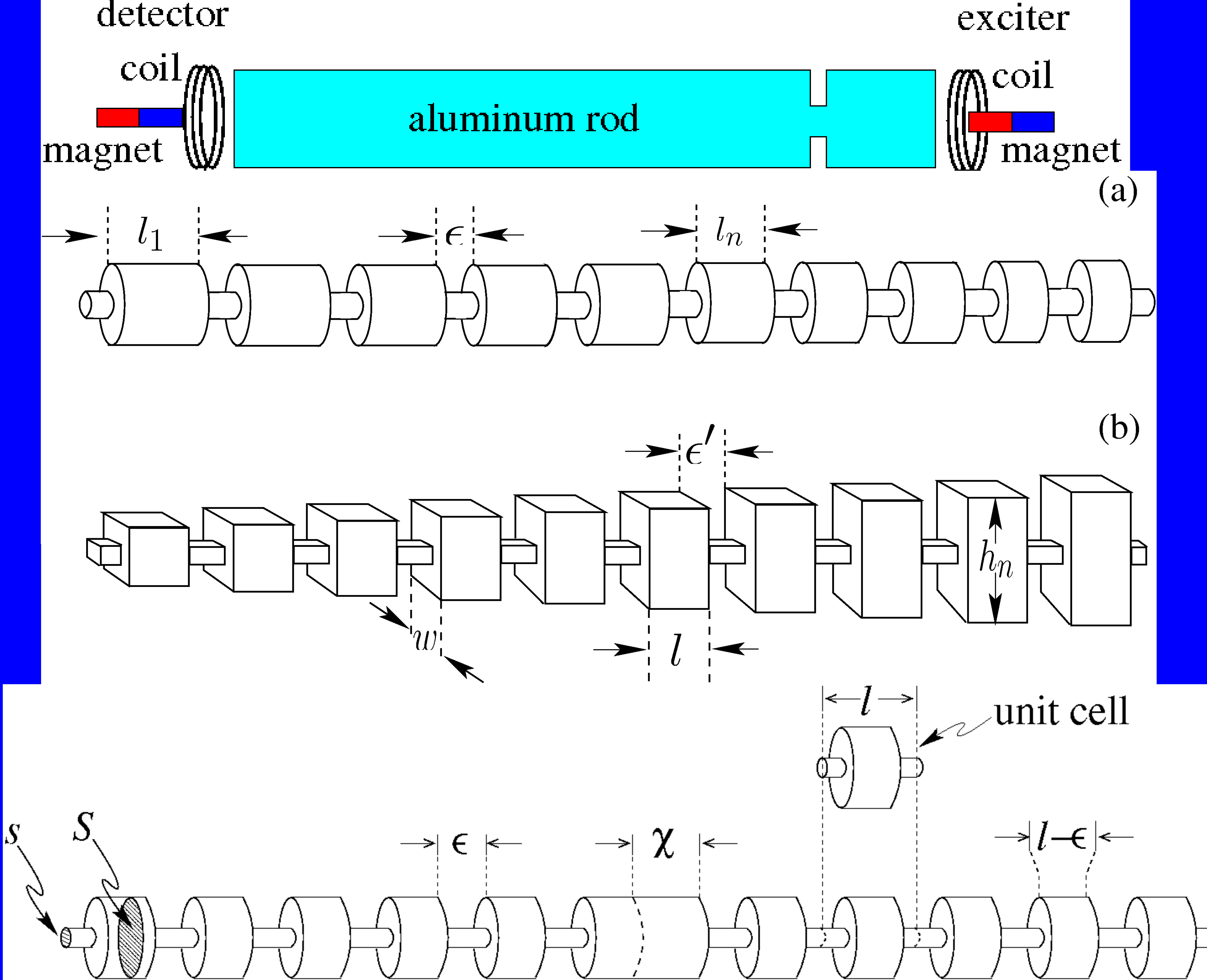
which is separable for a normal mode, when $\xi = \Psi(z)\cos(\omega t)$, with $\omega = 2\pi f$.

Here f is the frequency, G and E the shear and Young modulus, respectively, ρ the density, S the transversal area of the beam and I the moment of inertia.

The parameter κ is called the Timoshenko coefficient.

Experimental Setup




detector
coil
magnet
aluminum rod
exciter
coil
magnet
(a)
l_1
ϵ
l_n
(b)
ϵ'
h_n
w
l
l
unit cell
s
S
ϵ
χ
$l{-}\epsilon$

Some results in 1-D and 2-D systems

1) Design of EMATs: Am. J. Phys. 112 (2001) 1961.

2) Waves in periodic elastic systems: J. Acoust. Soc. Am. 112 (2002) 1961, J. Acoust. Soc. Am. 117, (2005) 2814.

3) Defects in periodic elastic systems: Physica E 19 (2003) 289.

4) Timoshenko's shear coefficient measured and 2nd TBT spectrum: J. Sound Vib. 279 (2005) 508 and J. Sound and Vibration, *in press*.

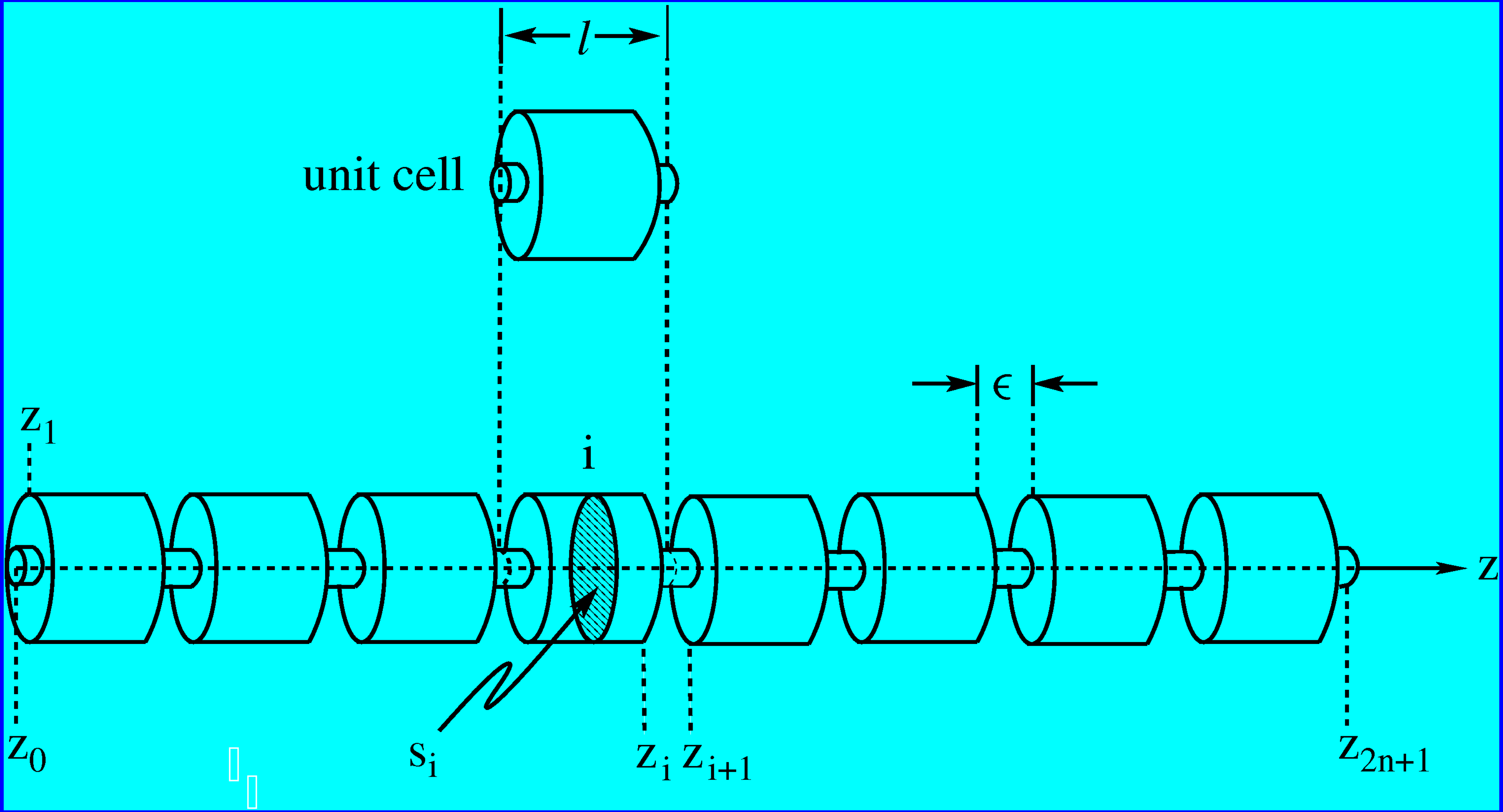
5) Poincaré map methodfor elastic systems: Physica E 30 (2005) 174.

