Status of B-physics on the lattice

Carlos Pena







XL IMFP-Flavour, Benasque, 27/05/12

outline

- Iattice QCD in the precision era
 - cost scaling: reach of lattice QCD computations.
 - O lattice techniques for B-physics.

- overview of results (CKM, UT) (simulations with light dynamical quarks only!)
 - O B-meson decay constants, mixing amplitudes, semileptonic form factors.
 - O summaries of lattice results: FLAG.

• a word on issues involving perturbation theory

hadronic effects in flavour physics

We are in an era of precision flavour physics. Hadronic effects can be ...

• mostly irrelevant: $\mu \rightarrow e \gamma, \ \mathbf{d}_{\mathrm{n}}$

- under good theoretical control: $K \to \pi \nu \bar{\nu}$
- relevant, difficult, but measured indirectly: $(g-2)_{\mu}$
- relevant and difficult to compute: $V_{xy}, K \to \pi\pi, \Delta m_{d,s}, \ldots$

Use first-principles technique to deal with low-energy hadronic physics: lattice QCD.

(Complement with other first-principles/systematic approaches: dispersion relations, effective theories.)

hadronic effects in flavour physics

Your typical weak hadronic matrix element: $\begin{aligned} & \sum_{k=1}^{|\mathcal{E}_K|} \approx C_e \frac{|\mathcal{E}_K|}{|\mathcal{E}_K|} \approx C_e \frac{|\mathcal{E}_K|}{|\mathcal{E}_K|} \approx C_e \frac{|\mathcal{E}_K|}{|\mathcal{E}_K|} \\ & = \sum_{k=1}^{|\mathcal{E}_K|} \frac{|\mathcal{E}_K|} \\ & = \sum_{k=1}^{|\mathcal{E}_K|} \frac{|\mathcal{E}_K$



First-principles approach to strongly coupled quantum field theories.

[Wilson 1974]



First-principles approach to strongly coupled quantum field theories.

[Wilson 1974]



- Take continuum, infinite volume limits.
- Tune irrelevant couplings to preserve symmetries, improve scaling to CL ...

$$S_{\text{lat}} = S_0 + aS_1 + a^2S_2 + \dots$$
$$\mathcal{O}_{\text{lat}} = \mathcal{O}_0 + a\mathcal{O}_1 + a^2\mathcal{O}_2 + \dots$$

several different lattice actions: universality

fermion actions: (improved) Wilson, (improved) staggered, domain-wall, perfect actions, Neuberger fermions, twisted-mass QCD,

First-principles approach to strongly coupled quantum field theories.



Many tools developed along the last 20+ years:

- Control scaling (Symanzik improvement).
- Non-perturbative renormalisation and matching (e.g. to effective theories).
- Lattice regularisations with exact chiral symmetry.

0 ...

First-principles approach to strongly coupled quantum field theories.



Crucial: control systematic uncertainties.

- Get rid of cutoffs ($a \rightarrow 0, L \rightarrow \infty$).
- Compute in / extrapolate to physical SSB regime (light quarks, isospin breaking).
 - Keep all relevant scales far from cutoffs.

What is the current physics reach of LQCD?





Main cost factor: reiterated inversion of lattice Dirac operator on fixed gauge field.



For a long time: serious difficulties in reaching light dynamical quark masses.



Main cost factor: reiterated inversion of lattice Dirac operator on fixed gauge field.





Mass preconditioning/domain decomposition, deflation \Rightarrow mild mass dependence.

Algorithm efficiency degrades rapidly below lattice spacings ~0.05 fm ("topology freezing"). [Del Debbio, Panagopoulos, Vicari 2002; Schaefer, Sommer, Virotta 2010]



Work with open boundary conditions?

[Lüscher, Schaefer 2011]

n.b.: 0.05 fm \times 4 GeV \approx 1

lattice QCD reach: simulation landscape



lattice QCD reach: simulation landscape



lattice QCD reach: simulation landscape





[BMW Collaboration 2008]

lattice QCD reach: a precision era



B-physics on the lattice: not quite there yet

b quark scale (naively) not accessible without introducing large am_b cutoff effects.



various approaches:

- Effective theories for HQ sector (NRQCD, non-perturbative HQET).
- O Relativistic quarks in charm region + static limit ⇒ HQET-inspired interpolations to b region.
- Use heavily improved actions and/or mass-dependent matching conditions to try and push relativistic quarks into b mass regime.

