Adiabatic preparation of a Heisenberg antiferromagnet using an optical superlattice

Michael Lubasch, Valentin Murg, Ulrich Schneider,
J. Ignacio Cirac and Mari-Carmen Bañuls

Max Planck Institute of Quantum Optics, Garching, Germany
University of Vienna, Austria
Ludwig-Maximilians-University Munich, Germany

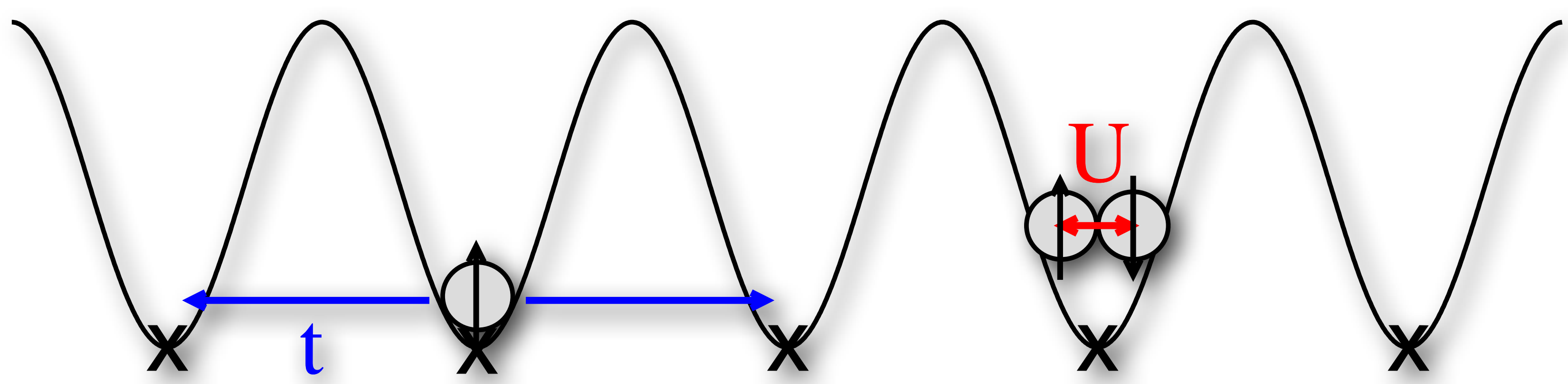
Main ideas

- Experimental proposal
- Adiabaticity conditions
  - for the total lattice
  - for a sublattice
- Effect of holes and harmonic trap

Motivation

Motivation

- recent experimental realization of **fermionic Hubbard model** in optical lattice [*Schneider et al.*, Science'08; *Jördens et al.*, Nature'08]

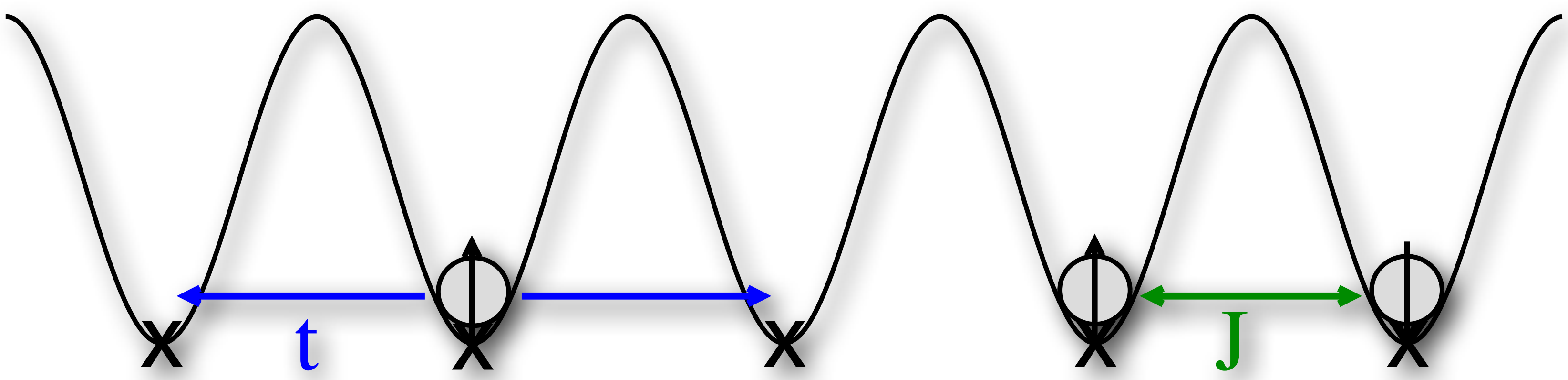


$$\hat{H} = -t \sum_{\langle l,m\rangle,\sigma} (c^\dagger_{l,\sigma} c_{m,\sigma} + c^\dagger_{m,\sigma} c_{l,\sigma}) + U \sum_l \hat{n}_{l,\uparrow} \hat{n}_{l,\downarrow}$$

Motivation

- limit of strong interactions $U \gg t$: **t-J model**

$$J := 4t^2/U$$

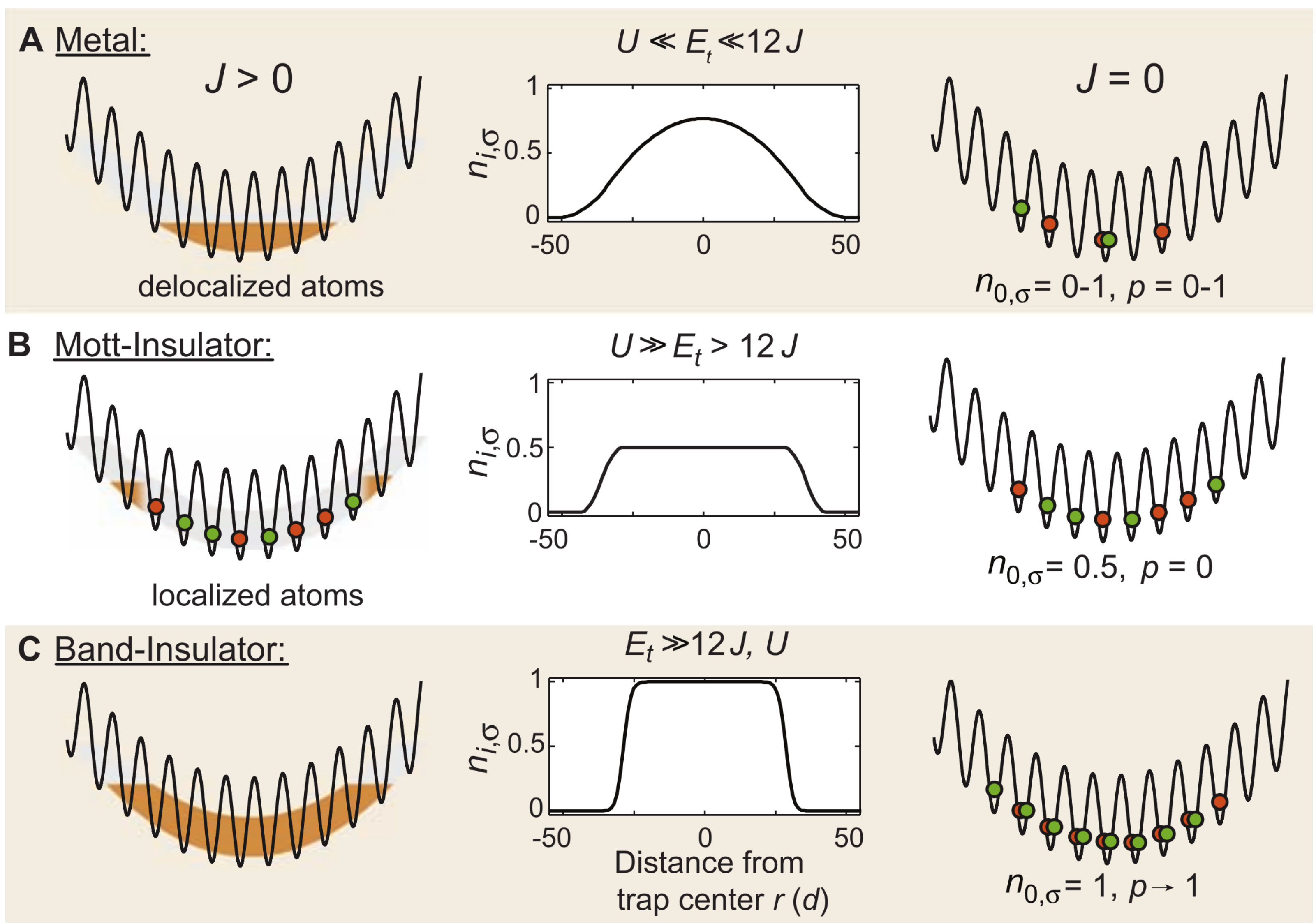


$$\hat{H} = -t \sum_{\langle l,m \rangle,\sigma} (\tilde{c}^\dagger_{l,\sigma}\tilde{c}_{m,\sigma} + \tilde{c}^\dagger_{m,\sigma}\tilde{c}_{l,\sigma}) + J \sum_{\langle l,m \rangle} (\vec{S}_l \cdot \vec{S}_m - \frac{\hat{n}_l \hat{n}_m}{4})$$

Motivation

- experimental realization of **various phases**:

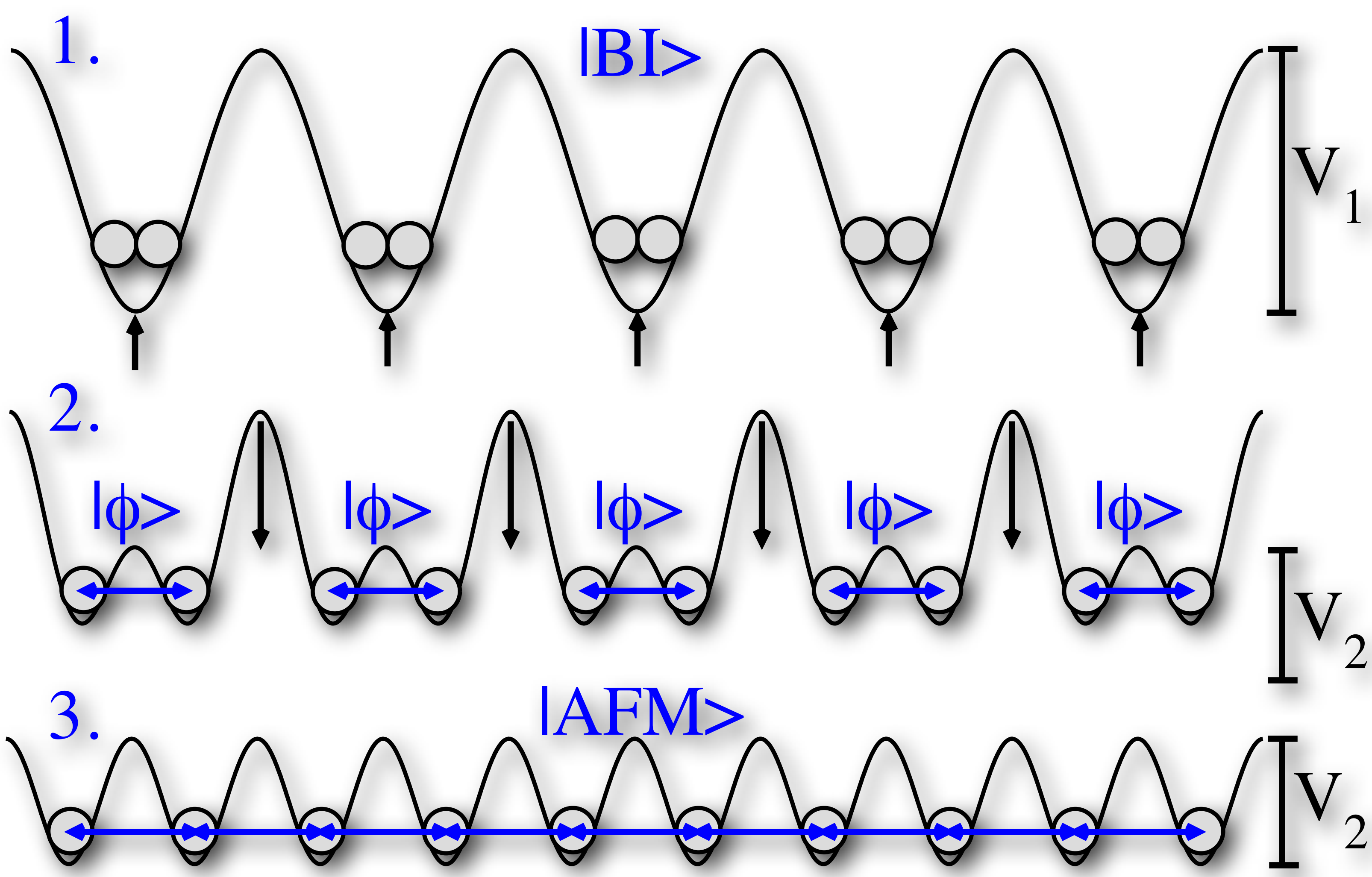
$$\hat{H} + V \sum_{l} (l - l_0)^2 \hat{n}_l$$



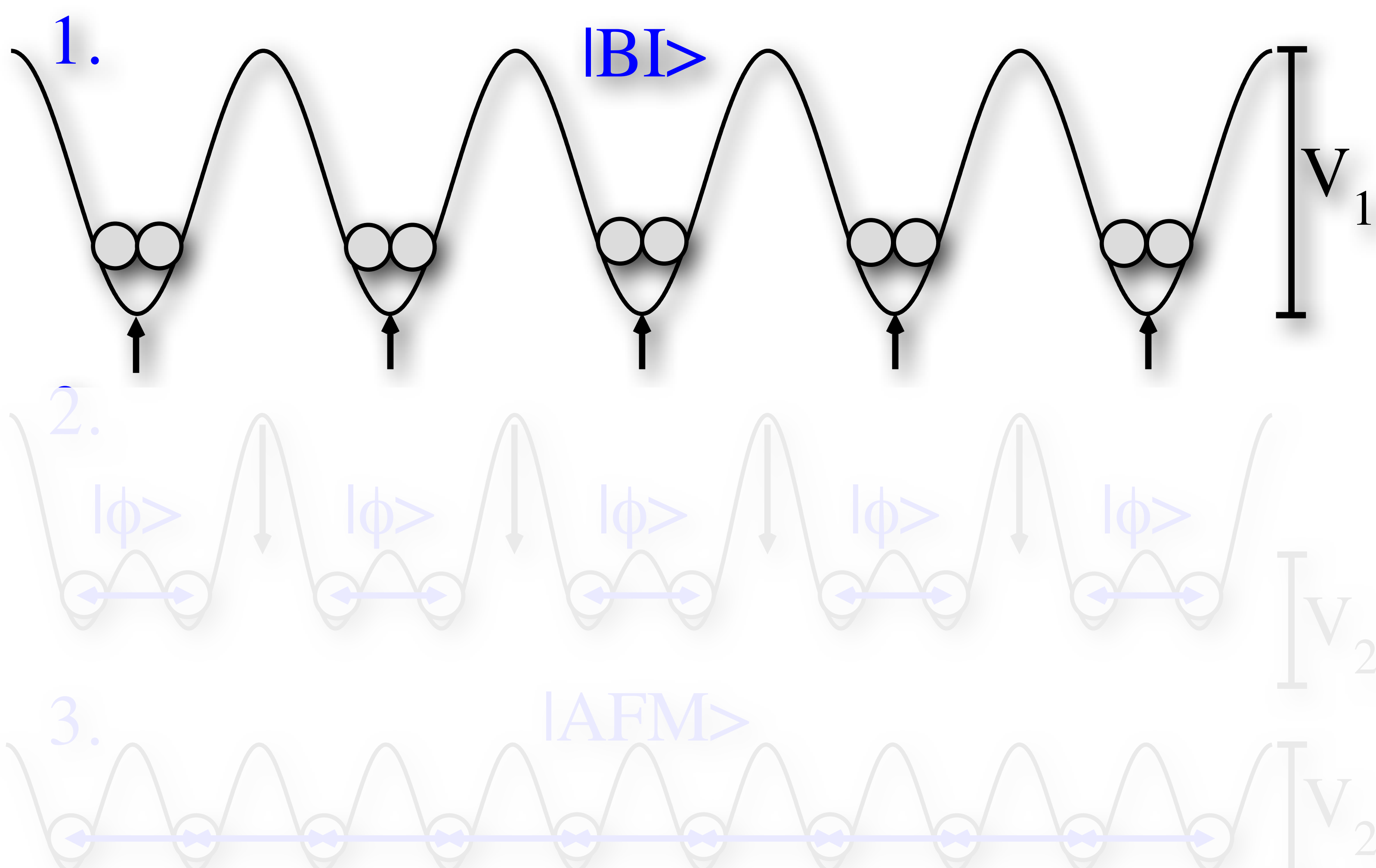
taken from [*Schneider et al.*, Science'08]

