f_B from the Lattice

Eduardo Follana

Universidad de Zaragoza

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Outline

- Motivation.
- Lattice QCD errors.
- Heavy quarks in lattice QCD.
- Results.
- Conclusions and Outlook.

Motivation

- Simple QCD matrix elements enter into weak decay rates (CKM, unitarity).

$$
\begin{aligned}
\mathcal{B}(B \to l\nu) &= \frac{G_F^2 |V_{ub}|^2 \tau_B}{8\pi} {f_B}^2 m_B m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2 \\
\langle 0 | A^\mu | B(p) \rangle &= f_B p_\mu
\end{aligned}
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$$\langle 0|A^\mu|B(p)\rangle = f_B p_\mu$$

- For neutral mesons

$$\mathcal{B}(B_s \to \mu^+\mu^-) = \frac{G_F^2\alpha^2|V_{tb}V_{ts}^*|^2\tau_{B_s}}{64\pi^3} f_{B_s}^2 m_{B_s}^3 \sqrt{1 - \frac{4m_\mu{}^2}{m_{B_s}{}^2}} \times \{\cdots\}$$

$\mathcal{B}(Bs \to \mu^+\mu^-) = 3.1(1.4)x10^-9$ (SM: error dominated by f_{B_s})
$\mathcal{B}(Bs \to \mu^+\mu^-) < 4.5(3.8)x10^-9$ LHCb(arXiv:1203.4493)

Motivation

- In the heavy quark sector (c and b) there are many gold-plated states in the spectrum. We can test our calculations.
- Precision is crucial for searches of BSM physics. We need good control over all systematic errors. Best if we have independent calculations for crosscheck.

Lattice calculation

- We introduce a space-time lattice, with length L and lattice spacing a.
- We discretize the (euclidean) action.
- High-dimensional integral $\Rightarrow$ Montecarlo integration.
- We eliminate the lattice.

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Fixing the parameters

The free parameters in the lattice formulation are fixed by setting a set of calculated quantities to their measured physical values. Quantities that can be accurately calculated from the lattice and are measured with good precision experimentally.

- Scale: lattice spacing a:
- Quark masses: $m_{u,d}, m_s, m_c,\ m_b$.
  Could be fixed, for example, by $m_\pi, m_K, m_{\eta_c}, m_{\eta_b}$.

Some systematic errors

- Finite volume: $m_\pi^{-1} \ll L$.
- Finite lattice spacing: discretization errors $\mathcal{O}(a^k)$.
  Simulations at different values of a and extrapolation to the continuum limit $a \to 0$.
  Improved actions and operators lead to smaller errors (asqtad, HISQ, TW, clover, ...)
- Renormalization constants: The lattice is an ultraviolet regulator. In general, we need to calculate renormalization constants to relate quantities calculated in the lattice with quantities calculated in a different scheme.
- Matching constants: When using effective field theories, we need to match such EFT to QCD.
- Chiral extrapolation: Usually we are not able to simulate at physical values of the light quark masses $m_{u,d}$. We simulate at a set of m_l and extrapolate $m_l \to m_{u,d}$.
  More important for hadrons with valence light quarks.
- Parameter determination: Errors in the determination of the lattice spacing, quark masses, etc.

Heavy Quarks on the Lattice

- The discretization errors grow with the quark mass as powers of am (tipically $(am)^2$ in most currently used formulations). For a lattice spacing of $a \approx 0.1$ fm, $am_c \approx 0.4$ and $am_b \approx 2.0$.
- For a direct simulation, we need:

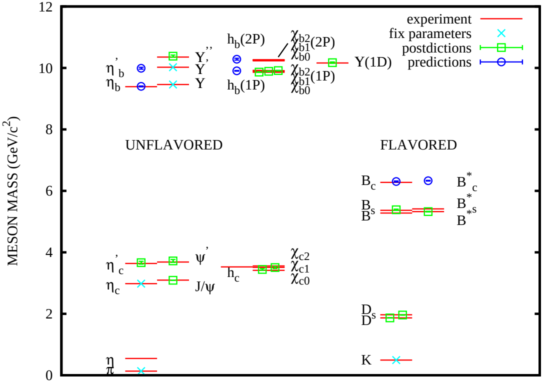
$$\begin{aligned} am_h &\ll 1 && \text{(heavy quarks)} \\ La &\gg m_\pi^{-1} && \text{(light quarks)} \end{aligned}$$

- Two scales. Difficult to do directly. Instead take advantage of the fact that m_h is large: $\Rightarrow$ effective field theory (NRQCD, HQET). Very successful for b physics.

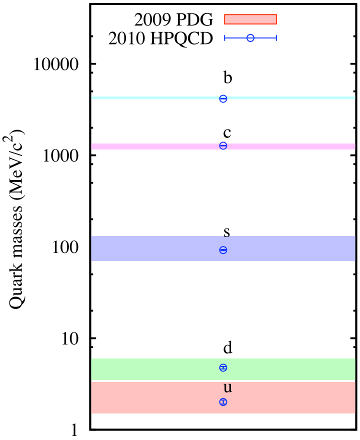
Relativistic Heavy Quarks

A relativistic formulation has several advantages:

- An effective theory needs matching to QCD: hard, source of systematic error difficult to reduce.
- If the action has enough symmetry, some quantities do not renormalize. For example, for staggered quarks, meson decay constants do not renormalize because of PCAC.
- Using improved actions (HISQ, TMW) and fine enough lattices, it is possible to get accurate results. This has been extensively tested for c quarks and works very well. Can reduce the errors to the few percent level. Worth trying for b.
- If we use the same action for heavy-heavy and heavy-light systems $\rightarrow$ extensive consistency checks.
  Error cancelation in many ratios.



