Flavour Physics from Lattice QCD: Strange and Charm Quark Sectors

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Flavour Mini-Workshop

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Lattice QCD & Flavour Physics

- Collective effort to test the Standard Model
- What quantities can be addressed on the lattice?

$$m_q, \quad M_h, \quad \langle h|\mathcal{O}|0\rangle, \quad \langle h|\mathcal{O}|h'\rangle, \quad \ldots \quad \langle h|\mathcal{O}|h_1 h_2\rangle$$

- Examples in the strange and charm quark sectors
- What are the current uncertainties?
- What is the impact of lattice QCD in heavy flavour physics?
- What are the possible improvements?

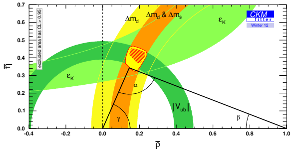
precision in lattice QCD

- control of systematic uncertainties
  - number of dynamical flavours (u,d,s,c,... quarks) $N_\mathrm{f} = 0;\ 2;\ 2+1;\ 2+1+1$
  - cutoff effects: lattice spacing a — $O(a)$ improvement, continuum limit
    broken symmetries at $a \neq 0$
    $m_q \ll 1/a$
  - range of quarks masses : simulation/physics — applicability of χPT, HQET
  - finite size effects (FSE): lattice size L — $m_{\mathrm{PS}} L \gg 1$
  - renormalisation — non-perturbative

- statistical errors
  - improvement in algorithms
  - autocorrelations
  - machines

CKM matrix & lattice QCD

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



lattice determinations in strange and charm sector

CKM	process	lattice	precision (%)
$\lvert V_{us}\rvert$	$K \to \ell\nu$	f_K	$\leq$ 1.5
	$K \to \pi\ell\nu$	$f_+^{K\pi}(q^2=0)$	1.0
$\lvert V_{us}\rvert/\lvert V_{ud}\rvert$	$K \to \mu\nu/\pi \to \mu\nu$	f_K/f_π	$\leq$ 1.5
$\lvert V_{cd}\rvert$	$D \to \ell\nu$	f_{D_s}/f_D	4.0
	$D \to \pi\ell\nu$	$f_{+,0}^{D\pi}(0)$	$\sim$ 10.0
$\lvert V_{cs}\rvert$	$D_s \to \ell\nu$	f_{D_s}	2.5
	$D \to K\ell\nu$	$f_+^{DK}(0)$	$\sim$ 7.0
$V_{tq}^* V_{tq'}$; $V_{cq}^* V_{cq'}$	ϵ_K	$\hat{B}_K$	4.0
		$K \to \pi\pi$	$\to$ 30.0

- quark masses, BSM four-fermion operators, ...

quark masses

$$m_s \qquad\qquad m_c$$

quark masses

- fundamental parameters of the SM
- appear in decay rates and induce symmetry breaking
- PDG : large uncertainties

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How to determine a quark mass in lattice QCD?

- not physical observables
- parameters of the lagrangian $\rightsquigarrow$ nead experimental input

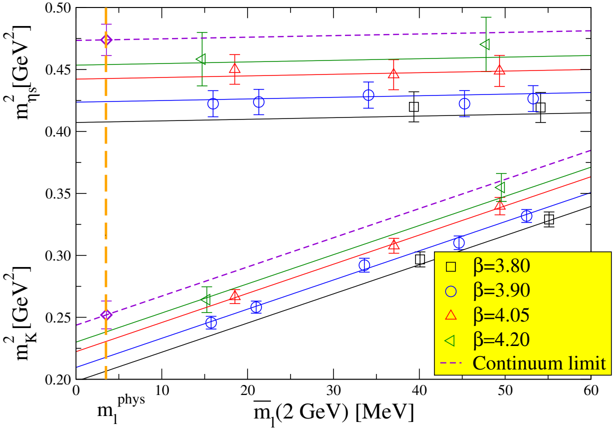
- use experimental measure of an observable depending on m_q — m_s : m_K , ...