Experimental proposal

Experimental proposal

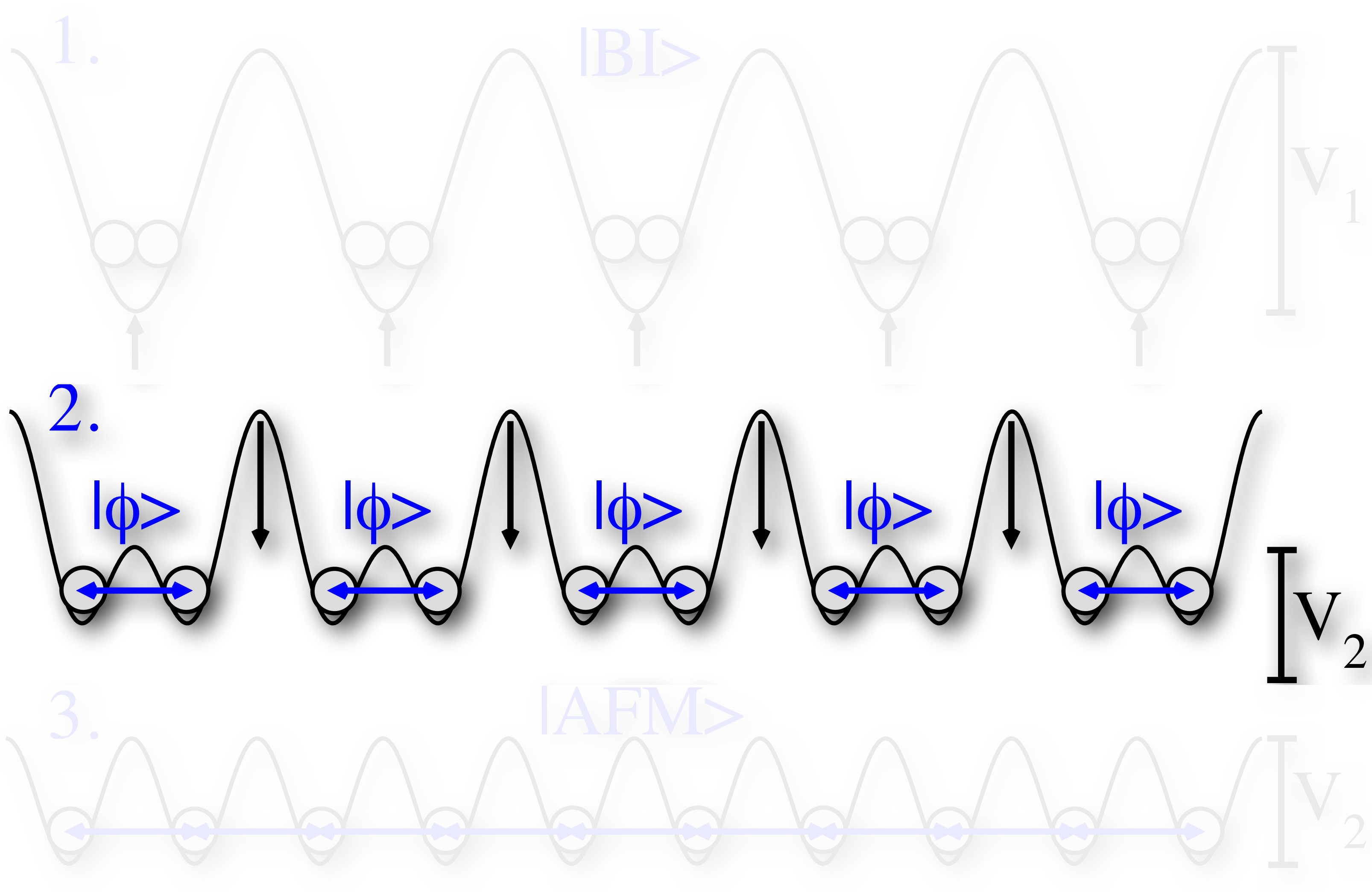


Experimental proposal



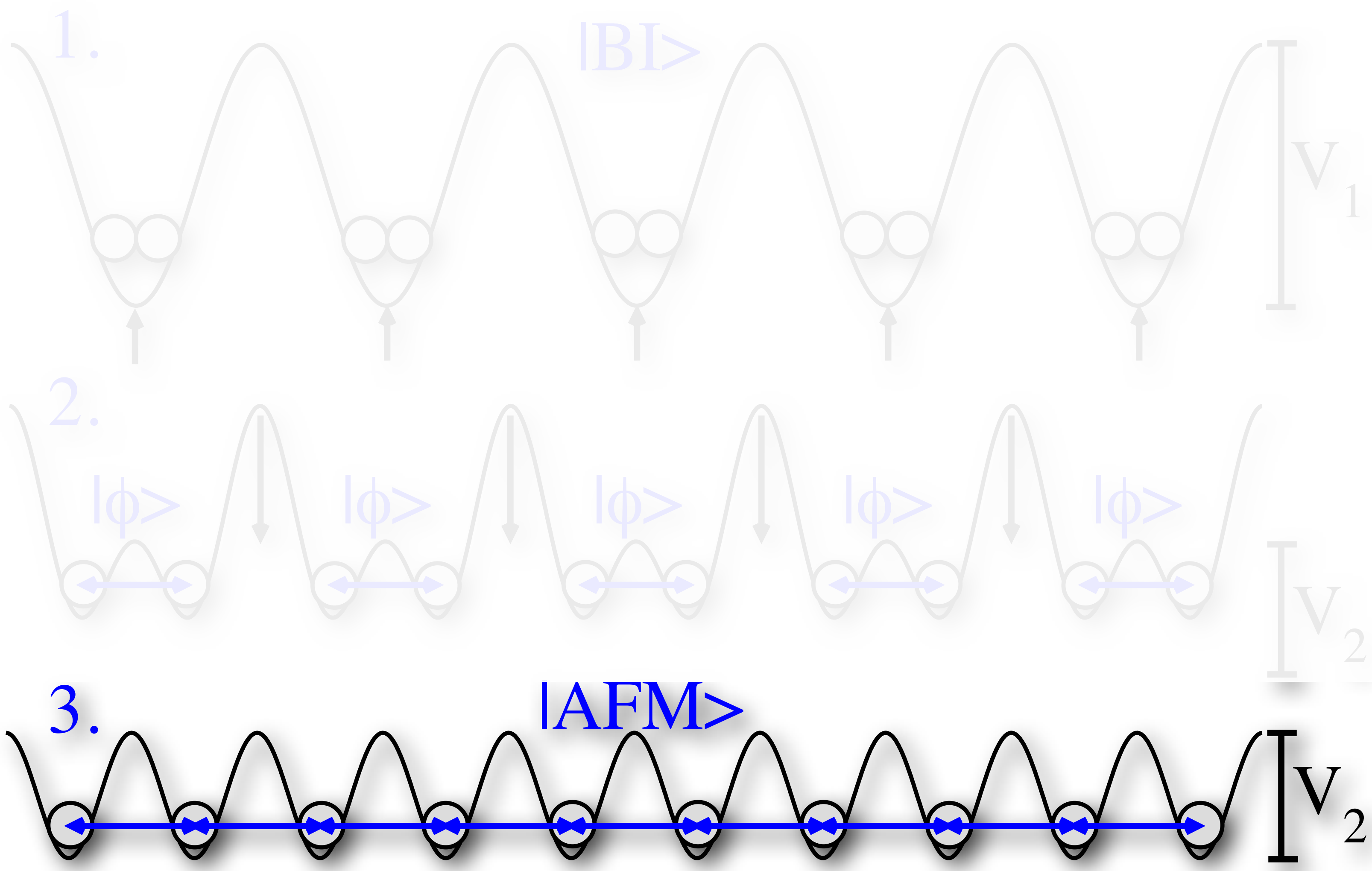
1. band insulating ground state |BI> [*Schneider et al.*, Science'08]

Experimental proposal



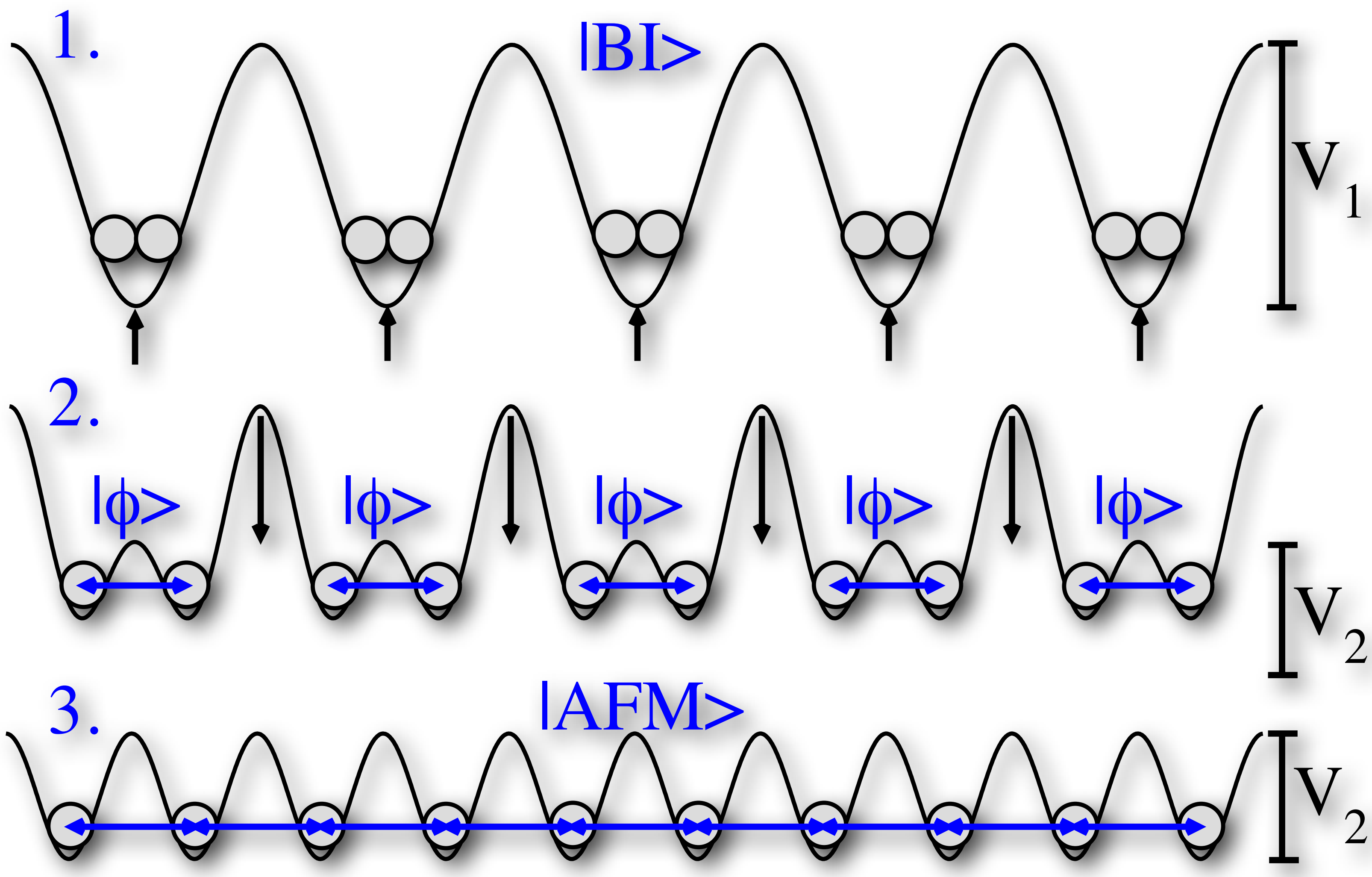
2. dimerized ground state [*Trotzky et al.*, PRL'10]

Experimental proposal

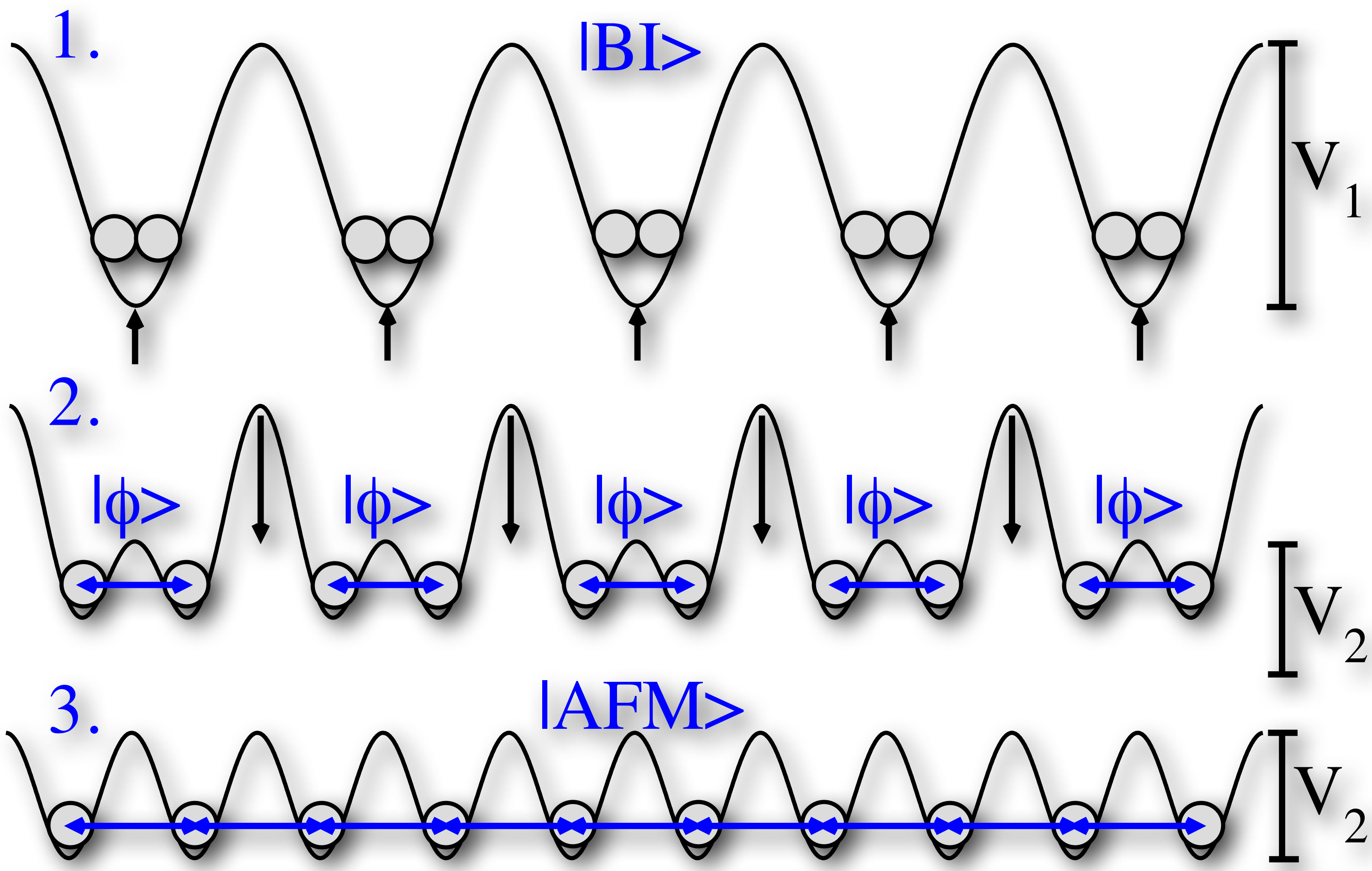


3. quantum Heisenberg antiferromagnet $|AFM\rangle$

Experimental proposal



Experimental proposal



Advantage over direct preparation of Mott insulator:
Band insulator has less entropy!

