Enhancing lepton flavor violation with the Z-penguin

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Based on:

M. Hirsch, F. Staub, A. Vicente, ArXiv:1202.1825 [hep-ph]

H.K. Dreiner, K. Nickel, F. Staub, A. Vicente, ArXiv:1204.5925 [hep-ph]
A. Abada, D. Das, A. Vicente, C. Weiland, ArXiv:1206.XXXX [hep-ph]

Outline of the talk

- $l_i \to 3l_j$ in the MSSM
- Some mass scaling considerations
- $l_i \to 3l_j$ in the MSSM revisited
- Other observables
- Beyond MSSM
- Final remarks

$l_i \to 3l_j$ in the MSSM

In supersymmetry, the additional degrees of freedom provided by the superparticles typically increase the flavor violating signals to observable levels.

The most popular example in the leptonic sector is the radiative decay $\mu \to e\gamma$ (why the most popular? see later...), but other interesting processes have been studied in the literature. For example:

$$l_i \to 3l_j$$

- J. Hisano et al., PRD 53 (1996) 2442
- E. Arganda and M.J. Herrero, PRD 73 (2006) 055003

$l_i \to 3l_j$ in the MSSM

A brief détour...

Experimental limits

$l_i \to l_j\gamma$	$l_i \to 3l_j$
$\text{Br}(\mu \to e\gamma) < 2.4 \cdot 10^{-12}$	$\text{Br}(\mu \to 3e) < 1.0 \cdot 10^{-12}$
$\text{Br}(\tau \to e\gamma) < 3.3 \cdot 10^{-8}$	$\text{Br}(\tau \to 3e) < 2.7 \cdot 10^{-8}$
$\text{Br}(\tau \to \mu\gamma) < 4.4 \cdot 10^{-8}$	$\text{Br}(\tau \to 3\mu) < 2.1 \cdot 10^{-8}$

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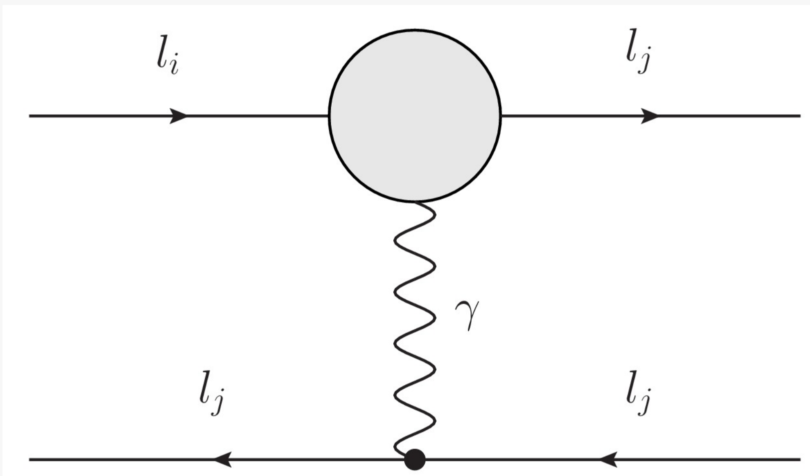
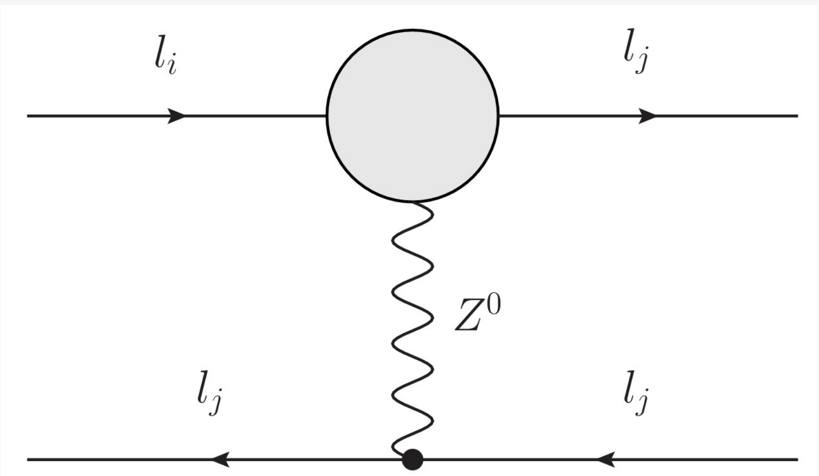
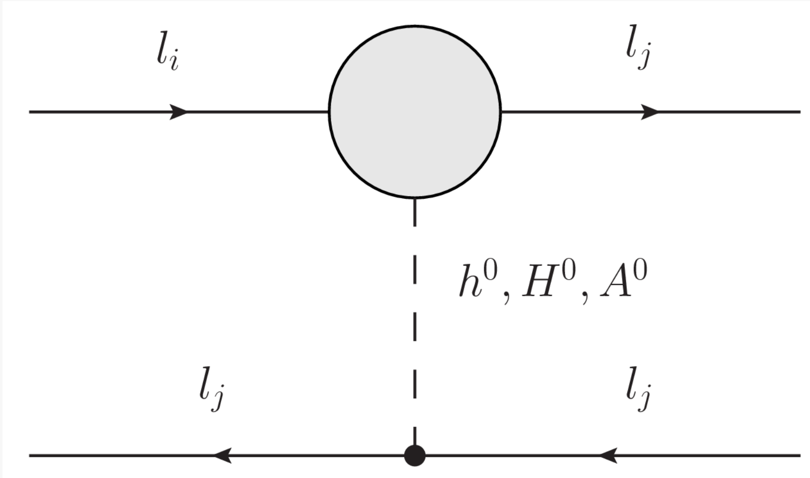
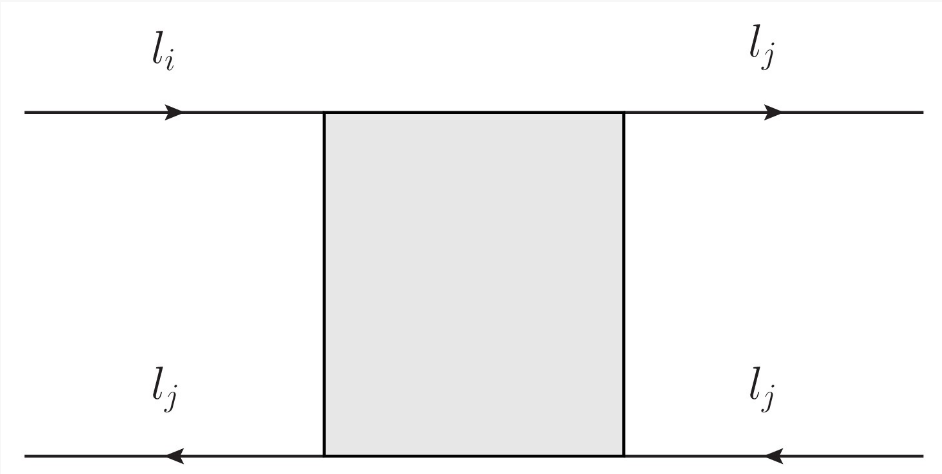
$l_i \to 3l_j$ in the MSSM