  match to lattice determination $\rightsquigarrow$ m_q^{bare} — interpolation

- renormalisation : — non-perturbative

m_s : ETMC $N_f = 2$

$a = \{0.054,\ 0.067,\ 0.085,\ 0.098\}$ fm from $f_\pi^{(\mathrm{exp})}$



light-quark mass and lattice spacing dependence

[ETMC, 1010.3659]

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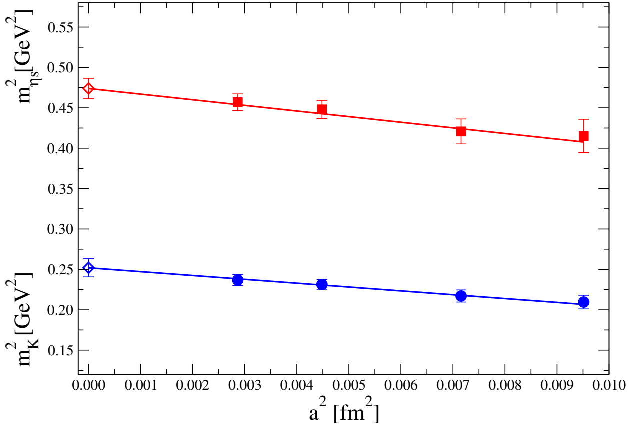

lattice spacing dependence

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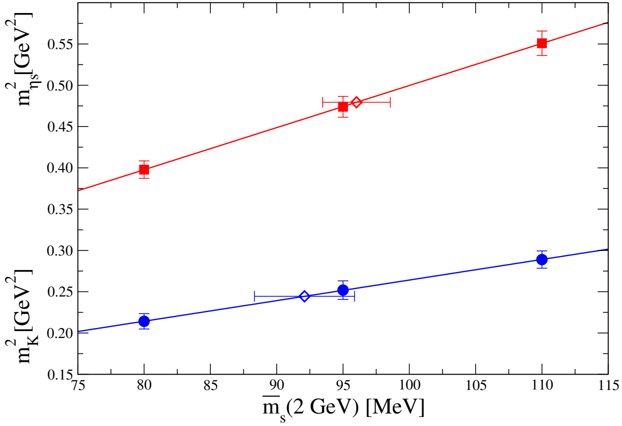
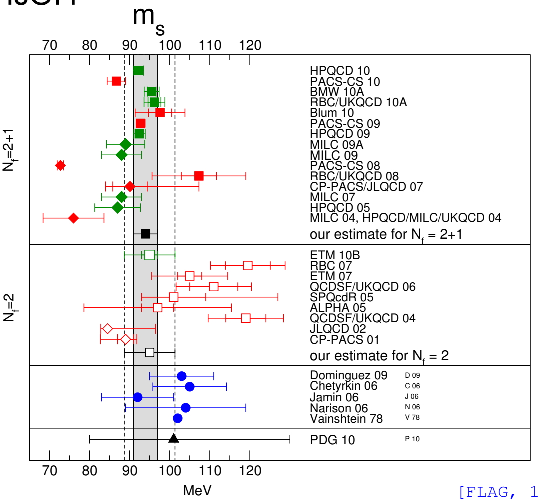


strange-quark mass dependence

[ETMC, 1010.3659]

$\rightsquigarrow$ $m_s[\overline{\mathrm{MS}}, \mu = 2\,\mathrm{GeV}] = 95(6)\,\mathrm{MeV}$ (6%)

m_s : comparison



[FLAG, 1011.4408]

averages:

$m_s[\overline{\text{MS}}, 2\text{GeV}] = 94(3)$ MeV (3%) $N_f = 2+1$

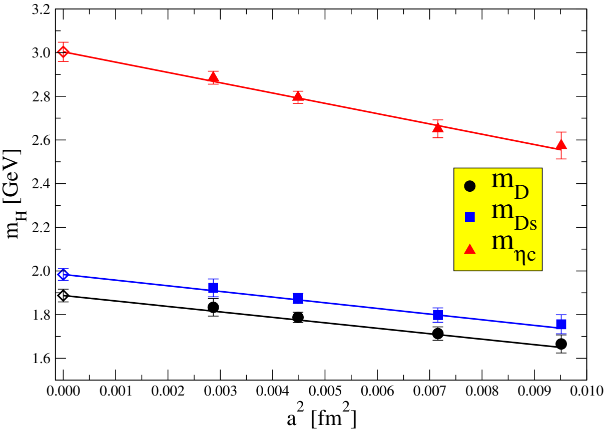
95(6) MeV (6%) $N_f = 2$

update:

102(3) MeV (3%) [ALPHA, 1205.5380] $N_f = 2$

m_c : ETMC $N_f = 2$

$a = \{0.054,\ 0.067,\ 0.085,\ 0.098\}$ fm from $f_\pi^{(\mathrm{exp})}$

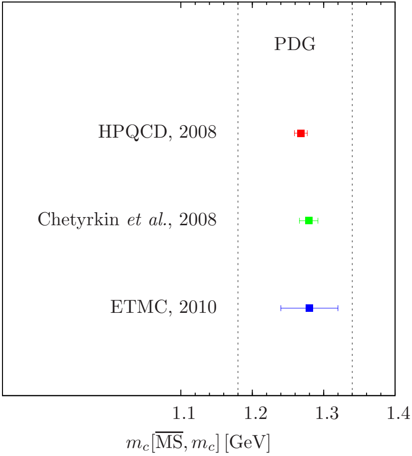


lattice spacing dependence
[ETMC, 1010.3659]

$\rightsquigarrow$ $m_c[\overline{\mathrm{MS}}, \mu = m_c] = 1.28(4)$ GeV (3%) $m_c/m_s = 12.0(3)$ (2.5%)

m_c

non exhaustive comparison of recent results



$m_c/m_s = 12.0(3)$	(2.5%)	[ETMC, 1010.3659]	$N_f = 2$
$11.3(5)$	(4.0%)	[Dürr & Koutsou, 1108.1650]	$N_f = 2$
$11.9(2)$	(1.4%)	[HPQCD, 0910.3102]	$N_f = 2+1$

quark masses

what improvements are needed?

- reduce the uncertainty coming from perturbation theory
- matching non-perturbative scheme (Schrödinger Functional, RI-MOM) to $\overline{\mathrm{MS}}$
- use of mass-independent scheme with $N_{\mathrm{f}} = 4$

[ALPHA, 1006.0672]
[ETMC, 1112.1540]

K decays

$$K \to \ell\nu$$

$$K \to \pi\,\ell\nu$$

$$V_{us}$$

unitarity of the CKM matrix: first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + O\left(\frac{M_W^2}{\Lambda_{\rm NP}^2}\right)$$

Relative contributions

- $|V_{ud}| \approx 0.974$: rel. error $\delta \sim 0.02\%$ — nuclear β decays
- $|V_{us}| \approx 0.225$: $\delta \sim 0.50\% \div 1\%$ — $K_{\ell 3}$ and $K_{\ell 2}$ decays
- $|V_{ub}| \approx 0.004$: small

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Determinations of $|V_{us}|$

- semileptonic $K_{\ell 3}$ decays: $K \to \pi \ell \nu$

$$\Gamma(K_{\ell 3(\gamma)}) \propto |V_{us}|^2 \, f_+(0)^2$$

  - $\delta(|V_{us}| f_+(0)) \sim 0.20\%$
  - $f_+^{K^0\pi^-}(0)$: hadronic matrix element at $q^2 = 0$ $\rightsquigarrow$ sub-percent precision in LQCD

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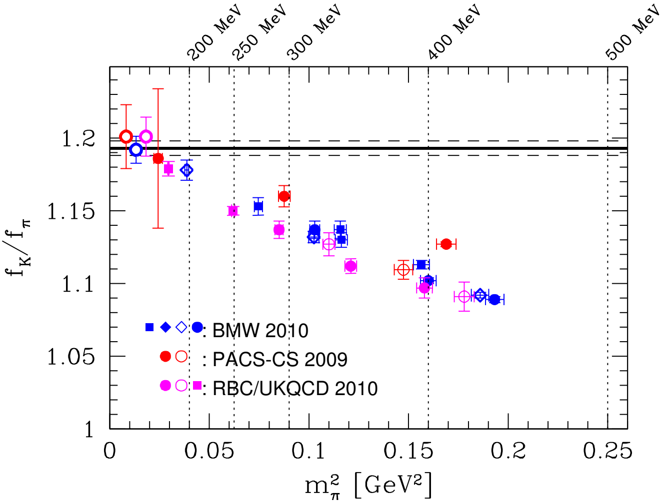
- leptonic $K_{\ell 2}$ decays: $K \to \ell \nu$