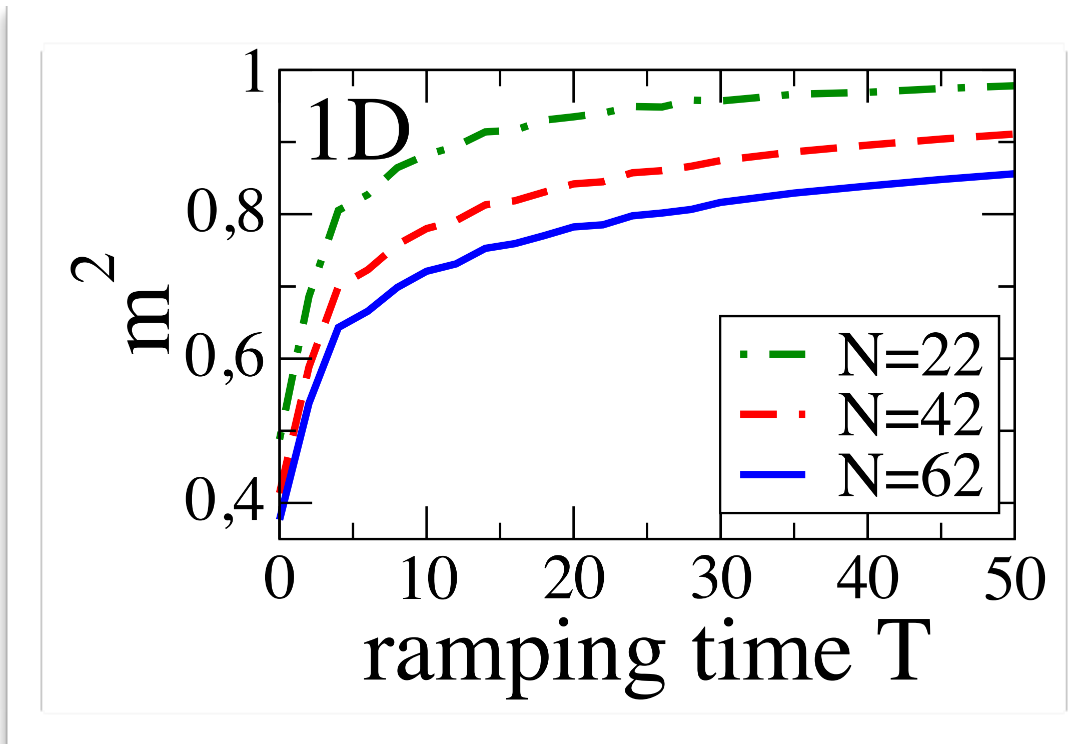
Adiabaticity conditions for the total lattice

Adiabaticity on total lattice

1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{N^2}\sum_{l,m=1}^{N}(-1)^{l+m}\langle \vec{S}_l \cdot \vec{S}_m\rangle \; : \; m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$

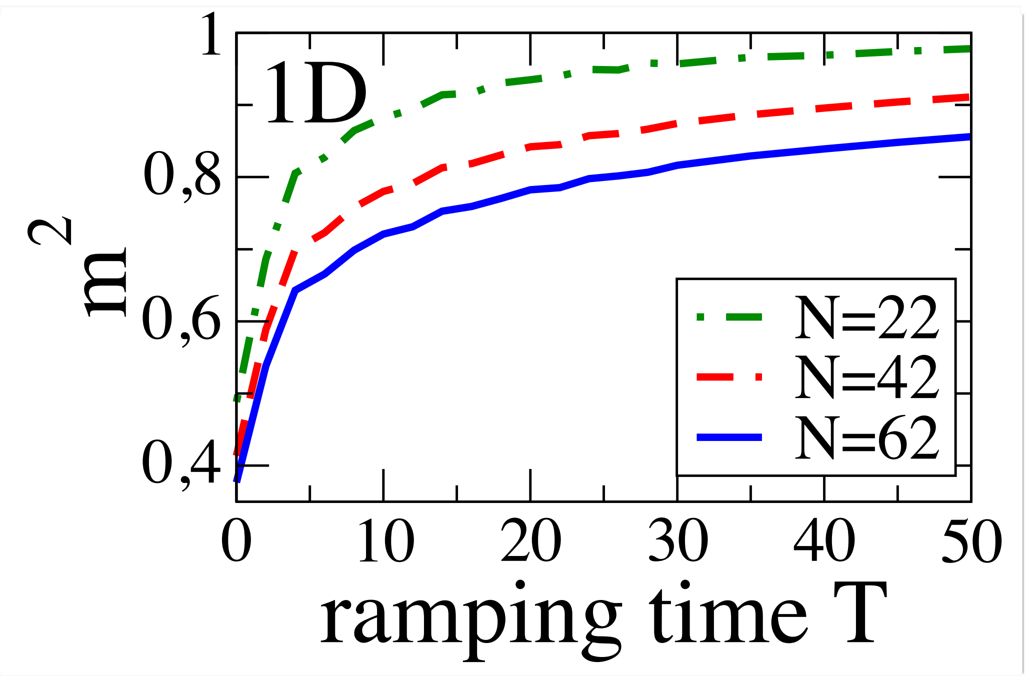


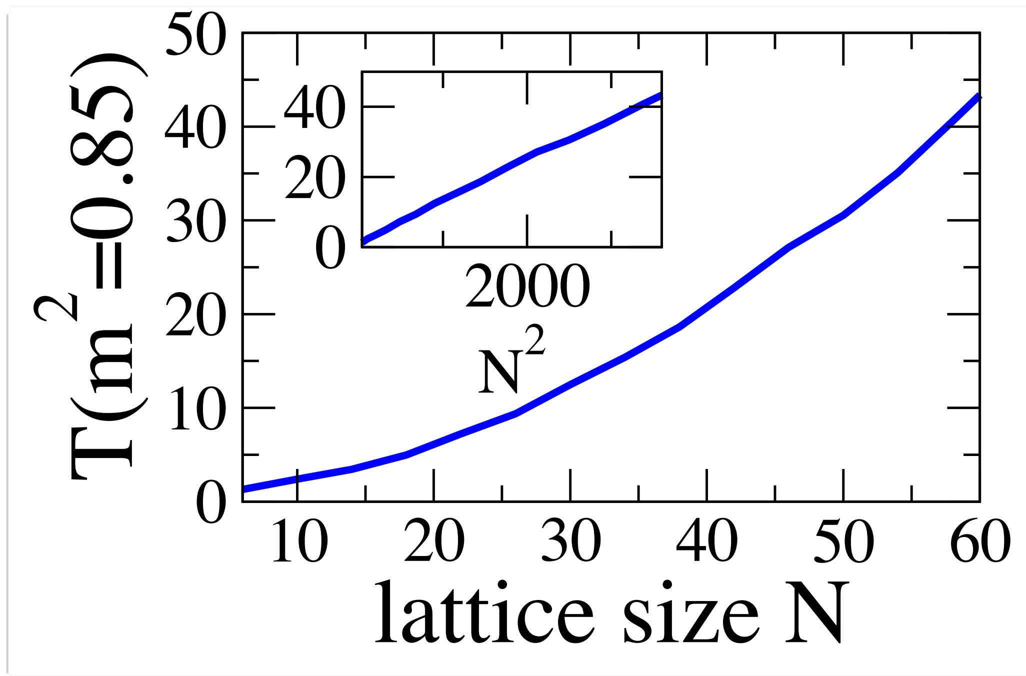
Adiabaticity on total lattice

1D

- experimental observable: **squared staggered magnetization**

$$M_{\mathrm{stag}}^2 = \frac{1}{N^2}\sum_{l,m=1}^{N}(-1)^{l+m}\langle \vec{S}_l \cdot \vec{S}_m\rangle \; : \; m^2(T) := M_{\mathrm{stag}}^2(T)/M_{\mathrm{stag,AFM}}^2$$





- Landau-Zener formula: $T \propto 1/\Delta^2$
- 1D: gap closes at end of protocol [*Matsumoto et al.*, PRB'01]

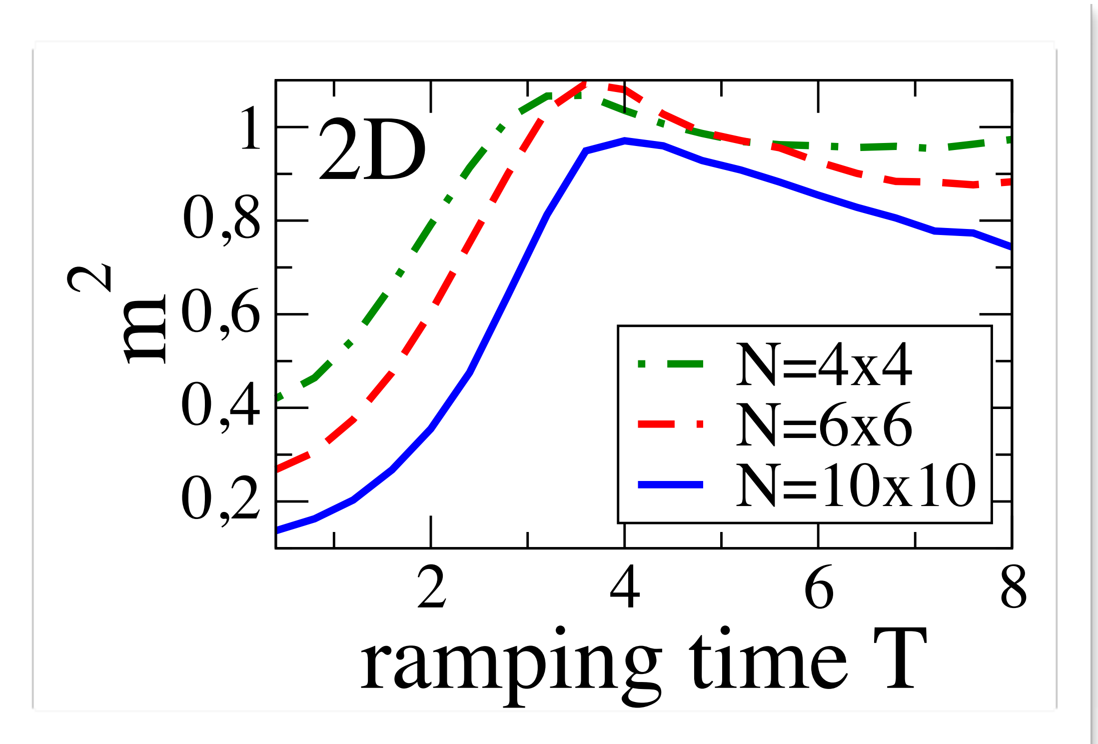
$$\Delta \propto 1/N \quad \rightarrow \quad T \propto N^2$$

Adiabaticity on total lattice

2D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{N^2} \sum_{l,m=1}^{N} (-1)^{l+m} \langle \vec{S}_l \cdot \vec{S}_m \rangle \; : \; m^2(T) := M^2_{\text{stag}}(T) / M^2_{\text{stag,AFM}}$$

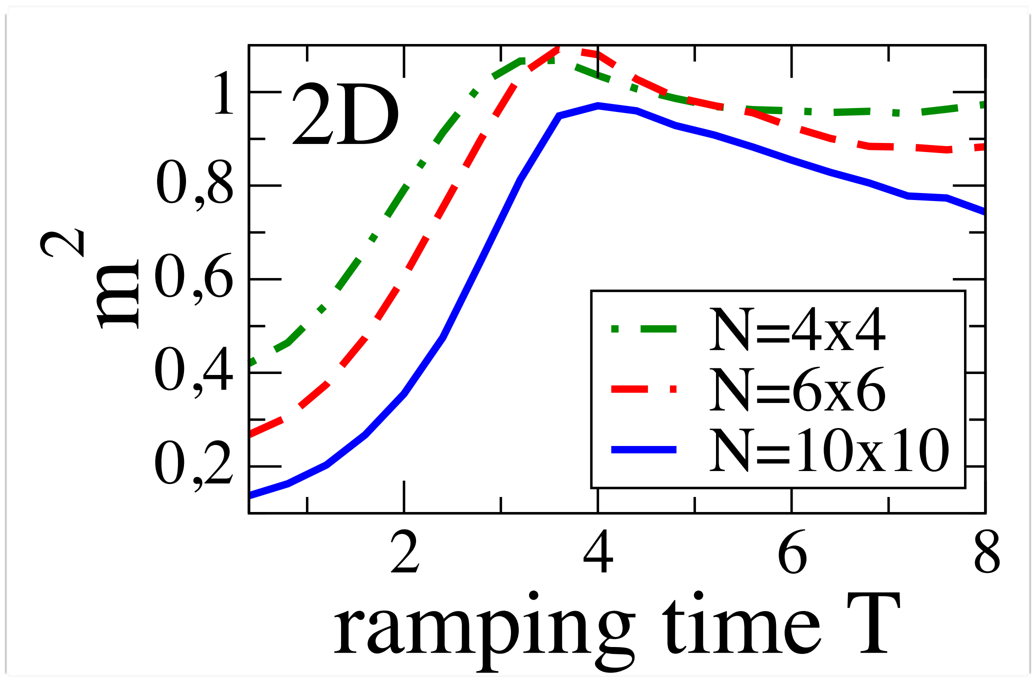


Adiabaticity on total lattice

2D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{N^2}\sum_{l,m=1}^{N}(-1)^{l+m}\langle\vec{S}_l\cdot\vec{S}_m\rangle \; : \; m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$



high magnetization in short ramping time

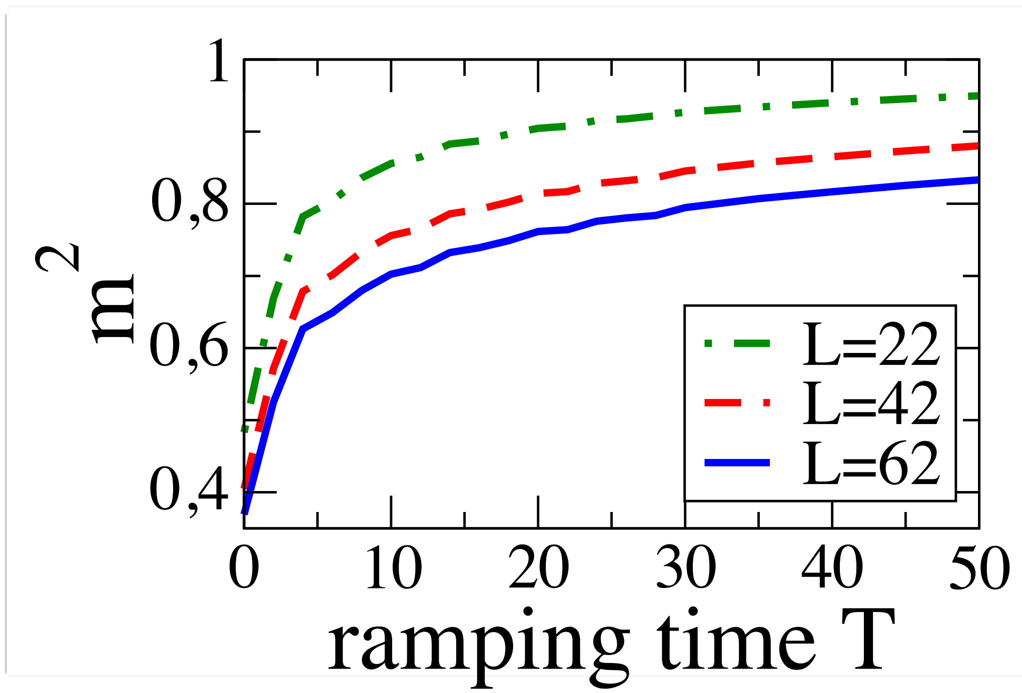
Adiabaticity conditions for a sublattice

Adiabaticity on sublattice

1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{L^2} \sum_{l,m=1}^{L} (-1)^{l+m} \langle \vec{S}_l \cdot \vec{S}_m \rangle \; : m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$

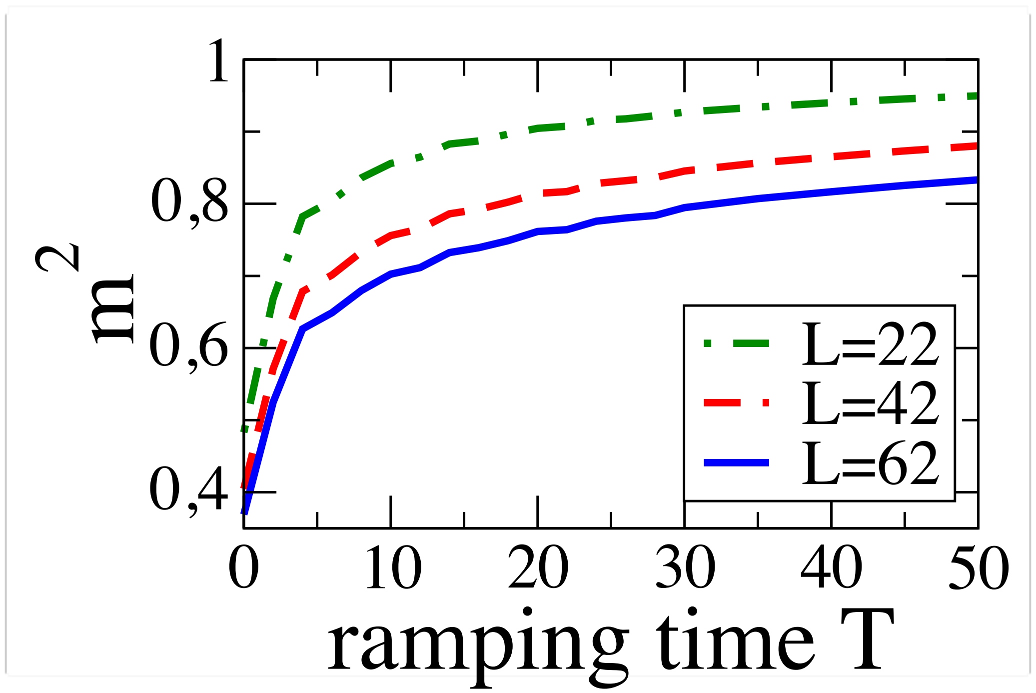


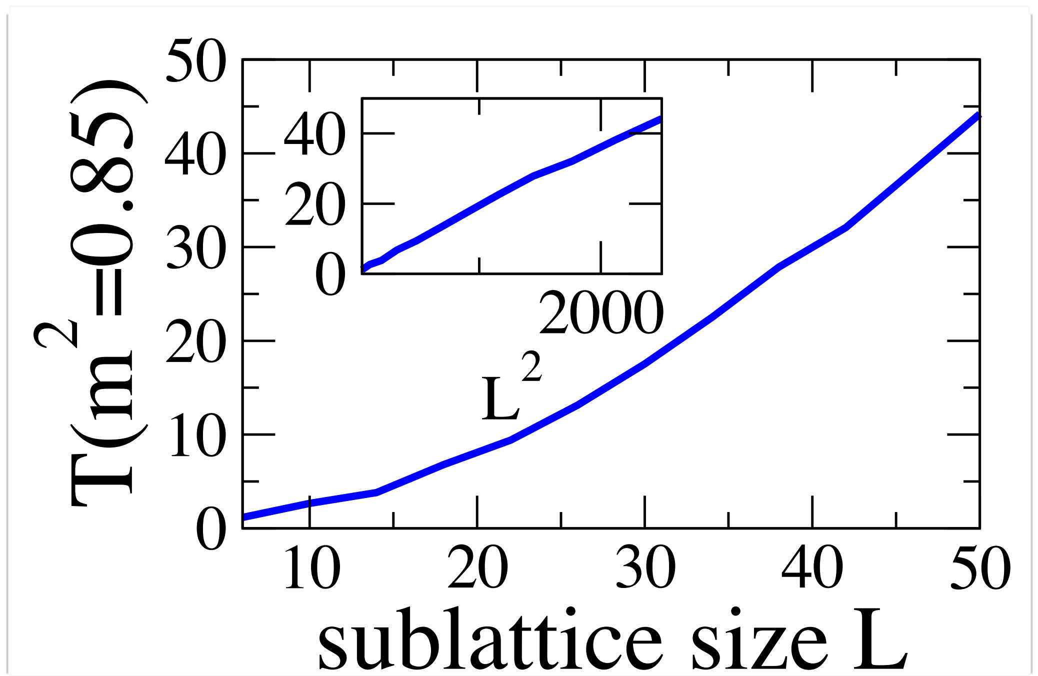
Adiabaticity on sublattice

1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{L^2} \sum_{l,m=1}^{L} (-1)^{l+m} \langle \vec{S}_l \cdot \vec{S}_m \rangle \; : m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$





