Towards Unconventional Symmetries in Tensor Network States

Oliver Buerschaper

Perimeter Institute for Theoretical Physics

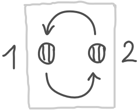
Networking Tensor Networks, Benasque 2012

Motivation

Symmetry

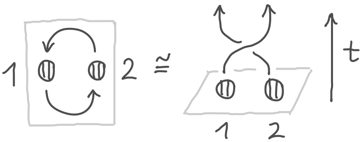
Conclusion

Anyonic Excitations



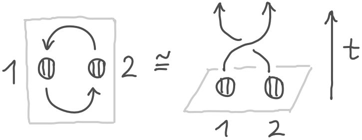
Wilczek, PRL 1982

Anyonic Excitations



Wilczek, PRL 1982

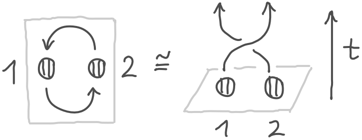
Anyonic Excitations



$|\Psi_{12}\rangle$

Wilczek, PRL 1982

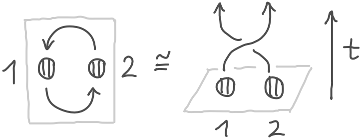
Anyonic Excitations



$$|\Psi_{12}\rangle \longmapsto U|\Psi_{12}\rangle$$

Wilczek, PRL 1982

Anyonic Excitations

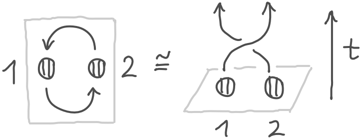


$$|\Psi_{12}\rangle \longmapsto U|\Psi_{12}\rangle$$

bosons $+1$

Wilczek, PRL 1982

Anyonic Excitations

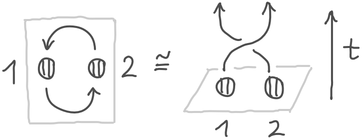


$$|\Psi_{12}\rangle \mapsto U|\Psi_{12}\rangle$$

bosons $+1$
fermions -1

Wilczek, PRL 1982

Anyonic Excitations



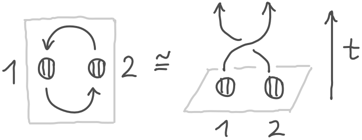
$$|\psi_{12}\rangle \mapsto U|\psi_{12}\rangle$$

bosons $+1$
fermions -1

$e^{i\theta}$ Abelian anyons

Wilczek, PRL 1982

Anyonic Excitations



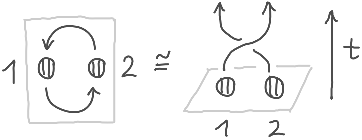
$$|\Psi_{12}\rangle \mapsto U|\Psi_{12}\rangle$$

bosons $+1$
fermions -1

$e^{i\theta}$ Abelian anyons
U non-Abelian anyons

Wilczek, PRL 1982

Anyonic Excitations



new particles!