$$\frac{\Gamma(K^{\pm}_{\ell 2(\gamma)})}{\Gamma(\pi^{\pm}_{\ell 2(\gamma)})} \propto \left|\frac{V_{us}}{V_{ud}}\right|^2 \left(\frac{f_K}{f_\pi}\right)^2$$

  - $\delta(V_{us}/V_{ud} \times f_K/f_\pi) \sim 0.20\%$
  - f_K/f_π : ratio of decay constants $\rightsquigarrow$ sub-percent precision in LQCD is needed

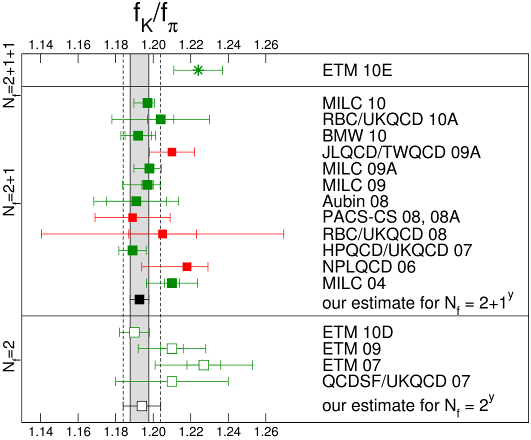
f_K / f_π



light-quark mass dependence [H. Wittig, lat11, 1201.4774]

SU(3) breaking

f_K/f_π



[FLAG, 1011.4408]

$$f_K/f_\pi = 1.193(05) \quad [0.4\%] \qquad N_f = 2+1$$

$$1.210(18) \quad [1.5\%] \qquad N_f = 2$$

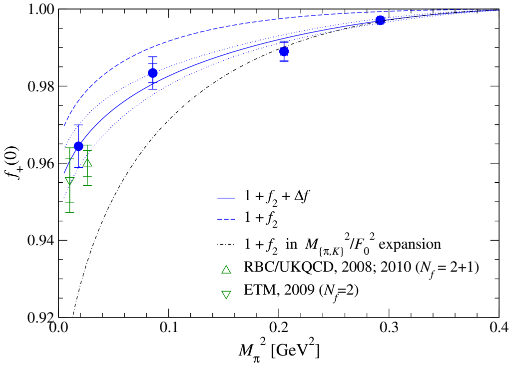
. . . preliminary [ETMC, 1012.0200] $N_f = 2+1+1$

$K_{\ell 3}$: $K \to \pi \ell \nu$

$$\Gamma(K_{\ell 3(\gamma)}) \propto |V_{us}|^2 \; f_+(q^2=0)^2 \quad \rightsquigarrow \quad \langle \pi(p') \,|\, V_\mu \,|\, K(p) \rangle$$

- interpolation to $q^2 = 0$
- SU(3) breaking : $f_+(0) = 1 + f_2 + f_4$
- Ademollo-Gatto theorem : no LECs of χPT in f_2 : $f_2 = -0.0226$

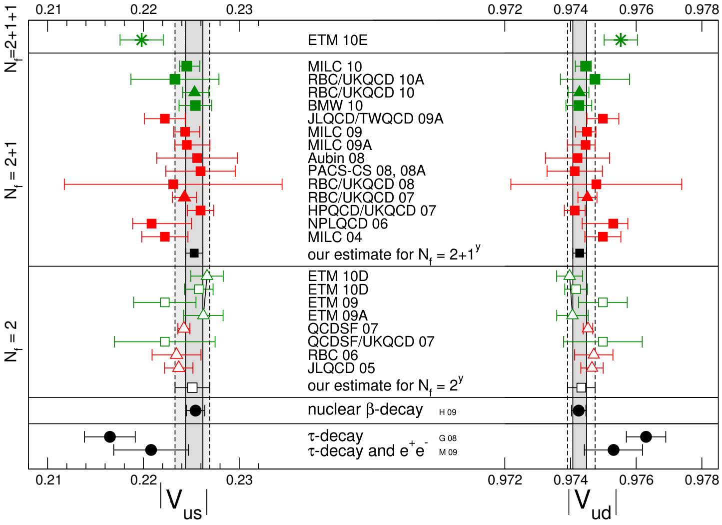
compute deviations from $f_+(0) - 1 - f_2$