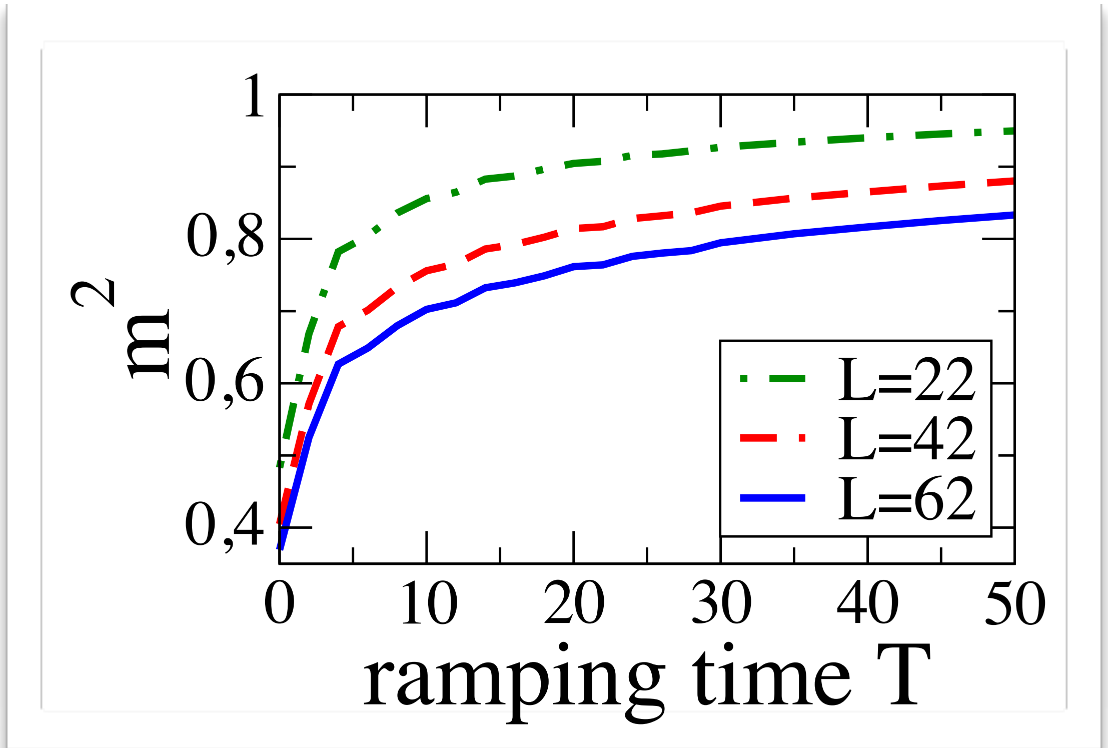
- effective local gap: $\Delta \propto 1/L \quad \rightarrow \quad T \propto L^2$

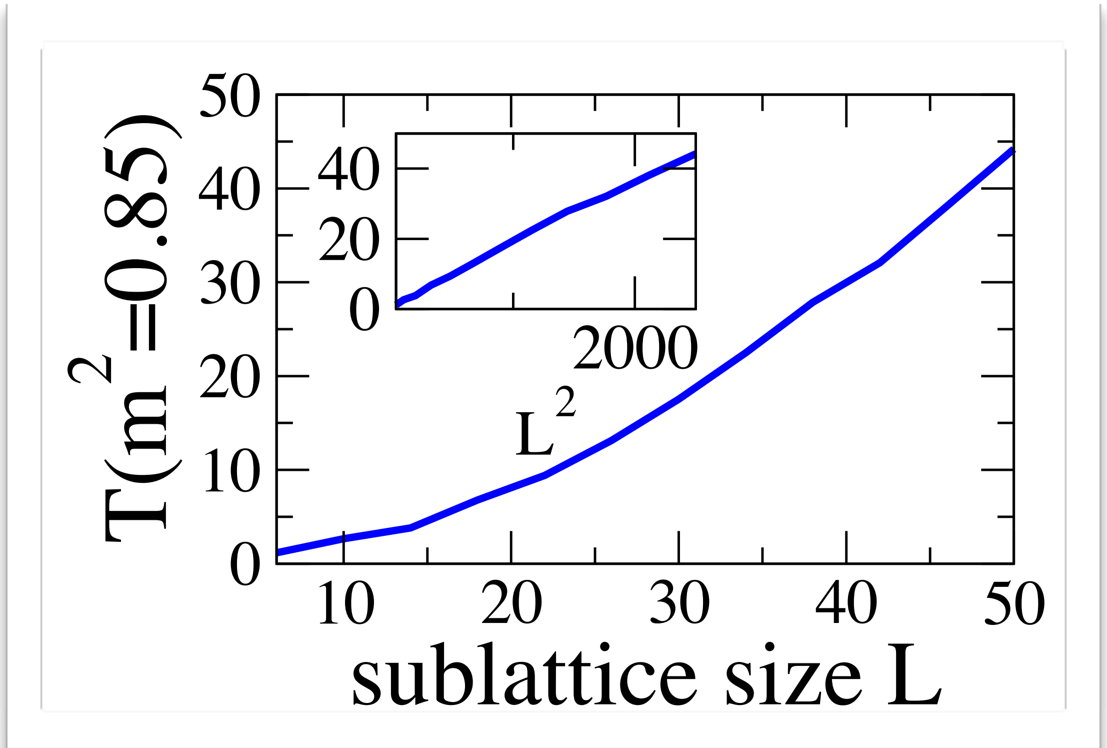
Adiabaticity on sublattice

1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{L^2} \sum_{l,m=1}^{L} (-1)^{l+m} \langle \vec{S}_l \cdot \vec{S}_m \rangle \; : m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$





- effective local gap: $\Delta \propto 1/L \quad \rightarrow \quad T \propto L^2$

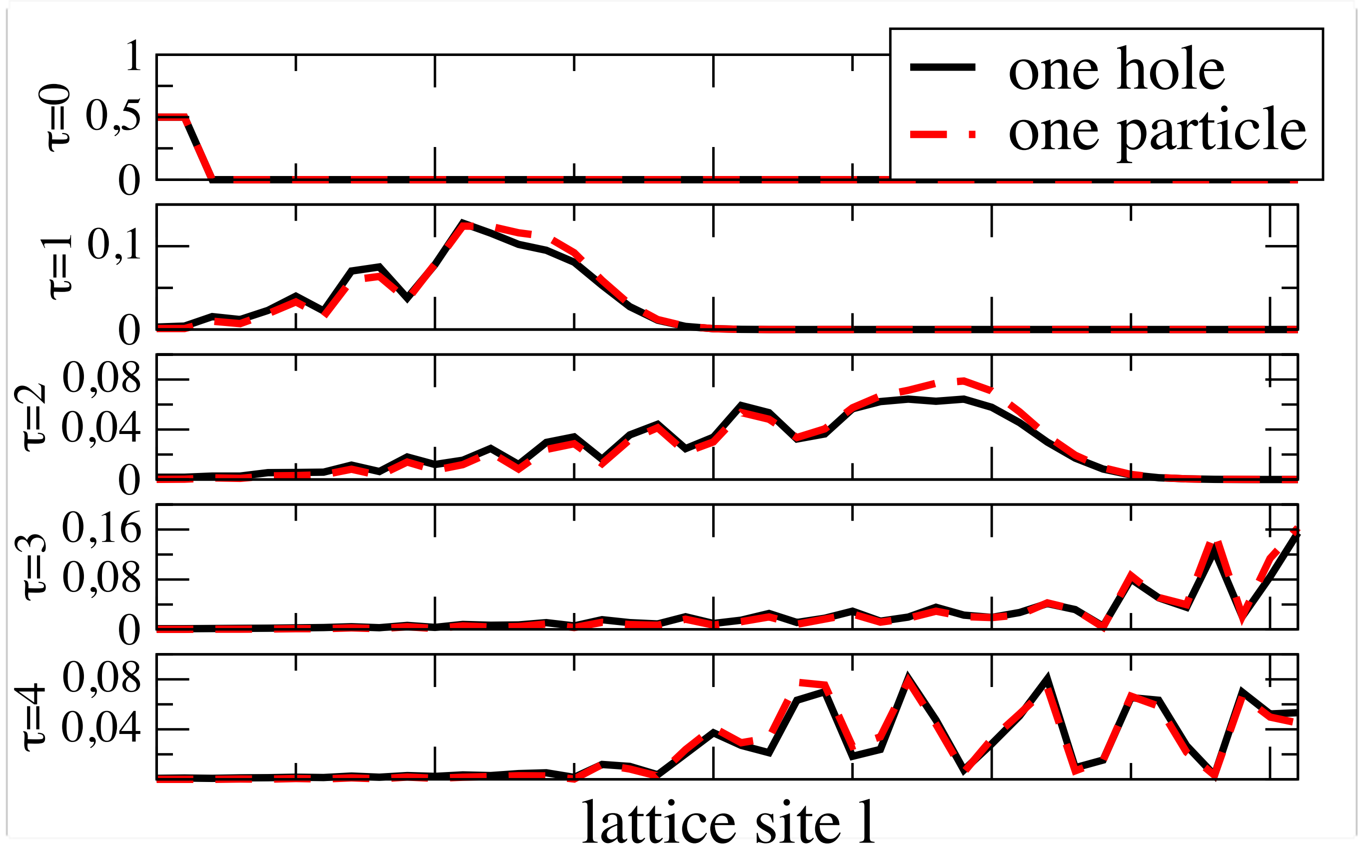
high magnetization in short ramping time on small part

Effect of holes

Effect of holes

1D

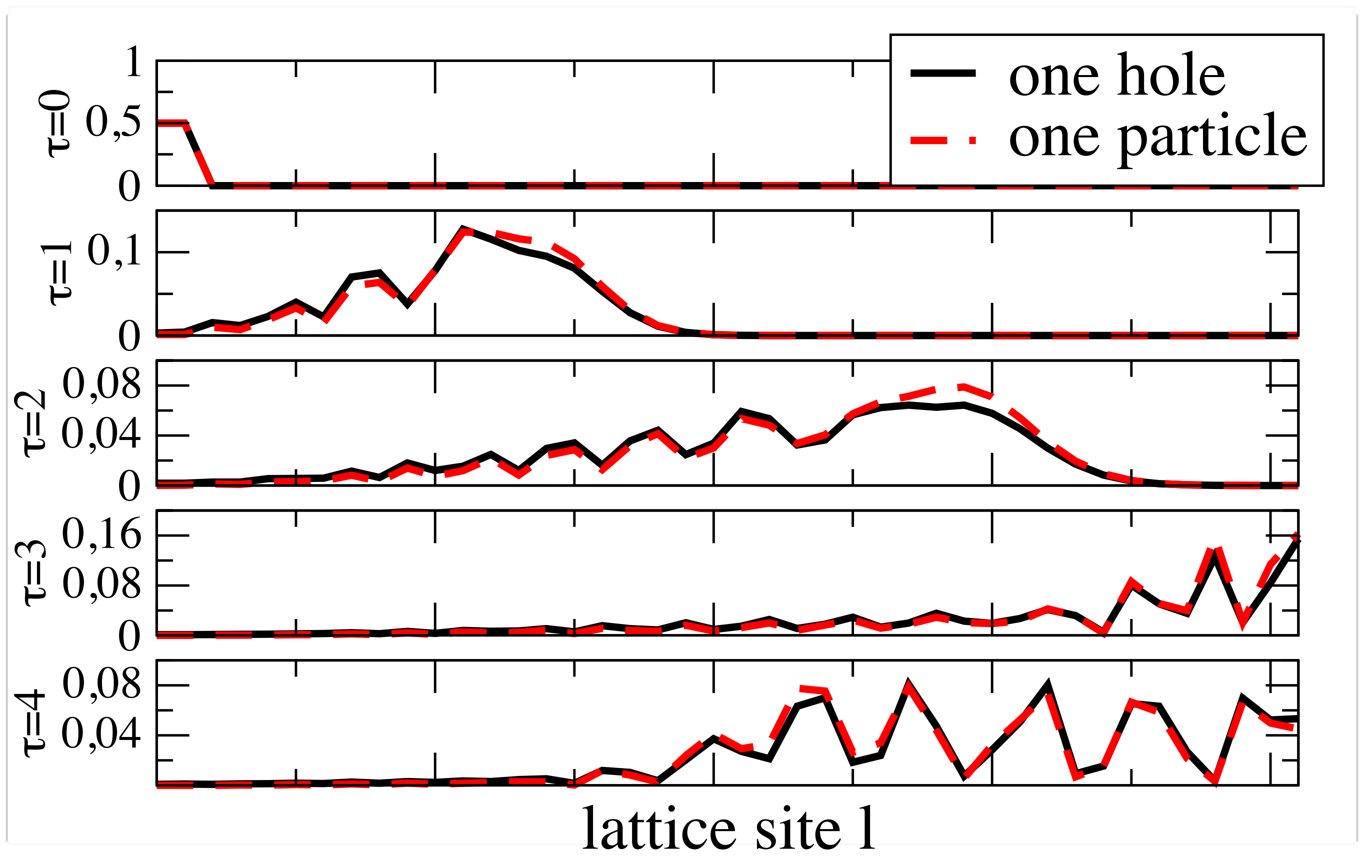
$$\hat{H} = -t \sum_{\langle l,m \rangle,\sigma} (\tilde{c}^{\dagger}_{l,\sigma}\tilde{c}_{m,\sigma} + \tilde{c}^{\dagger}_{m,\sigma}\tilde{c}_{l,\sigma}) + \hat{H}_{\text{spin}}$$



Effect of holes

$$\hat{H} = -t \sum_{\langle l,m \rangle,\sigma} (\tilde{c}^\dagger_{l,\sigma}\tilde{c}_{m,\sigma} + \tilde{c}^\dagger_{m,\sigma}\tilde{c}_{l,\sigma}) + \hat{H}_{\text{spin}}$$

1D

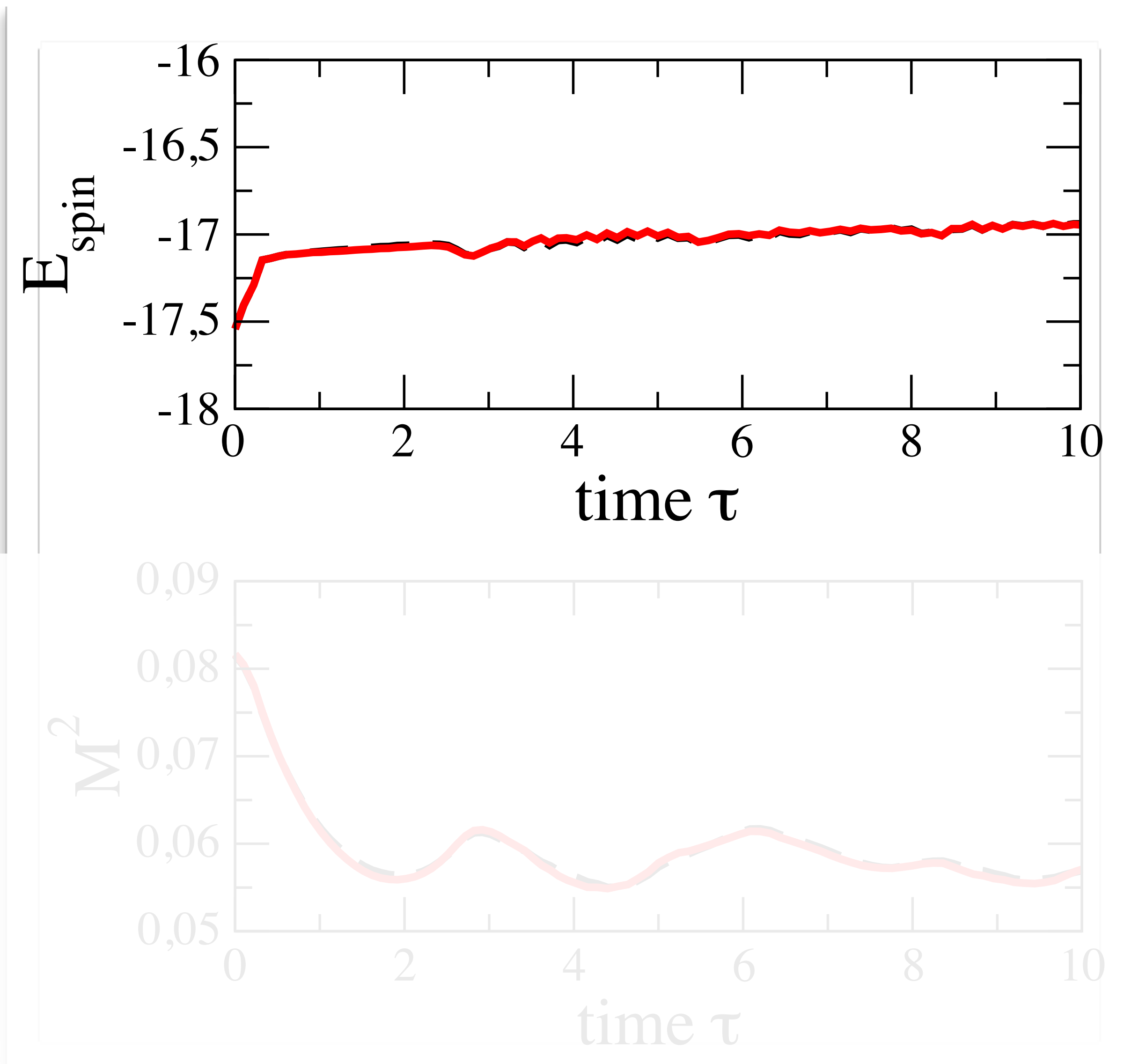


hole spreads as free particle: velocity $v = 2t$

Effect of holes

$$\hat{H} = -t \sum_{\langle l,m\rangle,\sigma} (\tilde{c}^\dagger_{l,\sigma}\tilde{c}_{m,\sigma} + \tilde{c}^\dagger_{m,\sigma}\tilde{c}_{l,\sigma}) + \hat{H}_{\text{spin}}$$