B-physics on the lattice: not quite there yet

• NRQCD: combined expansion in v^2 , Λ/m_b , a, perturbative matching to QCD. + easy to carry out to high orders, allows to work at large lattice spacing. – only works in scaling window $a\Lambda \ll 1, \ m_{
m h}a\gtrsim 1$, no continuum limit.

[HPOCD]

- npHQET: expansion in $m_{\rm h}^{-1}$, matched non-perturbatively to QCD (using small V).
 - + continuum limit exists at any order in the expansion, systematic tool.
 - difficult to go beyond $1/m_{\rm h}$ order (\Rightarrow percent systematic uncertainties).

ALPHA]

- combined: (smartly) interpolate between charm region and static limit. + well-controlled systematics in either end.
 - systematics associated to true mass dependence hard to control.

[ETMC, ALPHA]

relativistic b-quark: (HQET-inspired) tuning of counterterms to improve scaling. 0 + easy to carry out to high orders in the O(a) improvement philosophy. - systematics difficult to test (perturbative matching, "true" mass dependence).

results: $f_{B_{(s)}}$



results: $f_{B_{(s)}}$



$$BR(B \to \ell \nu) = \frac{G_F^2 m_B}{8\pi} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2} \right) f_B^2 |V_{ub}|^2 \tau_B$$

 $= 1.68 \pm 0.31$

collaboration	$N_{ m f}$	$f_B \;({ m MeV})$	$f_{B_s} \ ({ m MeV})$	method
HPQCD 11-12	2+1	191(9)	227(10)	HISQ+rSTAG
FNAL/MILC II	2+I	197(9)	242(10)	FNAL+rSTAG
HPQCD 09	2+1	97(3)	240(16)	NRQCD+rSTAG
ALPHA I I	2	174(11)(2)		npHQET+iW
ETMC I I	2	195(12)	232(10)	interp+tmQCD

results: $f_{B_{(s)}}$



results: $f_{B_{(s)}}^2 B_{B_{(s)}}$ and ξ



 $\langle \bar{B}^0 | (\bar{b}\gamma^{\rm L}_{\mu}d) (\bar{b}\gamma^{\rm L}_{\mu}d) | B^0 \rangle_{\rm RGI} = \frac{8}{3} m_B^2 f_B^2 \hat{B}_B$

$$\xi = \frac{f_{B_s} \sqrt{\hat{B}_{B_s}}}{f_B \sqrt{\hat{B}_B}} \quad \propto \frac{\Delta M_{B_s}}{\Delta M_B}$$



results: $f_{B_{(s)}}^2 B_{B_{(s)}}$ and ξ



 $\langle \bar{B}^0 | (\bar{b}\gamma^{\rm L}_{\mu}d) (\bar{b}\gamma^{\rm L}_{\mu}d) | B^0 \rangle_{\rm RGI} = \frac{8}{3} m_B^2 f_B^2 \hat{B}_B$

$$\xi = \frac{f_{B_s} \sqrt{\hat{B}_{B_s}}}{f_B \sqrt{\hat{B}_B}} \quad \propto \frac{\Delta M_{B_s}}{\Delta M_B}$$



non-lattice:
$$B_{B_s} = 1.220^{+0.103}_{-0.044}$$

collaboration	$f_B \sqrt{\hat{B}_B}$	$f_{B_s}\sqrt{\hat{B}_{B_s}}$	\hat{B}_B	\hat{B}_{B_s}	method
FNAL/MILC I I	252(23)	239(18)	—		FNAL+rSTAG
HPQCD 09	216(9)(12)	266(6)(17)	1.27(10)	l.33(5)(3)	NRQCD+rSTAG
HPQCD 06		281(21)		1.17(17)	NRQCD+rSTAG