6) Wannier-Stark ladders in elastic rods: Phys. Rev. Lett. 97, (2006) 114301 and J. Mech. Mat. and Struct. 2, (2007) 1629.

7) Test of the classical thin plate theory and the plane wave expansion method: J. Sound and Vibration 329 (2010) 5105-5115.

8) Doorway states in the time domain EPL, *to appear*.

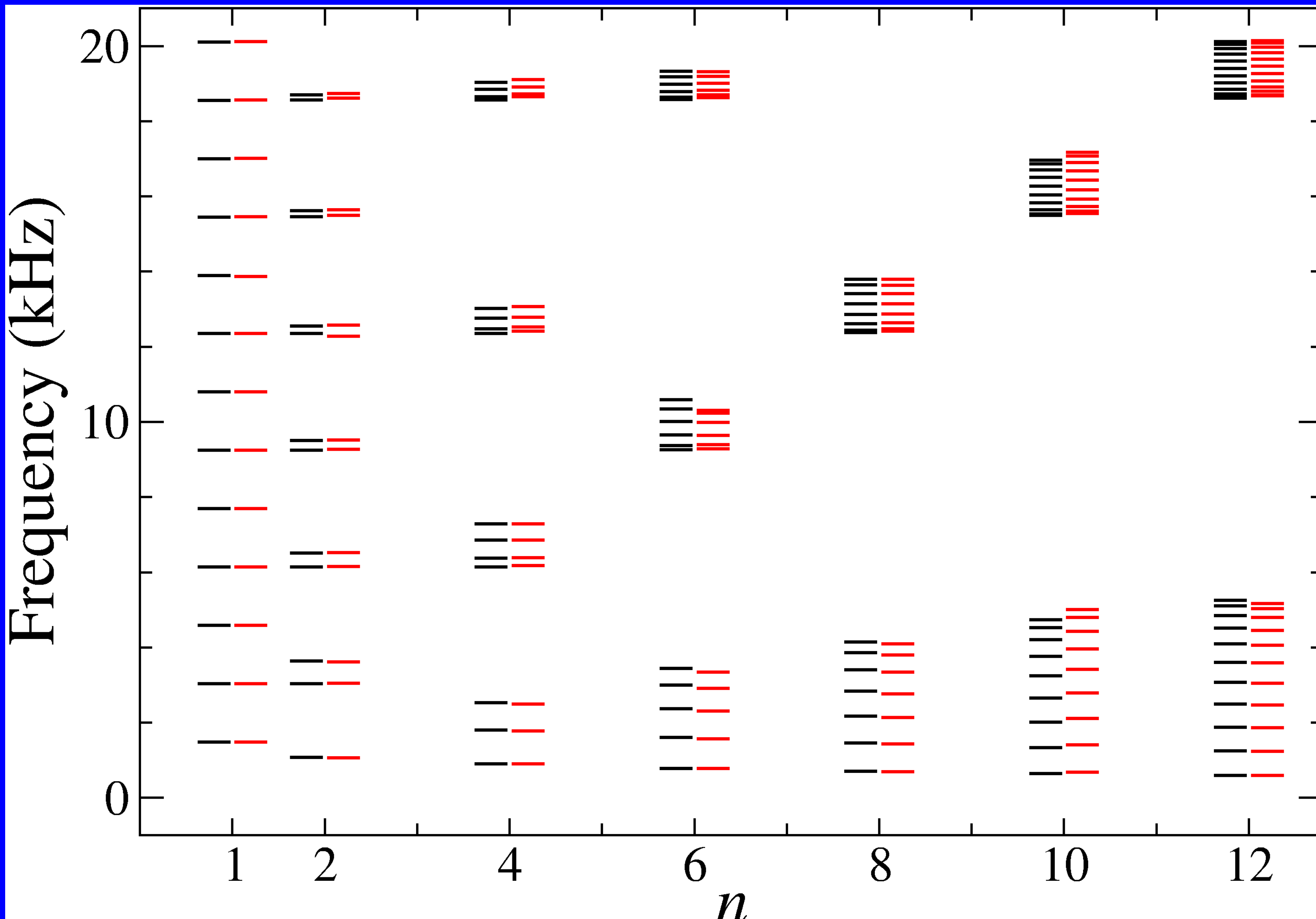
Locally periodic rods



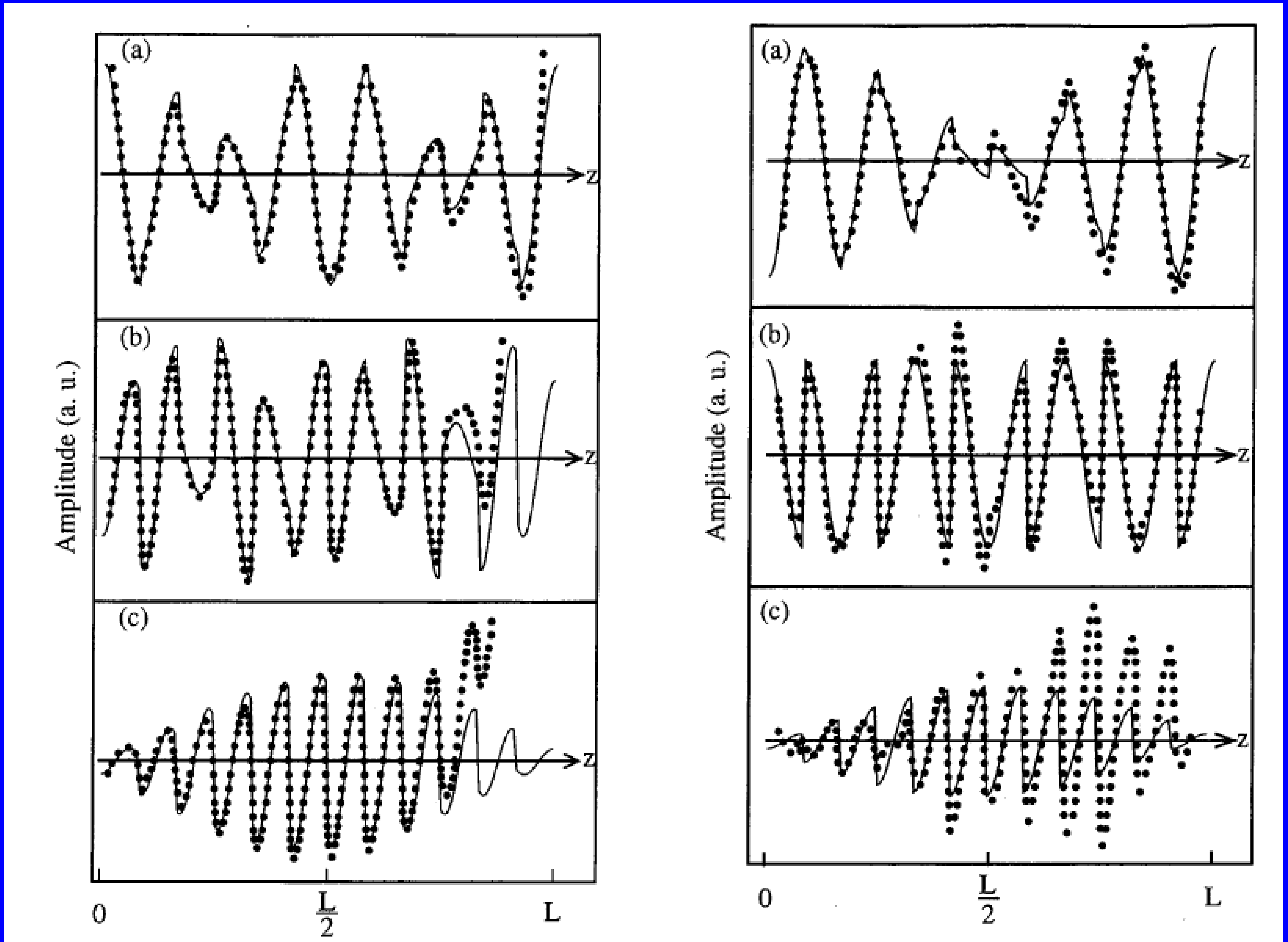