1D



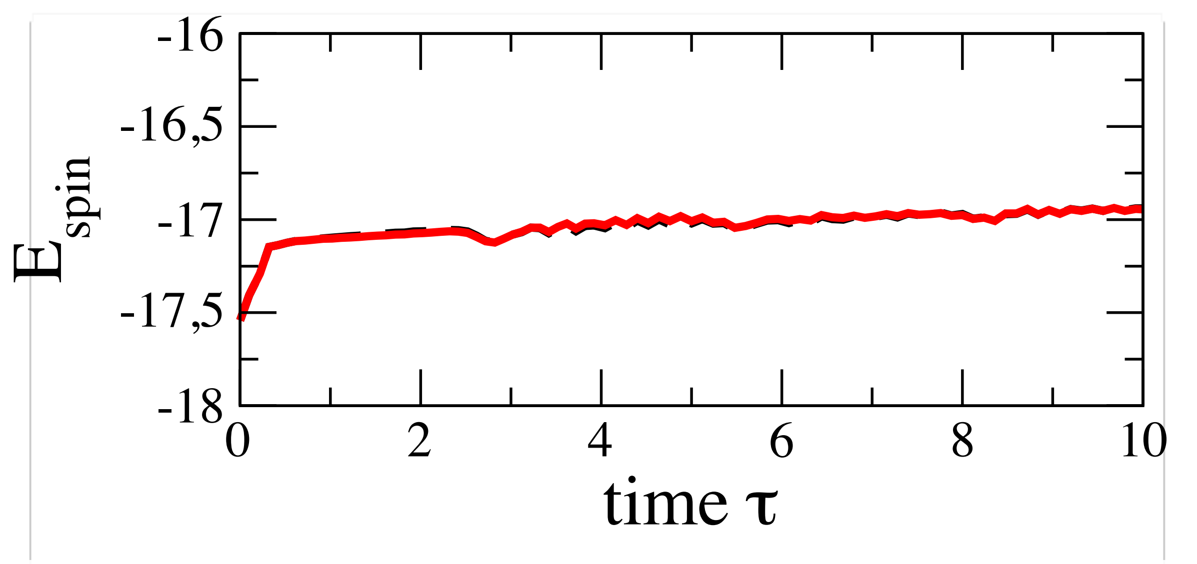
- energy increase: $\Delta E_{\text{spin}} \approx |\langle \vec{S}_l \cdot \vec{S}_{l+1}\rangle|$

Effect of holes

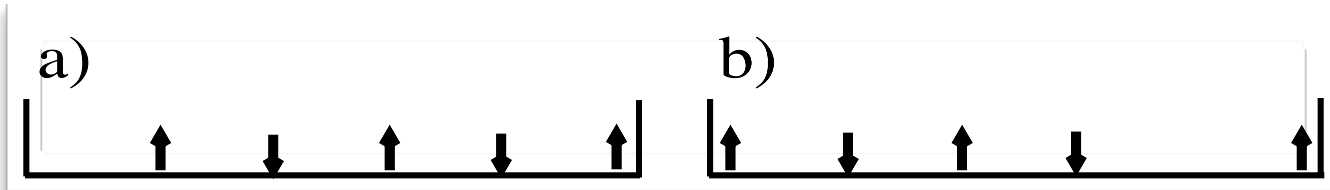
$$\hat{H} = -t \sum_{\langle l,m\rangle,\sigma} (\tilde{c}^\dagger_{l,\sigma}\tilde{c}_{m,\sigma} + \tilde{c}^\dagger_{m,\sigma}\tilde{c}_{l,\sigma}) + \hat{H}_{\text{spin}}$$

$$J \sum_{\langle l,m\rangle} \vec{S}_l \cdot \vec{S}_m$$

1D



- energy increase:
  $\Delta E_{\text{spin}} \approx |\langle \vec{S}_l \cdot \vec{S}_{l+1}\rangle|$

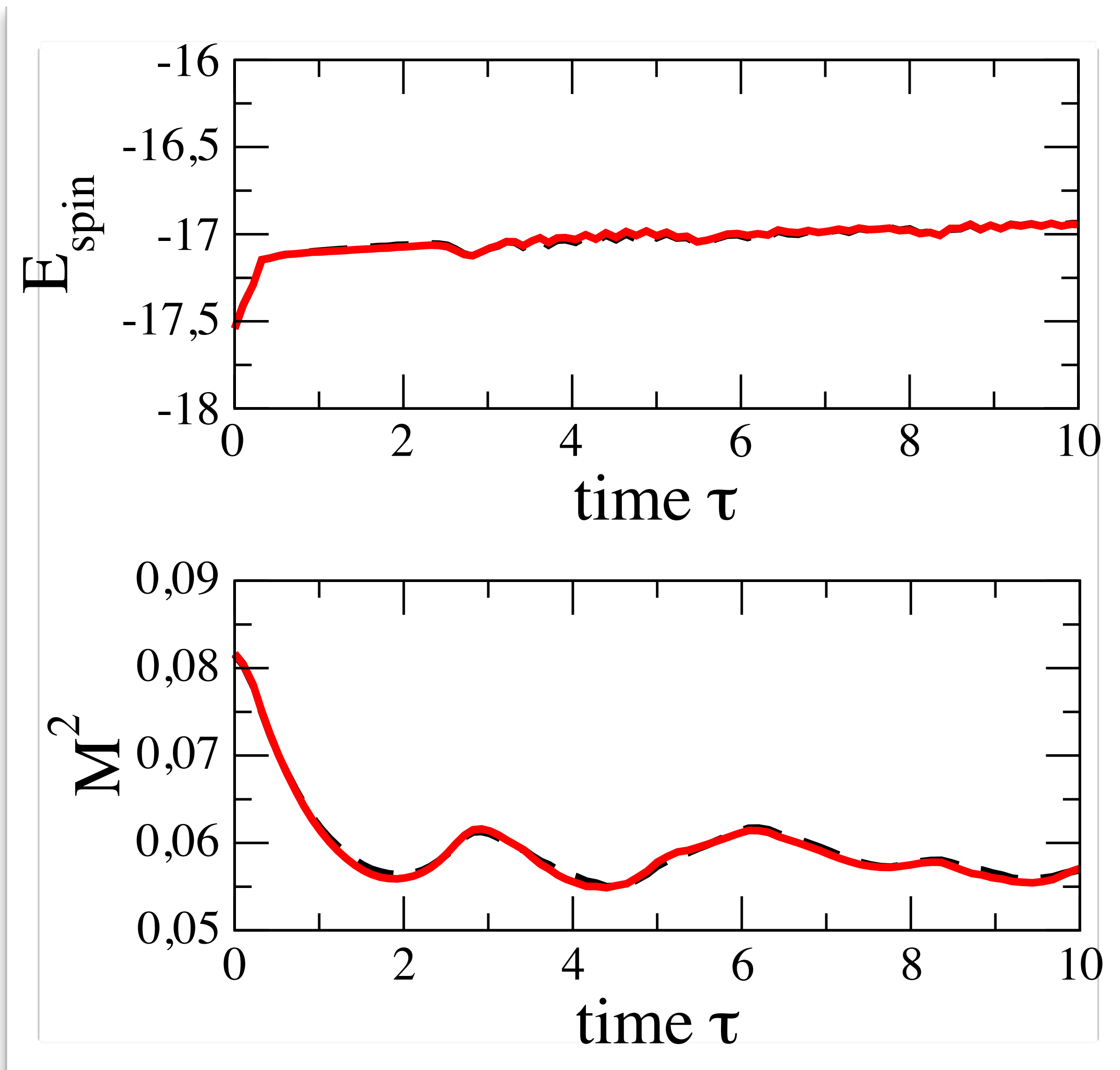


Effect of holes

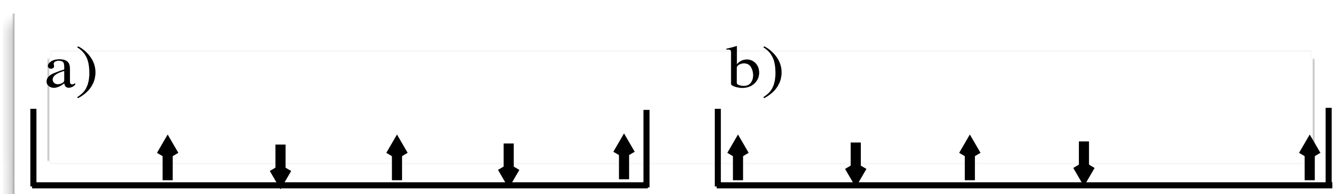
$$\hat{H} = -t \sum_{\langle l,m \rangle,\sigma} (\tilde{c}^{\dagger}_{l,\sigma}\tilde{c}_{m,\sigma} + \tilde{c}^{\dagger}_{m,\sigma}\tilde{c}_{l,\sigma}) + \hat{H}_{\text{spin}}$$

1D

$$J \sum_{\langle l,m \rangle} \vec{S}_l \cdot \vec{S}_m$$



- energy increase:
  $\Delta E_{\text{spin}} \approx |\langle \vec{S}_l \cdot \vec{S}_{l+1} \rangle|$



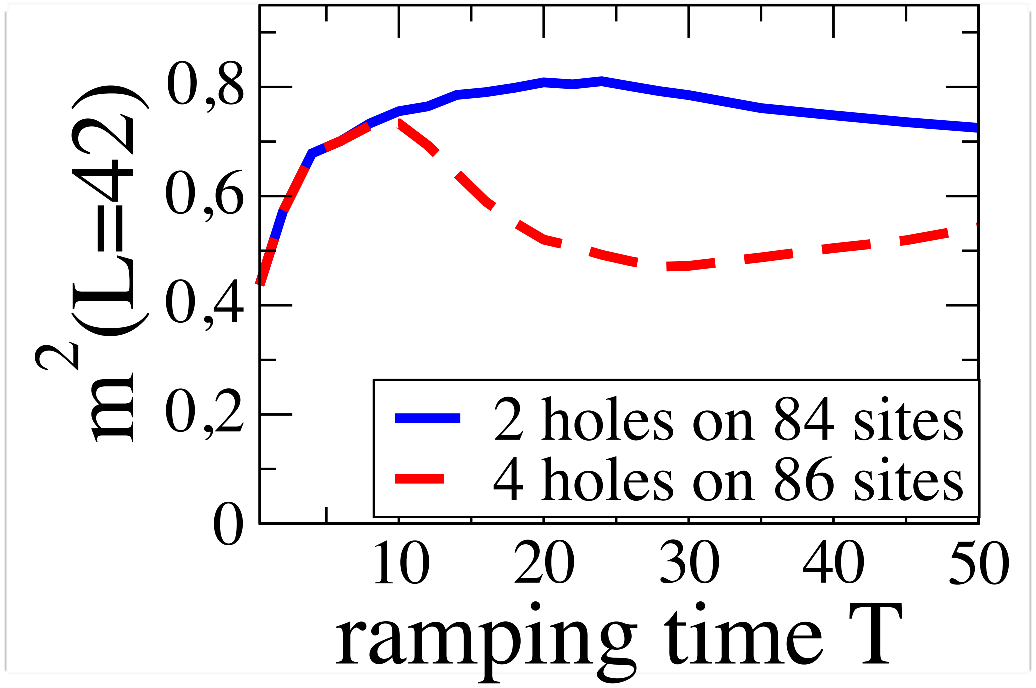
- drastic magnetization reduction

Effect of holes

1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{N^2} \sum_{l,m=1}^{N} (-1)^{l+m} \langle \vec{S}_l \cdot \vec{S}_m \rangle \; : m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$

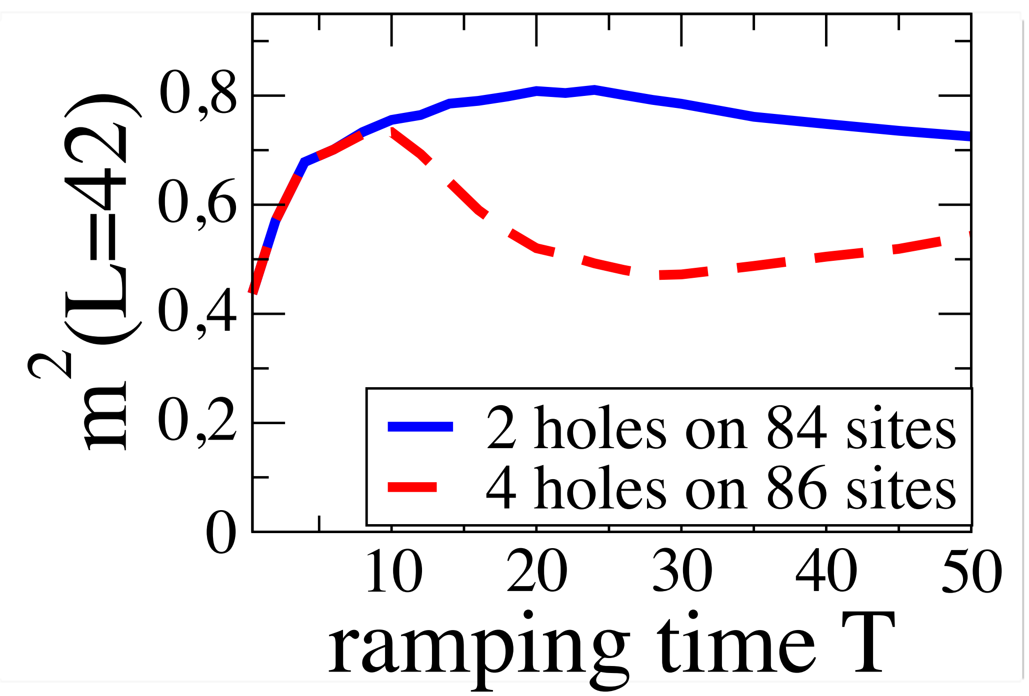


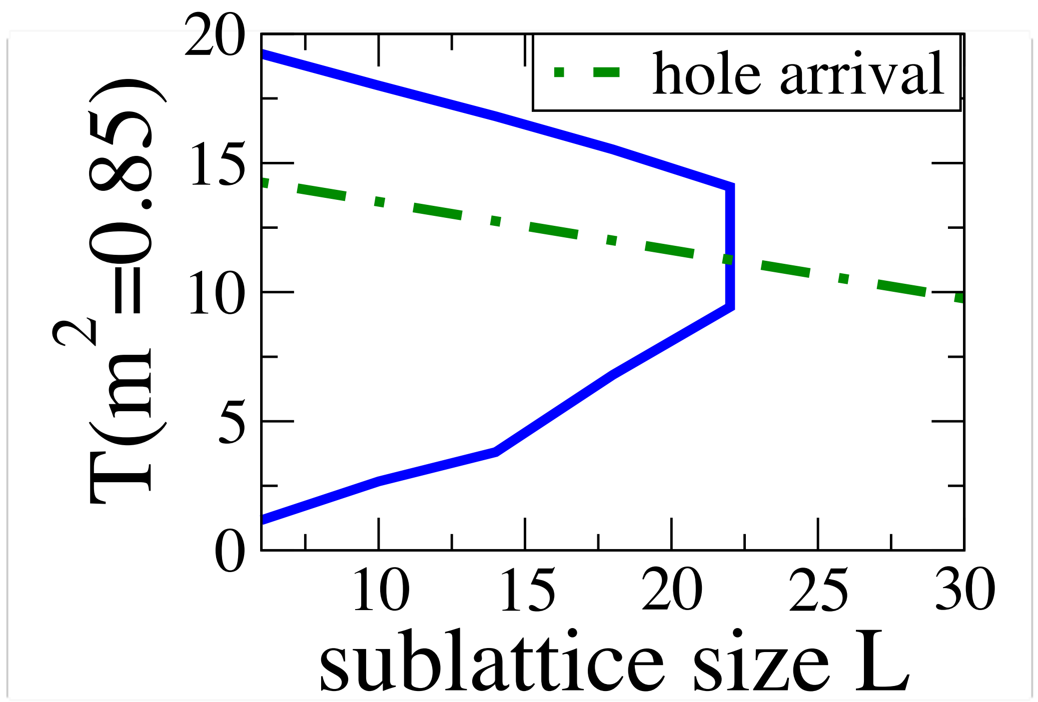
Effect of holes

1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{N^2} \sum_{l,m=1}^{N} (-1)^{l+m} \langle \vec{S}_l \cdot \vec{S}_m \rangle \; : m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$