[[]CKMFitter 2012]

results: $f_{B_{(s)}}^2 B_{B_{(s)}}$ and ξ



 $\langle \bar{B}^0 | (\bar{b}\gamma^{\rm L}_{\mu}d) (\bar{b}\gamma^{\rm L}_{\mu}d) | B^0 \rangle_{\rm RGI} = \frac{8}{3} m_B^2 f_B^2 \hat{B}_B$

$$\xi = \frac{f_{B_s} \sqrt{\hat{B}_{B_s}}}{f_B \sqrt{\hat{B}_B}} \quad \propto \frac{\Delta M_{B_s}}{\Delta M_B}$$



non-lattice:
$$B_{B_s}/B_B = 1.134^{+0.074}_{-0.093}$$

[CKMFitter 2012]

collaboration	ξ	\hat{B}_{B_s}/\hat{B}_B	method
RBC/UKQCD 10	1.13(12)		static+DWF
HPQCD 09	1.258(25)(21)	l.05(7)	NRQCD+rSTAG
FNAL/MILC 09	1.205(37)(34)		FNAL+rSTAG

Errors not far from 5% ballpark, even if cutting-edge lattice (lightest dynamical masses, finest lattice spacings) not included.

Careful, global assessment of involved systematics crucial.

Errors not far from 5% ballpark, even if cutting-edge lattice (lightest dynamical masses, finest lattice spacings) not included.

Careful, global assessment of involved systematics crucial.

what about Γ_{12} ?



charm kicking back: keeping it as an active d.o.f. starts to be relevant for B_K. [cf. Buras, Guadagnoli, Isidori 10]

bonus: if properly tackled, would allow to study D mixing.







Analyticity





Simultaneous solution: use dispersion relations, analyticity, unitarity to find wellbehaved parametrisation.

[several works 2005-...]

 χ^2 /d.o.f. = 0.59 χ^2 /d.o.f. = 0.59 simultaneous 4-parameter z-fit — simultaneous 4-parameter z-fit • Fermilab-MILC lattice data Fermilab-MILC lattice data 10 0.035 * BABAR data rescaled by $|V_{ub}|$ from z-fit ϕ * BABAR data rescaled by $|V_{ub}|$ from z-fit 0.03 $p_+\phi_+f_+$ $^{+}(q^{2})$ 0.025 0.02 2 0.015 0.01 -0.3 10 25 -0.2-0.1 0.1 0.2 $q^2 (\text{GeV}^2)$ \boldsymbol{z} $z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \rightarrow f_+(q^2) = \frac{1}{B(q^2)\phi(q^2, t_0)} \left| \sum_{n \ge 0} a_n(t_0) \, z(q^2, t_0)^n \right|_{n \ge 0}$

Simultaneous solution: use dispersion relations, analyticity, unitarity to find wellbehaved parametrisation.

[several works 2005-...]





Simultaneous solution: use dispersion relations, analyticity, unitarity to find wellbehaved parametrisation. [Okubo et al. 71; Bourrely et al. 81]

[several works 2005-...]

HFAG Ave. (BLNP) $4.32 \pm 0.16 + 0.22 - 0.23$ Ball-Zwicky $q^2 < 16$ HFAG Ave. (DGE) $4.46 \pm 0.16 + 0.18 - 0.17$ $3.34 \pm 0.12 + 0.55 - 0.37$ HFAG Ave. (GGOU) $4.34 \pm 0.16 + 0.15 - 0.22$ HPQCD $q^2 > 16$ HFAG Ave. (ADFR) $4.16 \pm 0.14 + 0.25 - 0.22$ $3.40 \pm 0.20 \pm 0.59 - 0.39$ HFAG Ave. (BLL) $4.87 \pm 0.24 \pm 0.38$ FNAL $q^2 > 16$ BABAR (LLR) $4.43 \pm 0.45 \pm 0.29$ $3.62 \pm 0.22 \pm 0.63 - 0.41$ BABAR endpoint (LLR) $4.28 \pm 0.29 \pm 0.48$ BABAR endpoint (LNP) $4.40 \pm 0.30 \pm 0.47$ HFAG HFAG ICHEP08 End Of 200 3 4 2 0 4 $|V_{ub}| [\times 10^{-3}]$ $|V_{ub}| [\times 10^{-3}]$

2

Lots of room for improvement by incorporating existing finer lattices with lighter dynamical quarks into the analysis.