$$|\Psi_{12}\rangle \mapsto U|\Psi_{12}\rangle$$

bosons $+1$

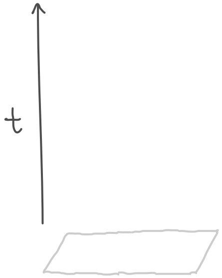
fermions -1

$e^{i\theta}$ Abelian anyons

U non-Abelian anyons

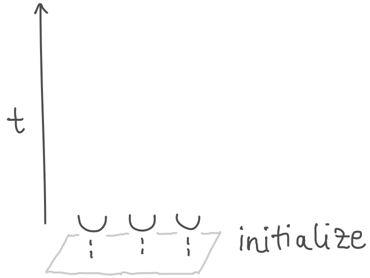
Wilczek, PRL 1982

Topological Quantum Computation



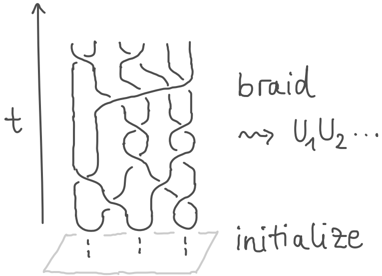
Kitaev, arXiv 1997/AoP 2003

Topological Quantum Computation



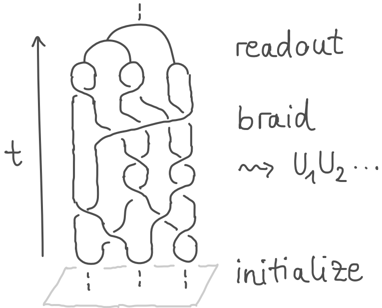
Kitaev, arXiv 1997/AoP 2003

Topological Quantum Computation



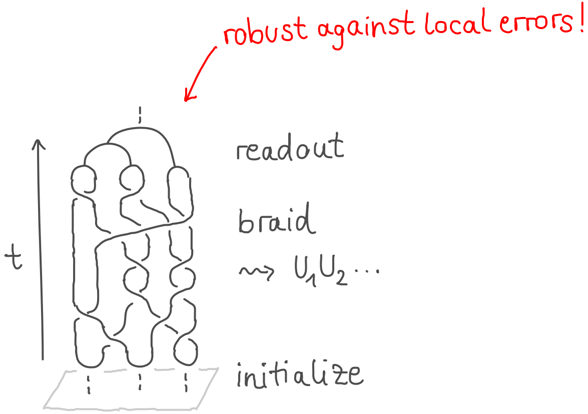
Kitaev, arXiv 1997/AoP 2003

Topological Quantum Computation



Kitaev, arXiv 1997/AoP 2003

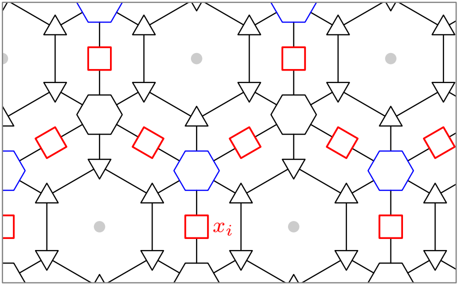
Topological Quantum Computation



Kitaev, arXiv 1997/AoP 2003

A Family of Topological Wavefunctions

$$\langle x_1, \ldots, x_N | \psi \rangle =$$



Buerschaper, Aguado, Vidal, PRB 2009
Gu, Levin, Swingle, Wen, PRB 2009

Characterizing Topological Order Locally

Find necessary and sufficient conditions on local tensors for topologically ordered quantum many-body states.

Questions & Hints

1. Which tensor variations are physical?

Questions & Hints

1. Which tensor variations are physical?
2. Invariance under (physical) local unitaries?

Questions & Hints

1. Which tensor variations are physical?
2. Invariance under (physical) local unitaries?
3. Robustness under renormalization?

Questions & Hints

1. Which tensor variations are physical?
2. Invariance under (physical) local unitaries?
3. Robustness under renormalization?
4. 1D: Classification of MPS via virtual (projective) G-symmetry

Chen, Gu, Wen, PRB 2011
Schuch, Pérez-García, Cirac, PRB 2011

Physical Symmetry?

Intrinsic topological order: no (robust) local physical symmetry

Physical Symmetry?

Intrinsic topological order: no (robust) local physical symmetry

$$H(\lambda) = H_0 + \lambda V$$

Bravyi, Hastings, Michalakis, JMP 2010
Michalakis, Pytel, arXiv 2011

Physical Symmetry?

Intrinsic topological order: no (robust) local physical symmetry

$$H(\lambda) = H_0 + \lambda V$$

fixed point
(frustration-free)

Bravyi, Hastings, Michalakis, JMP 2010
Michalakis, Pytel, arXiv 2011

Physical Symmetry?