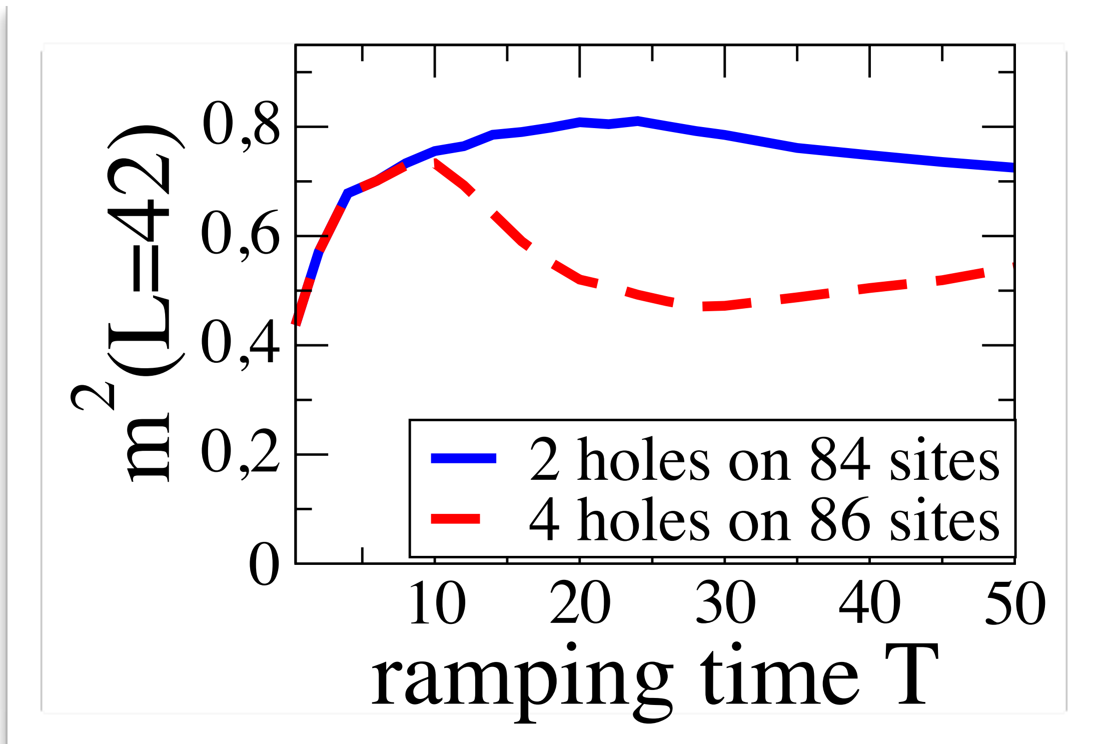


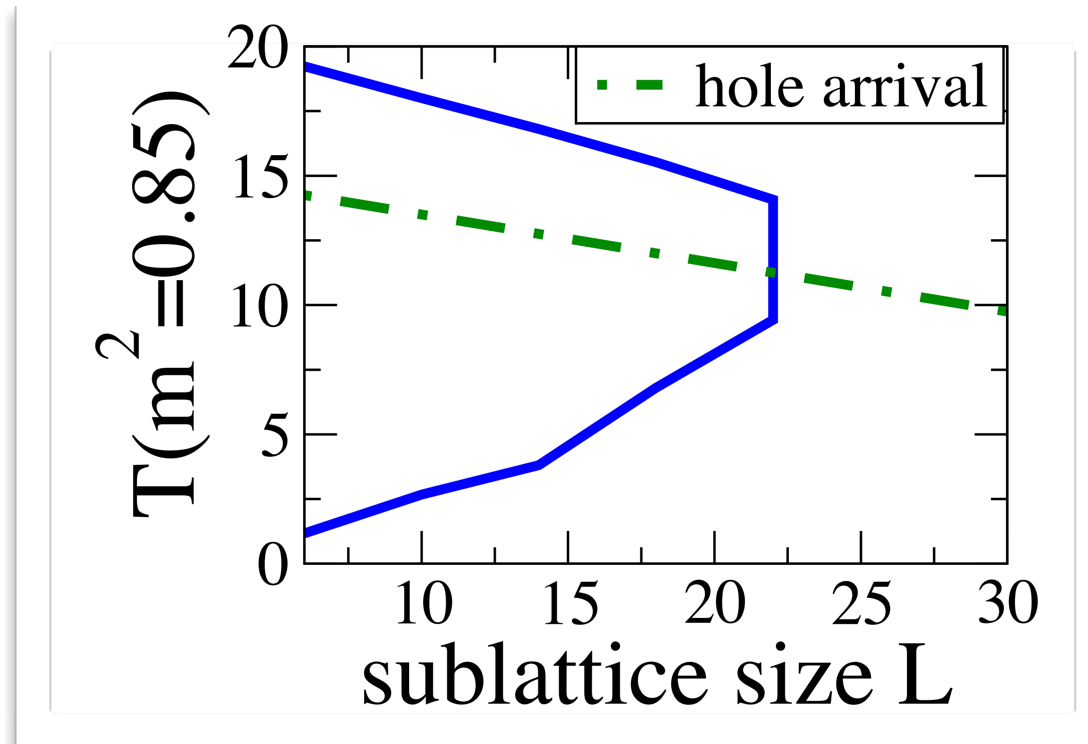
Effect of holes

1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{N^2}\sum_{l,m=1}^{N}(-1)^{l+m}\langle \vec{S}_l\cdot\vec{S}_m\rangle \;:\; m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$





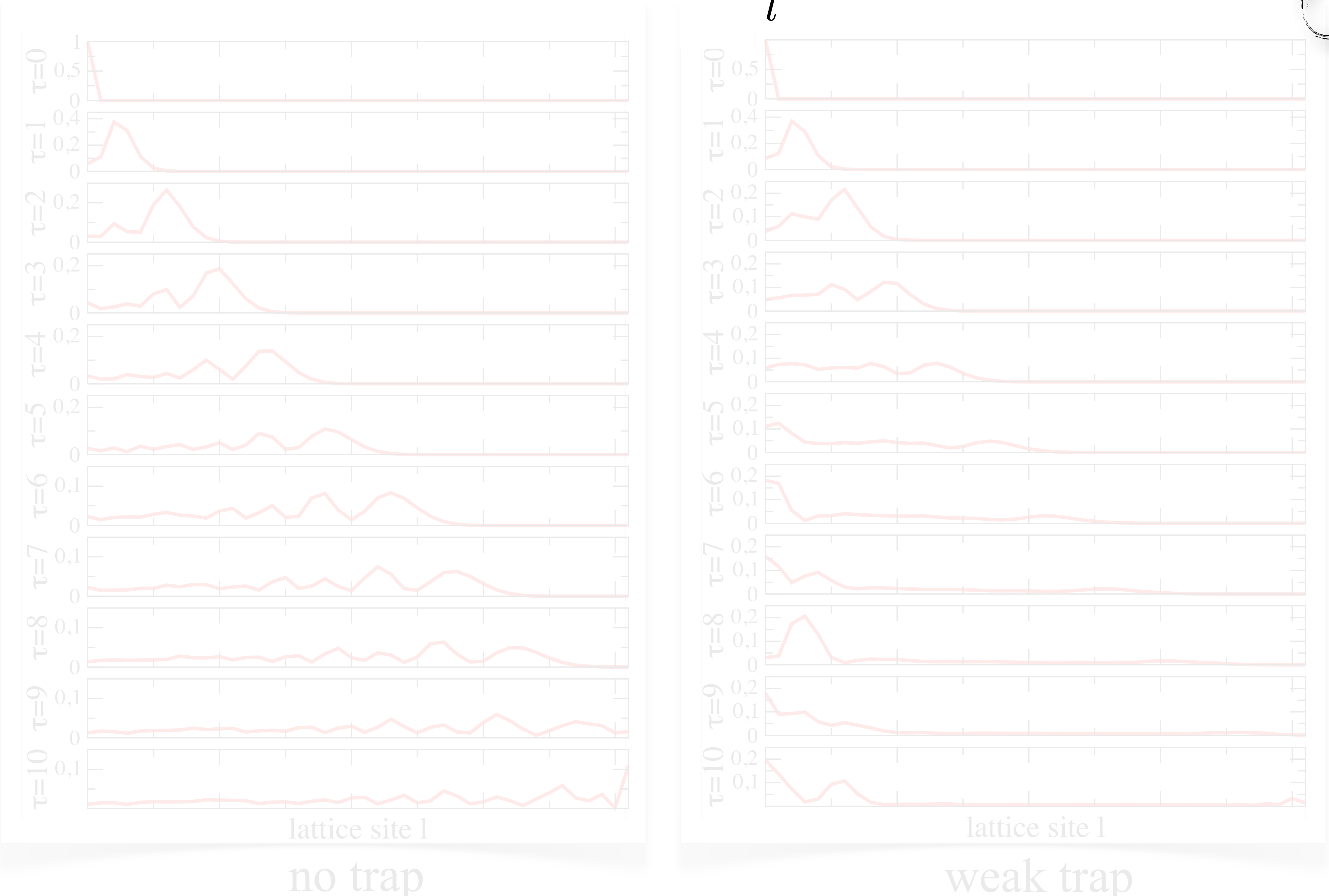
drastic magnetization reduction

Harmonic trap

Harmonic trap

$$\hat{H} = \hat{H}_{\rm kin} + \hat{H}_{\rm spin} + V \sum_l (l - l_0)^2 \hat{n}_l$$

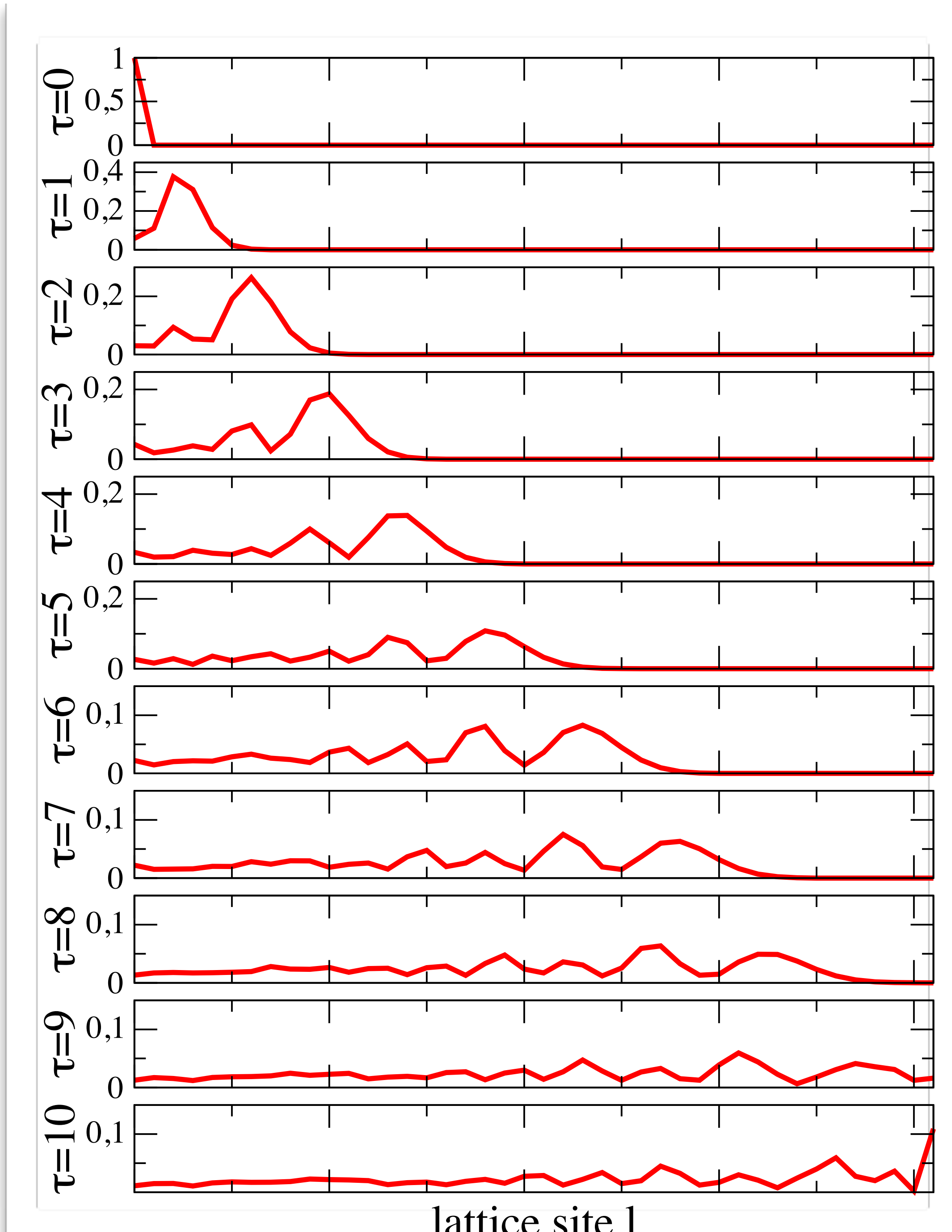
1D



Harmonic trap

$$\hat{H} = \hat{H}_{\text{kin}} + \hat{H}_{\text{spin}} + V\sum_{l}(l-l_0)^2\hat{n}_l$$