$l_i \to 3l_j$ in the MSSM

$$\begin{aligned}
\Gamma &= \frac{e^4}{512\pi^3} m_{l_j}^5 \Big[\left|A_1^L\right|^2 + \left|A_1^R\right|^2 - 2\left(A_1^L A_2^{R*} + A_2^L A_1^{R*} + h.c.\right) \\
&+ \left(\left|A_2^L\right|^2 + \left|A_2^R\right|^2\right)\left(\frac{16}{3}\log\frac{m_{l_j}}{m_{l_i}} - \frac{22}{3}\right) \\
&+ \frac{1}{6}\left(\left|B_1^L\right|^2 + \left|B_1^R\right|^2\right) + \frac{1}{3}\left(\left|\hat{B}_2^L\right|^2 + \left|\hat{B}_2^R\right|^2\right) \\
&+ \frac{1}{24}\left(\left|\hat{B}_3^L\right|^2 + \left|\hat{B}_3^R\right|^2\right) + 6\left(\left|B_4^L\right|^2 + \left|B_4^R\right|^2\right) \\
&- \frac{1}{2}\left(\hat{B}_3^L B_4^{L*} + \hat{B}_3^R B_4^{R*} + h.c.\right) \\
&+ \frac{1}{3}\left\{2\left(\left|F_{LL}\right|^2 + \left|F_{RR}\right|^2\right) + \left|F_{LR}\right|^2 + \left|F_{RL}\right|^2\right\} \\
&+ \text{interference terms}\Big]
\end{aligned}$$

$l_i \to 3l_j$ in the MSSM

$$
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&- \frac{1}{2}\left(\hat{B}_3^L B_4^{L*} + \hat{B}_3^R B_4^{R*} + h.c.\right) \\
&+ \frac{1}{3}\left\{2\left(\left|F_{LL}\right|^2 + \left|F_{RR}\right|^2\right) + \left|F_{LR}\right|^2 + \left|F_{RL}\right|^2\right\} \\
&+ \text{interference terms}\Big]
\end{aligned}
$$

$l_i \to 3l_j$ in the MSSM

What contribution dominates?

$l_i \to 3l_j$ in the MSSM

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- In most parts of parameter space: Photon penguins

J. Hisano et al., PRD 53 (1996) 2442
E. Arganda and M.J. Herrero, PRD 73 (2006) 055003

$$\frac{BR(l_i \to 3l_j)}{BR(l_ji \to l_j\gamma)} = \frac{\alpha}{3\pi}\left(\log\frac{m_{l_i}^2}{m_{l_j}^2} - \frac{11}{4}\right) \quad \Rightarrow \quad BR(l_i \to l_j\gamma) \gg BR(l_i \to 3l_j)$$

... and that, together with the good experimental bound, has made $\mu \to e\gamma$ so attractive for the pheno community

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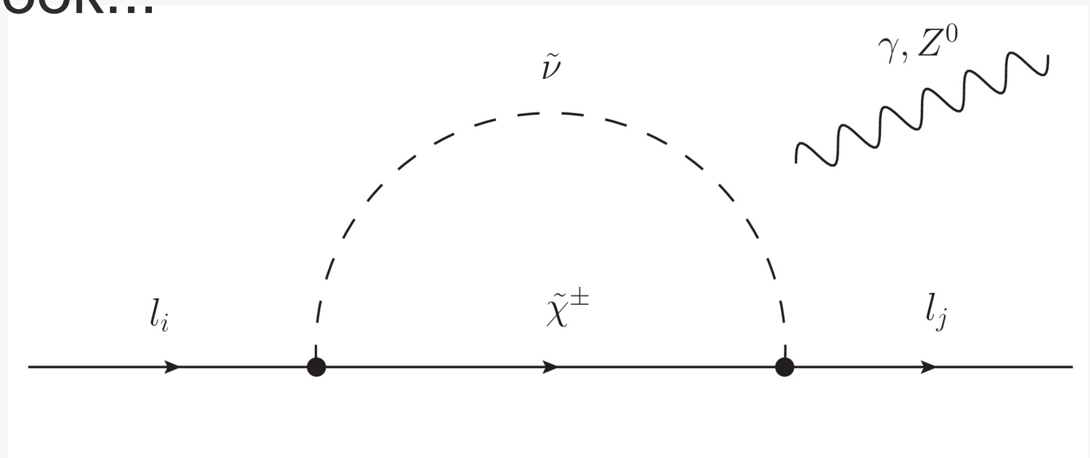
- For large $\tan\beta$ and a light pseudoscalar: Higgs penguins

K.S. Babu, C. Kolda, PRL 89 (2002) 241802

Mass scaling considerations

Let us give a more detailed look...

Consider the γ- and Z-penguins originated by chargino-sneutrino loops.



One finds:

$$A_a^{(c)L,R} = \frac{1}{m_{\tilde{\nu}}^2} \mathcal{O}_{A_a}^{L,R} s(x^2) \qquad F_X = \frac{1}{g^2 \sin^2 \theta_W m_Z^2} \mathcal{O}_{F_X}^{L,R} t(x^2)$$

γ - penguin **Z - penguin**

Mass scaling considerations

In fact, the mass scalings

$$A \sim m_{SUSY}^{-2} \qquad\qquad F \sim m_Z^{-2}$$

are quite intuitive. These are the lowest mass scales in the penguins (recall, for example, the H-penguins $\sim m_H^{-2}$)

Then, by doing a very simple estimate...

$$\frac{F}{A} \sim \frac{m_{SUSY}^2}{g^2 \sin^2 \theta_W \, m_Z^2} \sim 500 \qquad \text{for } m_{SUSY} \sim 300\text{GeV}$$

And, remember... $\Gamma(l_i \to 3 l_j) \propto A^2 \, , \, F^2$

Mass scaling considerations

These considerations lead us to the expectation

$$F \gg A$$

So...