Consist of *n* unit cells

When *n* increases, a band spectrum emerges

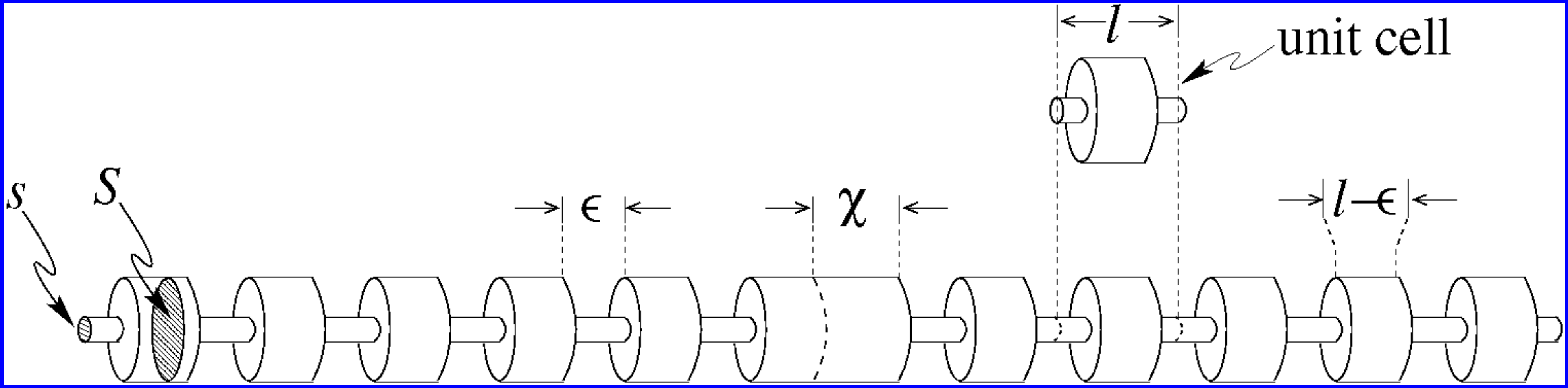
J. Acoust. Soc. Am. 112 (2002) 1961



The wave amplitudes are extended



Introducing a topological defect



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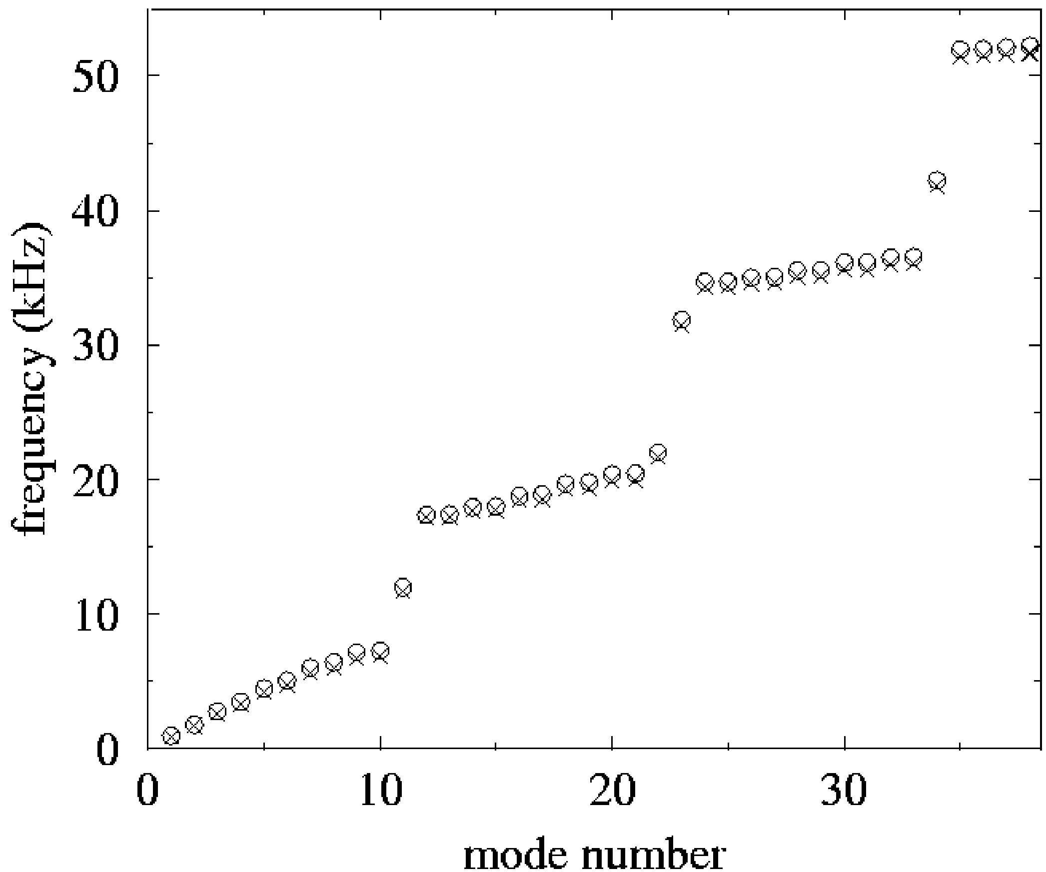
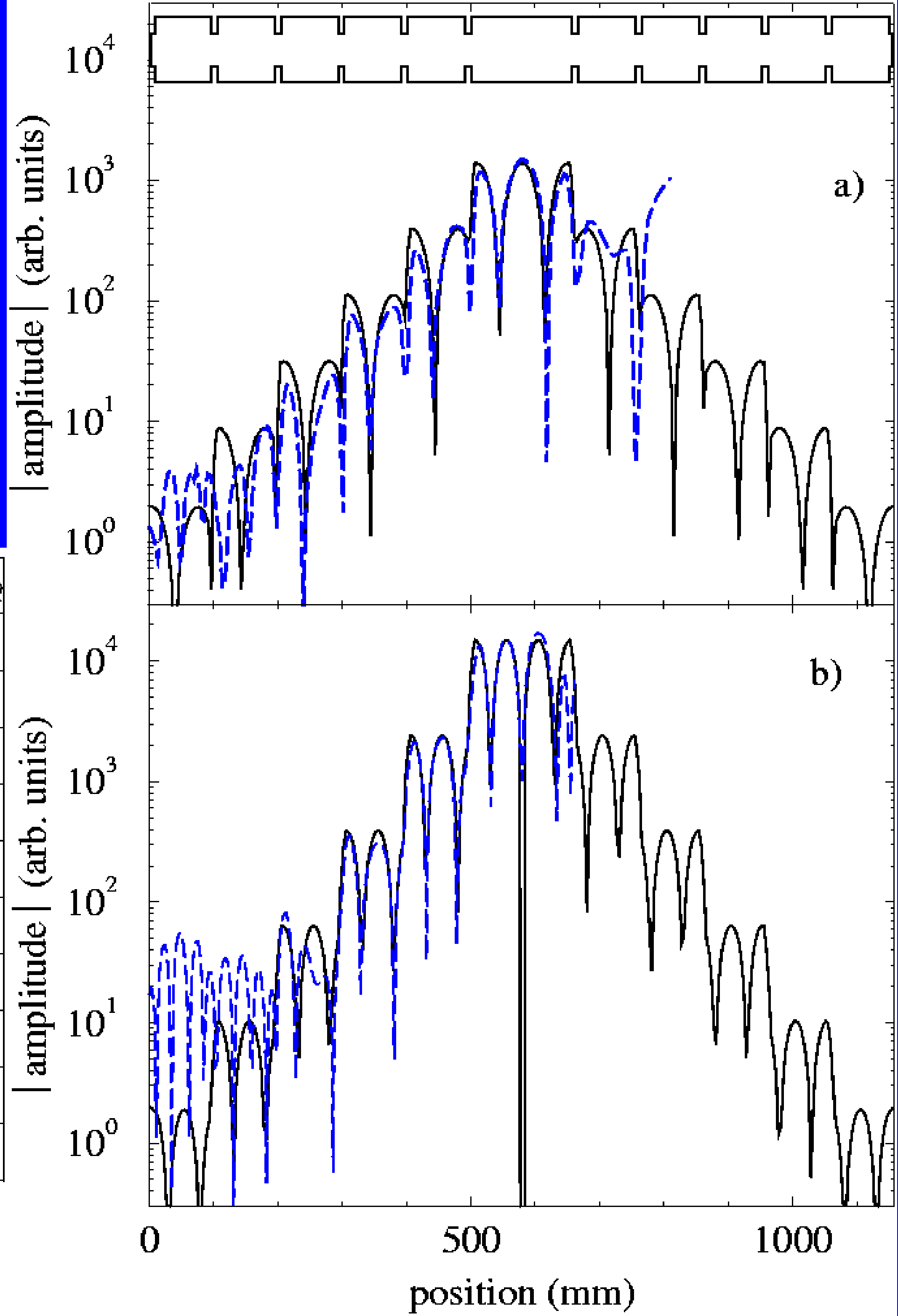
The topological defect breaks the long-range symmetry