light-quark mass dependence

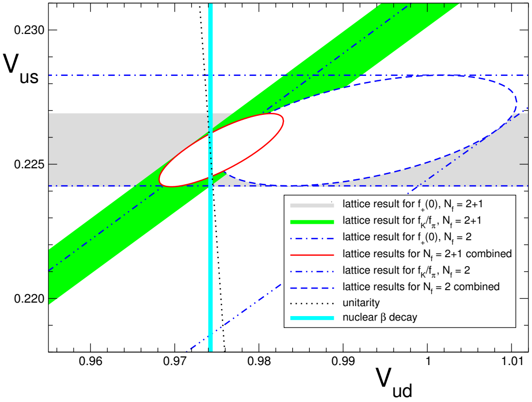
[JLQCD, 1112.5259]

V_{us} from $K_{\ell 2}$ and $K_{\ell 3}$



[FLAG, 1011.4408]

CKM : unitarity first row



lattice + Kaon branching fractions

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = \quad 0.0020(150)$$

lattice + $|V_{us}|$ from β-decay

$f_+(0)$: $\quad 0.0000(7)$

f_K/f_π : $\quad -0.0001(6)$

[FLAG, 1011.4408]

V_{us}

what improvements are needed? KLOE-2

incorporate isospin breaking effects on the lattice

- QED : $q_u \neq q_d$
- QCD : $m_u \neq m_d$
- example: removing QED $\rightsquigarrow$ $\hat{M}_{K^+} - \hat{M}_{K^0} \approx -6\,\mathrm{MeV}$ [1%]

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- example: removing QED $\rightsquigarrow$ $\hat{M}_{K^+} - \hat{M}_{K^0} \approx -6\,\text{MeV}$ [1%]
- expansion in $m_d - m_u$ [RM123, 1110.6294]

$$\mathcal{L}_m = \frac{m_u + m_d}{2}(\bar{u}u + \bar{d}d) - \frac{m_d - m_u}{2}(\bar{u}u - \bar{d}d)$$
$$= m_{ud}\,\bar{q}q - \Delta m_{ud}\,\bar{q}\tau_3 q$$

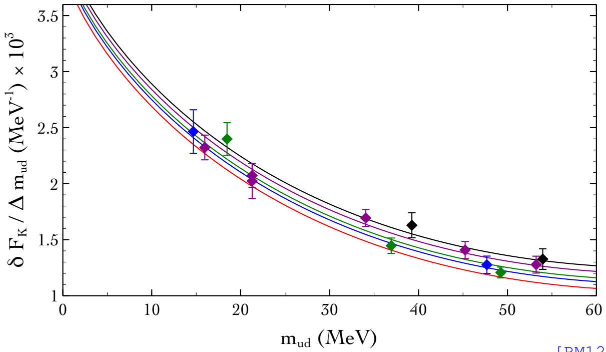
- expand the path integral

$$\langle \mathcal{O} \rangle = \frac{\int D\phi\, \mathcal{O}\, e^{-S}}{\int D\phi\, e^{-S}}$$
$$= \frac{\int D\phi\, \mathcal{O}\,(1 + \Delta m_{ud}\, \hat{S}_3)\, e^{-S_0}}{\int D\phi\, (1 + \Delta m_{ud}\, \hat{S}_3)\, e^{-S_0}} + \ldots = \frac{\langle \mathcal{O} \rangle_0 + \Delta m_{ud}\, \langle \mathcal{O} \hat{S}_3 \rangle_0}{1 + \Delta m_{ud}\, \langle \hat{S}_3 \rangle_0} + \ldots$$

$$\hat{S}_3 = \sum_x [\bar{q}\tau_3 q](x)$$

V_{us} : isospin breaking

$$\delta F_K \equiv \left. \frac{F_{K^0} - F_{K^+}}{F_K} \right|_{\mathrm{QCD}}$$



