Small violations of Bell inequalities by random states

Raphael C. Drumond

(in collaboration with R. I. Oliveira,

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Question

- For a fixed quantum system and a property P of interest:

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"How many" quantum states satisfy P?

Examples

- Entanglement

e.g.: P. Hayden, D. W. Leung, A. Winter, *Comm. Math. Phys*. 265(1) 95, (2006).

- Ergodicity: $\lim_{T\to\infty} \frac{1}{T} \int_0^T \langle A \rangle_{|\psi(t)\rangle} (t) dt = \langle A \rangle_{\rho_c}$

e.g.: N. Linden, S. Popescu, A.J. Short, A. Winter, Phys. Rev. E 79, 061103 (2009).

- (1-way) Quantum computation

e.g.: J. Bremner, C. Mora, A. Winter, *Phys. Rev. Lett*. 102, 190502 (2009).

“How many”

- Measure (or Volume or Probability Measure)

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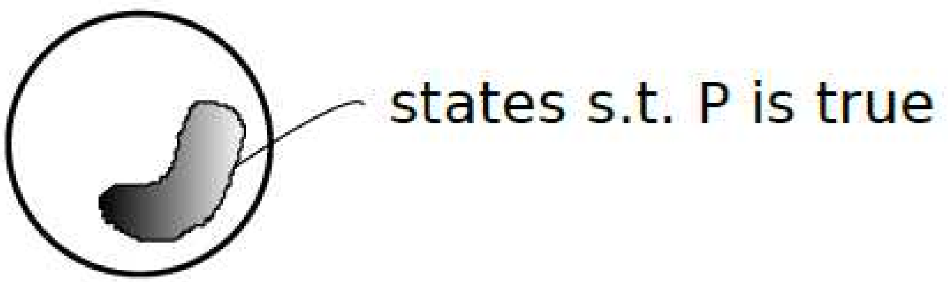
- For pure states of a finite dimensional Hilbert space we think them as high dimensional spheres:



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- Mixed states: ?

•What is the typical degree?

Question’

- What is the measure (probability) of the set (event) constitued by the pure states that violate some Bell inequality by at least *v*?

WWZB inequalities

- Consider N systems where, on each of them, one can measure two observables $A_{0,j}$ and $A_{1,j}$, with outcomes +1 and -1. We have, for LHV models:

$$-1 \leq \sum_{X \in \{0,1\}^N} S(X) \prod_{j=1}^{N} \frac{(A_0^j + (-1)^{x_j} A_1^j)}{2} \leq 1$$

R. F. Werner and M. M. Wolf, Phys. Rev. A 64, 032112 (2001).

M. Zukowski and C. Brukner, Phys. Rev. Lett. 88, 210401 (2002).

Violations by pure states

$$Q_{NL}(|\psi\rangle, \mathcal{Q}) \equiv \sum_{X \in \{0,1\}^N} |\langle\psi| \bigotimes_{j=1}^{N} \frac{(A_0^j + (-1)^{x_j} A_1^j)}{2} |\psi\rangle|$$

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- For appropriate states and observables:

$$Q(|\psi\rangle, \mathcal{Q}) \propto 2^{\frac{N}{2}}$$

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$$\mathcal{A}_v = \{|\psi\rangle : \sup_{\mathcal{Q}} Q_{NL}(\psi, \mathcal{Q}) > v\}$$

- What is

$$\mathbb{P}(\mathcal{A}_v)$$

Our result:

- For system of N parts, each with *d*-dimensional Hilbert space:

$$\mathbb{P}(\mathcal{A}_v) \leq 2\left(\frac{N 2^{N+1} d^2}{\delta} + 2\right)^{2d^2 N} e^{-\frac{(v-\delta-c_{d,N})^2 (d/2)^N}{9\pi^3}}$$

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- For *d*=2: $v \geq cN$ $\mathbb{P}(\mathcal{A}_v) \to 0 \text{ as } N \to \infty$

- For *d*>2 $v \geq 1$ $\mathbb{P}(\mathcal{A}_v) \to 0 \text{ as } N \to \infty$

Idea of the proof

We want to estimate the measure of:

$$\{|\psi\rangle : \sup_x f_x(|\psi\rangle) > v\} = \bigcup_x \{|\psi\rangle : f_x(|\psi\rangle) > v\}$$

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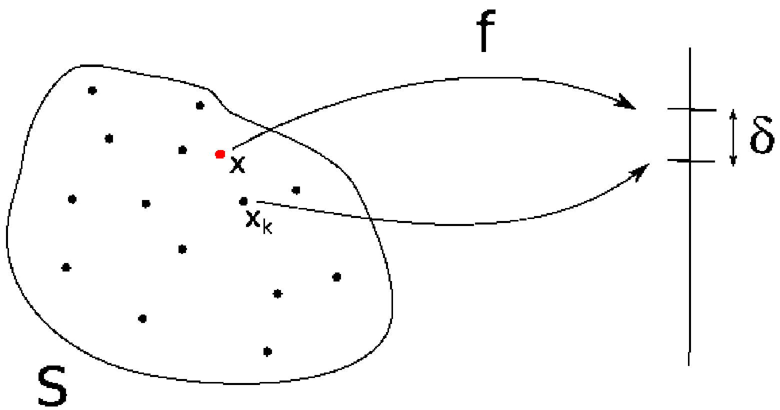

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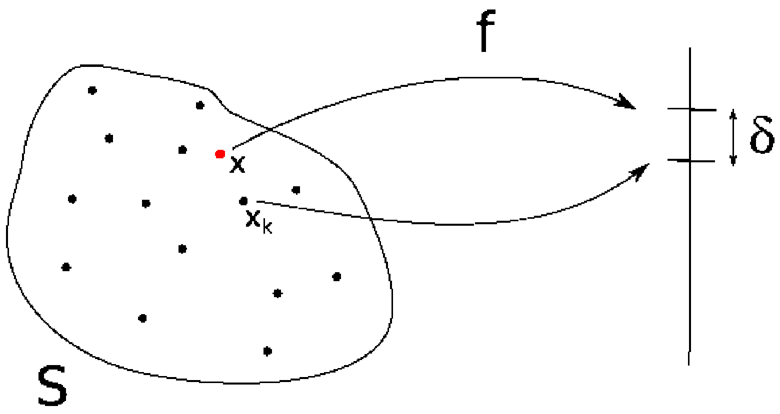

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To sum up:

non-linear inequality+epsilon-net+Lévy's lemma=bound

Noise and the case *d*=2

- For white noise:

$$\rho_2 \mapsto (1-\lambda)\rho_2 + \lambda \frac{I}{2}$$

Noise and the case *d=2*

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$$\rho_2 \mapsto (1-\lambda)\rho_2 + \lambda\frac{I}{2}$$

- For any $\lambda > 0 \quad v > 1$

$$\mathbb{P}(\mathcal{A}_v^\lambda) \to 0$$

Further work:

- Generalization to an arbitrary scenario

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- Generalization to an arbitrary scenario

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- Result valid as long as $d>>n,m$

- What happens if $d<<n,m$?

Thank You!