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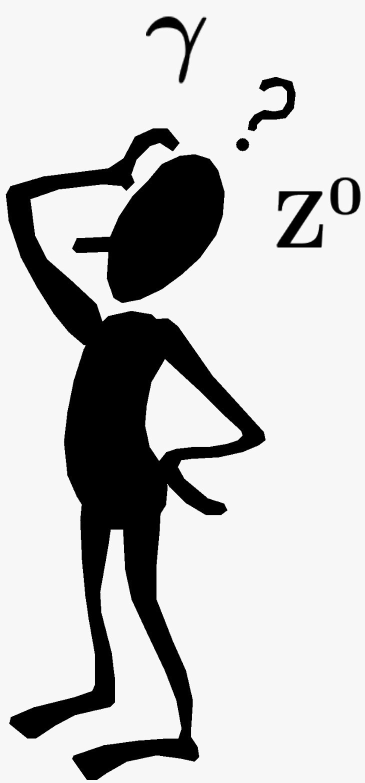


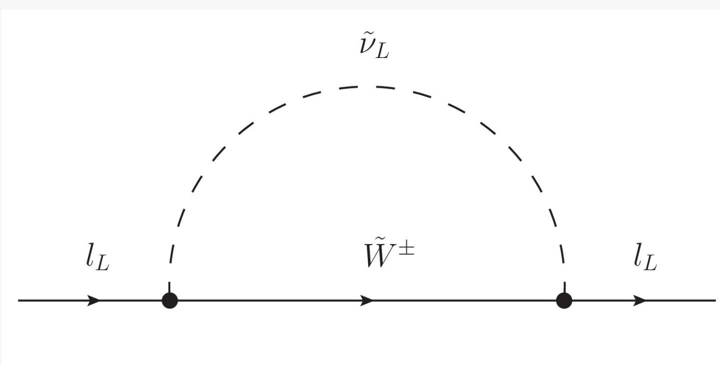
Why the Z-penguins are not the dominant contribution in the MSSM?

$l_i \to 3l_j$ in the MSSM revisited

Consider F_L, the dominant contribution within the Z-penguins, obtained when the external leptons are L-handed, and make an expansion on the chargino mixing angle.

$$F_L = F_L^{(0)} + \frac{1}{2}\theta^2_{\tilde{\chi}^\pm} F_L^{(2)} + \dots$$

$=$



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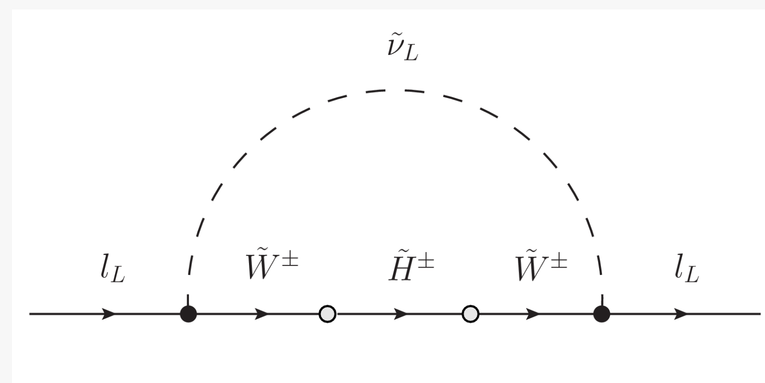
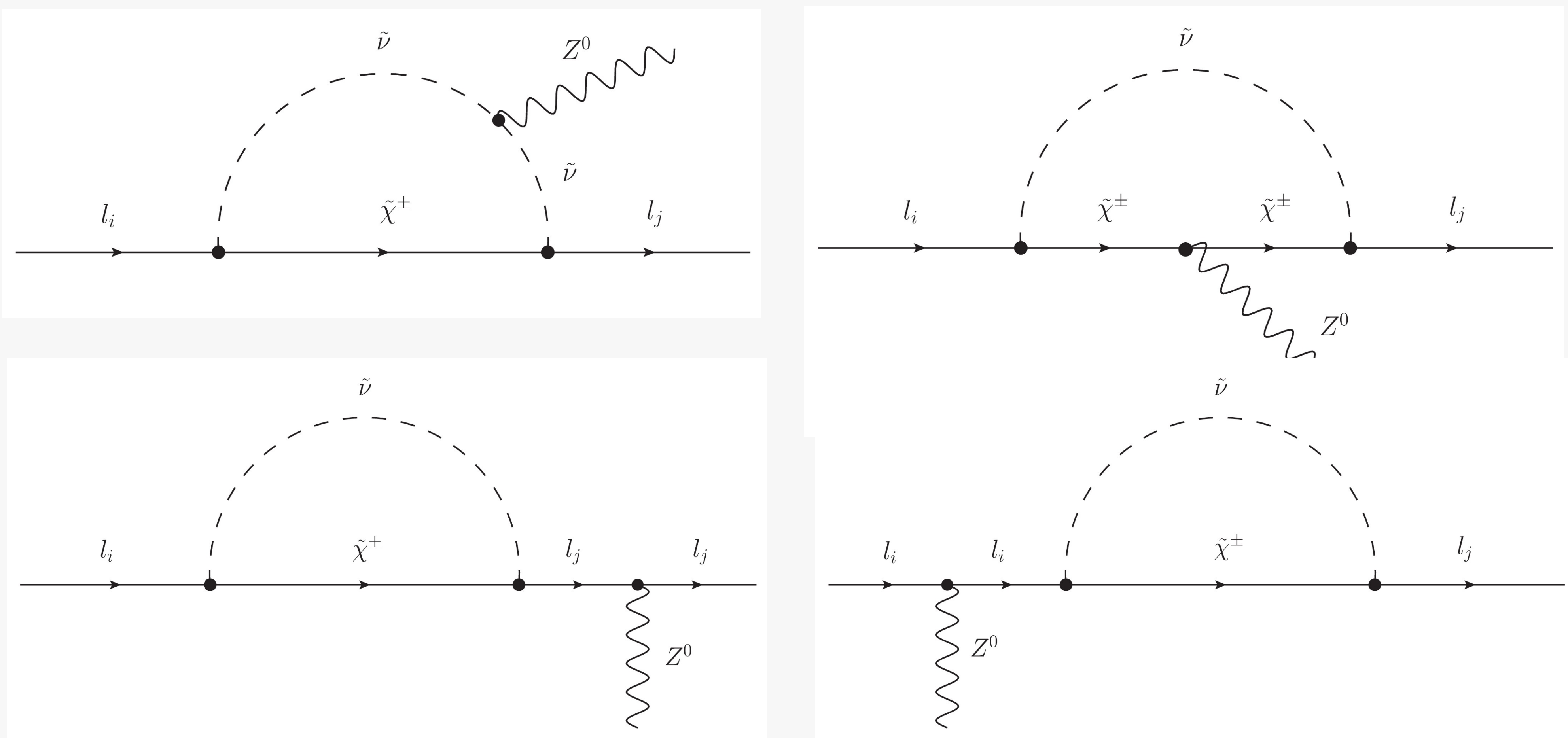


$+ \dots$

Important: There is no order 1!

