Shadows of rotating black holes

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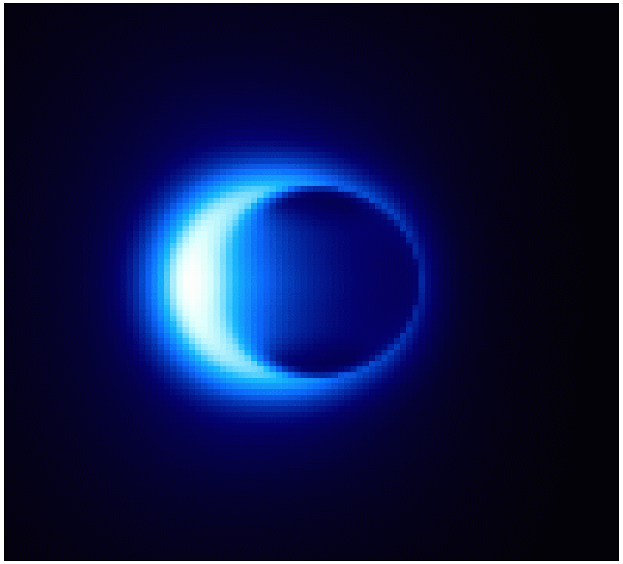


ERE2013, September 8th-13th, Benasque, Spain

Image: eventhorizontelescope.org

Motivation

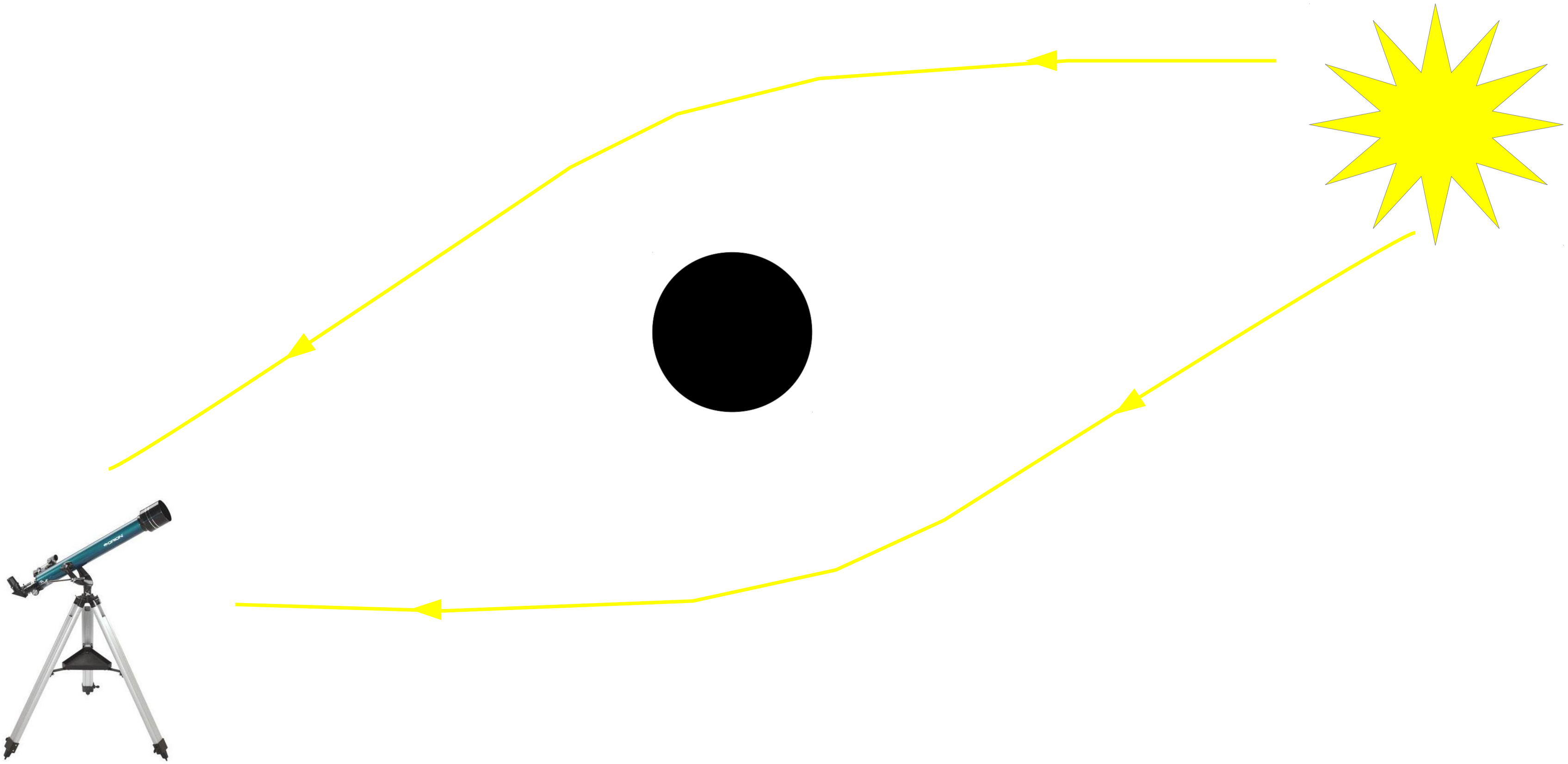
The apparent shape of a black hole (or shadow) corresponds to a full description of the near horizon region, without any theoretical assumption concerning the underlying theory or astrophysical processes in the black hole surroundings.