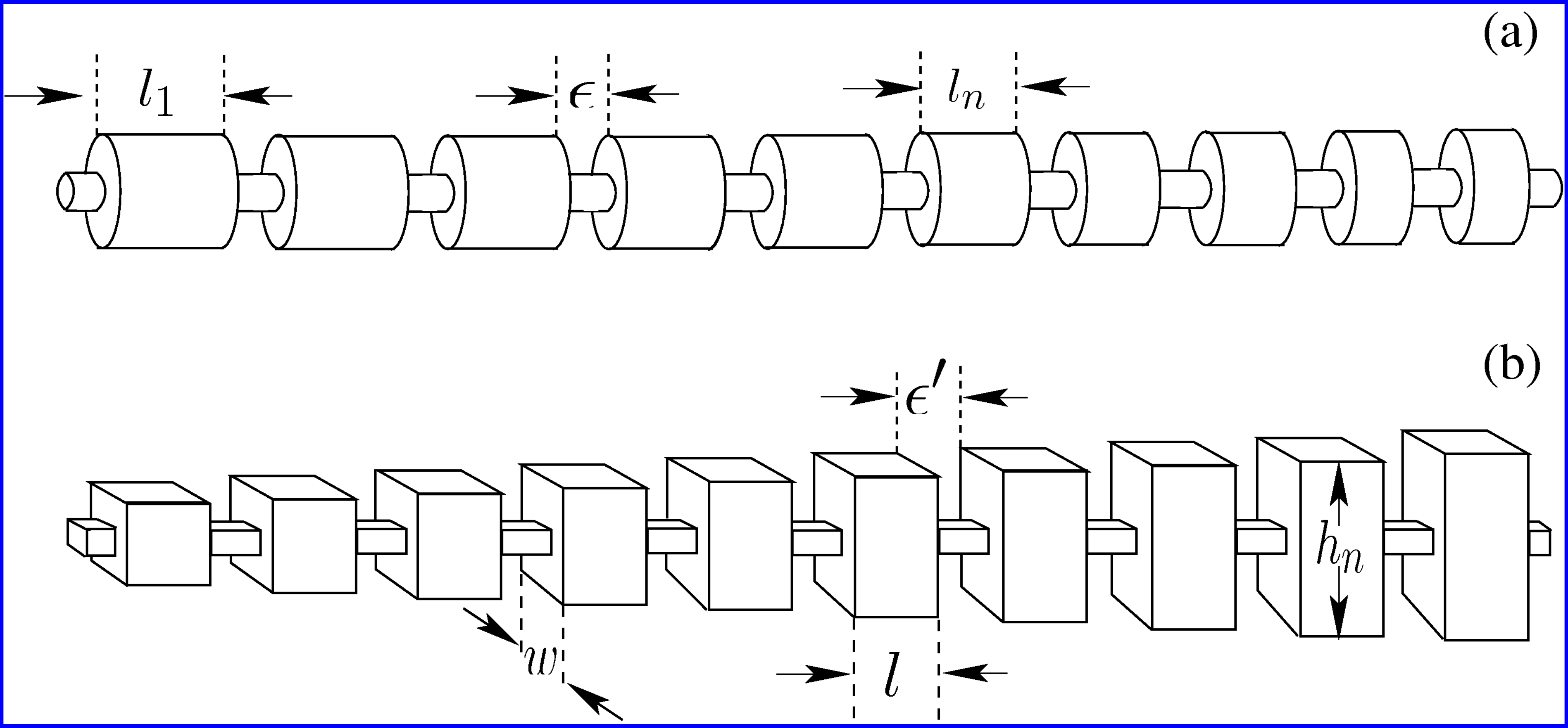
Two localized states appear in the gap

Physica E 19 (2003) 289.