$l_i \to 3l_j$ in the MSSM revisited

$$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} + \mathbf{F_4}$$



$l_i \to 3l_j$ in the MSSM revisited

When you sum the four diagrams that contribute to $F_L^{(0)}$:

$$\begin{aligned} F_L^{(0)} &= F_{L,1}^{(0)} + F_{L,2}^{(0)} + F_{L,3}^{(0)} + F_{L,4}^{(0)} \\ &= \frac{1}{2} g^3 c_W Z_V^{ki} Z_V^{kj*} X_1^k + \frac{1}{2} g^2 g' s_W Z_V^{ki} Z_V^{kj*} X_2^k \end{aligned}$$

X_1^k and X_2^k are combinations of PV functions, with different combinations of chargino and sneutrino masses. However, one finds that the masses cancel out and they just become numerical constants. Therefore...

$l_i \to 3l_j$ in the MSSM revisited

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$$\Rightarrow \quad F_L^{(0)} \propto \sum_k Z_V^{ki} Z_V^{kj*} = 0$$

It vanishes exactly!

Side comment: This cancellation was also found in Lunghi et al. Nucl. Phys. B 568 (2000) 120 when looking into $B \to X_s l^+ l^-$ in supersymmetry

$l_i \to 3l_j$ in the MSSM revisited

In **conclusion**, the Z-penguins are not dominant in the MSSM because the leading-order term vanishes and the first non-zero contribution is suppressed by two chargino insertions. This cancellation is not found in the photon penguins.

How can we break the cancellation?

- Additional states that mix with the sneutrinos
- New lepton couplings

$l_i \to 3l_j$ can be greatly enhanced!

$l_i \to 3l_j$ in the MSSM revisited

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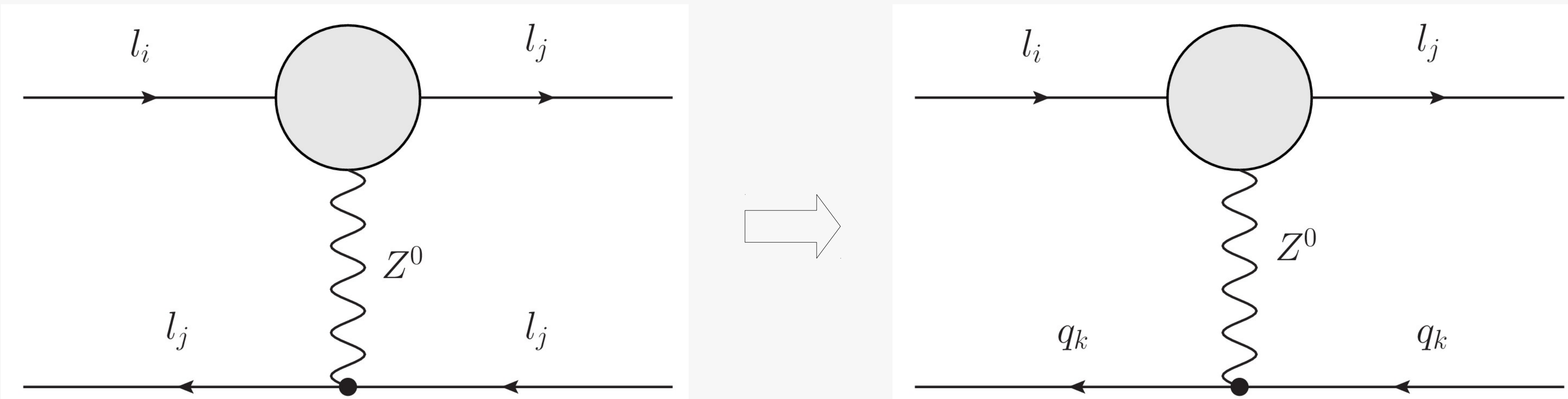
In fact... only $l_i \to 3l_j$?

Other observables



$\mu - e$ conversion in nuclei

$\tau \to P^0 l_i$

Same $Z^0 - l_i - l_j$ 1-loop effective coupling!

References:

E. Arganda, M.J. Herrero and A.M. Teixeira, JHEP 0710 (2007) 104
E. Arganda, M.J. Herrero and J. Portolés, JHEP 0806 (2008) 079

Beyond the MSSM

Example 1: MSSM + Trilinear R-parity violation

Beyond the MSSM

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$$W_{\not R} = W_{MSSM} + \frac{1}{2}\lambda_{ijk}\hat{L}_i\hat{L}_j\hat{E}^c_k$$

- Sneutrino-lepton loops are also possible in this case
- The new couplings break the cancellation since

$$\sum_k Z_V^{ki} Z_V^{kj*} \lambda_{jki}\lambda_{jkj} \neq 0$$

Great enhancement due to Z-boson penguins!

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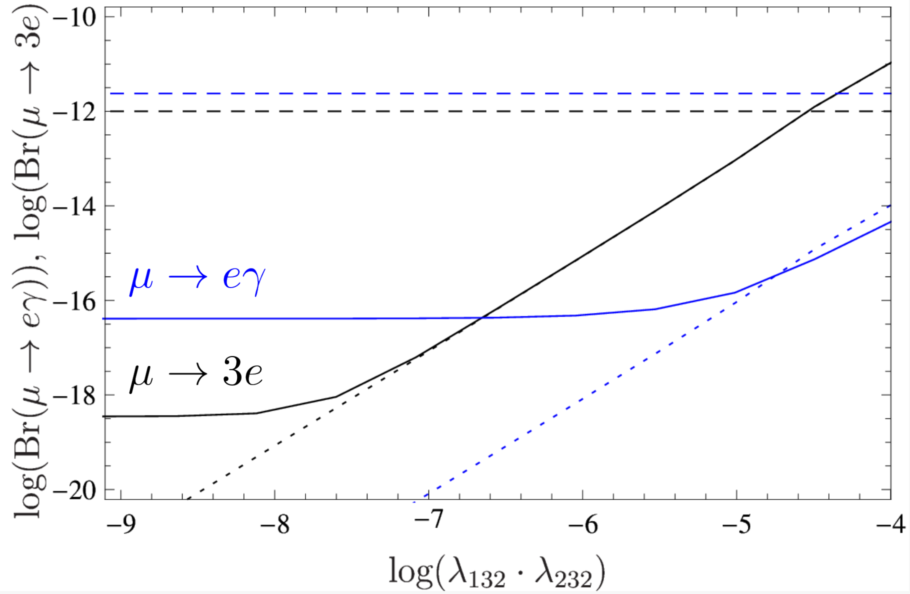

Bound in the literature: $\lambda_{132} \cdot \lambda_{232} \lesssim 0.2$

3 orders of magnitude improvement!