Outline

- *Black hole shadow in GR (Kerr geometry)*
  - *Procedure*
  - *Shape of the shadow*
- *Rotating braneworld black hole (or naked singularity)*
  - *Null geodesics*
  - *Some results and observational profile*
- *Discussion*

Black hole shadow in GR

Basic idea



- Nonrotating black hole (Schwarzschild): a black disc which radius corresponds to the apparent position of the photon sphere
- Rotating black hole (Kerr): a little more bizarre shape...

The Kerr metric (in BL coordinates)

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\sin^2\theta}{\Sigma}\left[adt - (r^2+a^2)d\phi\right]^2$$

with $\Delta = r^2 + a^2 - 2Mr$ $\Sigma = r^2 + a^2\cos^2\theta$

- Stationary and axially-symmetric (E and L_z constants of motion)
- Horizons at $\Delta(r_+) = 0$, i.e. $r_+ = M + \sqrt{M^2 - a^2}$

So: If $a \leq M$ we have a BH, otherwise we have a NS.

The Hamilton-Jacobi equation is separable because of the existence of a fourth constant of motion, in addition to E, L_z , and the mass (*Carter 1968*). Thus, the geodesic motion is fully integrable by quadratures.

The procedure (tedious calculation...):

- Write the HJ equation governing geodesic motion $\frac{\partial S}{\partial \lambda} = -\frac{1}{2} g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu}$

- Specialize for the metric corresponding to the Kerr geometry

- Use separability $S = \frac{1}{2} m^2 \lambda - Et + L_z \phi + S_r(r) + S_\theta(\theta)$

- Obtain decoupled equations (using $\frac{dS_r}{dr} = p_r = g_{rr}\dot{r}$ and $\frac{dS_\theta}{d\theta} = p_\theta = g_{\theta\theta}\dot{\theta}$)

$$\Sigma \frac{dr}{d\lambda} = \sqrt{\mathcal{R}} \quad \text{and} \quad \Sigma \frac{d\theta}{d\lambda} = \sqrt{\Theta}$$

Where $\mathcal{R} = \left[(r^2 + a^2)E - aL_z\right]^2 - \Delta\left[\mathcal{K} + (L_z - aE)^2\right]$ and $\Theta = \mathcal{K} + \cos^2\theta \left(a^2 E^2 - \frac{L_z^2}{\sin^2\theta}\right)$

Carter constant

- The equations for *t* and *Φ*

$$\Sigma \frac{dt}{d\lambda} = a(L_z - aE\sin^2\theta) + \frac{r^2 + a^2}{\Delta}\left[(r^2 + a^2)E - aL_z\right] \quad \text{and} \quad \Sigma \frac{d\phi}{d\lambda} = \left(\frac{L_z}{\sin^2\theta} - aE\right) + \frac{a}{\Delta}\left[(r^2 + a^2)E - aL_z\right]$$

The equations determining the unstable photon orbits of constant radius (photon sphere) are

$$\mathcal{R}(r) = 0 = d\mathcal{R}(r)/dr$$

and the impact parameters that fulfill this equations are (*M =1*, $\xi = L_z/E$, $\eta = \mathcal{K}/E^2$)

$$\xi(r) = \frac{r^2 - r\Delta - a^2}{a(r-1)} \quad \text{and} \quad \eta(r) = \frac{r^3\left[4\Delta - r(r-1)^2\right]}{a^2(r-1)^2}$$

This parameters can be related to the ("celestial") coordinates of the image as seen by an observer at infinity