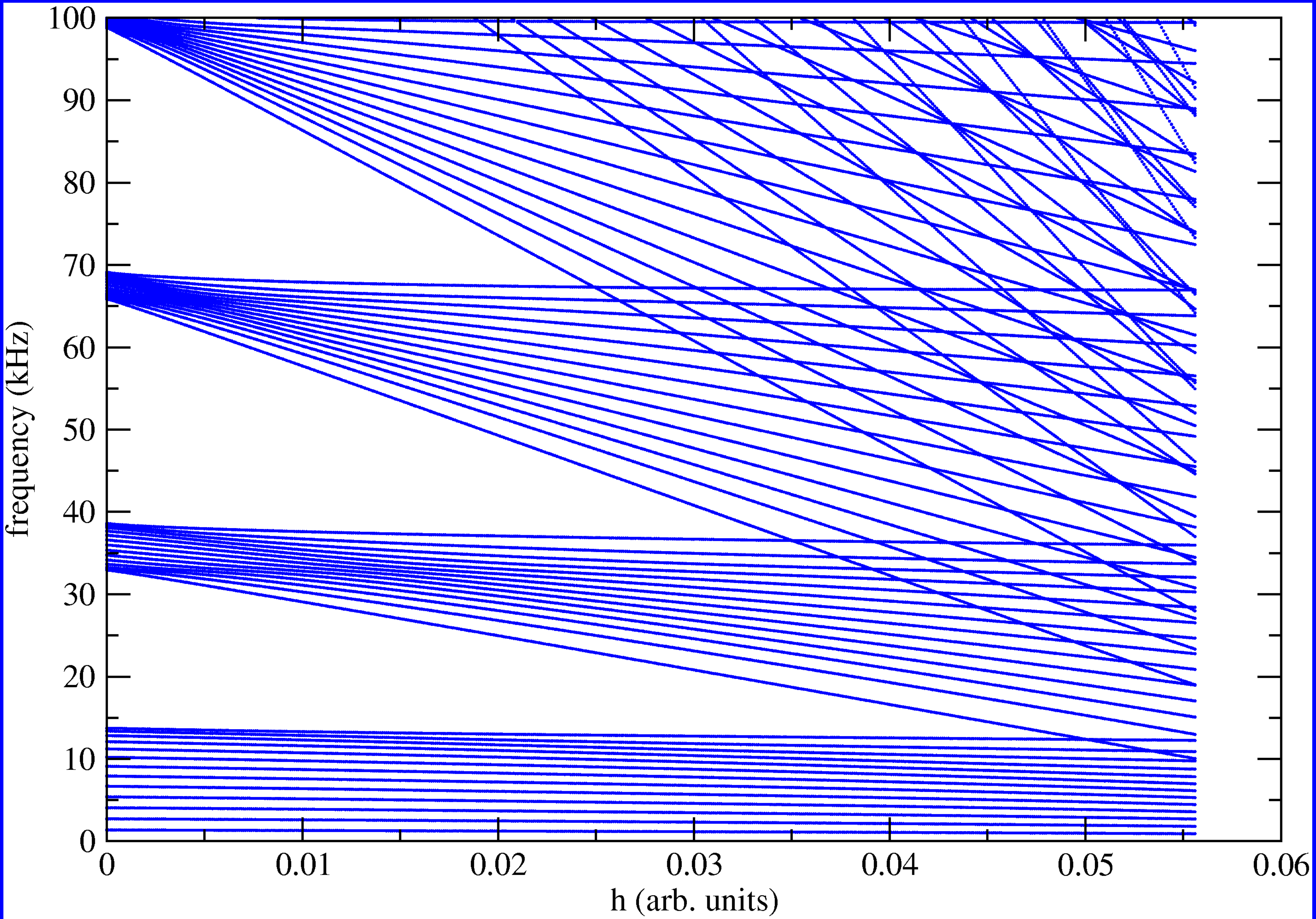


Wannier-Stark ladders

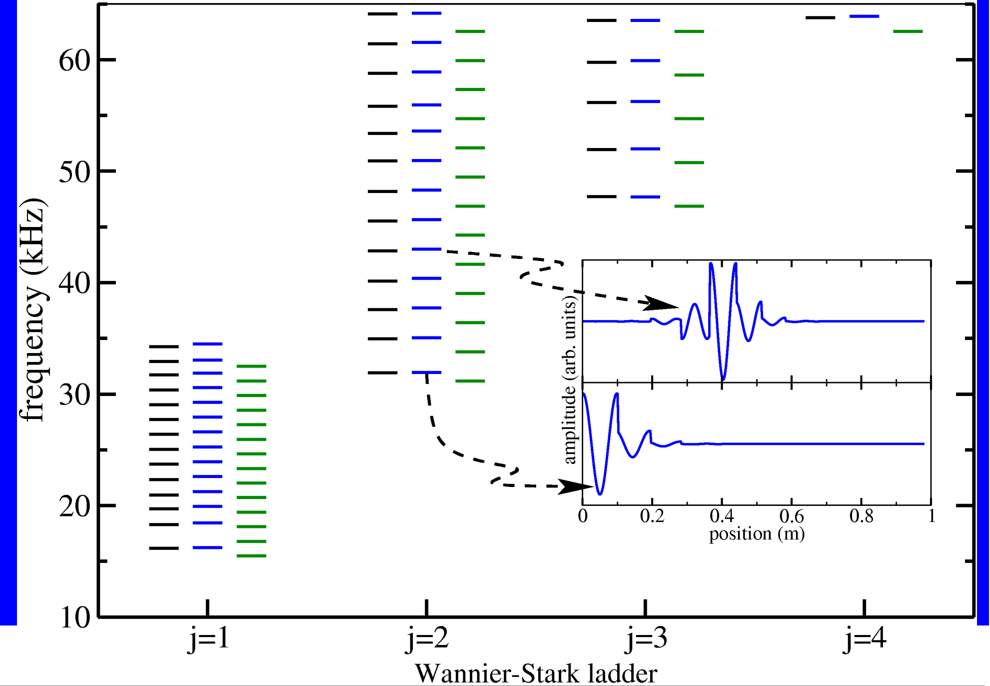


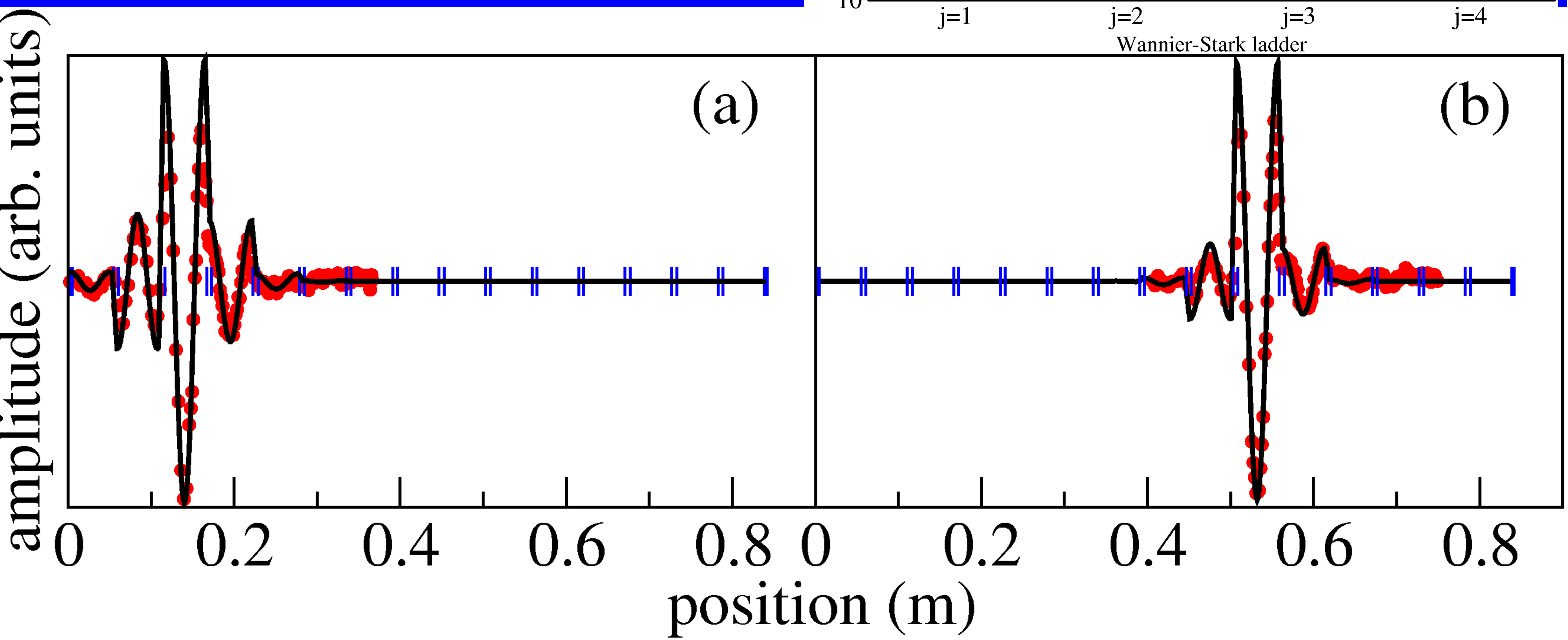
A renormalization of each cell is done according to $L^j = L^0/(1+j\,h)$