1D



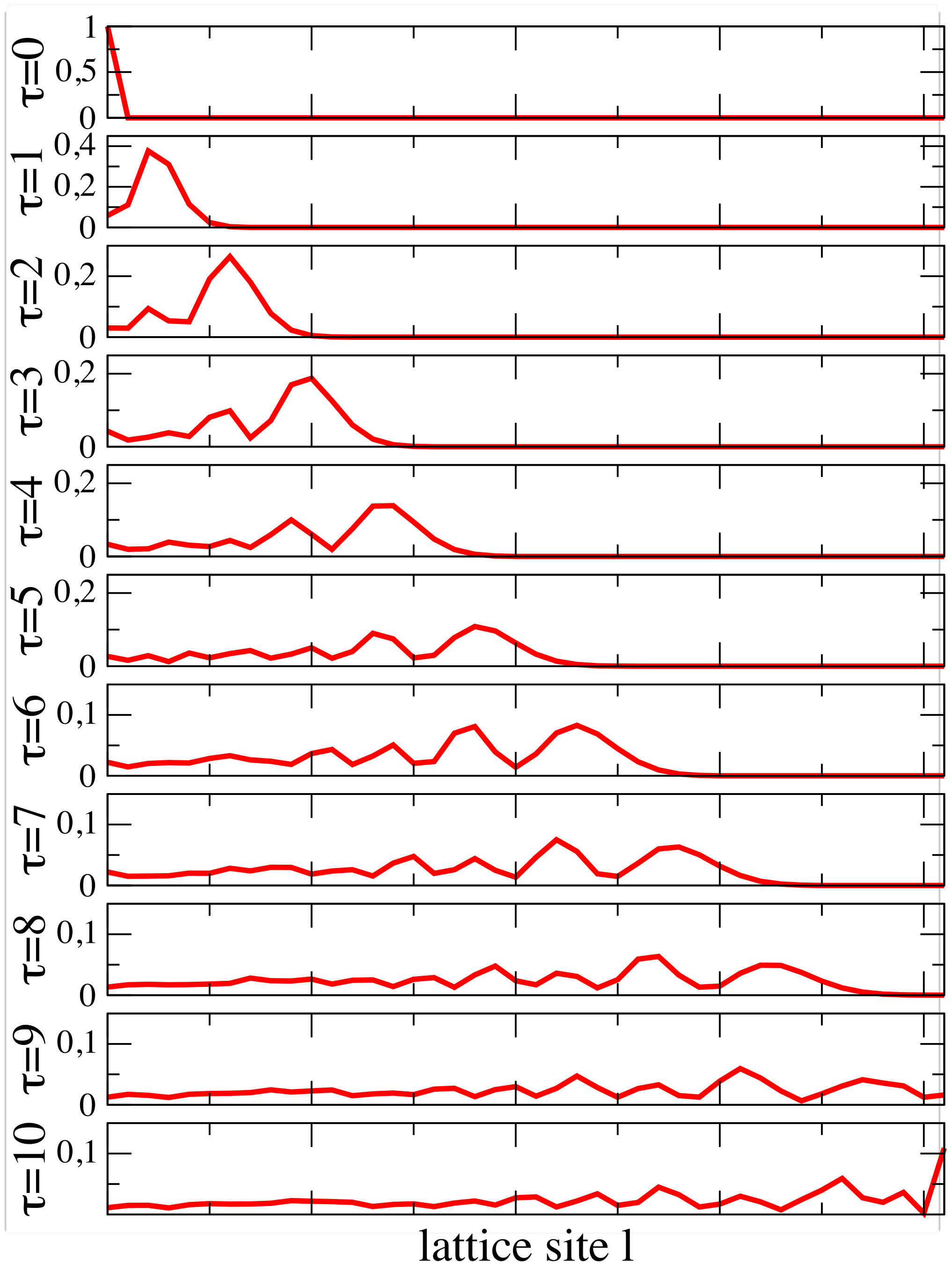
no trap

weak trap

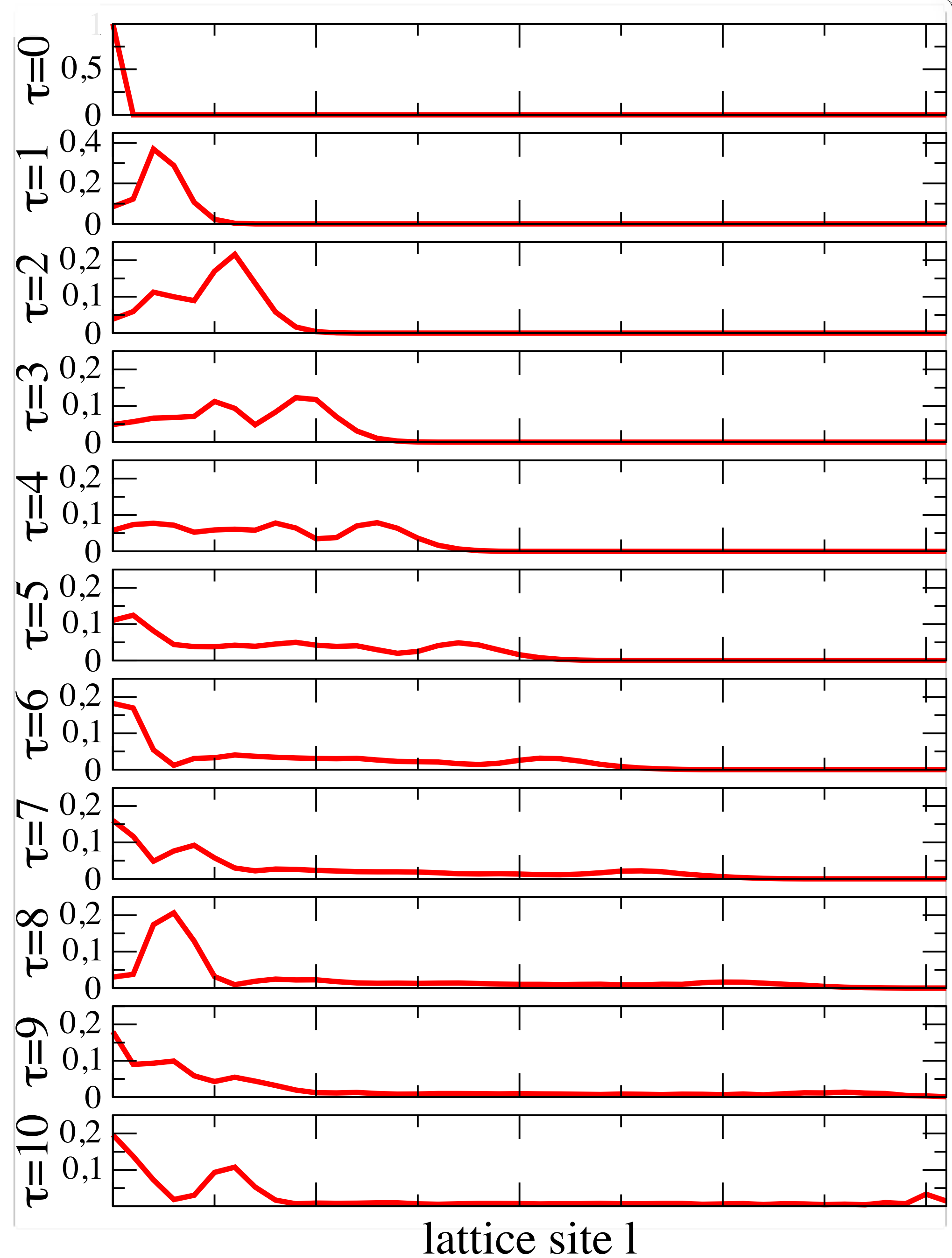
Harmonic trap

$$\hat{H} = \hat{H}_{\text{kin}} + \hat{H}_{\text{spin}} + V \sum_l (l - l_0)^2 \hat{n}_l$$

1D



no trap



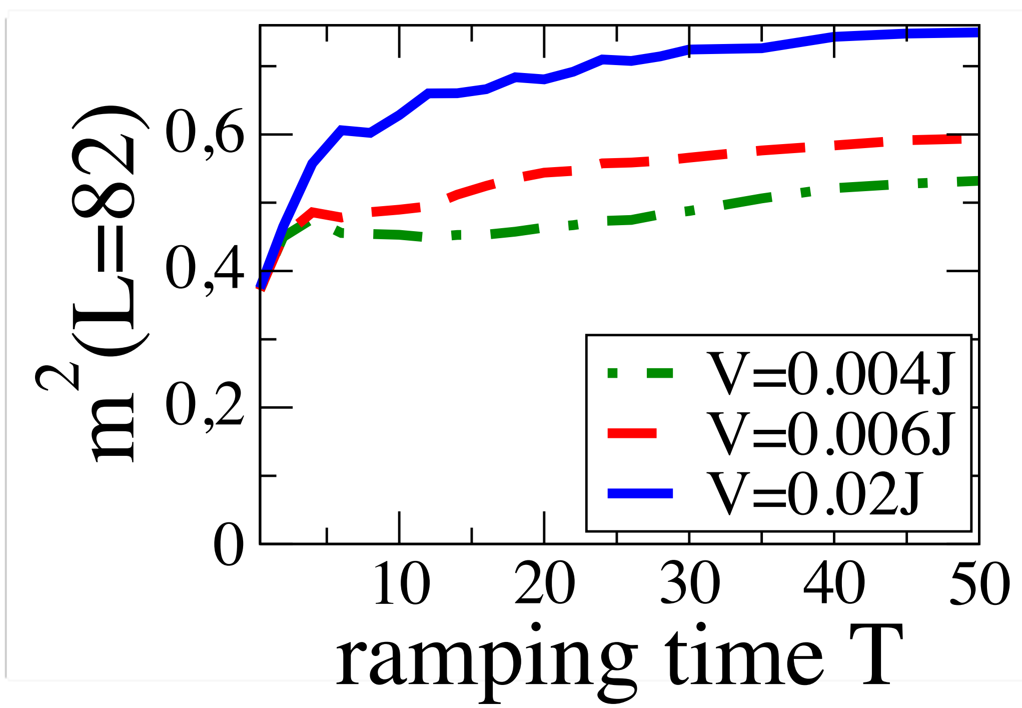
weak trap

Harmonic trap

1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{N^2}\sum_{l,m=1}^{N}(-1)^{l+m}\langle \vec{S}_l \cdot \vec{S}_m \rangle \,:\, m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$

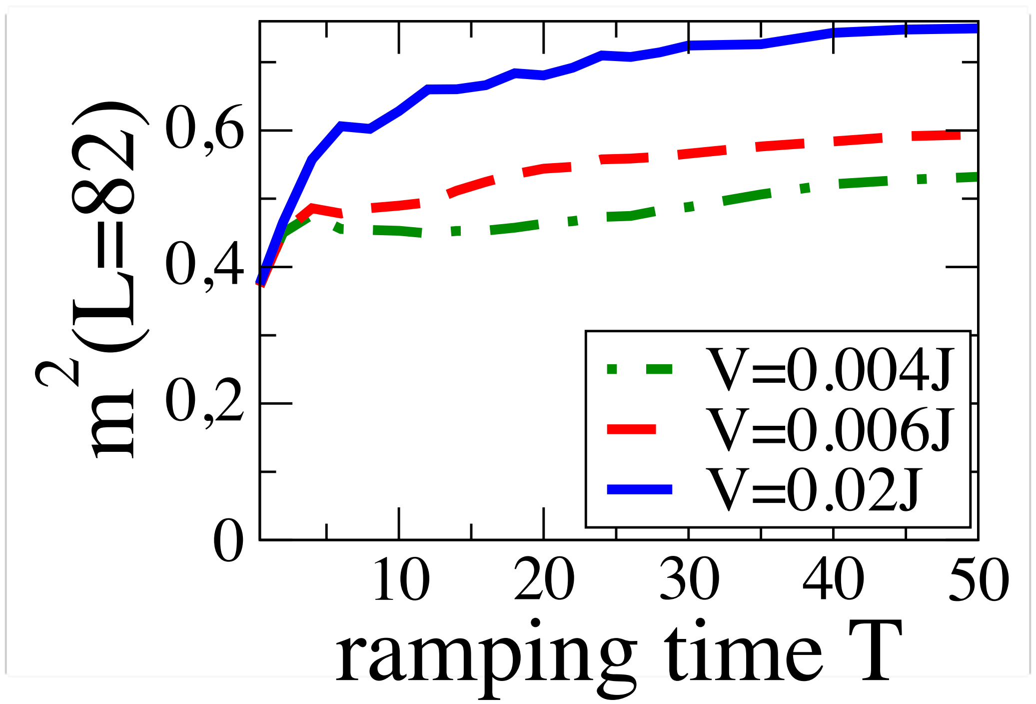


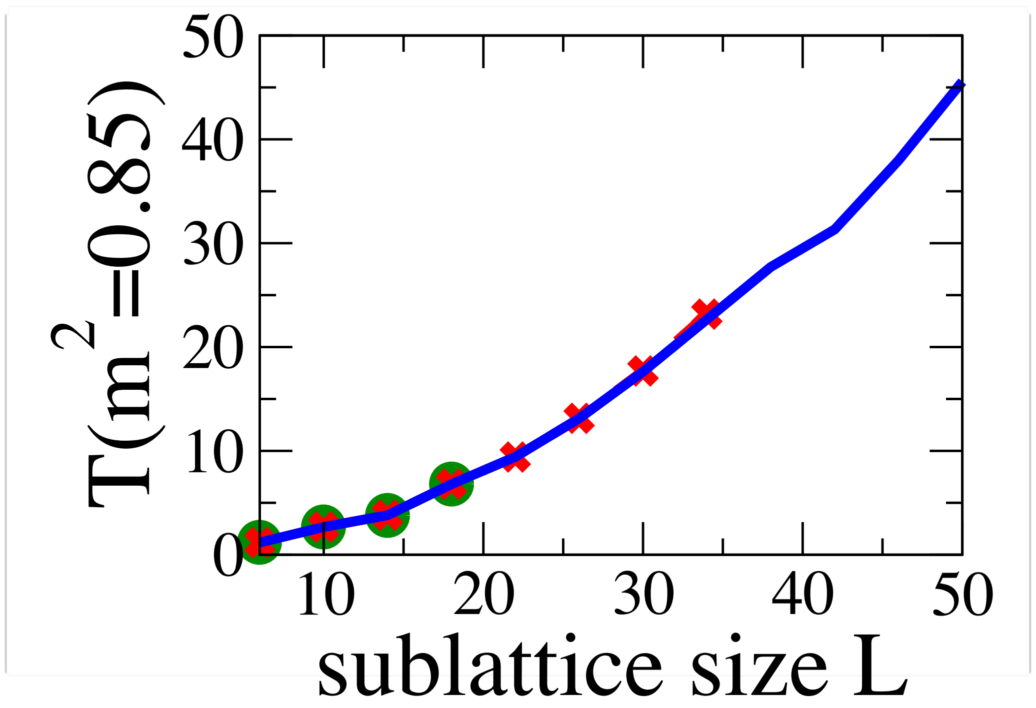
Harmonic trap

1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\text{stag}} = \frac{1}{N^2} \sum_{l,m=1}^{N} (-1)^{l+m} \langle \vec{S}_l \cdot \vec{S}_m \rangle \; : m^2(T) := M^2_{\text{stag}}(T)/M^2_{\text{stag,AFM}}$$



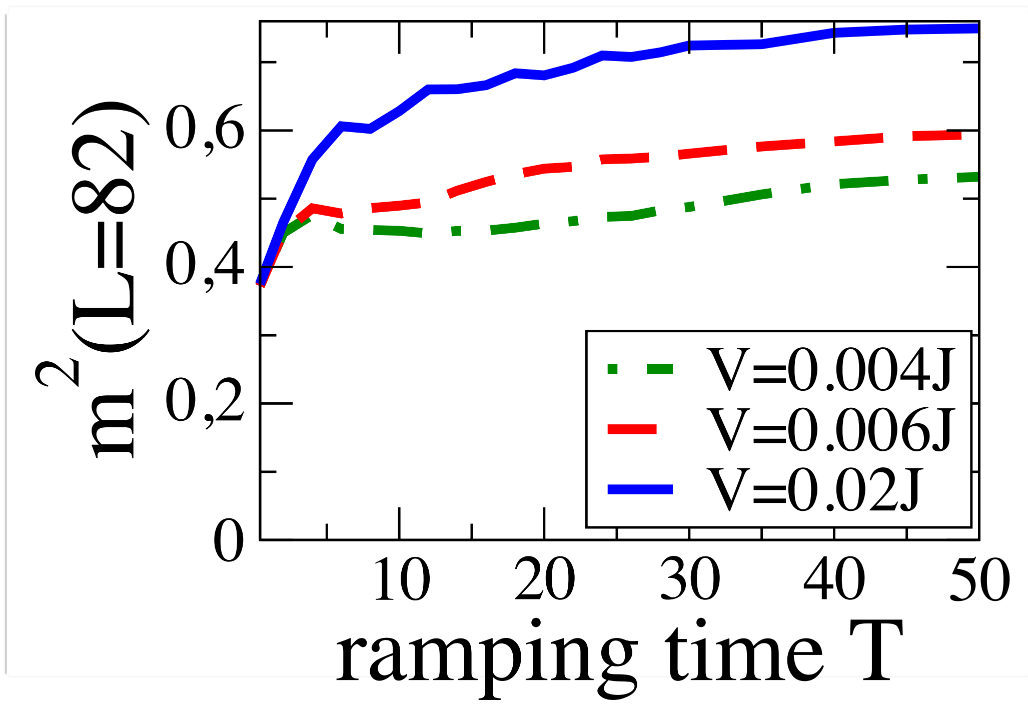


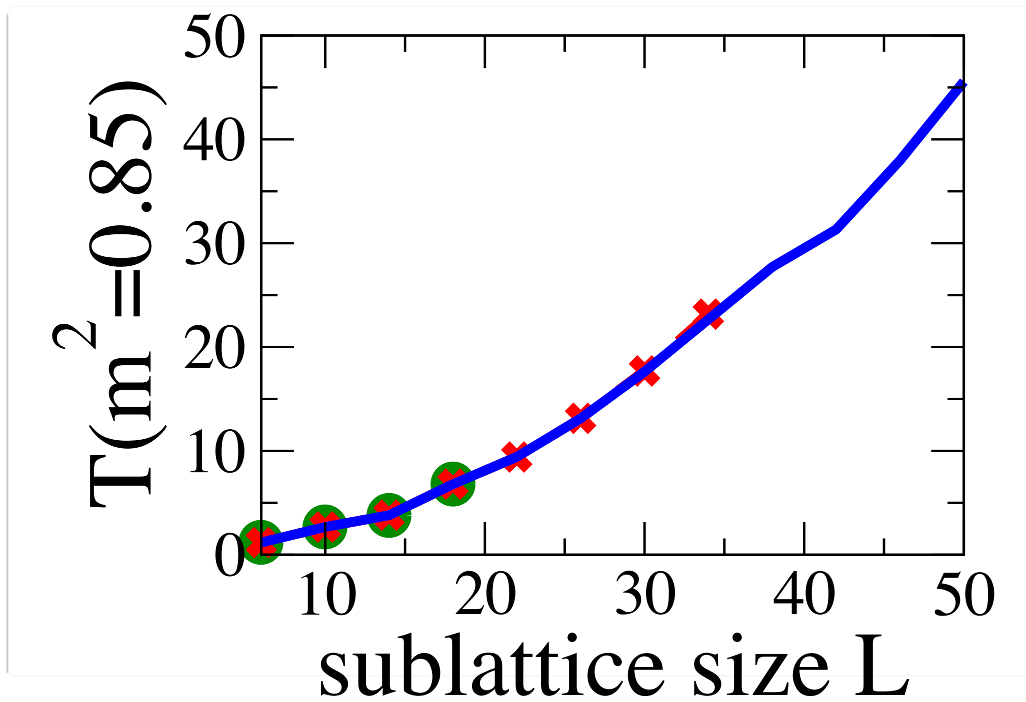
Harmonic trap

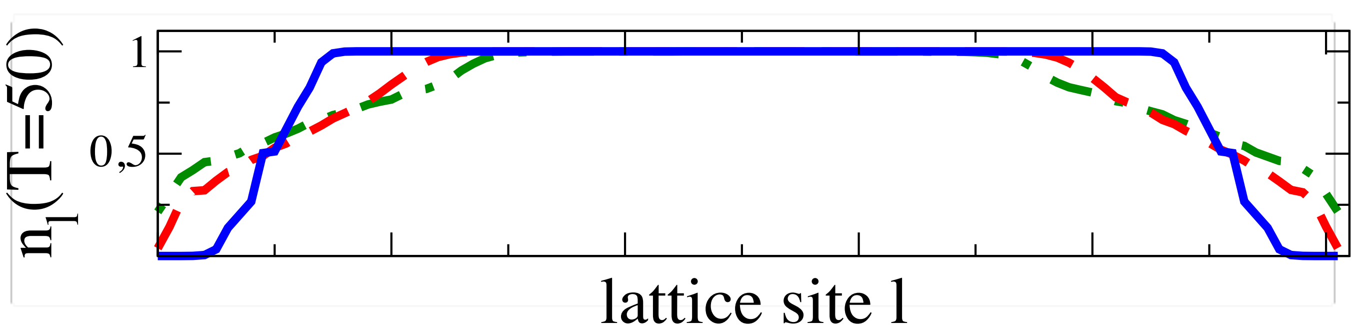
1D

- experimental observable: **squared staggered magnetization**

$$M^2_{\rm stag} = \frac{1}{N^2}\sum_{l,m=1}^{N}(-1)^{l+m}\langle \vec{S}_l\cdot\vec{S}_m\rangle \; : \; m^2(T) := M^2_{\rm stag}(T)/M^2_{\rm stag,AFM}$$