Beyond the MSSM

H.K. Dreiner, K. Nickel, F. Staub, AV, ArXiv:1204.5925 [hep-ph]

$$\mathcal{W}_{\not R} = \mathcal{W}_{MSSM} + \frac{1}{2}\lambda_{ijk}\hat{L}_i\hat{L}_j\hat{E}^c_k + \frac{1}{2}\lambda'_{ijk}\hat{L}_i\hat{Q}_j\hat{D}^c_k$$

Goal: set new bounds on R-parity violating couplings using Z-boson mediated LFV processes...

$1)\ l_i \to 3l_j$ $2)\ \mu - e$ conversion in nuclei $3)\ \tau \to P^0 l_i$

... and for completeness...

$4)\ l_i \to l_j\gamma$ $5)\ Z^0 \to l_i l_j$

Beyond the MSSM

H.K. Dreiner, K. Nickel, F. Staub, AV, ArXiv:1204.5925 [hep-ph]

Benchmark point BP0

$$m_0 = 100\,\mathrm{GeV}$$
$$M_{1/2} = 100\,\mathrm{GeV}$$
$$A_0 = 0$$
$$\tan\beta = 10$$
$$\mathrm{sign}(\mu) = +$$

Very light spectrum, included only for comparison!

Coupling	$l_i \to l_j \gamma$	$l_i \to 3 l_j$	$\tau \to l P / \mu - e$	$Z^0 \to l_i l_j$
$\lambda_{123}\lambda_{133}$	3.2×10^{-2}	1.3×10^{-2}	2.8×10^{-2}	2.8
$\lambda_{123}\lambda_{233}$	2.7×10^{-2}	1.4×10^{-2}	2.4×10^{-2}	7.9
$\lambda_{132}\lambda_{232}$	9.1×10^{-5}	3.7×10^{-5}	1.1×10^{-5}	3.5
$\lambda_{133}\lambda_{233}$	4.4×10^{-5}	3.7×10^{-5}	8.4×10^{-6}	3.3
$\lambda_{231}\lambda_{232}$	3.5×10^{-5}	2.4×10^{-5}	4.6×10^{-6}	2.7
$\lambda'_{122}\lambda'_{222}$	1.5×10^{-5}	9.8×10^{-5}	2.4×10^{-5}	1.3×10^{-1}
$\lambda'_{123}\lambda'_{223}$	1.5×10^{-5}	$1. \times 10^{-4}$	2.5×10^{-5}	1.3×10^{-1}
$\lambda'_{132}\lambda'_{232}$	1.5×10^{-5}	1.1×10^{-4}	2.4×10^{-5}	1.1×10^{-1}
$\lambda'_{133}\lambda'_{233}$	1.5×10^{-5}	1.1×10^{-4}	2.6×10^{-5}	1.1×10^{-1}
$\lambda'_{133}\lambda'_{333}$	4.2×10^{-3}	1.9×10^{-2}	7.9×10^{-2}	2.7×10^{-1}
$\lambda'_{233}\lambda'_{333}$	4.9×10^{-3}	2.6×10^{-2}	9.9×10^{-2}	3.0×10^{-1}

Limits on R-parity violating couplings

Beyond the MSSM

H.K. Dreiner, K. Nickel, F. Staub, AV, ArXiv:1204.5925 [hep-ph]

Benchmark point BP2

$$m_0 = 750\,\mathrm{GeV}$$
$$M_{1/2} = 350\,\mathrm{GeV}$$
$$A_0 = 0$$
$$\tan\beta = 10$$
$$\mathrm{sign}(\mu) = +$$

Realistic spectrum (allowed by LHC searches)

Coupling	$l_i \to l_j \gamma$	$l_i \to 3 l_j$	$\tau \to l_i P / \mu - e$	$Z^0 \to l_i l_j$
$\lambda_{123}\lambda_{133}$	1.8×10^{1}	1.2×10^{-2}	2.8×10^{-2}	1.4×10^{1}
$\lambda_{123}\lambda_{233}$	1.3×10^{1}	1.4×10^{-2}	2.4×10^{-2}	$4. \times 10^{1}$
$\lambda_{132}\lambda_{232}$	2.4×10^{-1}	3.5×10^{-5}	8.4×10^{-6}	1.7×10^{1}
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$\lambda'_{122}\lambda'_{222}$	4.5×10^{-4}	3.9×10^{-5}	9.3×10^{-6}	7.5×10^{-1}
$\lambda'_{123}\lambda'_{223}$	4.6×10^{-4}	3.9×10^{-5}	9.3×10^{-6}	7.5×10^{-1}
$\lambda'_{132}\lambda'_{232}$	4.9×10^{-4}	4.1×10^{-5}	9.8×10^{-6}	1.4
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$\lambda'_{133}\lambda'_{333}$	1.3×10^{-1}	1.6×10^{-2}	2.8×10^{-2}	3.3
$\lambda'_{233}\lambda'_{333}$	1.5×10^{-1}	1.4×10^{-2}	3.3×10^{-2}	3.6

Limits on R-parity violating couplings

Beyond the MSSM

Example 2: Supersymmetric inverse seesaw

A. Abada, D. Das, AV, C. Weiland, ArXiv:1206.XXXX [hep-ph]

$$\mathcal{W}_{IS} = \mathcal{W}_{MSSM} + Y_\nu^{ij} \widehat{\nu}_i^c \widehat{L}_j \widehat{H}_u + M_{R_{ij}} \widehat{\nu}_i^c \widehat{X}_j + \frac{1}{2} \mu_{X_{ij}} \widehat{X}_i \widehat{X}_j$$