[RM123, 1110.6294]

input : $\left[M^2_{K^0} - M^2_{K^+} \right]^{QCD} = 6.05(63) \times 10^3 \text{ MeV}^2$ exp^t + χPT + lattice [FLAG, 1011.4408]

results :

$$\frac{1}{2}\left[m_d - m_u\right]^{QCD}(\overline{MS}, 2GeV) = 1.18(4)(12)\,\text{MeV}$$

$$\left[\frac{F_{K^+}/F_{\pi^+}}{F_K/F_\pi} - 1 \right]^{QCD} = -0.0039(3)(2)$$

$-0.0022(6)$ χPT [Cirigliano & Neufeld, 1102.0563]

neutral meson mixing : bag parameters

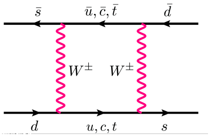
$$B_K$$

Kaon bag parameter B_K : $K^0 - \bar{K}^0$ oscillations

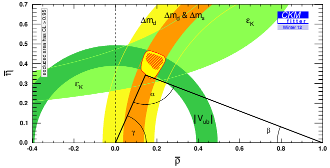
CKM matrix and Unitarity Triangle

K–sector : indirect CP violation via ϵ_K

- $$\epsilon_K = \frac{A\,(K_L \to (\pi\pi)_{I=0})}{A\,(K_S \to (\pi\pi)_{I=0})}$$
  $$\simeq \kappa_\epsilon \frac{e^{i\phi_\epsilon}}{2\sqrt{2}M_K \Delta M_K} \mathrm{Im}\{\langle \bar{K}^0|\mathcal{H}_{\mathrm{eff}}(\Delta S = 2)|K^0\rangle\}$$



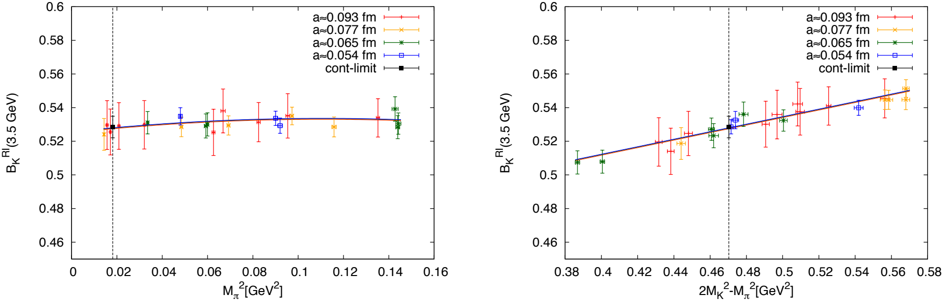
- B_K : Kaon bag parameter
  $$\langle \bar{K}^0|\mathcal{O}^{\Delta S=2}|K^0\rangle = \frac{8}{3} B_K f_K^2 m_K^2$$
  $$\mathcal{O}^{\Delta S=2} = (\bar{s}\gamma_\mu^L d)(\bar{s}\gamma_\mu^L d)$$



- among the largest uncertainties in the UTA :
  $$\delta(\epsilon_K)|_{\mathrm{exp}} \approx 0.5\%$$
  $$\delta(B_K)|_{\mathrm{latt}} \approx 4.0\%$$
  $$\delta(|V_{cb}|^4)|_{\mathrm{incl.}} \approx 4 \times 2.0\%$$

B_K : BMW

- non-perturbative renormalisation
- Wilson fermions : mixing with operators of wrong chirality

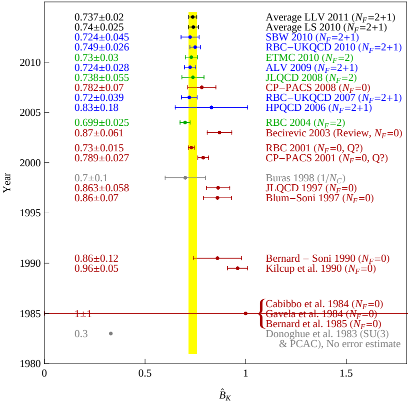


light and strange quark mass dependence

[BMW, 1106.3230]