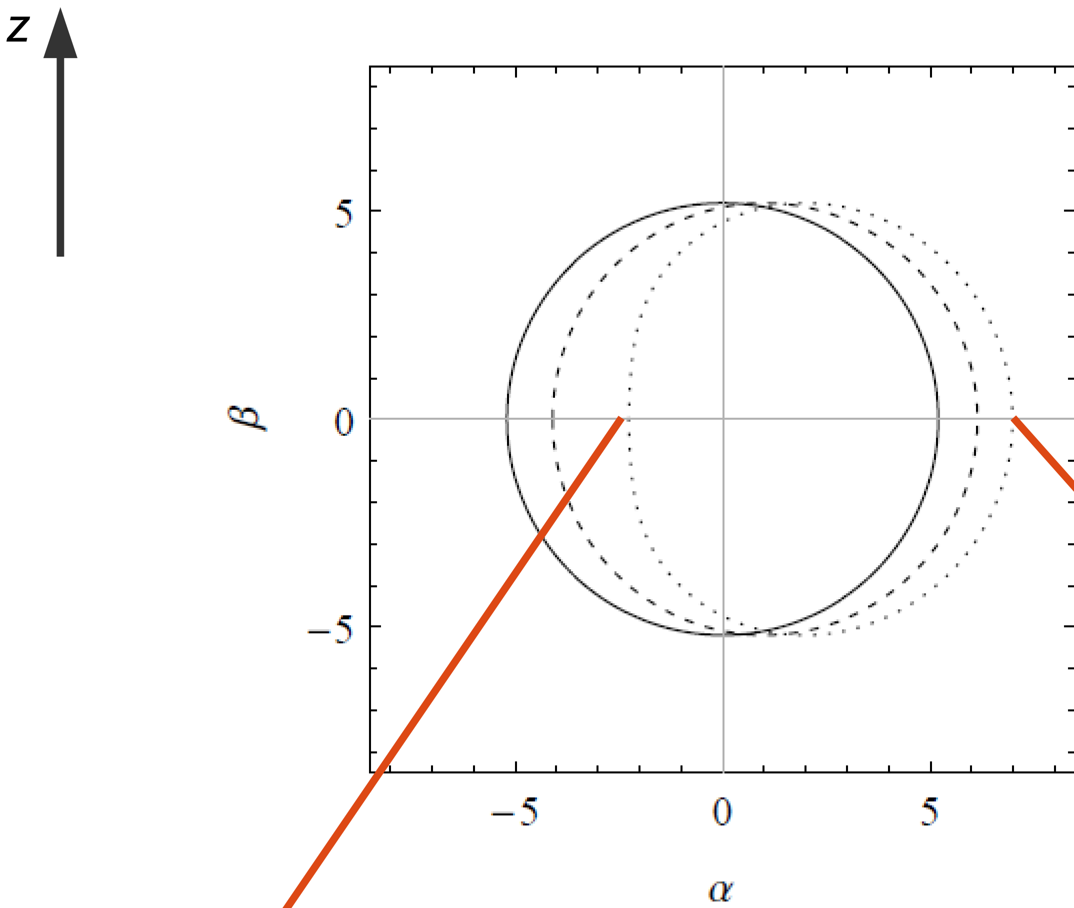
$$\alpha = -\xi \csc\theta_0 \quad \text{and} \quad \beta = \pm\sqrt{\eta + a^2\cos^2\theta_0 - \xi^2\cot^2\theta_0}$$

In the equatorial plane ($\theta_0 = \pi/2$)

$$\alpha = -\xi \qquad \text{and} \qquad \beta = \pm\sqrt{\eta}$$

The apparent shape of the black hole is obtained plotting β vs α

Black hole shadow in GR / The Kerr case



Full line: *a = 0*

Dashed: *a = 0.5*

Dotted: *a = 0.99*

(counterclockwise rotation around the *z* axis)