Wannier-Stark ladders spectrum



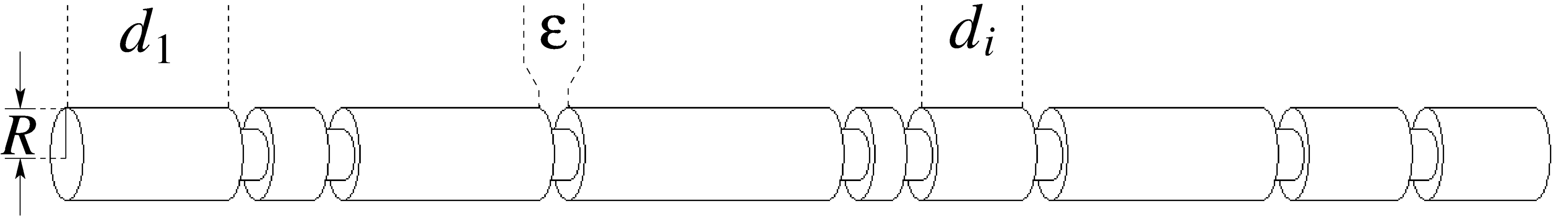
Wannier-Stark ladders wave amplitudes





L. Gutierrez, A. Díaz-de-Anda, J. Flores, R.A. Méndez-Sánchez, G. Monsivais and A. Morales, Physical Review Letters, 97 (2006) 114301.

Disordered rods

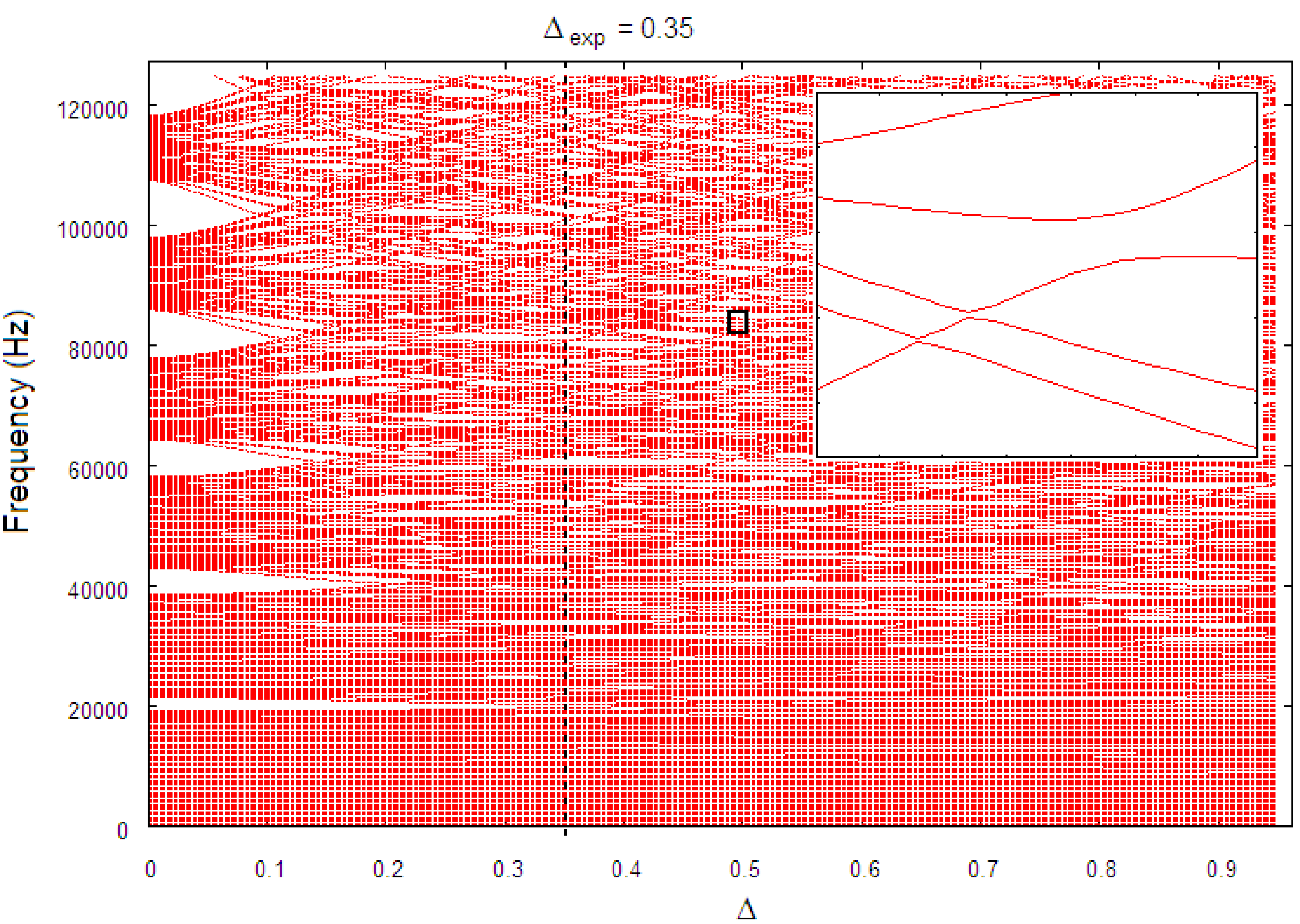


$\{d_i\}$ uncorrelated random numbers with a uniform distribution in the interval $[d(1-\Delta), d(1+\Delta)]$.