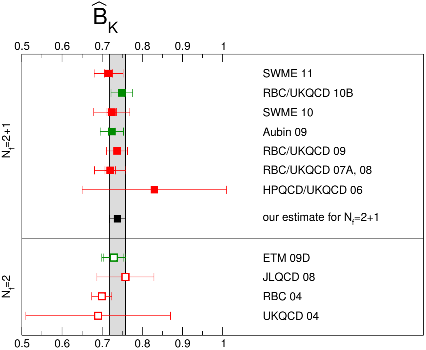
$\hat{B}_K$

- historical perspective



[Lunghi & Soni, 2011]

- recent determinations



[FLAG, 1011.4408]

	$\hat{B}_K =$ 0.738(20) [2.7%]		$N_f = 2+1$
	0.729(30) [4.1%]		$N_f = 2$
[ALPHA, 0902.1074]	0.730(30) [4.1%]		$N_f = 0$
PRELIMINARY [ETMC, 1111.1262]	0.747(18)		$N_f = 2+1+1$
update : [BMW, 1106.3230]	0.773(12) [1.4%]		$N_f = 2+1$

ϵ_K

what improvements are needed?

$$\epsilon_K = \exp(i\phi_\epsilon)\ \sin(\phi_\epsilon)\ \left[\frac{\mathrm{Im}[\langle\bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle]}{\Delta M_K} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right]$$

- long-distance contributions
- $K \to (\pi\pi)_{I=0} \rightsquigarrow \mathrm{Im}(A_0)$
- $K \to (\pi\pi)_{I=2}$ is easier: $\rightsquigarrow \mathrm{Im}(A_2)$
- indirect : chiral effective theory
- direct : QCD in a finite volume [Lellouch & Lüscher, hep-lat/0003023]
- [RBC−UKQCD, 1111.1699]

  value of $\mathrm{Re}(A_2)$ [18%] in good agreement with exp$^{\rm t}$ [4%]

  $\mathrm{Im}(A_2) = -(6.83 \pm 0.51 \pm 1.30)\, 10^{-13}\,\mathrm{GeV}$ [21%]

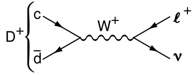
  meanwhile use $\mathrm{Re}(A_{0,2})^{\rm exp.}$ and $(\epsilon'/\epsilon)|_{\rm exp.}$ to estimate $\mathrm{Im}(A_0)$

$(\kappa_\epsilon)_{\rm abs} = 0.923(06)$

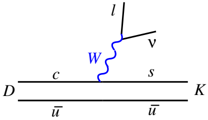
[Buras, Guadagnoli, Isidori, 1002.3612] 0.940(20)

D decays

$$D_s\,,\quad D \to \ell\nu$$



$$D \to K(\pi)\,\ell\nu$$

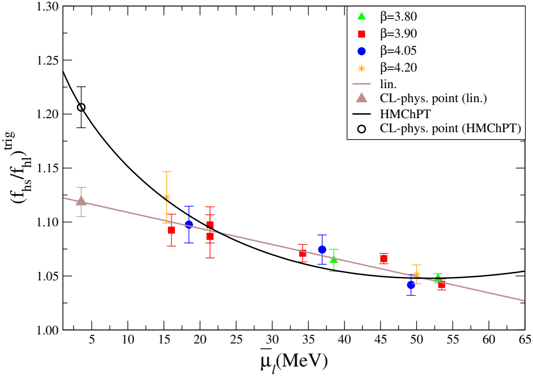


$N_f = 2$: f_D and f_{D_s}

$$\Gamma(D_s \to \ell\nu) \propto |V_{cs}|^2\ f_{D_s}^2 \quad \text{with} \quad f_{D_s} p_\mu = \langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s(p)\rangle$$