Retrograde circular photon orbit

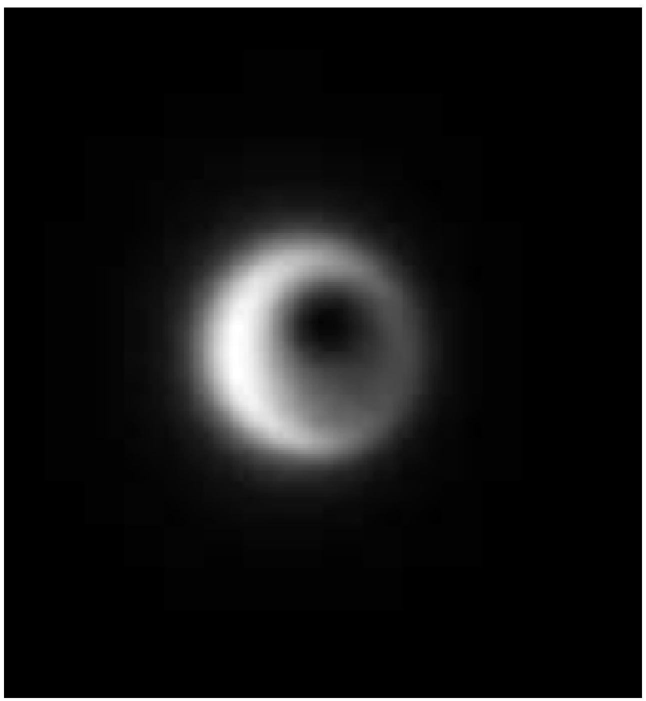
Prograde circular photon orbit

Galactic center black hole

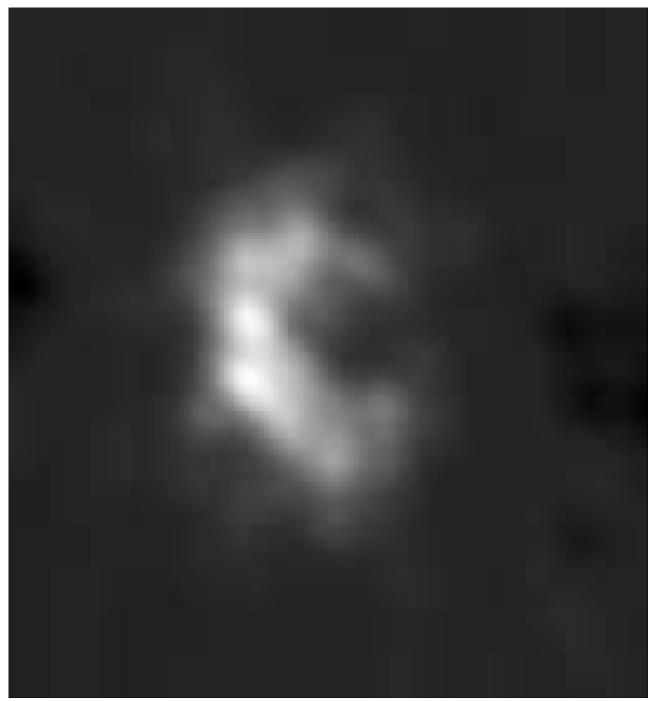
- The apparent angular radius of the supermassive Galactic black hole is about 27 μas.

- Appearence of Sgr A* at wavelength of 0.8 mm (simulated images):

Perfect telescope



Array of 7 VLBI stations



Array of 13 VLBI stations

Figure: V.L. Fish, S.S. Doeleman, IAU Symposium 261, 1304 (2009)

Another example: rotating braneworld black hole

Based on: L. Amarilla and EFE, Phys. Rev. D 85, 064019 (2012) [arXiv:1112.6349]

- In braneworld cosmologies the ordinary matter is on a three dimensional space (the *brane*) which is embedded in a larger space (the *bulk*) where only gravity can propagate.
- It has been proposed to solve the hierarchy problem.
- Motivation: string theory, M-theory, ...
- Randall–Sundrum model: a positive tension brane in an asymptotically AdS 5-dimensional bulk.
- The presence of the extra dimension would modify the properties of black holes.
- Recently, rotating black hole solutions with a tidal charge were studied by Aliev et al. (2005).

The braneworld rotating black hole (naked singularity) metric (in BL coordinates)

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\sin^2\theta}{\Sigma}\left[adt - (r^2+a^2)d\phi\right]^2$$

Where $\Delta = r^2 + a^2 - 2Mr + Q, \quad \Sigma = r^2 + a^2\cos^2\theta$

Tidal charge: imprint of the gravitational effects from the bulk space

- The tidal charge can be either positive or negative.
- Horizons at $\Delta(r_+) = 0$, i.e. $r_+ = M + \sqrt{M^2 - a^2 - Q}$

So: If $Q \leq Q_c = M^2 - a^2$ we have a BH. Otherwise we have a NS.

Remember: in Kerr–Newman, the electric charge appears squared.

Null geodesics

The Hamilton-Jacobi equation is also separable in this case.
Thus, the geodesic motion is fully integrable by quadratures

$$\Sigma \frac{dt}{d\lambda} = a(L_z - aE\sin^2\theta) + \frac{r^2+a^2}{\Delta}\left[(r^2+a^2)E - aL_z\right]$$
$$\Sigma \frac{d\phi}{d\lambda} = \left(\frac{L_z}{\sin^2\theta} - aE\right) + \frac{a}{\Delta}\left[(r^2+a^2)E - aL_z\right]$$
$$\Sigma \frac{dr}{d\lambda} = \sqrt{\mathcal{R}}$$
$$\Sigma \frac{d\theta}{d\lambda} = \sqrt{\Theta}$$

where $\mathcal{R} = \left[(r^2+a^2)E - aL_z\right]^2 - \Delta\left[\mathcal{K} + (L_z - aE)^2\right]$ and $\Theta = \mathcal{K} + \cos^2\theta\left(a^2E^2 - \frac{L_z^2}{\sin^2\theta}\right)$

Again, the impact parameters that fulfill the conditions $\mathcal{R}(r) = 0 = d\mathcal{R}(r)/dr$ are (*M =1*)

$$\xi(r) = \xi_{\mathrm{K}}(r) - \frac{2Qr}{a(r-1)} \quad \text{and} \quad \eta(r) = \eta_{\mathrm{K}}(r) - \frac{4Qr^2(\Delta - r)}{a^2(r-1)^2}$$