Intrinsic topological order: no (robust) local physical symmetry

$$H(\lambda) = H_0 + \lambda V$$

fixed point (frustration-free)

arbitrary local perturbation (bounded)

Bravyi, Hastings, Michalakis, JMP 2010
Michalakis, Pytel, arXiv 2011

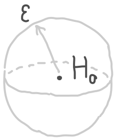
Physical Symmetry?

Intrinsic topological order: no (robust) local physical symmetry

$$H(\lambda) = H_0 + \lambda V$$

fixed point (frustration-free)

arbitrary local perturbation (bounded)



topological order is robust

Bravyi, Hastings, Michalakis, JMP 2010
Michalakis, Pytel, arXiv 2011

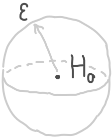
Physical Symmetry?

Intrinsic topological order: no (robust) local physical symmetry

$$H(\lambda) = H_0 + \lambda V$$

fixed point (frustration-free) → H_0

arbitrary local perturbation (bounded) → V



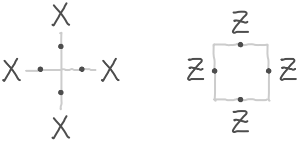
topological order is robust $\longleftrightarrow$ (accidental) symmetry at H_0 may be destroyed

Bravyi, Hastings, Michalakis, JMP 2010
Michalakis, Pytel, arXiv 2011

A Simple Example of Virtual Symmetry

Toric code

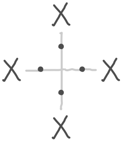
$$H = -\sum_s A(s) - \sum_p B(p)$$

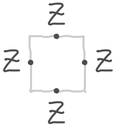


A Simple Example of Virtual Symmetry

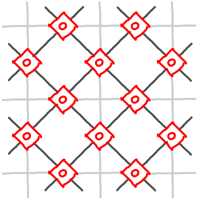
Toric code

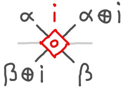
$$H = -\sum_s A(s) - \sum_p B(p)$$





Tensor network



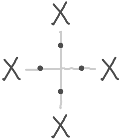


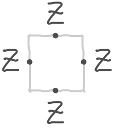
$= 1$

A Simple Example of Virtual Symmetry

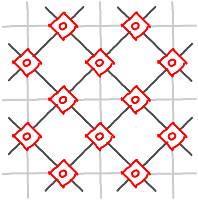
Toric code