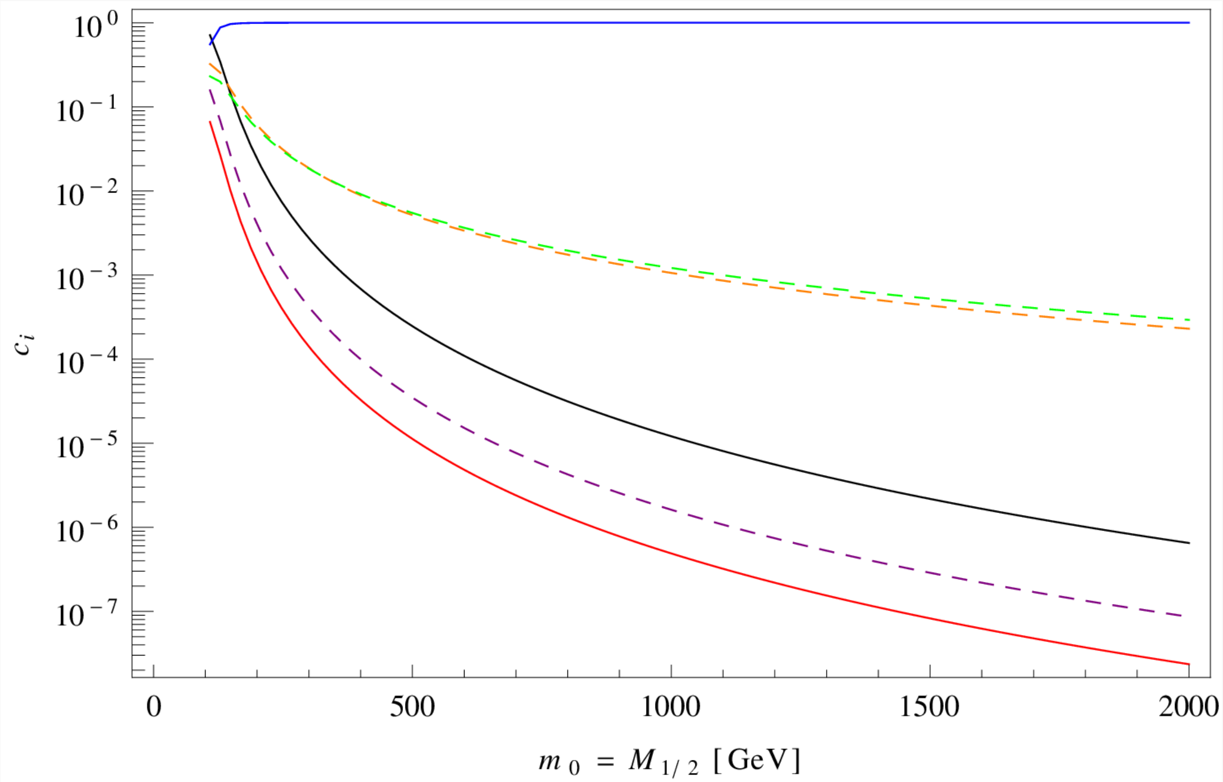
- Simple extension of the MSSM that includes non-zero neutrino masses

$$m_\nu \simeq \frac{v_u^2}{2} Y_\nu^T (M_R^T)^{-1} \mu_X M_R^{-1} Y_\nu = \frac{v_u^2}{2} Y_\nu^T M^{-1} Y_\nu$$

- The suppression by μ_X allows to have (in principle) $Y_\nu \sim \mathcal{O}(1)$

Goal: revisit LFV in the Inverse Seesaw

Beyond the MSSM



Relative contributions to $\mu \to 3e$

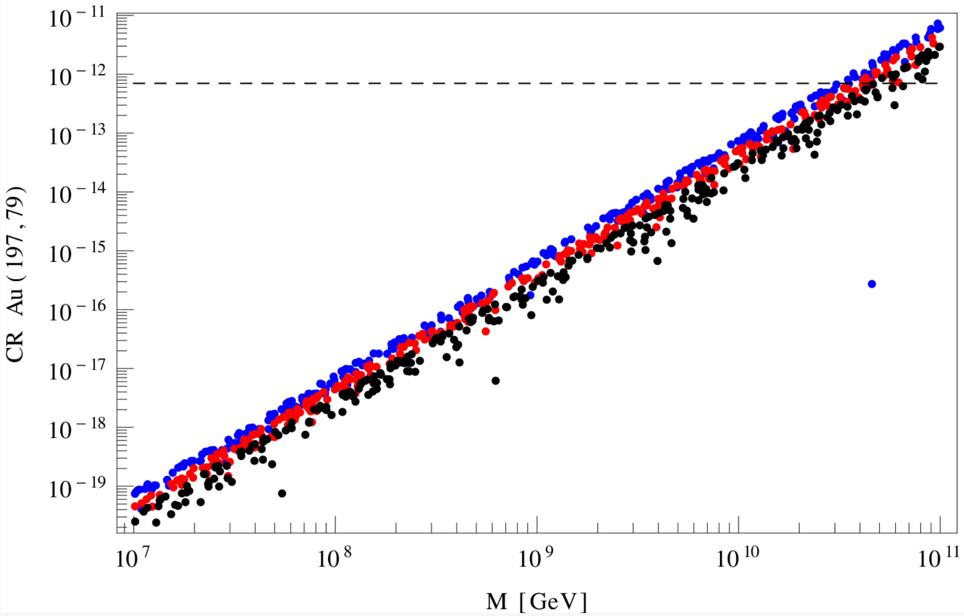
Blue : Z
Black : photon
Red : Higgs + Box

(dashed lines show interference terms)

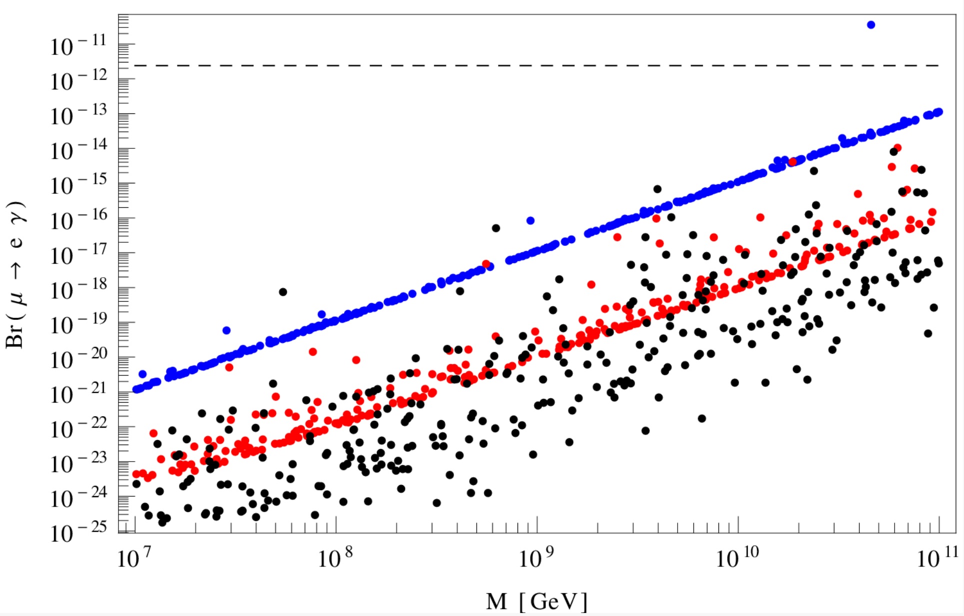
- In this figure $M_R = 10\,\mathrm{TeV}$ and $M = 10^{10}\,\mathrm{GeV}$
- **Photonic contributions** are competitive ONLY for low m_{SUSY}