Observational profile and some results

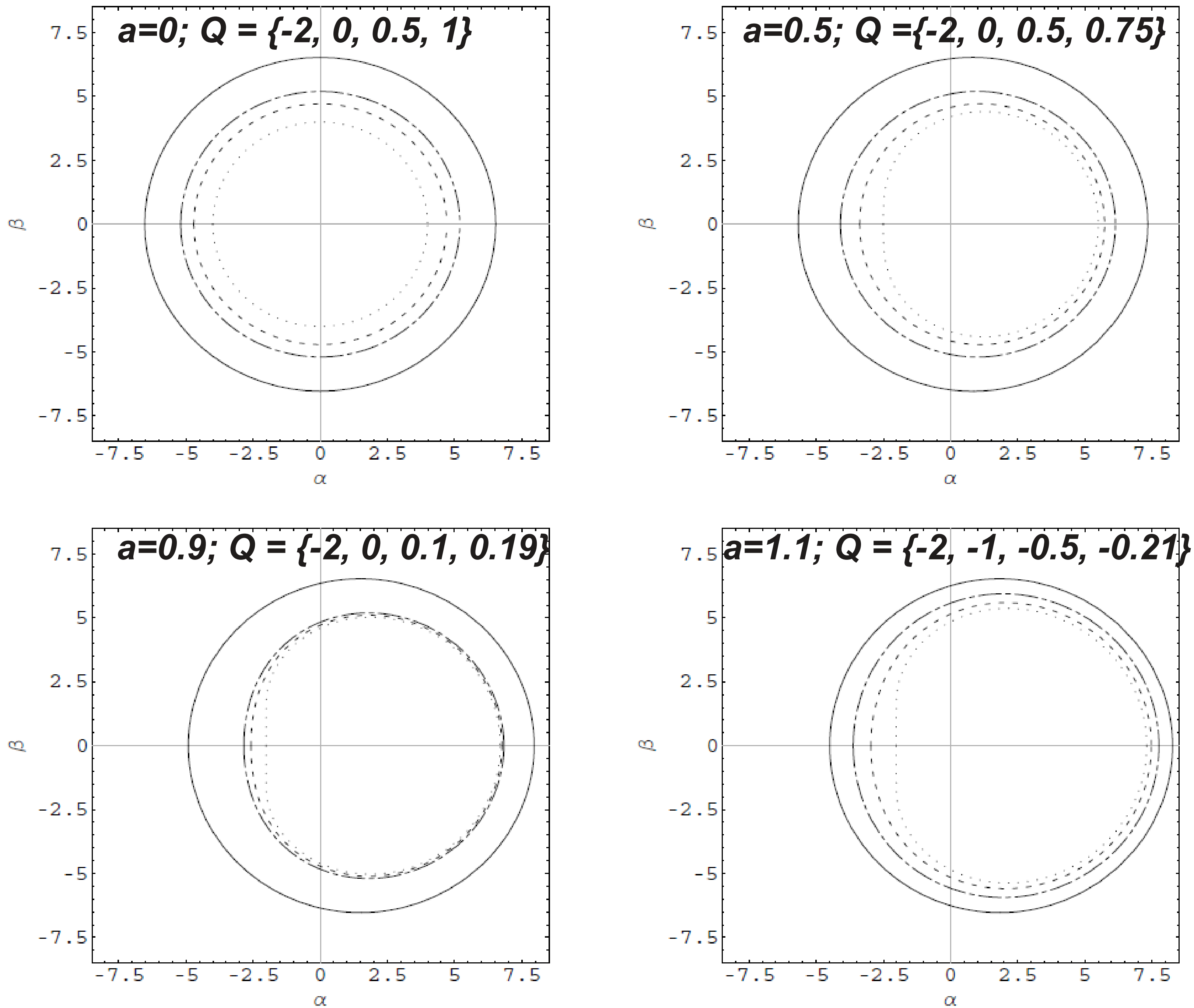
Remember: The apparent shape of the black hole or naked singularity is obtained with the plot β vs α.

In the equatorial plane ($\theta_0 = \pi/2$): $\alpha = -\xi$ and $\beta = \pm\sqrt{\eta}$

Our work...