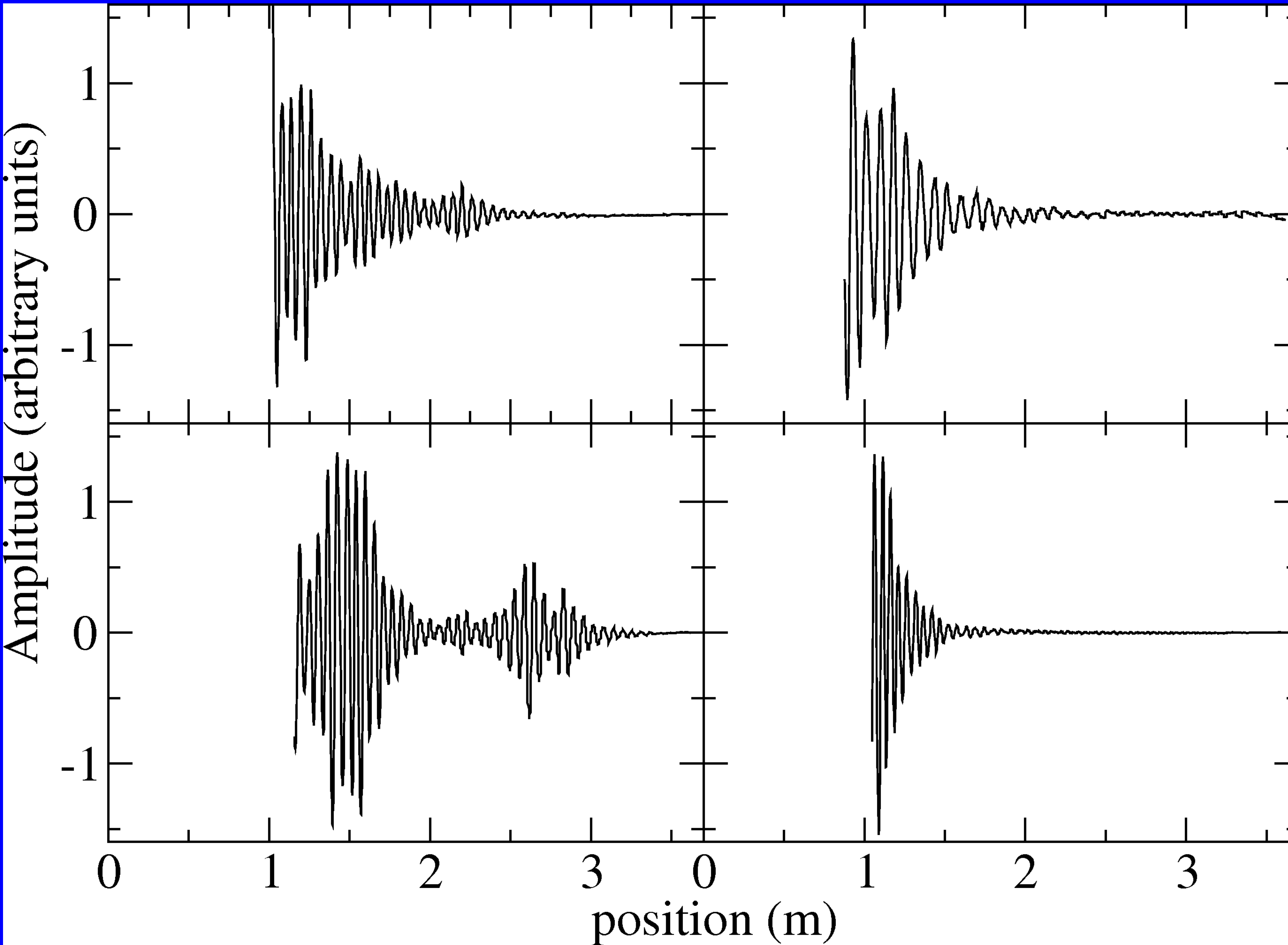
$d = \langle d_i \rangle$ is the average of d_i.

Δ measures the disorder, i. e. $0 < \Delta / < d/2$.