 $B \rightarrow D^{(*)}$: less kinematical trouble, can extract $|V_{cb}|$ from one single combination of form factors at zero recoil.

$$\frac{d\Gamma(\bar{B} \to Dl\bar{v})}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (M_B + M_D)^2 (w^2 - 1)^{3/2} |V_{cb}|^2 |\mathscr{G}(w)|^2,$$

$$\frac{d\Gamma(\bar{B} \to D^* l\bar{v})}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (M_B - M_{D^*})^2 (w^2 - 1)^{1/2} |V_{cb}|^2 \chi(w) |\mathscr{F}(w)|^2$$

$$q^2 = M_B^2 + M_{D^{(*)}}^2 - 2wM_BM_{D^{(*)}}$$

collaboration	$\mathcal{F}(1)$	$\mathcal{G}(1)$	method
FNAL/MILC 08-10	0.908(17)		FNAL+rSTAG
FNAL/MILC 04		1.074(24)	FNAL+rSTAG





[BaBar, arXiv:1205.5442]

Update by various collaborations due soon. Progress most welcome in view of recently reported BaBar excess.

(Requires good control of τ decay channel \longrightarrow scalar form factor.)

FLAG

Users of lattice results should be (made) aware of potential systematic effects. Lattice groups should provide qualified global averages of lattice results.

\Rightarrow FLAG

FLAG-I

Users of lattice results should be (made) aware of potential systematic effects. Lattice groups should provide qualified global averages of lattice results.

FLAG = FLAVIAnet Lattice Averaging Group

FLAG-1 members:

G Colangelo (BERN) S Dürr (Jülich, BMW) A Jüttner (CERN, RBC/UKQCD) L Lellouch (Marseille, BMW) H Leutwyler (Bern) V Lubicz (Rome 3, ETM) S Necco (CERN, Alpha-CLS) C Sachrajda (Southampton, RBC/UKQCD) S Simula (Rome 3, ETM) A Vladikas (Rome 2, Alpha and ETM) U Wenger (Bern, ETM) H Wittig (Mainz, Alpha-CLS)



FLAG-I

Users of lattice results should be (made) aware of potential systematic effects. Lattice groups should provide qualified global averages of lattice results.

FLAG = FLAVIAnet Lattice Averaging Group



First review appeared on Nov 2010, updated May 2011 (EPJC).

regularly updated web: http://itpwiki.unibe.ch/flag

what does FLAG offer?

FLAG-1: concentrate on light hadron physics.

- light quark masses
- LECs
- decay constants
- pion and kaon form factors
- kaon bag parameter

For each quantity provide:

- complete list of references, summary of formulae/notations, ...
- summary of essential aspects of each computation
- averages (if sensible)
- "lattice dictionary" for non-experts

what does FLAG offer?

			6/ic	ital Statu	the boy	Núnue Dúnue	un ettedoi.
	N _f		'nq	Ľ		с ^О	action
$\overline{0.9599(34)(^{+31}_{-47})(14)}$	2+1	RBC/UKQCD 10	Α	•	*		DWF
0.9644(33)(34)(14)	2+1	RBC/UKQCD 07	Α	•	*		DWF
0.9544(68) _{stat}	2	ETM 10D	С	•	*		max. tmQCD
0.9560(57)(62)	2	ETM 09A	Ρ	•	•	•	max. tmQCD
0.9647(15) _{stat}	2	QCDSF 07	С		*		clover (NP)
0.968(9)(6)	2	RBC 06	Α		*		DWF
0.967(6)	2	JLQCD 05	С		*		clover (NP)

1.197(2)(+3)	2+1	MILC 10	C	•	*	*	KSMILC
1.204(7)(25)	2+1	RBC/UKQCD 10A	Р		•	*	DWF
1.192(7)(6)	2+1	BMW 10	Α	*	*	*	tISW
1.210(12) _{stat}	2+1	JLQCD/TWQCD	C	•			overlap
1.198(1)(⁺⁶)	2+1	MILC 09A	C	*	*	*	KS _{MILC}
1.191(16)(16)	2+1	ALVdW 08	C	*	•	•	KS _{MILC} /DWF
1.189(20)	2+1	PACS-CS 08	Р	*			clover (NP)
1.18(1)(1)	2+1	BMW 08	C	*	*	*	impr. Wilson
1.189(2)(7)	2+1	HPQCD/UKQCD 08	Α	*	•	*	KS ^{HISQ}
1.205(18)(62)	2+1	RBC/UKQCD 07	Α	•	*		DWF
1.218(2)(⁺¹¹ ₋₂₄)	2+1	NPLQCD 07	Α	•			KS_{MILC}/DWF
1.210(6)(15)(9)	2	ETM 09	Α	•	•	*	max. tmQCD
1.21(3)	2	QCDSF/UKQCD 07	C	•	\star	•	clover (NP)

 $f_{+}(0)$

 f_K/f_π

what does FLAG offer?