Beyond the MSSM

$\mu - e$ conversion in gold



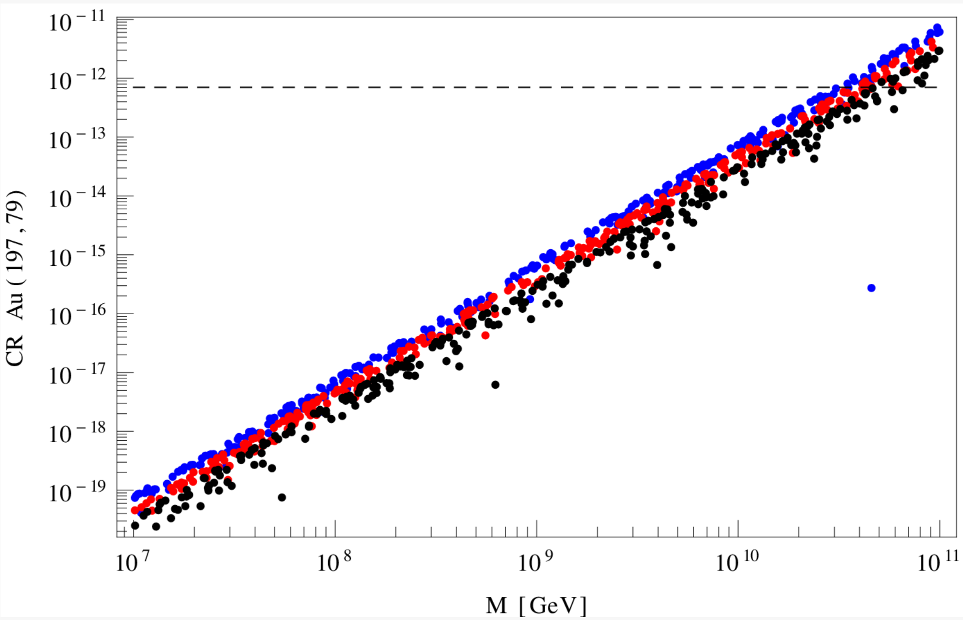
$\mu \to e\gamma$



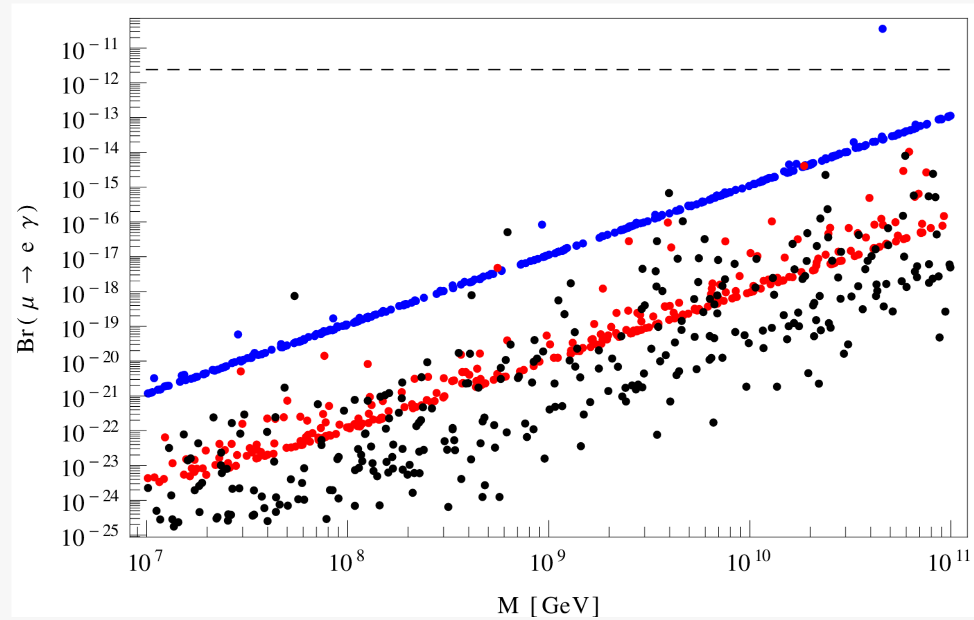
- $m_0, M_{1/2}$ randomly taken in [0,3] TeV. The Z-mediated observables have very little dependence: non-decoupling behavior.
- Colors: $M_R = 100\,\text{GeV}$, $M_R = 1\,\text{TeV}$, $M_R = 10\,\text{TeV}$

Beyond the MSSM

$\mu - e$ conversion in gold



$\mu \to e\gamma$



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- Colors: $M_R = 100\,\text{GeV}, M_R = 1\,\text{TeV}, M_R = 10\,\text{TeV}$

$$M \lesssim 4 \cdot 10^{10}\,\text{GeV} \Rightarrow \left(Y_\nu^\dagger Y_\nu\right)_{21} \lesssim 10^{-5} \Rightarrow$$ Order 1 Yukawas are not allowed!

Final remarks

- In the MSSM the Z-penguin contributions are usually neglected or regarded as sub-dominant. And that's totally correct!
- However, in many extensions of the lepton sector the Z-penguin becomes dominant, enhancing the signal by many orders of magnitude.
- In fact, one can find $\mathrm{CR}_{\mu-e}\,,\ \mathrm{Br}(\mu \to 3e) \gg \mathrm{Br}(\mu \to e\gamma)$
- LFV studies should be re-considered and bounds re-evaluated.

Thank you!