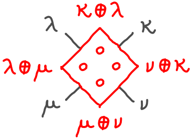
$$H = -\sum_s A(s) - \sum_p B(p)$$





Tensor network

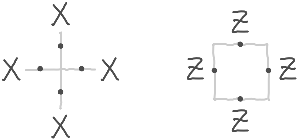




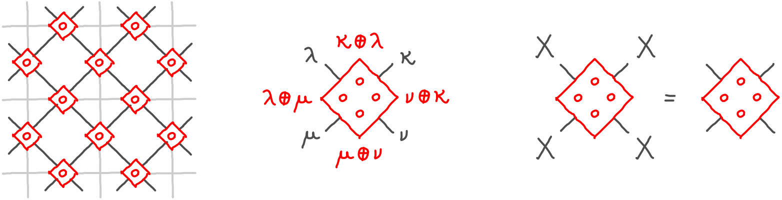
A Simple Example of Virtual Symmetry

Toric code

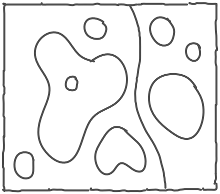
$$H = -\sum_s A(s) - \sum_p B(p)$$



Tensor network



Intrinsic Topological Order and Virtual Symmetry I

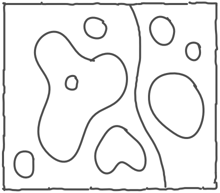


toric code

$D(\mathbb{Z}_2)$

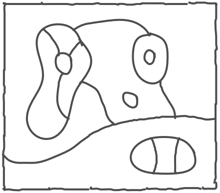
G-symmetry

Intrinsic Topological Order and Virtual Symmetry I



toric code

$D(\mathbb{Z}_2)$

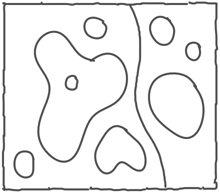


Fibonacci SN

$\tau \times \tau = 1 + \tau$

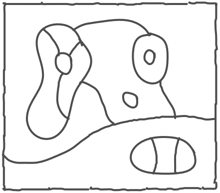
G-symmetry $\subset$?-symmetry

Intrinsic Topological Order and Virtual Symmetry I



toric code
$D(\mathbb{Z}_2)$

$\cdots$

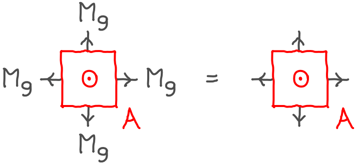


Fibonacci SN
$\tau \times \tau = 1 + \tau$

G-symmetry $\subset$ $\cdots$ $\subset$?-symmetry

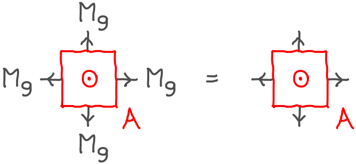
Virtual G-Symmetry

1. Symmetry

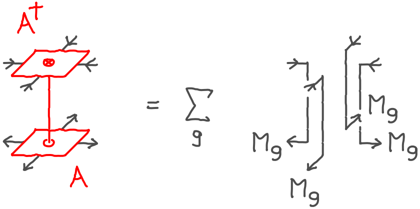


Virtual G-Symmetry

1. Symmetry

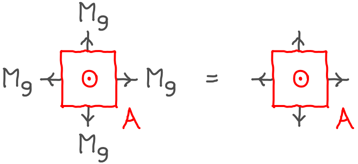


2. Invertability (up to symmetry)

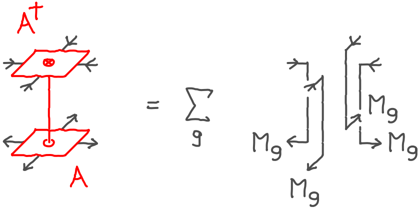


Virtual G-Symmetry

1. Symmetry



2. Invertability (up to symmetry)



physical
$\cong$ virtual/symmetry

Virtual G-Symmetry

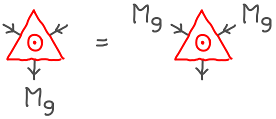
- Parent Hamiltonian with finite ground state degeneracy

Virtual G-Symmetry

- Parent Hamiltonian with finite ground state degeneracy
- Locally indistinguishable ground states