FLAG-2

Expand to cover most quantities, include representatives of *all* large collaborations, and distribute over Europe/US/Japan.

(n.b.: merge with parallel effort by Laiho, Lunghi, Van de Water)

```
advisory board: S. Aoki (J), C. Bernard (US), C. Sachrajda (EU)
editorial board: G. Colangelo, H. Leutwyler, A. Vladikas, U. Wenger
working groups:
                                        T. Blum, L. Lellouch, V. Lubicz
   quark masses
                                      A. Jüttner, T. Kaneko, S. Simula
   V_{ud}, V_{us}
   LECs
                                         S. Dürr, H. Fukaya, S. Necco
   B_K
                                         J. Laiho, S. Sharpe, H. Wittig
   \alpha_{\mathbf{s}}
                                 T. Onogi, J. Shigemitsu, R. Sommer
   f_B, B_B, f_D
                               Y.Aoki, M. Della Morte, A. El Khadra
   B, D \to H \ell \nu
                                      E. Lunghi, CP, R.Van de Water
```

target: next review by end 2012/early 2013; new published review every 2nd year; regular web updates in between.

FLAG-2

Expand to cover most quantities, include representatives of *all* large collaborations, and distribute over Europe/US/Japan.

(n.b.: merge with parallel effort by Laiho, Lunghi, Van de Water)

```
advisory board: S. Aoki (J), C. Bernard (US), C. Sachrajda (EU)
editorial board: G. Colangelo, H. Leutwyler, A. Vladikas, U. Wenger
working groups:
   quark masses
                                        T. Blum, L. Lellouch, V. Lubicz
                                      A. Jüttner, T. Kaneko, S. Simula
   V_{ud}, V_{us}
   LECs
                                         S. Dürr, H. Fukaya, S. Necco
   B_K
                                         J. Laiho, S. Sharpe, H. Wittig
   \alpha_{\mathbf{s}}
                                 T. Onogi, J. Shigemitsu, R. Sommer
   f_B, B_B, f_D
                               Y.Aoki, M. Della Morte, A. El Khadra
   B, D \to H \ell \nu
                                       E. Lunghi, CP, R.Van de Water
```

Preliminary results: plenary talk by G. Colangelo at Lattice 2012 (end June).

perturbation theory strikes back

Use of effective theories in lattice B-physics almost always involves some use of perturbation theory.

- matching NRQCD, HQET to QCD
- O(a) improvement (removing cutoff effects in relativistic actions)
- RG running between the QCD, b, and W scales

perturbation theory strikes back



convergence of perturbation theory is sometimes poor, even at higher orders

perturbation theory strikes back



These issues can be brought under control within npHQET \Rightarrow comprehensive B-physics programme by the ALPHA Collaboration.

conclusions and outlook

- Lattice QCD has become a precision tool. B-physics still requires some crucial help from effective theories, though.
- Results for basic quantities have to be taken seriously, and have UT impact.
 - reach of effective theories, perturbative artifacts (at various levels)?
 - O most collaborations do light physics first, B-physics lagging somewhat behind
 - O progress will be fast in the next 2-3 years
- Effort from the lattice community to provide clean state-of-the-art input for phenomenology.
 - **O** FLAG: next review due early 2013.

Lots of uncharted territory (rare/non-leptonic decays? cf. K).

rare B decays?

Results start to be available for some less obvious processes, e.g. $B \to K^{(*)}$.

Example of lattice QCD keeping pace with experimental developments? FNAL-MILC update on ratios $(B_s \rightarrow D_s)/(B \rightarrow D)$ needed to normalise fragmentation fractions used in $B_s^0 \rightarrow \mu^+ \mu^-$ analysis.

[FNAL-MILC, arXiv:1202.6346]



$$\frac{f_s}{f_d} = 0.286(16)_{\text{stat}}(21)_{\text{syst}}(26)_{\text{latt}}(22)_{\text{NE}}$$