$a = \{0.054,\ 0.067,\ 0.085,\ 0.098\}$ fm

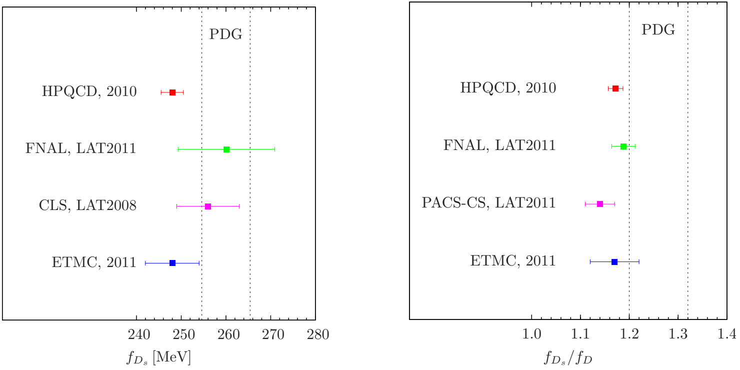
light-quark mass dependence : f_{D_s}/f_D vs. m_ℓ



[ETMC, 1107.1441]

f_D and f_{D_s} : comparison

$$\Gamma(D_s \to \ell\nu) \propto |V_{cs}|^2\ f_{D_s}^2 \qquad \text{with} \qquad f_{D_s} p_\mu = \langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s(p)\rangle$$

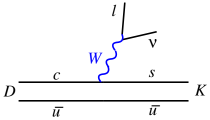


SU(3) breaking : f_{D_s}/f_D

unitarity : second row $|V_{cd}|$ (6%) $|V_{cs}|$ (4%)

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 \approx 2(7) \times 10^{-2}$$

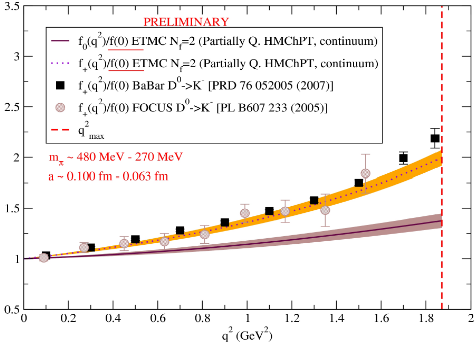
semileptonic $D \to K$



$$\frac{d\Gamma}{dq^2}(D \to K\ell\nu_\ell) \propto |V_{cs}|^2 \; f_+(q^2)^2$$

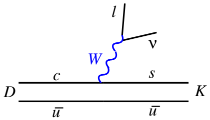
$$\langle K(k)|\bar{q}\gamma_\mu c|D(p)\rangle = \left(p_\mu + k_\mu - q_\mu \frac{m_D^2 - m_K^2}{q^2}\right) f_+(q^2) + q_\mu \frac{m_D^2 - m_K^2}{q^2} f_0(q^2)$$

momentum transfer dependence of $f_+(q^2)/f_+(0)$



[ETMC, 1104.0869; PRELIMINARY]

semileptonic $D \to K$



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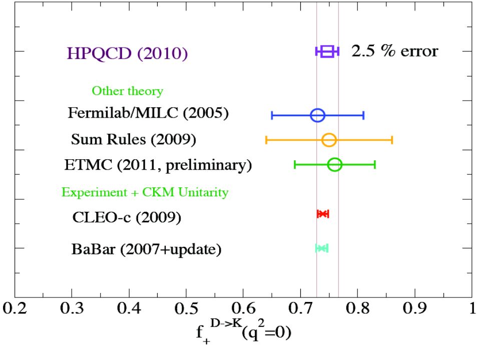

[Na et al., LAT2011]

Need further improvements to constrain $|V_{cd}|$ and $|V_{cs}|$

conclusions

strange and charm quark physics from the lattice

types of observables :

- precise : accuracy $\lesssim 1\%$

  $K_{\ell 2}$, $K_{\ell 3}$ $\rightsquigarrow$ isospin breaking

- tricky : few %

  m_a, m_c

  B_K

  f_{D_s}, f_D, $f_{+,0}(0)$

- challenging : $> 10\%$

  $K \to \pi\pi$

- terra incognita

  $D \to h_1 h_2$

- other quantities currently being studied on the lattice :

  BSM operators relevant for $K^0 - \bar{K}^0$ mixing

  charmonium spectrum and radiative decays, f_{D*}, $g_{D^* D\pi}$, B_D, ...