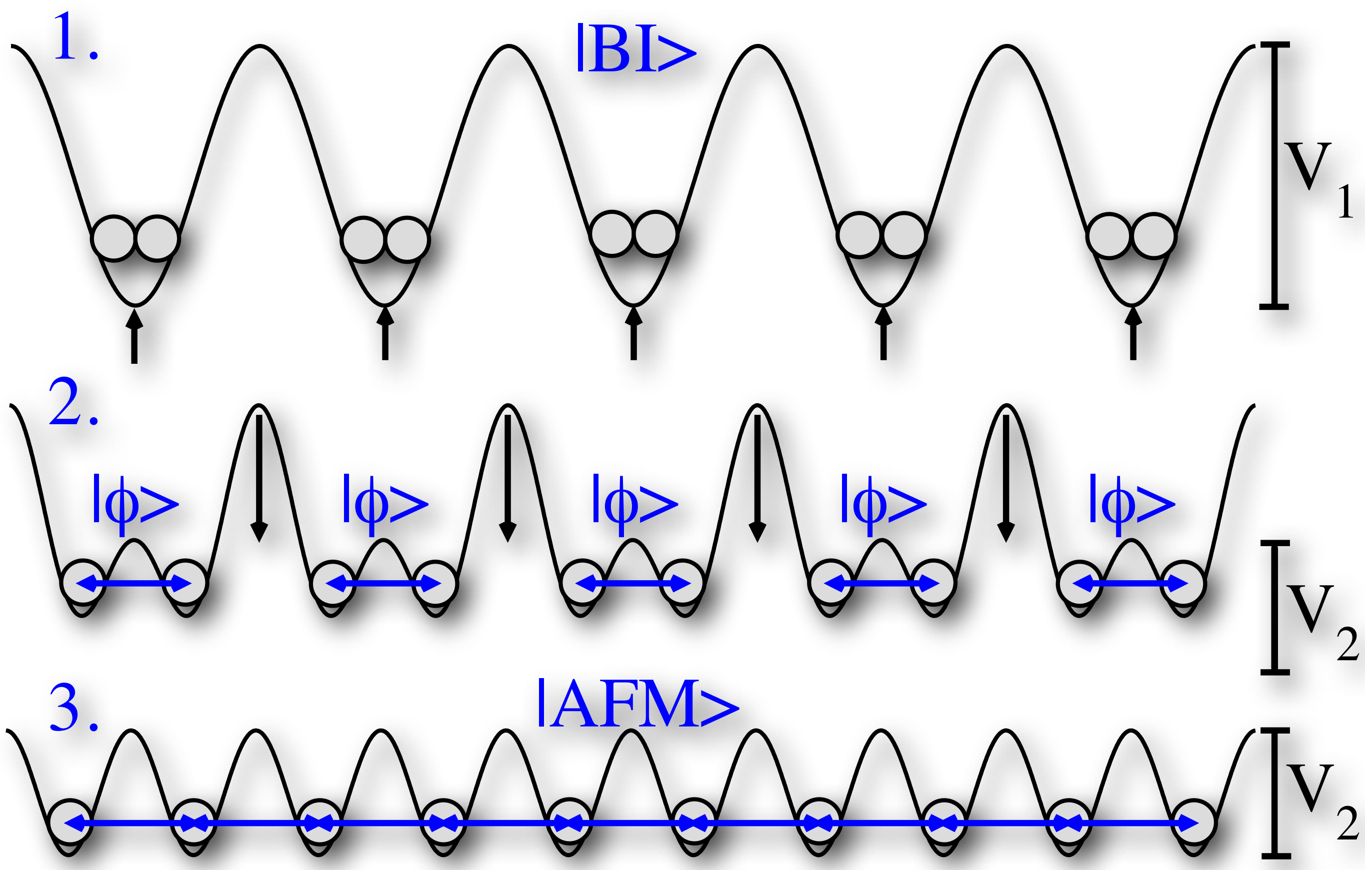


strong trap: hole-free case

Conclusions

Conclusions

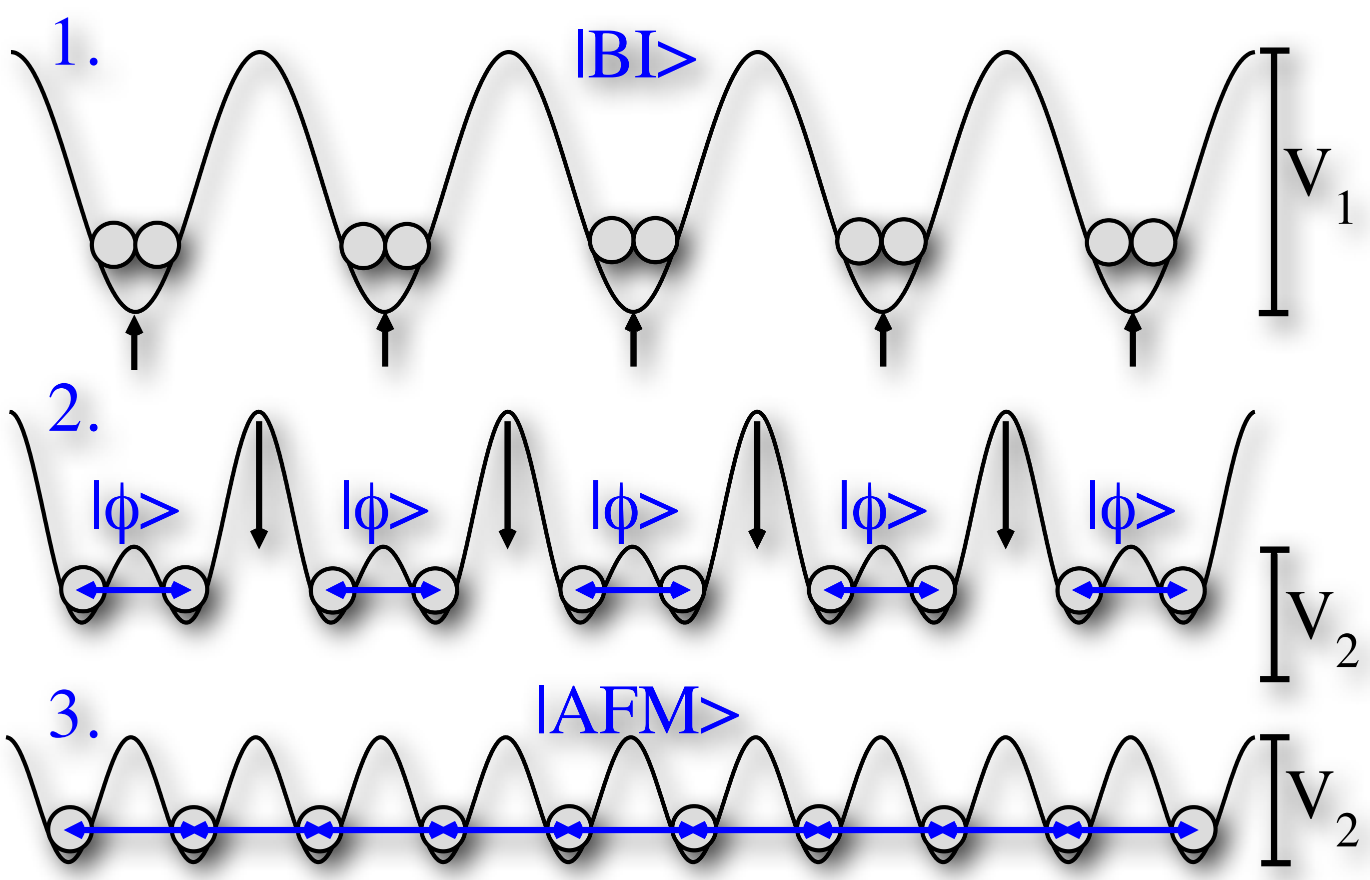
- feasible timescales



Conclusions

- feasible timescales

- sublattice adiabaticity: governed only by its size

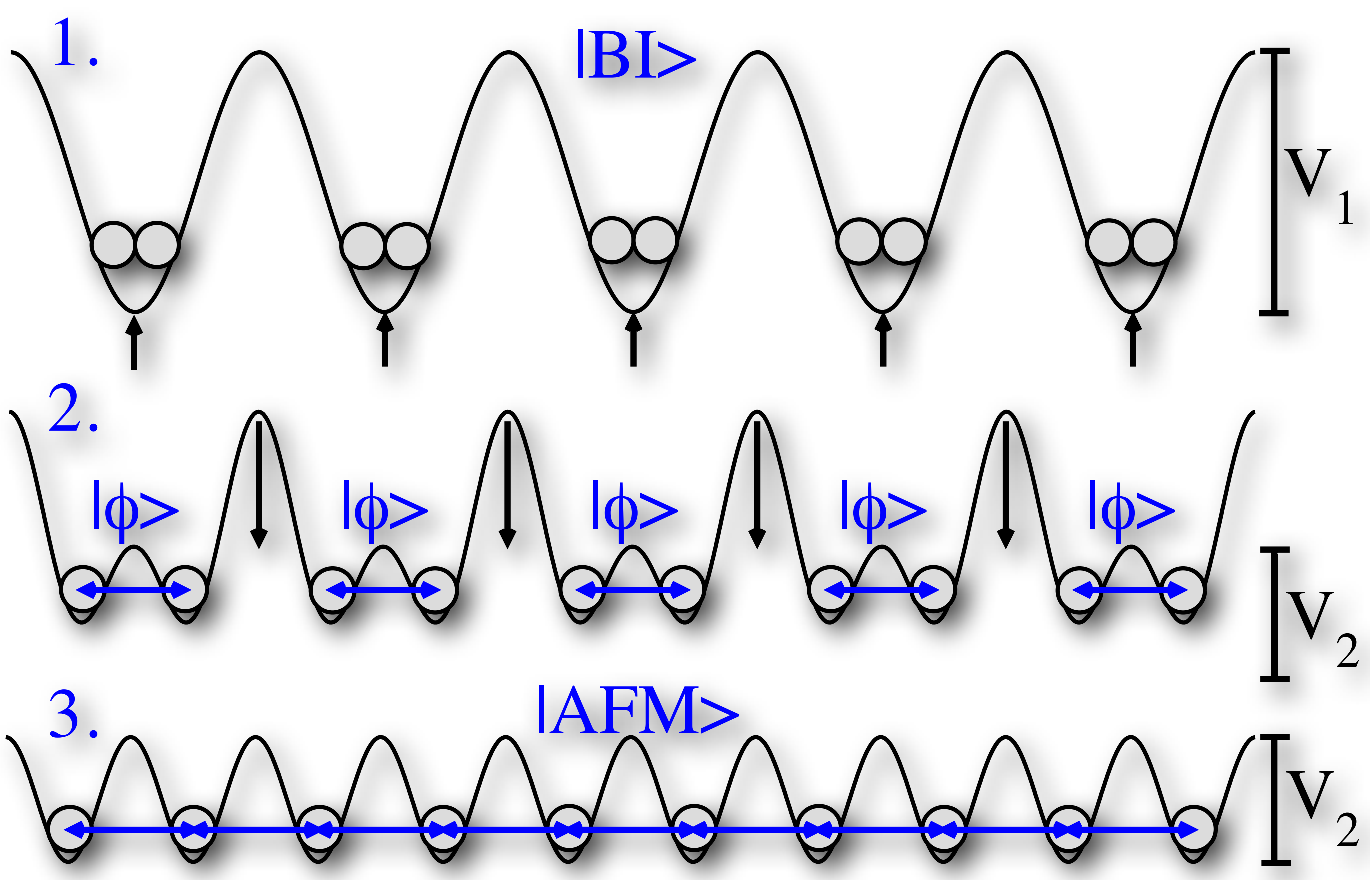


Conclusions

- feasible timescales

- sublattice adiabaticity: governed only by its size

- destructive effect of holes: controlled by harmonic trap

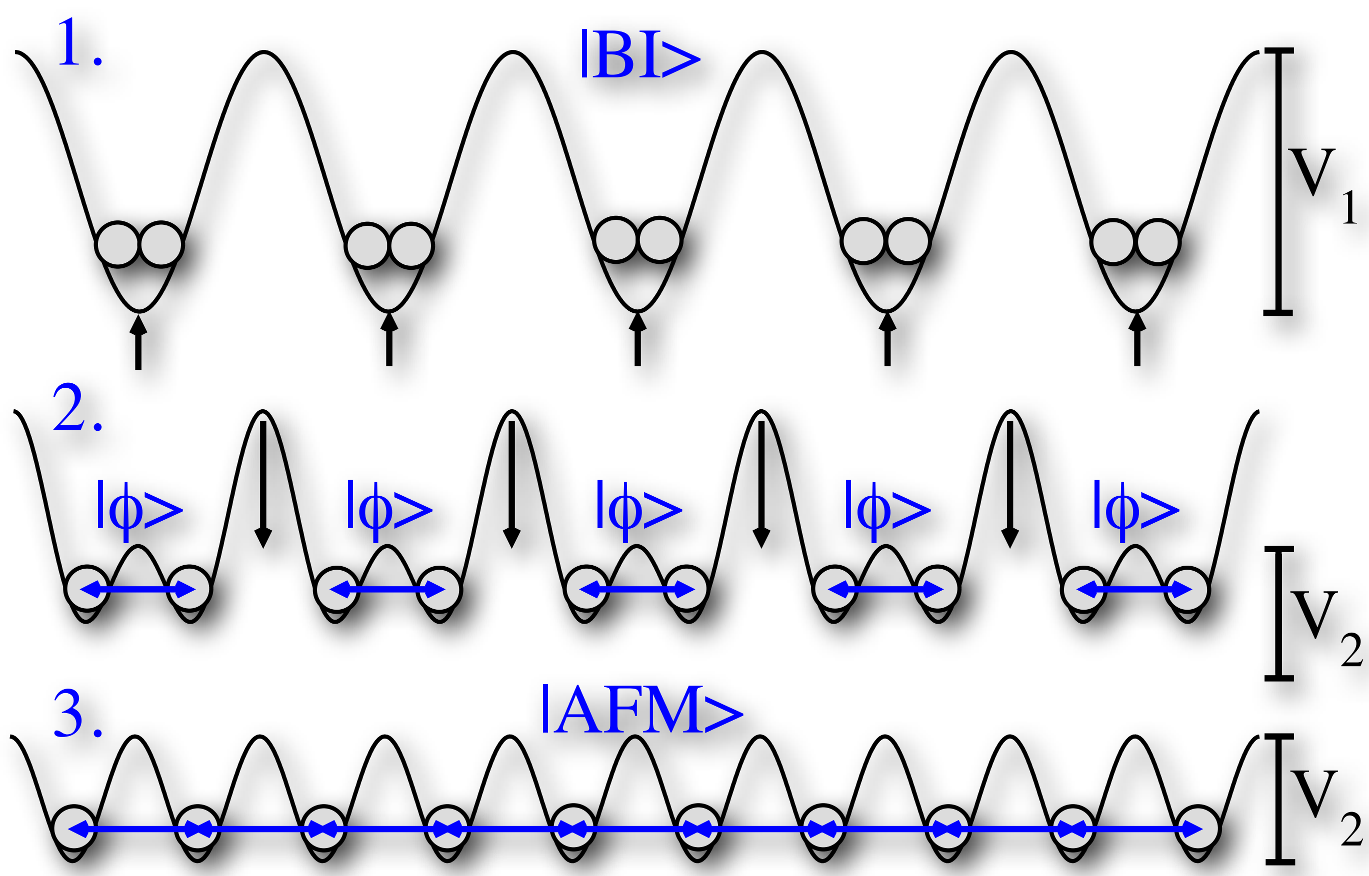


Conclusions

- feasible timescales

- sublattice adiabaticity: governed only by its size

- destructive effect of holes: controlled by harmonic trap



- details: [*M. Lubasch, V. Murg, U. Schneider, J. I. Cirac, M.-C. Bañuls,* PRL 107, 165301 (2011)]