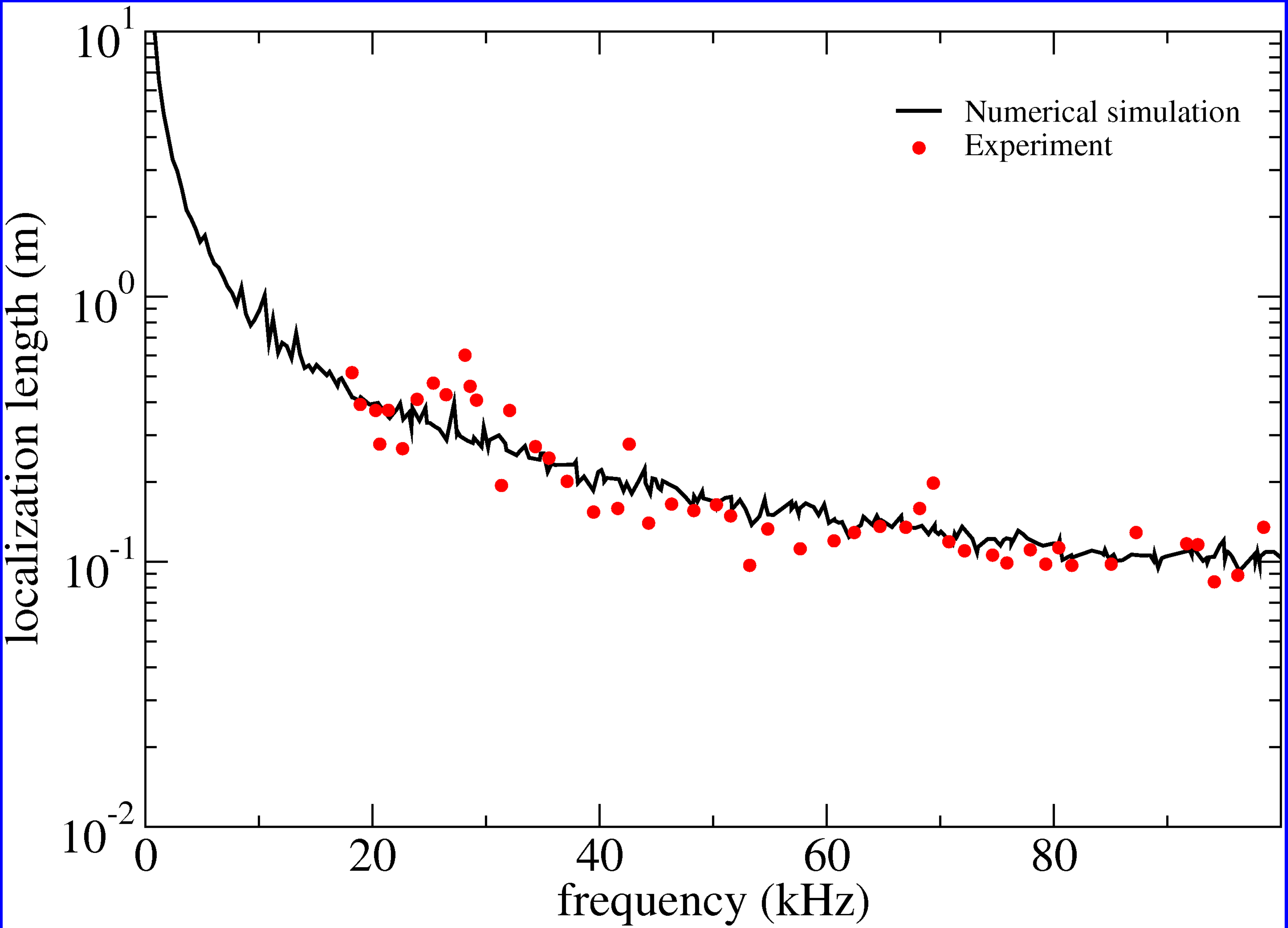
Increasing the disorder



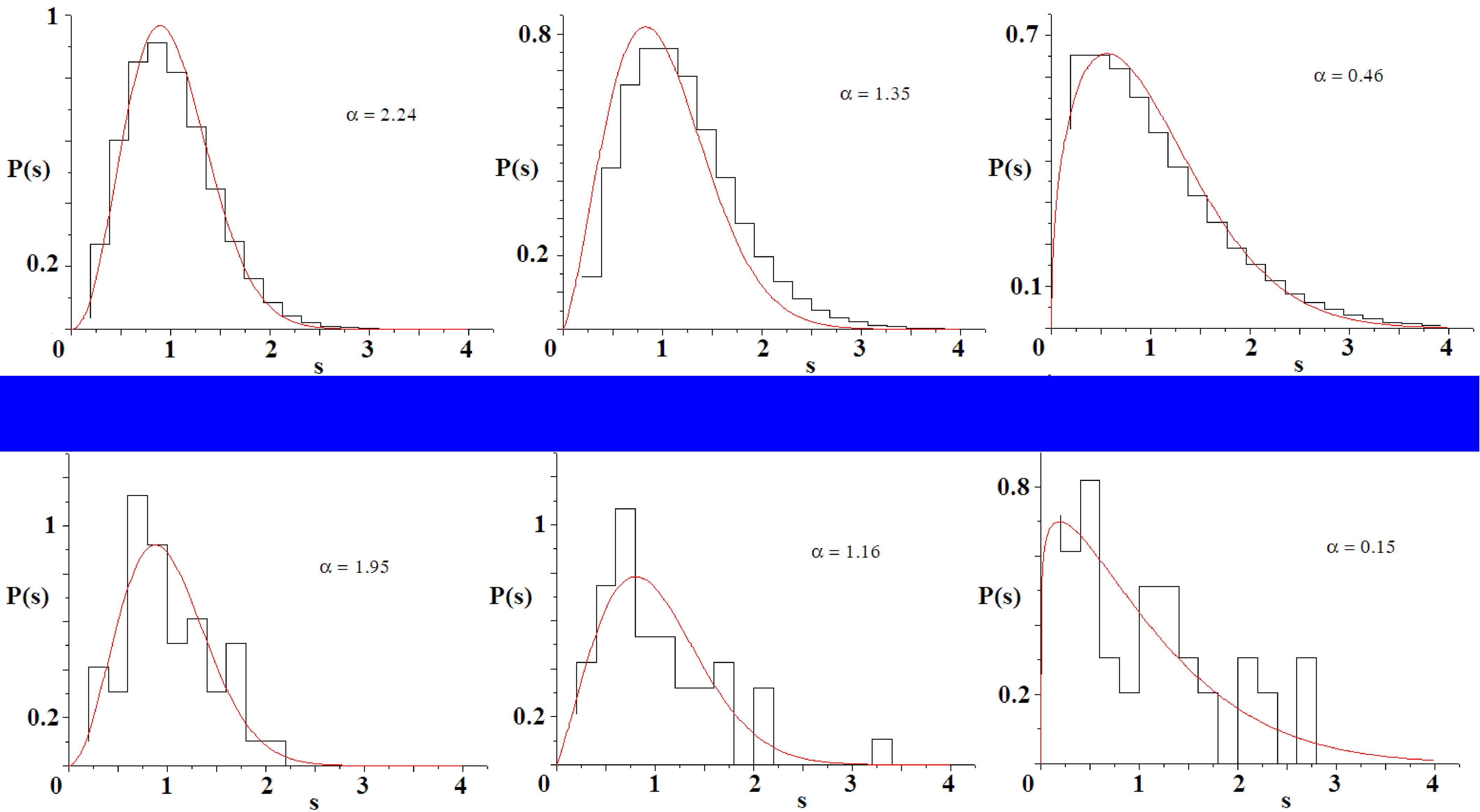
Experimental wave amplitudes



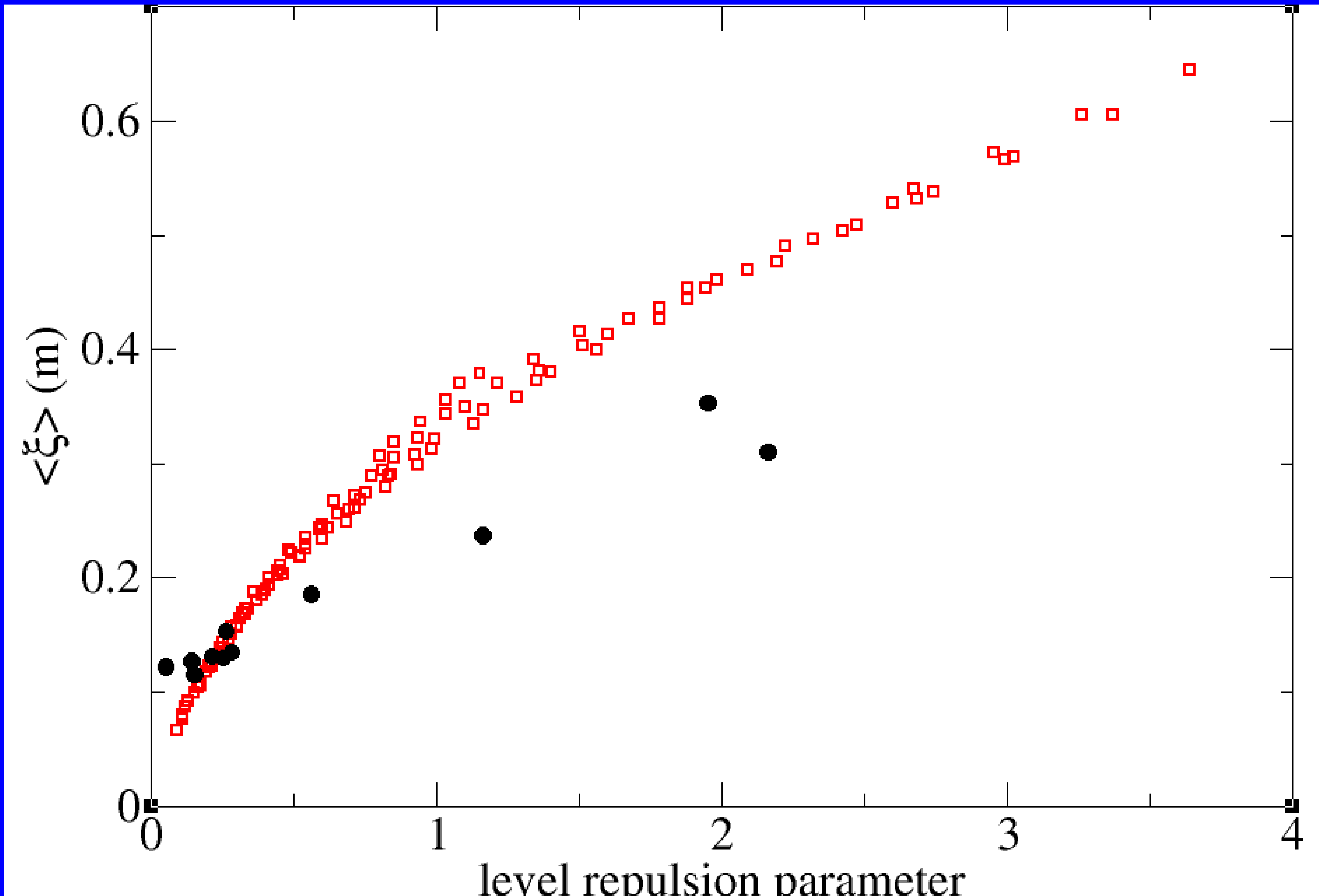
Localization length



Nearest-neighbor spacing distribution



Localization length *vs.* level repulsion parameter



Conclusions

-We measure Anderson localization in disordered elastic rods.

-From the measured spectrum the level repulsion was obtained

-The localization length was obtained from the measured wave amplitudes

-The localization length is a linear function of the level repulsion only in a range