Backup slides

Photon penguin contributions

E. Arganda and M.J. Herrero, PRD 73 (2006) 055003

$$A_a^{L,R} = A_a^{(n)L.R} + A_a^{(c)L,R}, \quad a = 1, 2$$

$$\begin{aligned}
A_1^{(n)L} &= \frac{1}{576\pi^2} N_{iAX}^{R} N_{jAX}^{R*} \frac{1}{m_{\tilde{l}_X}^2} \frac{2 - 9x_{AX} + 18x_{AX}^2 - 11x_A^3 + 6x_{AX}^3 \log x_{AX}}{(1 - x_{AX})^4} \\
A_2^{(n)L} &= \frac{1}{32\pi^2} \frac{1}{m_{\tilde{l}_X}^2} \left[N_{iAX}^{L} N_{jAX}^{L*} \frac{1 - 6x_{AX} + 3x_{AX}^2 + 2x_{AX}^3 - 6x_{AX}^2 \log x_{AX}}{6(1 - x_{AX})^4} \right. \\
&+ N_{iAX}^{R} N_{jAX}^{R*} \frac{m_{l_i}}{m_{l_j}} \frac{1 - 6x_{AX} + 3x_{AX}^2 + 2x_{AX}^3 - 6x_{AX}^2 \log x_{AX}}{6(1 - x_{AX})^4} \\
&+ \left. N_{iAX}^{L} N_{jAX}^{R*} \frac{m_{\tilde{\chi}_A^0}}{m_{l_j}} \frac{1 - x_{AX}^2 + 2x_{AX} \log x_{AX}}{(1 - x_{AX})^3} \right] \\
A_a^{(n)R} &= A_a^{(n)L} \Big|_{L \leftrightarrow R}
\end{aligned}$$

$$\text{where } x_{AX} = m_{\tilde{\chi}_A^0}^2 / m_{\tilde{l}_X}^2$$

Photon penguin contributions

E. Arganda and M.J. Herrero, PRD 73 (2006) 055003

$$A_1^{(c)L} = -\frac{1}{576\pi^2} C^R_{iAX} C^{R*}_{jAX} \frac{1}{m^2_{\tilde{\nu}_X}} \frac{16 - 45x_{AX} + 36x^2_{AX} - 7x^3_A + 6(2 - 3x_{AX}) \log x_{AX}}{(1 - x_{AX})^4}$$

$$\begin{aligned} A_2^{(c)L} &= -\frac{1}{32\pi^2} \frac{1}{m^2_{\tilde{\nu}_X}} \left[C^L_{iAX} C^{L*}_{jAX} \frac{2 + 3x_{AX} - 6x^2_{AX} + x^3_{AX} + 6x_{AX} \log x_{AX}}{6\,(1 - x_{AX})^4} \right. \\ &+ C^R_{iAX} C^{R*}_{jAX} \frac{m_{l_i}}{m_{l_j}} \frac{2 + 3x_{AX} - 6x^2_{AX} + x^3_{AX} + 6x_{AX} \log x_{AX}}{6\,(1 - x_{AX})^4} \\ &+ \left. C^L_{iAX} C^{R*}_{jAX} \frac{m_{\tilde{\chi}^-_A}}{m_{l_j}} \frac{-3 + 4x_{AX} - x^2_{AX} - 2 \log x_{AX}}{(1 - x_{AX})^3} \right] \\ A_a^{(c)R} &= A_a^{(c)L} \Big|_{L \leftrightarrow R} \end{aligned}$$

$$\text{where } x_{AX} = m^2_{\tilde{\chi}^-_A} / m^2_{\tilde{\nu}_X}$$

Z-penguin contributions

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$$F_{L(R)} = F^{(n)}_{L(R)} + F^{(c)}_{L(R)}$$

$$\begin{aligned}
F_L^{(n)} &= -\frac{1}{16\pi^2}\Big\{N^R_{iBX}N^{R*}_{jAX}\Big[2E^{R(n)}_{BA}C_{24}(m^2_{\tilde{l}_X},m^2_{\tilde{\chi}^0_A},m^2_{\tilde{\chi}^0_B}) - E^{L(n)}_{BA}m_{\tilde{\chi}^0_A}m_{\tilde{\chi}^0_B}C_0(m^2_{\tilde{l}_X},m^2_{\tilde{\chi}^0_A},m^2_{\tilde{\chi}^0_B})\Big] \\
&+ N^R_{iAX}N^{R*}_{jAY}\Big[2Q^{\tilde{l}}_{XY}C_{24}(m^2_{\tilde{\chi}^0_A},m^2_{\tilde{l}_X},m^2_{\tilde{l}_Y})\Big] + N^R_{iAX}N^{R*}_{jAX}\Big[Z^{(l)}_L B_1(m^2_{\tilde{\chi}^0_A},m^2_{\tilde{l}_X})\Big]\Big\} \\
F_R^{(n)} &= F_L^{(n)}\Big|_{L\leftrightarrow R} \\
F_L^{(c)} &= -\frac{1}{16\pi^2}\Big\{C^R_{iBX}C^{R*}_{jAX}\Big[2E^{R(c)}_{BA}C_{24}(m^2_{\tilde{\nu}_X},m^2_{\tilde{\chi}^-_A},m^2_{\tilde{\chi}^-_B}) - E^{L(c)}_{BA}m_{\tilde{\chi}^-_A}m_{\tilde{\chi}^-_B}C_0(m^2_{\tilde{\nu}_X},m^2_{\tilde{\chi}^-_A},m^2_{\tilde{\chi}^-_B})\Big] \\
&+ C^R_{iAX}C^{R*}_{jAY}\Big[2Q^{\tilde{\nu}}_{XY}C_{24}(m^2_{\tilde{\chi}^-_A},m^2_{\tilde{\nu}_X},m^2_{\tilde{\nu}_Y})\Big] + C^R_{iAX}C^{R*}_{jAX}\Big[Z^{(l)}_L B_1(m^2_{\tilde{\chi}^-_A},m^2_{\tilde{\nu}_X})\Big]\Big\} \\
F_R^{(c)} &= F_L^{(c)}\Big|_{L\leftrightarrow R}
\end{aligned}$$

Z-penguin contributions

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However, note that in the decay width one has

$$
\begin{aligned}
F_{LL} &= \frac{F_L Z_L^{(l)}}{g^2 \sin^2\theta_W m_Z^2} \\
F_{RR} &= F_{LL}\big|_{L\leftrightarrow R} \\
F_{LR} &= \frac{F_L Z_R^{(l)}}{g^2 \sin^2\theta_W m_Z^2} \\
F_{RL} &= F_{LR}\big|_{L\leftrightarrow R}
\end{aligned}
$$

Other observables

Another brief détour...

Experimental limits

$\mu - e$ conversion	$\tau \to P^0 l_i$
$\mathrm{CR}_{\mathrm{Au}(197,79)} < 7 \cdot 10^{-13}$	$\mathrm{Br}(\tau \to \pi\mu) < 5.8 \cdot 10^{-8}$
$\mathrm{CR}_{\mathrm{Ti}(48,22)} < 4.3 \cdot 10^{-12}$	$\mathrm{Br}(\tau \to \eta\mu) < 5.1 \cdot 10^{-8}$
$< 10^{-18} - 10^{-16}$ (future)	...