MESON MASS (GeV/c^2)
0
2
4
6
8
10
12
UNFLAVORED
FLAVORED
experiment
fix parameters
postdictions
predictions
η'_b
η_b
Υ''
Υ'
Υ
$h_b(2P)$
$h_b(1P)$
χ_{b2} χ_{b1} (2P) χ_{b0}
χ_{b2} χ_{b1} (1P) χ_{b0}
$\Upsilon(1D)$
B_c
B^*_c
B_s
B
B^*_s
B^*
η'_c
η_c
ψ'
J/ψ
h_c
χ_{c2}
χ_{c1}
χ_{c0}
D_s
D
η
π
K



Quark masses (MeV/c^2)
2009 PDG
2010 HPQCD
1
10
100
1000
10000
b
c
s
d
u

Non-relativistic results

Two calculations on MILC $N_f = 2 + 1$ asqtad configurations. Two lattice spacings, a $\sim 0.12, 0.09$ fm.

HPQCD: NRQCD b quarks, HISQ light valence quarks.

$f_B = 191(9)$ MeV. 4.6%

$f_{B_s} = 227(10)$ MeV. 4.4%

$\frac{f_{B_s}}{f_B} = 1.188(18)$ 1.5%

FERMILAB/MILC: clover Wilson/Fermilab b, asqtad light valence quarks.

$f_B = 197(9)$ MeV. 4.6%

$f_{B_s} = 242(10)$ MeV. 4.1%

$\frac{f_{B_s}}{f_B} = 1.229(26)$ 2.1%

Some room for improvement (matching, statistics).

Alpha collaboration: On CLS $Nf = 2$ configurations. Three lattice spacings, a ~ 0.075 to ~ 0.05 fm. HQET for b, NP improved Wilson for the light valence quarks.

$f_B = 174(11)$ MeV 6.3%

Relativistic results I

ETMC: Twisted Wilson quarks. $N_f = 2$ configurations.

Four values of the lattice spacing, a $\sim$ .1 fm down to $\sim$ .054 fm.

Heavy quark: $m_h \sim m_c, 2.4 m_c$.
Uses also static point to constrain the extrapolation.

$f_{B_s} = 232(10)$ MeV. 4.3%

$\frac{f_{B_s}}{f_B} = 1.19(5)$. 4.2%

$f_B = f_{B_s} \left(\frac{f_B}{f_{B_s}} \right) = 195(12)$ 6.2%

Relativistic results II

HPQCD: MILC $N_f = 2 + 1$ asqtad configurations.

5 values of the lattice spacing, from a ~ 0.15 fm to ~ 0.045 fm.

HISQ valence quarks: m_s, $m_h \sim m_c, m_b$.

f_{H_s}, with H_s varying between D_s and B_s as we change the heavy quark mass.

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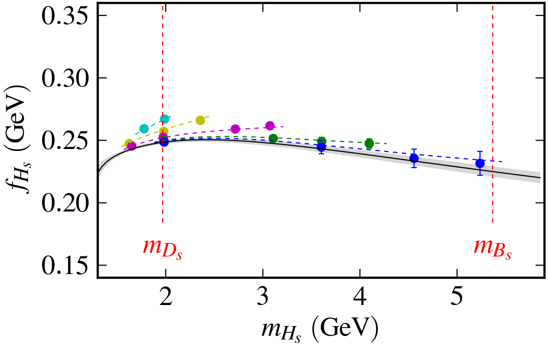

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f_{H_s}, with H_s varying between D_s and B_s as we change the heavy quark mass.

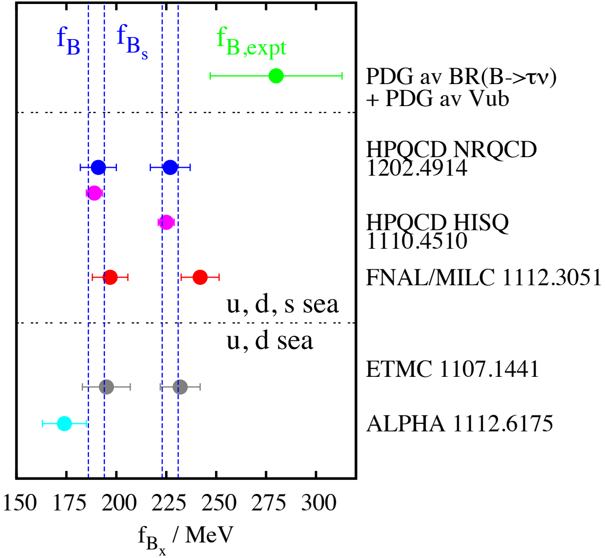
$f_{B_s} = 225(4)$ MeV. 1.8%

$f_B = f_{B_s}^{relat} (\frac{f_B}{f_{B_s}})^{NRQCD} = 189(4)$ MeV 2.1%

$f_{B_s} < f_{D_s}$: $\frac{f_{B_s}}{f_{D_s}} = 0.906(14)$

f_B could be calculated directly, but much more expensive.

Comparisons



$\sim 3\sigma$ tension with unitarity in the CKM matrix (arXiv:1204.0791, arXiv:1104.2117). f_B, V_{ub}? Hint of new physics?

Conclusions and Outlook

- The lattice is starting to produce a good enough f_B to impact on phenomenology (unitarity tests). We need to reduce the errors and have as many independent calculations as possible for crosscheck.
- We need to calculate as many quantities as possible, again for crosscheck of our lattice methods.
- Effective theory methods and relativistic ones can be complementary, at least for a time.
- To increase precision in relativistic calculations we will need to go to smaller lattice spacings.
  In principle straightforward (computing time), but there may be problems (topology freezing?).
- There is still much scope for improvement.