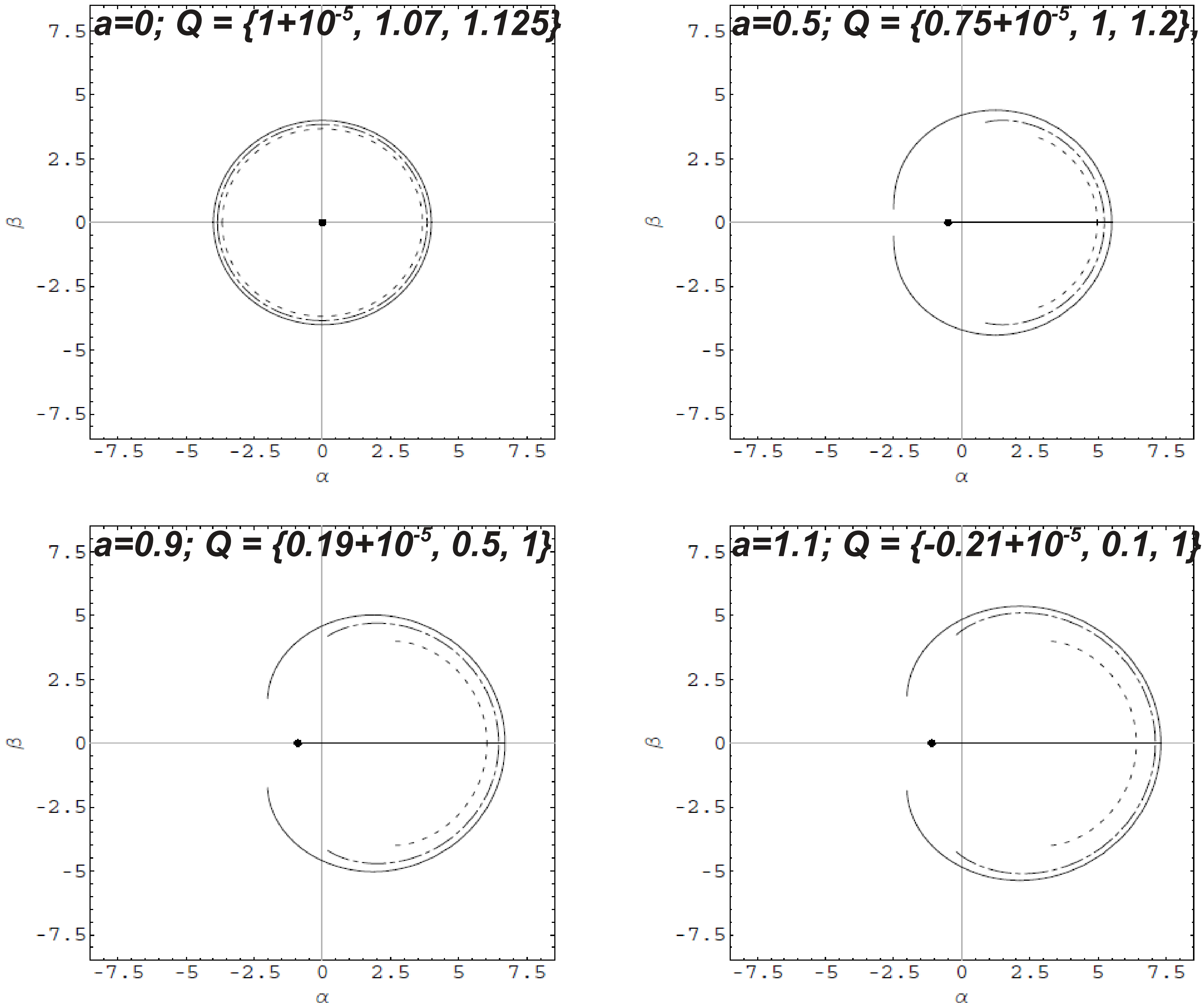
Observational profile and some results

L. Amarilla and EFE, Phys. Rev. D 85, 064019 (2012)



Observational profile and some results

Figure: L. Amarilla and EFE, Phys. Rev. D 85, 064019 (2012)



Observational profile and some results

In order to study the shape of the shadows we use the observables R_s and $\delta_s = D / R_s$ defined by Hioki and Maeda (2009)