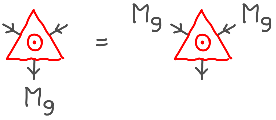
Unconventional Virtual Symmetries

Transparency



Unconventional Virtual Symmetries

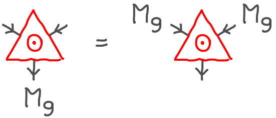
Transparency

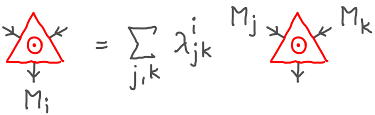


not entangled

Unconventional Virtual Symmetries

Transparency

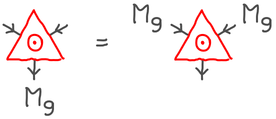




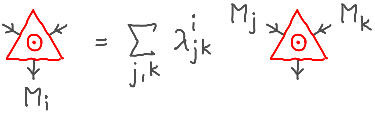
not entangled

Unconventional Virtual Symmetries

Transparency



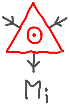
not entangled



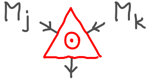
entangled

Unconventional Virtual Symmetries

$\mathbb{C}^G$- symmetry

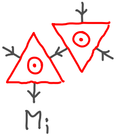


$$= \sum_{j,k\in G} \delta_i(jk)$$

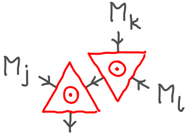


Unconventional Virtual Symmetries

H - symmetry

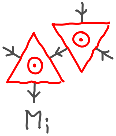


$$= \sum_{j,k,l} \left(\sum_m \lambda^i_{jm} \lambda^m_{kl} \right)$$

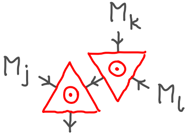


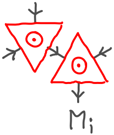
Unconventional Virtual Symmetries

H - symmetry

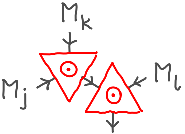


$$= \sum_{j,k,l} \left(\sum_{m} \lambda^{i}_{jm} \lambda^{m}_{kl} \right)$$

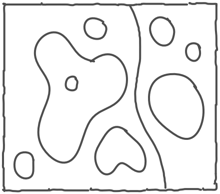




$$= \sum_{j,k,l} \left(\sum_{n} \lambda^{i}_{nl} \lambda^{n}_{jk} \right)$$



Intrinsic Topological Order and Virtual Symmetry II

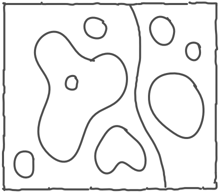


toric code

$D(\mathbb{Z}_2)$

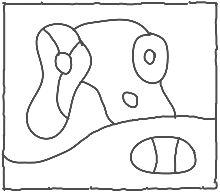
G-symmetry

Intrinsic Topological Order and Virtual Symmetry II



toric code

$D(\mathbb{Z}_2)$

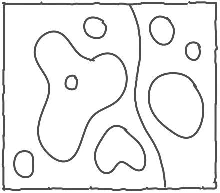


Fibonacci SN

$\tau \times \tau = 1 + \tau$

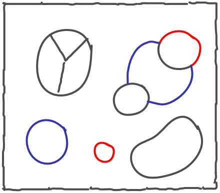
G-symmetry $\subset$?-symmetry

Intrinsic Topological Order and Virtual Symmetry II



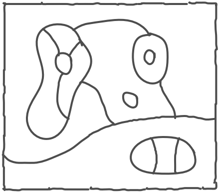
toric code

$D(\mathbb{Z}_2)$



quantum double

$D(H)$

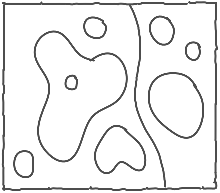


Fibonacci SN

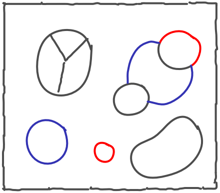
$\tau \times \tau = 1 + \tau$

G-symmetry $\subset$ H-symmetry $\subset$?-symmetry

Intrinsic Topological Order and Virtual Symmetry II

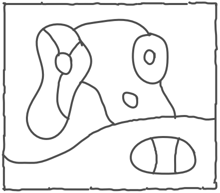


toric code
$D(\mathbb{Z}_2)$



quantum double
$D(H)$

…



Fibonacci SN
$\tau \times \tau = 1 + \tau$

G-symmetry $\subset$ H-symmetry $\subset \cdots \subset$?-symmetry

Summary

- Tensor networks are a powerful tool for characterizing topological order and quantum phases

Summary

- Tensor networks are a powerful tool for characterizing topological order and quantum phases
- 1D: virtual (projective) G-symmetry classifies MPS

Summary

- Tensor networks are a powerful tool for characterizing topological order and quantum phases
- 1D: virtual (projective) G-symmetry classifies MPS
- 2D: unconventional virtual symmetries are needed to classify PEPS

Summary

- Tensor networks are a powerful tool for characterizing topological order and quantum phases
- 1D: virtual (projective) G-symmetry classifies MPS
- 2D: unconventional virtual symmetries are needed to classify PEPS
- $\mathbb{C}^G$- and H-symmetries are such stepping stones

Collaboration

- David Pérez-García
- Liang Kong
- Miguel Aguado
- Matthias Christandl
- Martín Mombelli
- Guifré Vidal