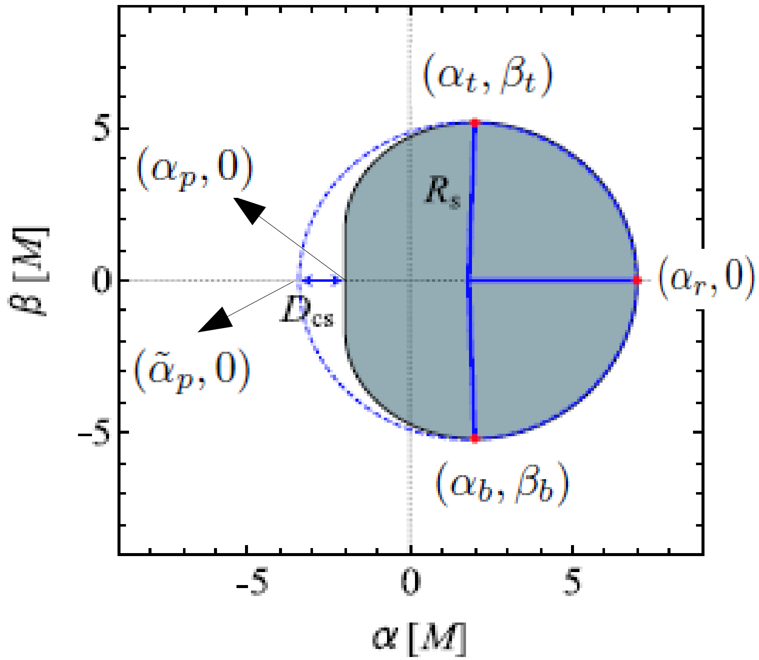


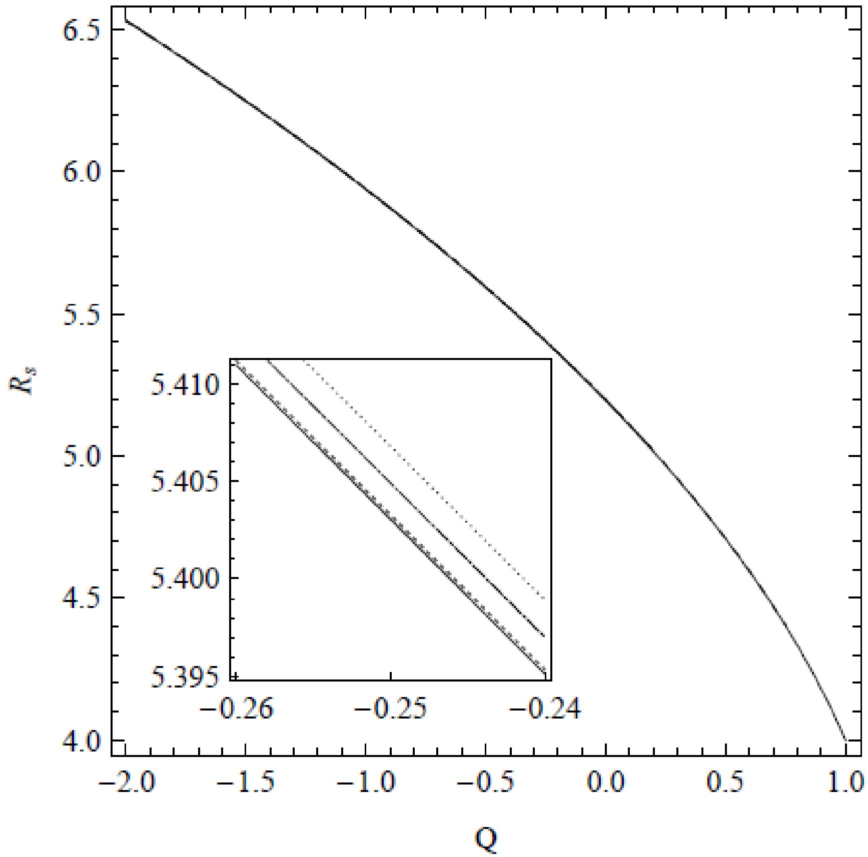
Figure: K. Hioki and K. Maeda, Phys. Rev. D 80, 024042 (2009)

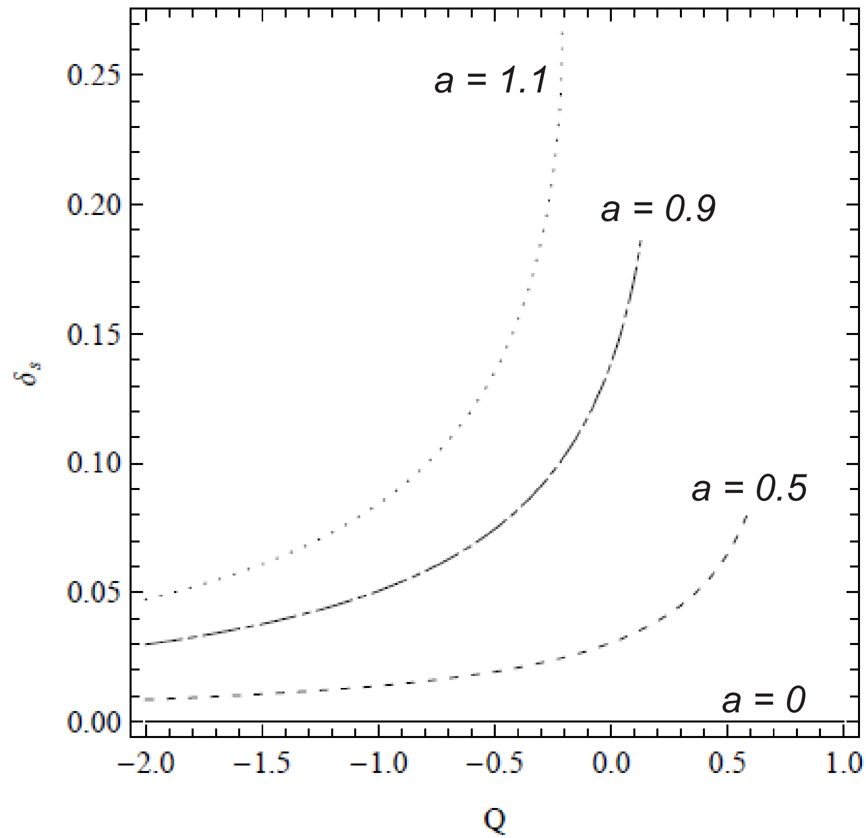
Using some geometry

$$R_s = \frac{(\alpha_t - \alpha_r)^2 + \beta_t^2}{2|\alpha_t - \alpha_r|} \qquad \delta_s = \frac{\tilde{\alpha}_p - \alpha_p}{R_s}$$

Observational profile and some results

Figures: L. Amarilla and EFE, Phys. Rev. D 85, 064019 (2012)





- The difference between the $a = 0$ curve and the $a = 1.1$ one is of order 10^{-3} (small variation in the size as a function of a)
- For $Q > 0$: reduction of R_S
- For $Q < 0$: enlargement of R_S

- Maximal distortion for $Q = Q_C$
- For fixed Q, the deformation of the shadow increases with a.
- For $Q < 0$: reduction of δ_S

Observational profile and some results

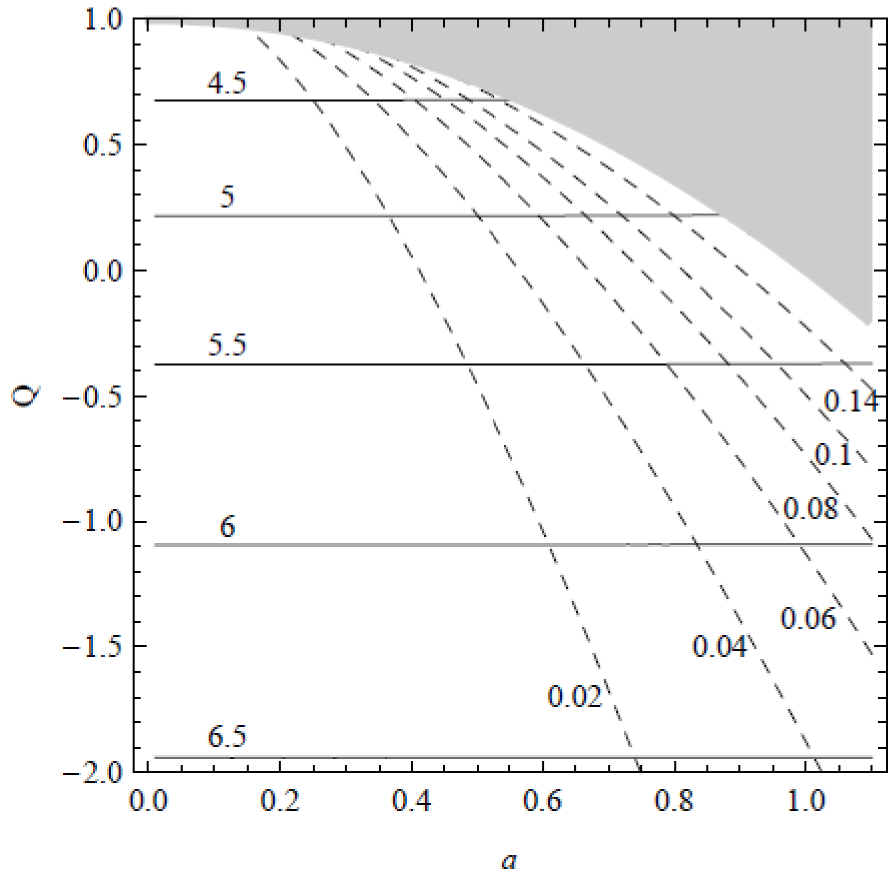


Figure: L. Amarilla and EFE, Phys. Rev. D 85, 064019 (2012)

R_s and δ_s come from observations. The point in the plane where the associated contour curves intersect each other gives the corresponding values of a and Q.

Discussion

Summarizing: For fixed *a*, the presence of a negative (positive) tidal charge leads to a larger (smaller) shadow than in the case of Kerr geometry; while a negative (positive) value of *Q* gives a less (more) distorted shadow.

Let's put some numbers

The angular size of the shadow can be estimated by R_S to obtain the angular radius $\theta_S = R_S\, M / D_0 = 9.87098 \times 10^{-6}\ R_S\, (M / M_\bullet)(1\ kpc / D_0)$.
For the black hole at the galactic center *Sgr A** : $M = 4.3 \times 10^6\, M_\bullet$ and $D_0 = 8.3 kpc$. Thus, for some illustrative values of *a* and *Q*, we have

a	0				0.9			
Q	-0.5	-0.1	0	0.1	-0.5	-0.1	0	0.1
$\theta_s(\mu as)$	28.605	27.006	26.572	26.120	28.612	27.018	26.586	26.136
$\delta_s(\%)$	0	0	0	0	7.45	11.8	13.9	17.2

L. Amarilla and EFE Phys. Rev. D 85, 064019 (2012)

Resolutions of less than 1 μas are needed in order to extract useful information from future observations of the shadow of the Galactic black hole.

Discussion

In the near future (~ *5 – 10 years*) observational facilities, most of them space-based, will be fully operational

- RadioAstron (in space since 2011): Radio ~ *1 – 10 μas*
- Millimetron: Radio ~ *0.3 μas @0.4 mm*
- Event Horizon Telescope: VLBI (sub)millimeter wavelength ~ *15 μas @345 GHz*
- MAXIM: X-ray interferometer ~ *0.1 μas*

For a more detailed description of the black hole shadow it would be necessary a second generation of instruments with improved resolution.

References

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